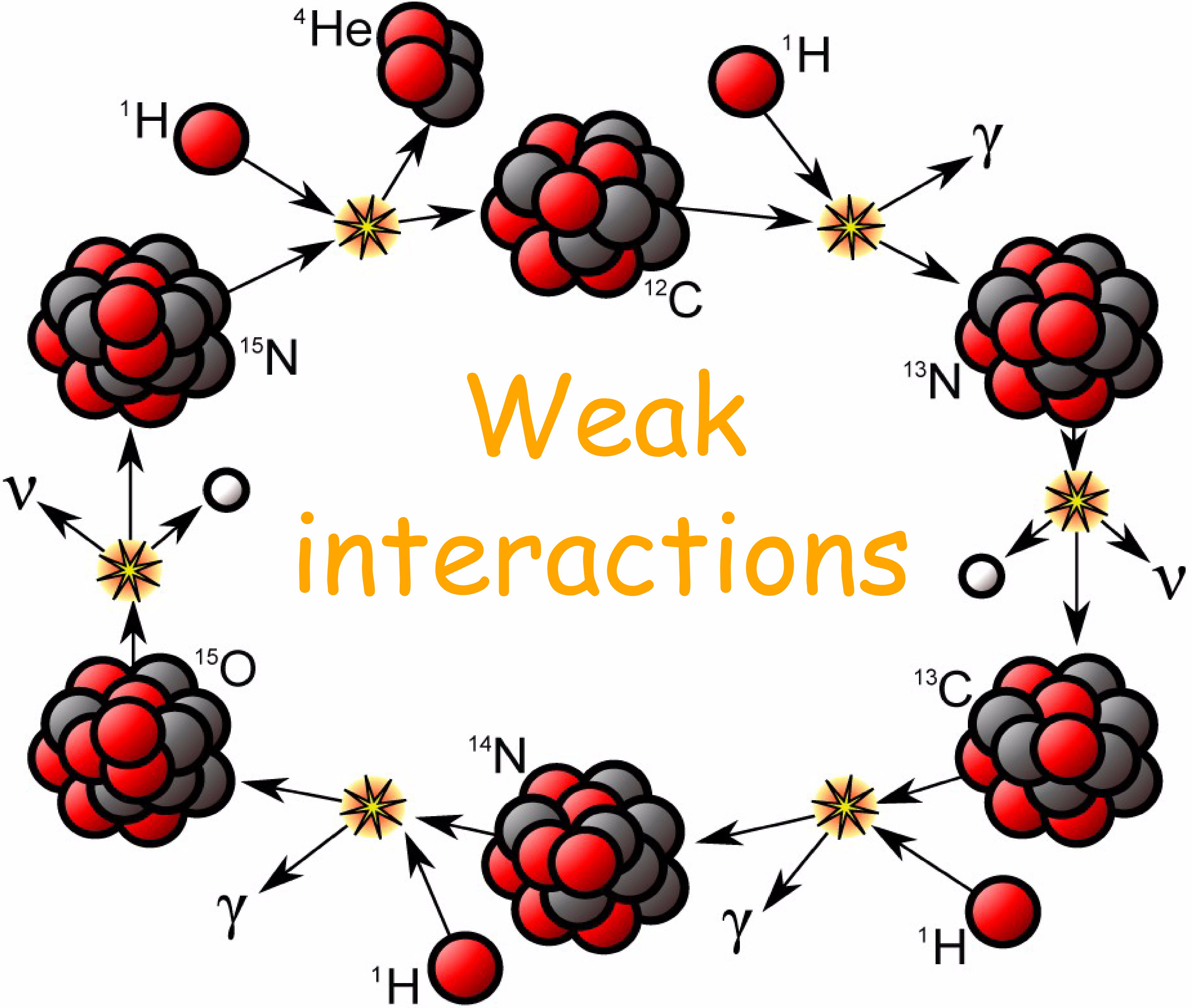
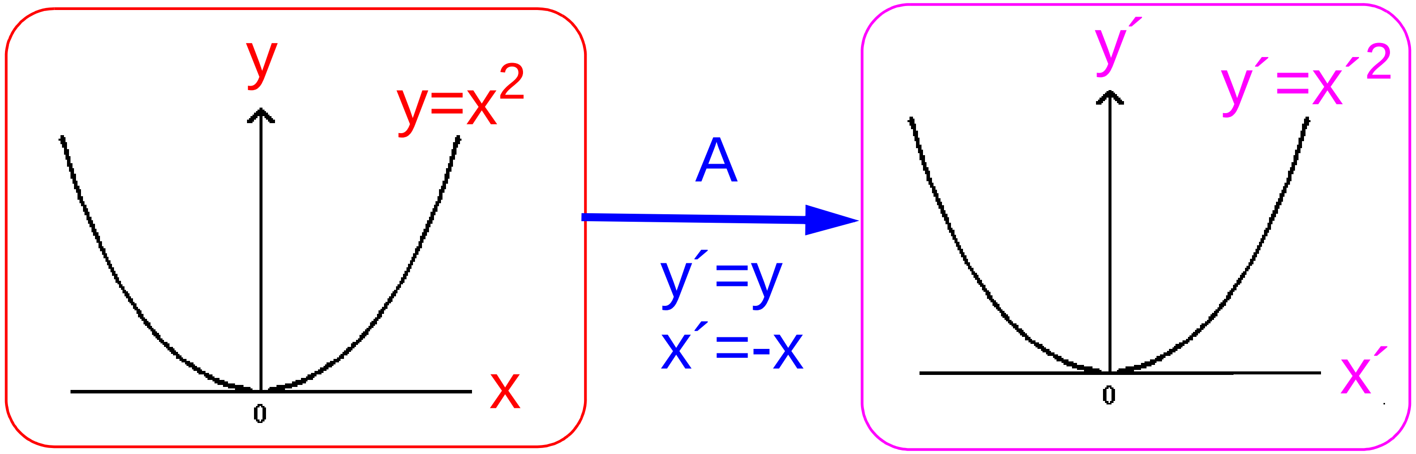
Weak interactions



The electroweak theory

➡ Gauge invariant theories.



The equation $y=x^2$ is symmetric or invariant under the transformation A, i.e. it looks the same before and after the transformation.

- **Gauge invariant theories ⇨ the main equations do not change when a gauge transformation is performed.**

- **Requiring gauge invariance ⇨ deduce the various interactions.**

The electroweak theory

➡ What is a gauge transformation ?

- **Different gauge transformations ⇔ different interactions.**

Example: Non-relativistic electromagnetism

- **The equation of motion: for a free non-relativistic particle:**

$$i\frac{\partial\psi(\vec{x},t)}{\partial t} = -\frac{1}{2m}\nabla^2\psi(\vec{x},t)$$

The free particle Schroedinger equation

- **Goal: modify this equation to describes particles that interact electromagnetically.**

The electroweak theory

- **Assumption:** the new equation is invariant under a

  U(1) phase transformation:

$$\psi(\vec{x}, t) \rightarrow \psi'(\vec{x}, t) = e^{iq\alpha(\vec{x}, t)}\psi(\vec{x}, t)$$

were $\alpha(\vec{x}, t)$ is an arbirary continous function.

- **Failure:** the transformed wavefunction $\psi'(\vec{x}, t)$ is not a solution to the Schroedinger equation.

The Gauge principle

- **Gauge principle:** to keep the invariance condition satisfied it is necessary to add a minimal field to the Schroedinger equation

  ⇨ an interaction will have to be introduced.

The electroweak theory

- **Introduce interaction:** require that the Schroedinger equation is also invariant under a gauge transformation of type

$$\bar{A} \rightarrow \bar{A}' = \bar{A} + \nabla\alpha$$

$$V \rightarrow V' = V - \frac{\partial\alpha}{\partial t}$$

where A and V are the vector and scalar potenials of the electro-magnetic field in which a particle with a charge q is moving.

- **Invariance:** U(1) phase transformation + gauge transformation

⇨ new equation

$$i\frac{\partial\psi(\vec{x},t)}{\partial t} = \left[\frac{1}{2m}(\bar{p} - q\bar{A}) + qV\right]\Psi(\vec{x},t)$$

The equation for a non-relativistic particle with charge q moving in an electromagnetic field.

The electroweak theory

⇨ The electro weak theory by Glashow, Weinberg and Salam.

- The EW or GWS model is a quantum field theory for both weak interactions and electromagnetic interactions.

- Weak isospin charge ⇨ I_3^W
  Weak hypercharge ⇨ Y^W
  Electric charge ⇨ Q

$$\left.\begin{array}{l} I_3^W \\ Y^W \\ Q \end{array}\right\} \quad Q = I_3^W + Y^W/2$$

- Gauge invariance ⇨ massless gauge particles ⇨ W^+, W^-, W^0, B^0

- The gauge particles interact with massless fermions.

The electroweak theory

- The Higgs field ⇨ generates mass to gauge bosons and fermions.
- W^+ and W^- ⇨ weak radioactive decay
- W^0 and B^0 ⇨ not observed experimentally
- Photon ⇨ gauge boson for the electromagnetic interaction

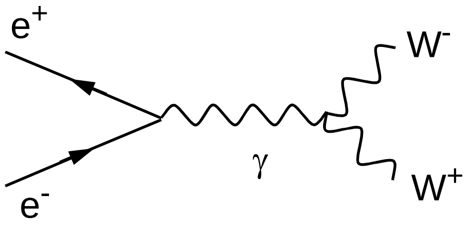
  Z^0 ⇨ gauge boson for the weak neutral current interaction

$$\boxed{\begin{aligned} \gamma &= B^0 \cos\theta_W + W^0 \sin\theta_W \\ Z^0 &= -B^0 \sin\theta_W + W^0 \cos\theta_W \end{aligned}}$$

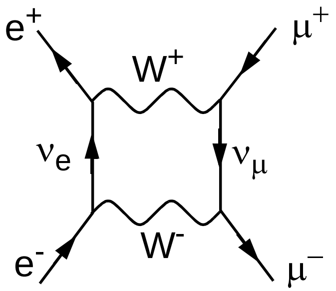
- θ_W ⇨ the weak mixing angle ⇨ a parameter (not predicted).

The electroweak theory

- **Only W-exchange** ⇨ **divergent processes** ⇨ **infinities**

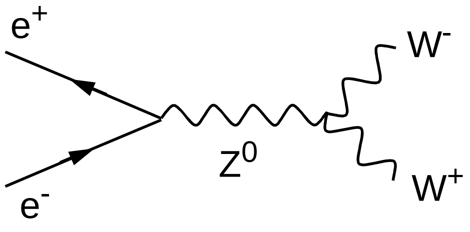


Non-divergent integrals

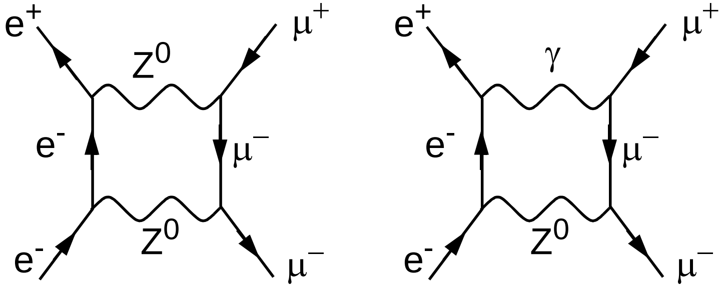


Divergent integrals

- **The Z⁰-boson** ⇨ **new diagrams cancel out the divergencies**



Non-divergent integrals



Divergent integrals

The electroweak theory

- Coupling constants in EW ⇨ e, g_W and g_Z
- "Strength parameters"

QED: $\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$ QCD: $\alpha_s = \frac{g_s^2}{4\pi} \approx \frac{1}{9}$ EW: $\alpha_W = \frac{g_W^2}{4\pi} \approx \frac{1}{250}$ $\alpha_Z = \frac{g_Z^2}{4\pi} \approx \frac{1}{850}$

- The coupling constants are not independent (in order for all the infinities to cancel out)

The unification condition.

$$e/\sqrt{8} = g_W \sin\theta_W = g_Z \cos\theta_W$$

Alternative

$$e = g\sin\theta_W = g'\cos\theta_W$$

$g = \sqrt{8}\, g_W$ $g' = \sqrt{8}\, g_Z$

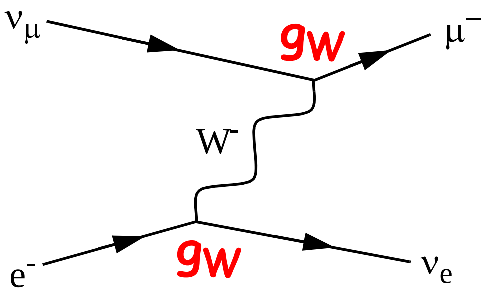
- The weak mixing angle (Weinberg angle) ⇨ $\cos\theta_W = \frac{M_W}{M_Z}$

The electroweak theory

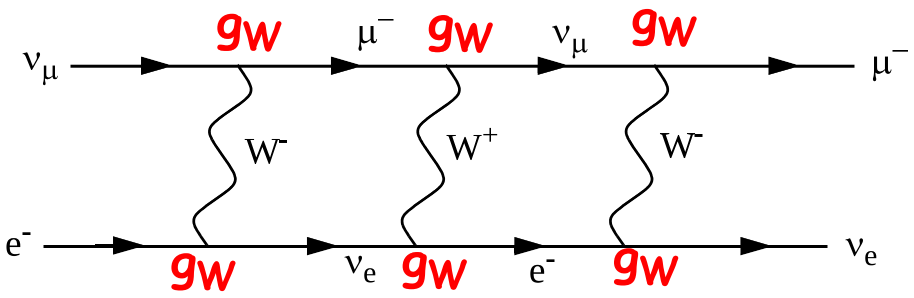
- The strength parameters ⇨ contribution of different processes (diagrams) to the cross-section.

- Example: muon-scattering on electrons $\nu_\mu + e^- \rightarrow \mu^- + \nu_e$

  Diagrams with increasing complexity can contribute:



Second order diagram



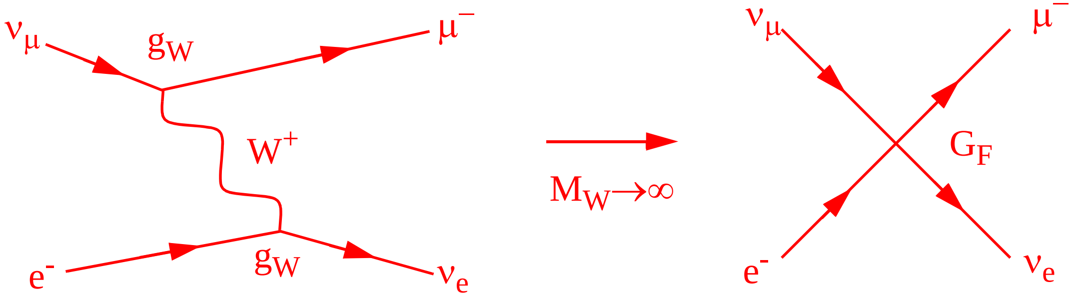
Sixth order diagram

- Second order diagram ⇨ cross section is proportional to a_W^2
- Sixth order diagram ⇨ cross section is proportional to a_W^6

The electroweak theory

➡ Point-like interactions and low-energy measurements

- Fermi's theory of weak interactions ⇨ four-fermion point-like interactions (without W and Z exchange).
- W bosons are heavy ⇨ charged current interactions can be approximated by a zero-range interaction at low energy



- Fermi coupling constant (G_F) ⇨ The strength of the zero-range interactions

next slide

Relationship between g_W and G_F:

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2}} = \frac{4\pi\alpha_W}{M_W^2}$$

The electroweak theory

- **A bit of algebra:**

The unification condition

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2}} \quad \text{and} \quad \sqrt{\frac{\pi\cdot\alpha_{em}}{2}} = g_W \sin\theta_W \quad \textbf{gives} \quad M_W^2 = \frac{g_W^2\sqrt{2}}{G_F} = \frac{\pi\alpha_{em}}{\sqrt{2}G_F\sin^2\theta_W}$$

$$\cos\theta_W = \frac{M_W}{M_Z} \quad \text{and} \quad M_W^2 = \frac{\pi\alpha_{em}}{\sqrt{2}G_F\sin^2\theta_W} \quad \textbf{gives} \quad M_Z^2 = \frac{\pi\alpha_{em}}{\sqrt{2}G_F\cos^2\theta_W\sin^2\theta_W}$$

Definition Weinberg angle

- G_Z ⇒ **neutral current coupling constant in the low energy zero-range approximation**

$$\boxed{\frac{G_Z}{\sqrt{2}} = \frac{g_Z^2}{M_Z^2}} \quad \textbf{which gives} \quad \frac{G_Z}{G_F} = \frac{\sqrt{2}g_Z^2 M_W^2}{\sqrt{2}g_W^2 M_Z^2} = \frac{\frac{\pi\alpha}{2\cos^2\theta_W}}{\frac{\pi\alpha}{2\sin^2\theta_W}}\cos^2\theta_W = \sin^2\theta_W$$

The electroweak theory

- Measurements of weak interaction rates at low energy ⇨ G_Z and G_F ⇨ $G_Z/G_F=\sin^2(\theta_W)$ ⇨

$$\sin^2\theta_W = 0,277 \pm 0.014$$

- Measurement of the weak mixing angle ⇨ predict the masses of the W and Z:

$$M_W = 78.3 \pm 2.4 \text{ GeV/c}^2$$
$$M_Z = 89.0 \pm 2.0 \text{ GeV/c}^2$$

- Discovery at CERN of W and Z at predicted masses ⇨ confirmation that the electroweak theory is correct.

The electroweak theory

- **Modern estimation of the Weinberg angle (from many experiments)**

$$\sin^2\theta_W = 0.2255 \pm 0.0021$$

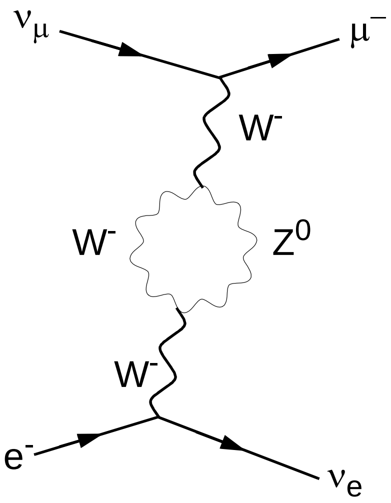
- **This value and the previous formulas give:**

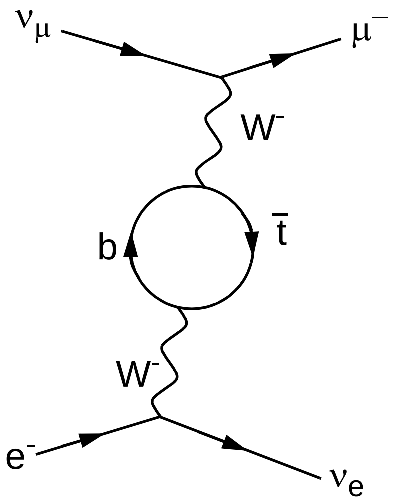
$$M_W = 78.5\ \mathrm{GeV/c^2} \qquad M_Z = 89.0\ \mathrm{GeV/c^2}$$

while direct measurements give

$$M_W = 80.4\ \mathrm{GeV/c^2} \qquad M_Z = 91.2\ \mathrm{GeV/c^2}$$

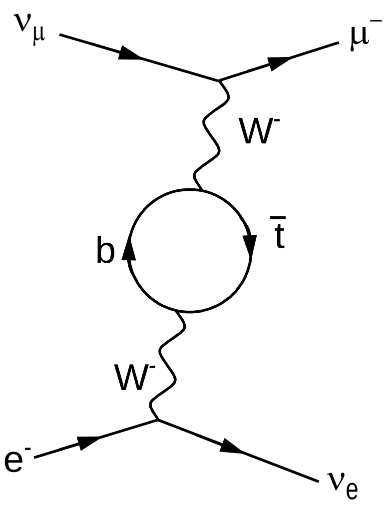
- **The** **difference** **is caused by higher-order diagrams (not taken into account in the low-energy formulas)**





The electroweak theory

- Higher-order corrections with top-quarks ⇨ measurement of electroweak processes ⇨ predict the top-quark mass



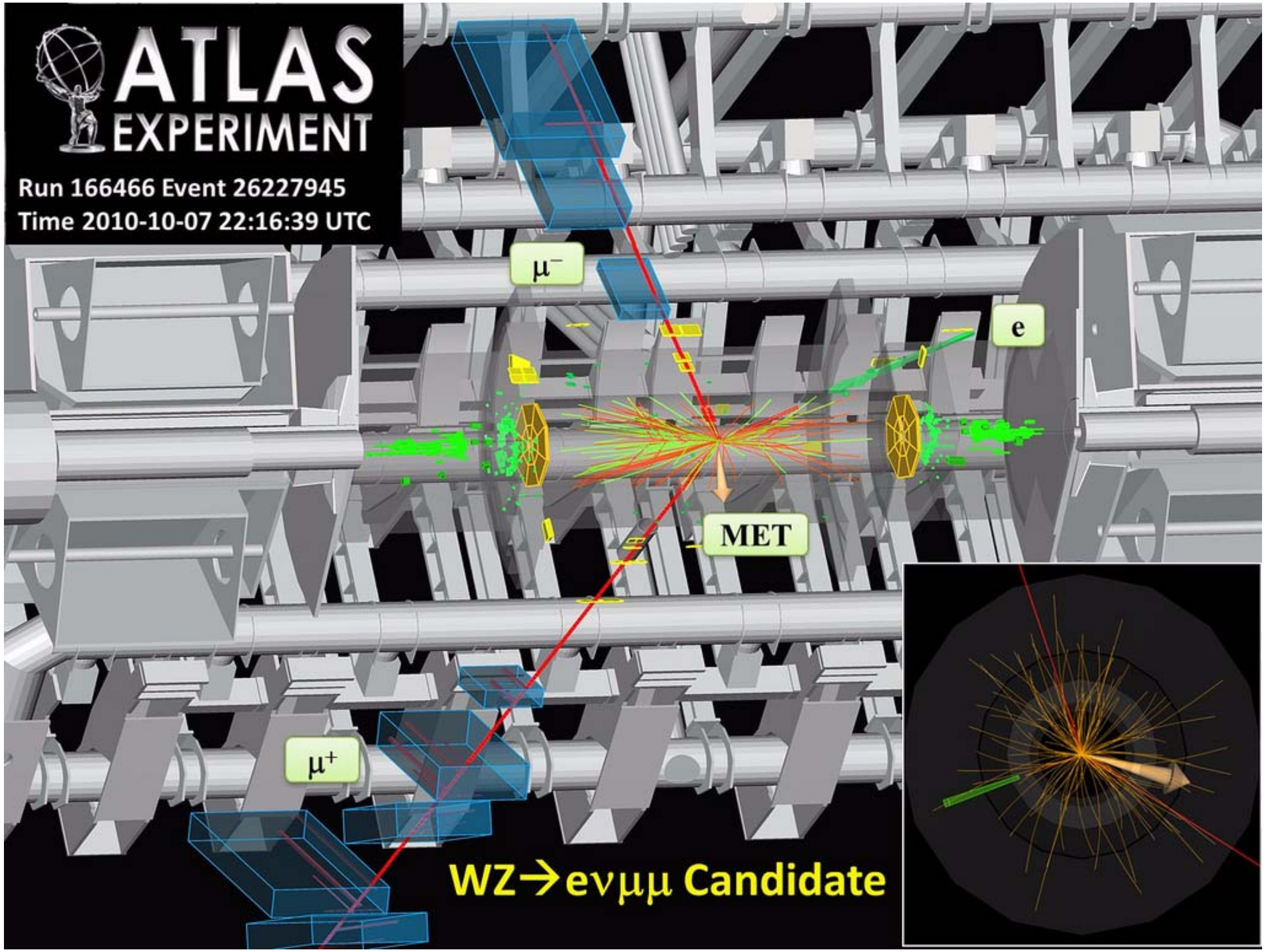
$$m_t = 170 \pm 30 \text{ GeV/c}^2$$

- The directly measured mass of the top quark at Fermilab by the CDF experiment gave a value

$$m_t = 176 \pm 5 \text{ GeV/c}^2$$

Perfect agreement with the low-energy prediction !

The W and Z bosons



The W and Z bosons

➡ The force carriers in weak interactions are spin-1 bosons (as in QED and QCD) that couple to quarks and leptons.

- W^+ , W^- and Z^0 ⇨ Intermediate vector bosons
  The force carriers of weak interactions
- W^+, W^- and Z^0 bosons are very massive particles
  m_W=80.4 GeV and m_Z=91.2 GeV ⇨
  Weak interactions have a very short range (2 x 10^{-3} fm).
- All observed low-energy weak processes (e.g. β-decay) ⇨
  charged current reactions ⇨ mediated by W^+ or W^- bosons.
- Electroweak prediction ⇨ neutral current reactions
  caused by the Z^0 boson should exist.

Discovery of the neutral current

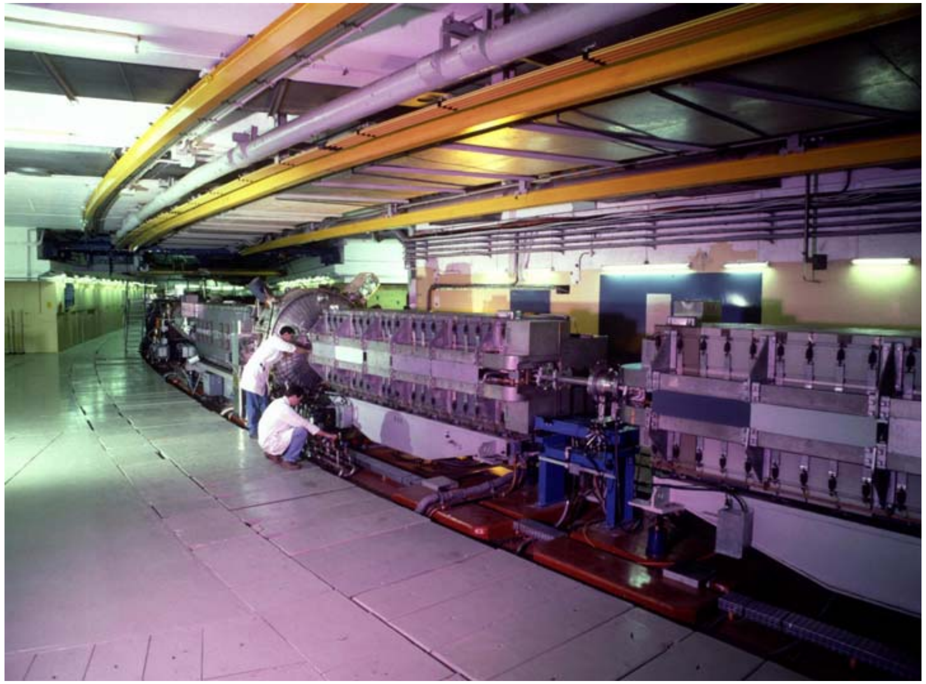
➡ Accelerator: The Proton Synchrotron (PS) at CERN

The PS accelerator: **Length = 628 m, Number of magnets = 277,**
Proton beam energy = 28 GeV

Neutrino beams:

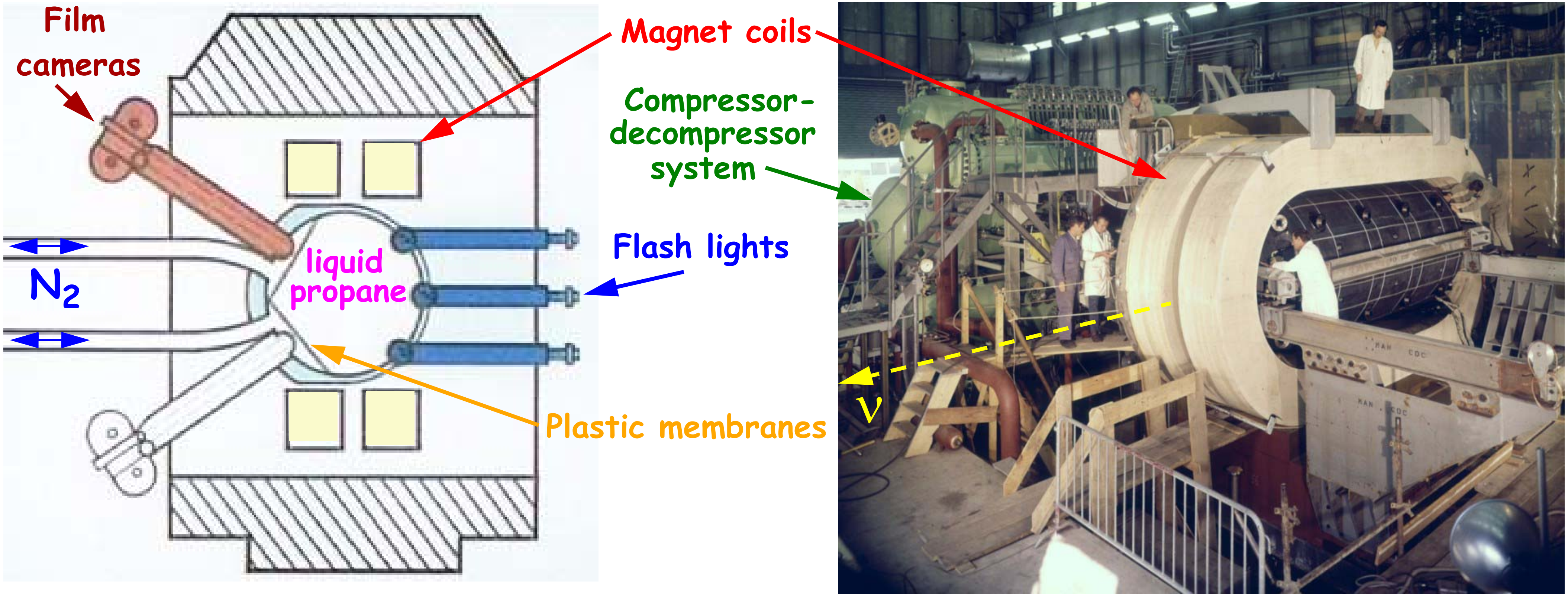
Step 1. Intense proton beam hits a target

Step 2. Charged pions and kaons decay $\pi^+ \rightarrow \mu^+ + \nu_\mu \rightarrow \mathbf{e}^+ + \nu_\mathbf{e} + \bar{\nu}_\mu + \nu_\mu.$



Discovery of the neutral current

- Neutral current events was first observed by the Gargamelle experiment at CERN in 1973.



Step 1. The liquid propane is at a temperature below its boiling point.

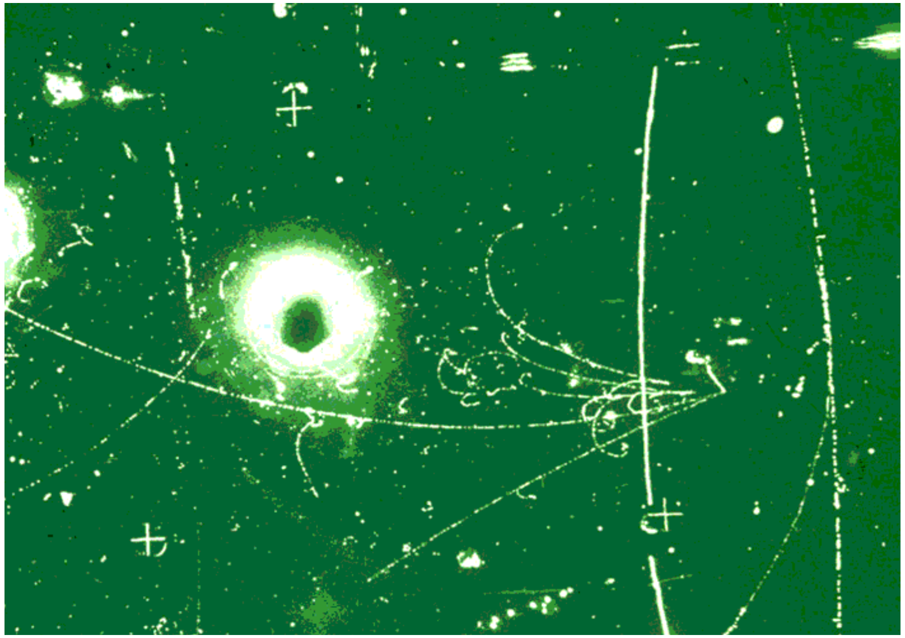
Step 2. When the ν enters the propane, its pressure is lowered to make it superheated.

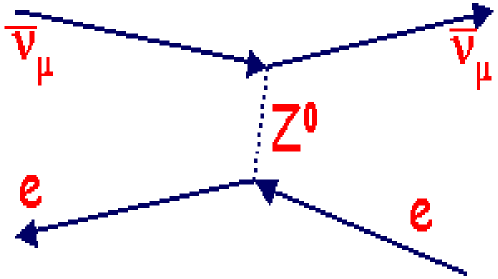
Step 3. Charged tracks ionize the propane and these ions create bubbles in the liquid.

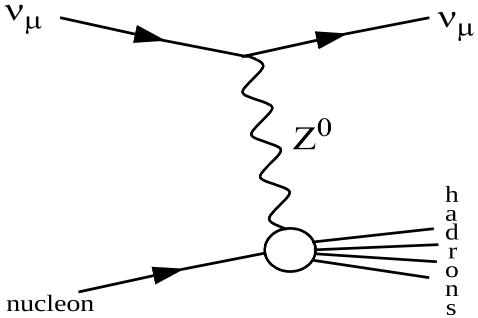
Step 4. The bubble tracks are photographed by film cameras.

Discovery of the neutral current

- ➡ Elastic and inelastic neutral current reactions possible.

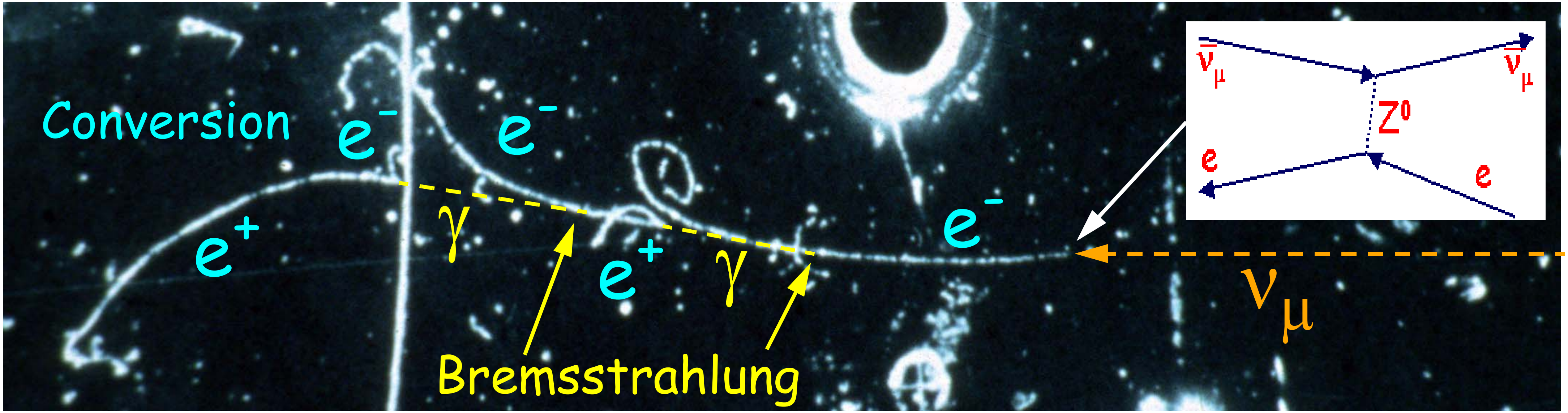






- Main **background** ⇨ **neutron** - nucleon interactions.

Discovery of the neutral current



One of the first neutral current reactions seen by the Gargamelle experiment.

The discovery of the W and Z bosons

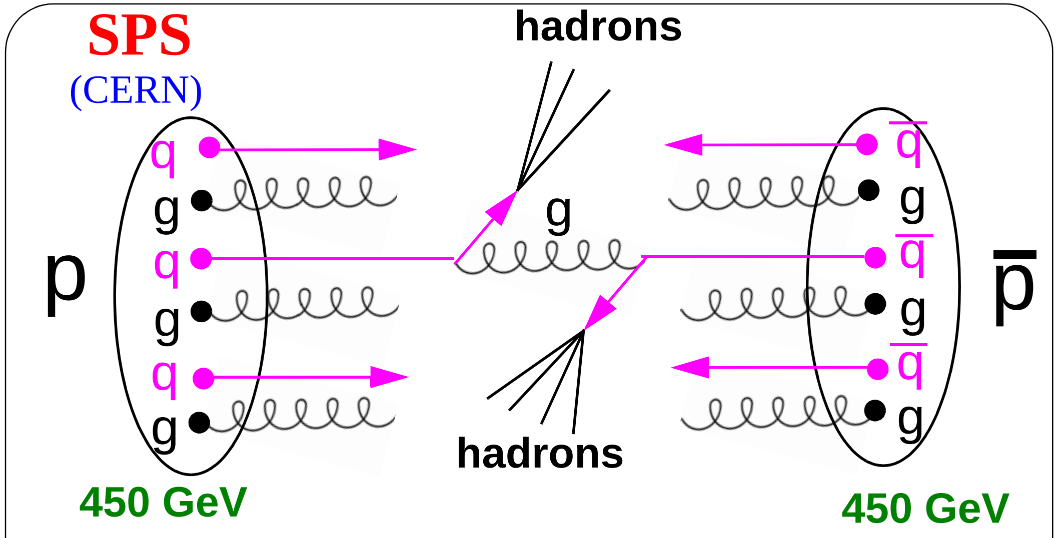
➡ The accelerator: The Super Proton Synchrotron

- **The W and Z were discovered by the UA1 and UA2 experiments.**

The SPS accelerator: Length = 6.9 km, Number of magnets = 960,
Fixed target = 400 ⟶ 450 GeV
$p\bar{p}$-collider = 540 ⟶ 630 ⟶ 900 GeV

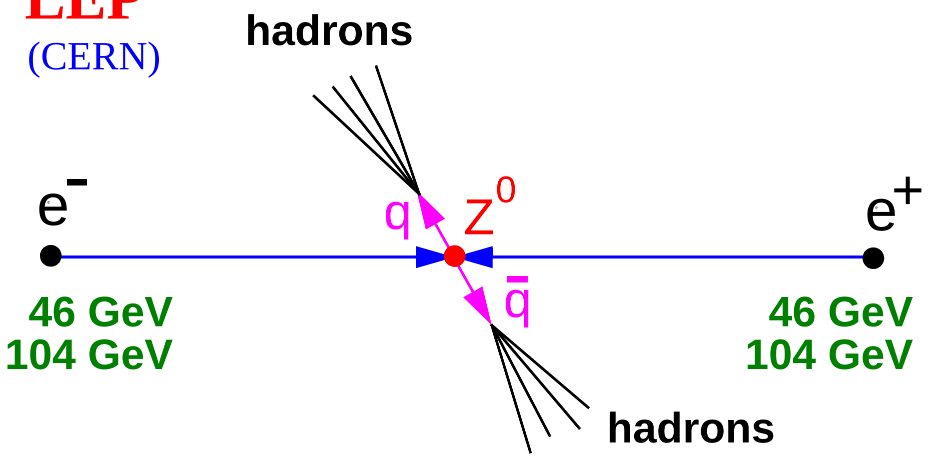
The discovery of the W and Z bosons

SPS
(CERN)



Length: 7 km (960 magnets)
Experiments: UA1, UA2

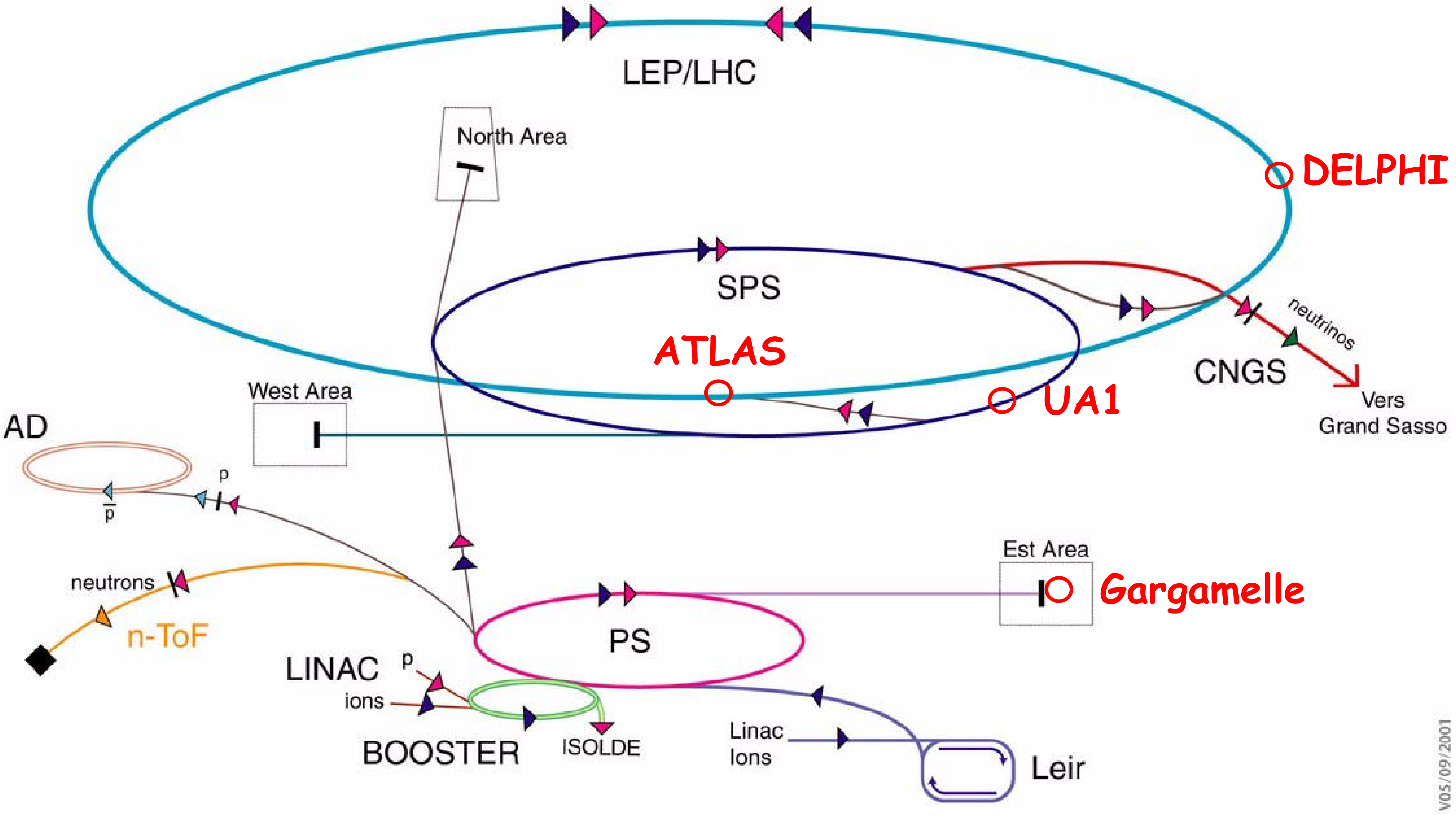
LEP
(CERN)



Length: 27 km (4184 magnets)
Experiments: DELPHI, OPAL, ALEPH, L3

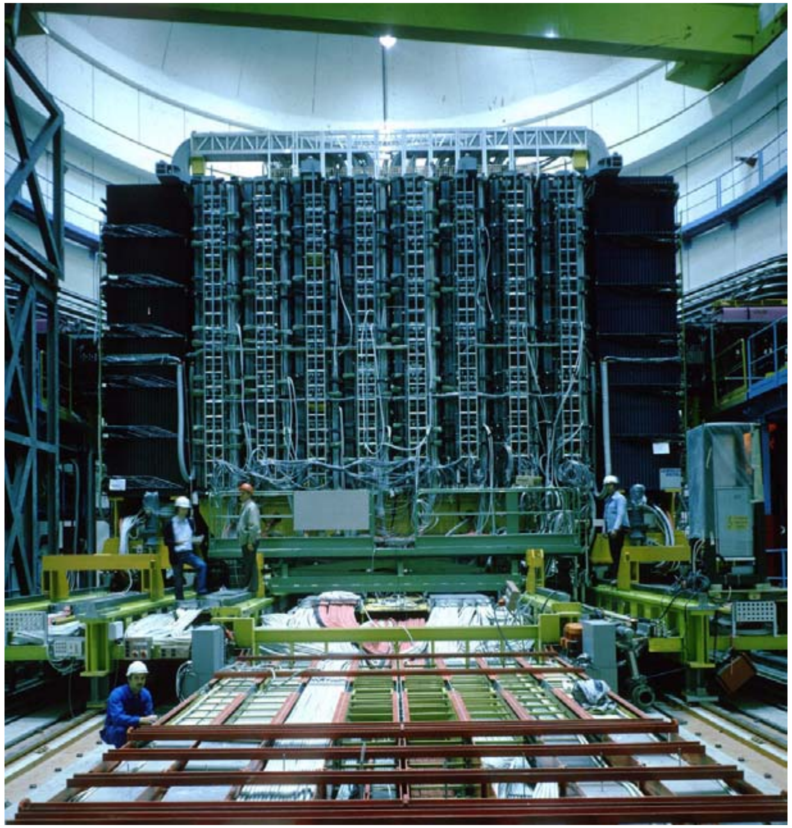
- The energy of the quarks and gluons carry only a fraction of the proton energy.
- To produce W- ans Z-bosons with a mass of 80-90 GeV ⇨ a collider with a beam energy of 270 GeV was needed.
- The beam energy was later increased to 450 GeV.

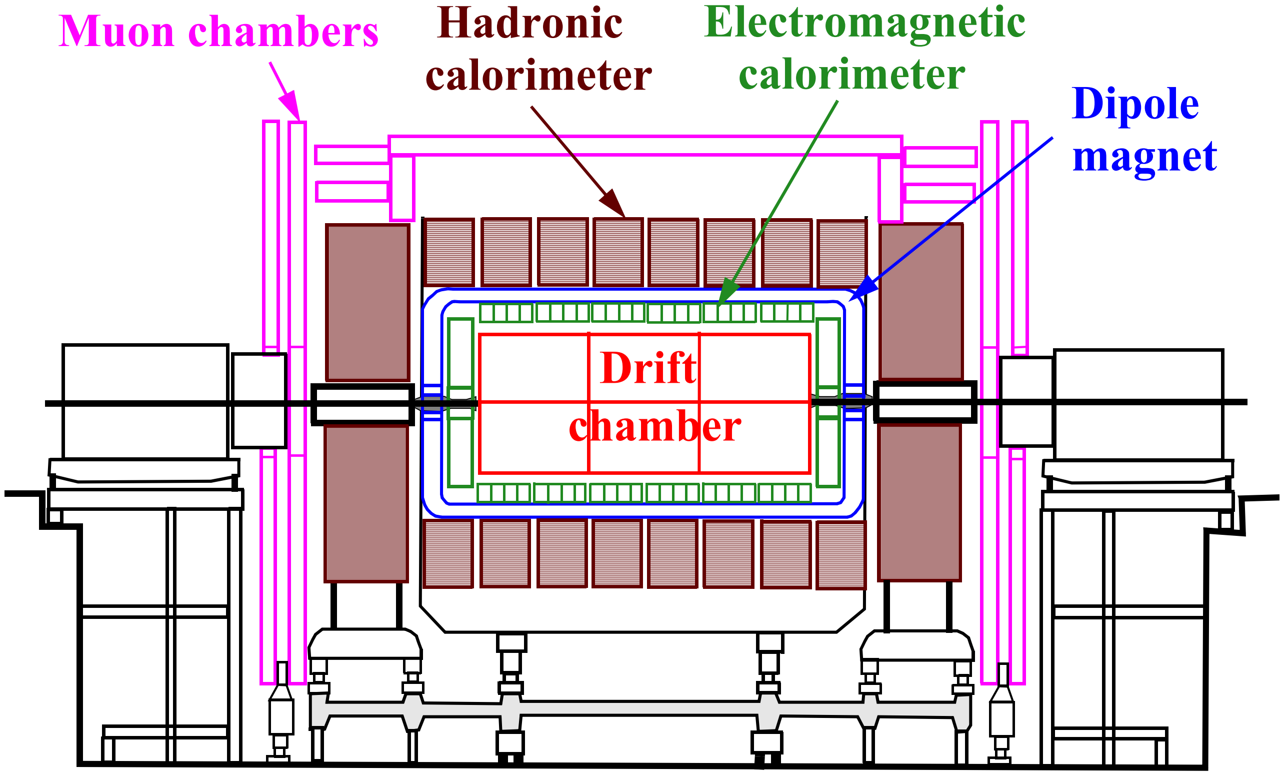
The discovery of the W and Z bosons



The discovery of the W and Z bosons

➡ The experiment: UA1





Tracking detector: Central wire chamber with 0.7 T dipole magnet.

Electromagnetic calorimeter: Lead/scintillator sandwich

Hadronic calorimeter: Iron/Scintillator sandwich

Muon detector: 8 planes of drift chambers.

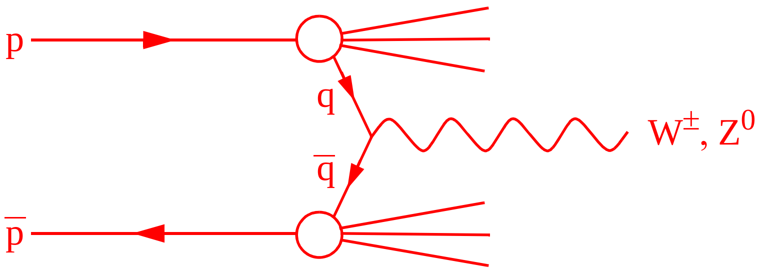
The discovery of the W and Z bosons

➡ Production of W and Z bosons

- The W and Z bosons are in proton colliders produced by quark-antiquark annihilations:

$$u + \bar{d} \rightarrow W^{+} \qquad d + \bar{u} \rightarrow W^{-}$$

$$u + \bar{u} \rightarrow Z^{0} \qquad d + \bar{d} \rightarrow Z^{0}$$



- The lifetime of both the W and the Z is about 3×10^{-25} s

⇨ they cannot be seen directly in the experiments.

The discovery of the W and Z bosons

➡ The decay of W and Z bosons

hadronic decays

$$\bar{p} + p \rightarrow W^{+} + X$$
$$\lfloor\!\!\longrightarrow q' + \bar{q}$$

$$\bar{p} + p \rightarrow W^{-} + X$$
$$\lfloor\!\!\longrightarrow q' + \bar{q}$$

$$\bar{p} + p \rightarrow Z^{0} + X$$
$$\lfloor\!\!\longrightarrow q + \bar{q}$$

leptonic decays

$$\bar{p} + p \rightarrow W^{+} + X$$
$$\lfloor\!\!\longrightarrow l^{+} + \nu_{l}$$

$$\bar{p} + p \rightarrow W^{-} + X$$
$$\lfloor\!\!\longrightarrow l^{-} + \bar{\nu}_{l}$$

$$\bar{p} + p \rightarrow Z^{0} + X$$
$$\lfloor\!\!\longrightarrow l^{+} + l^{-}$$

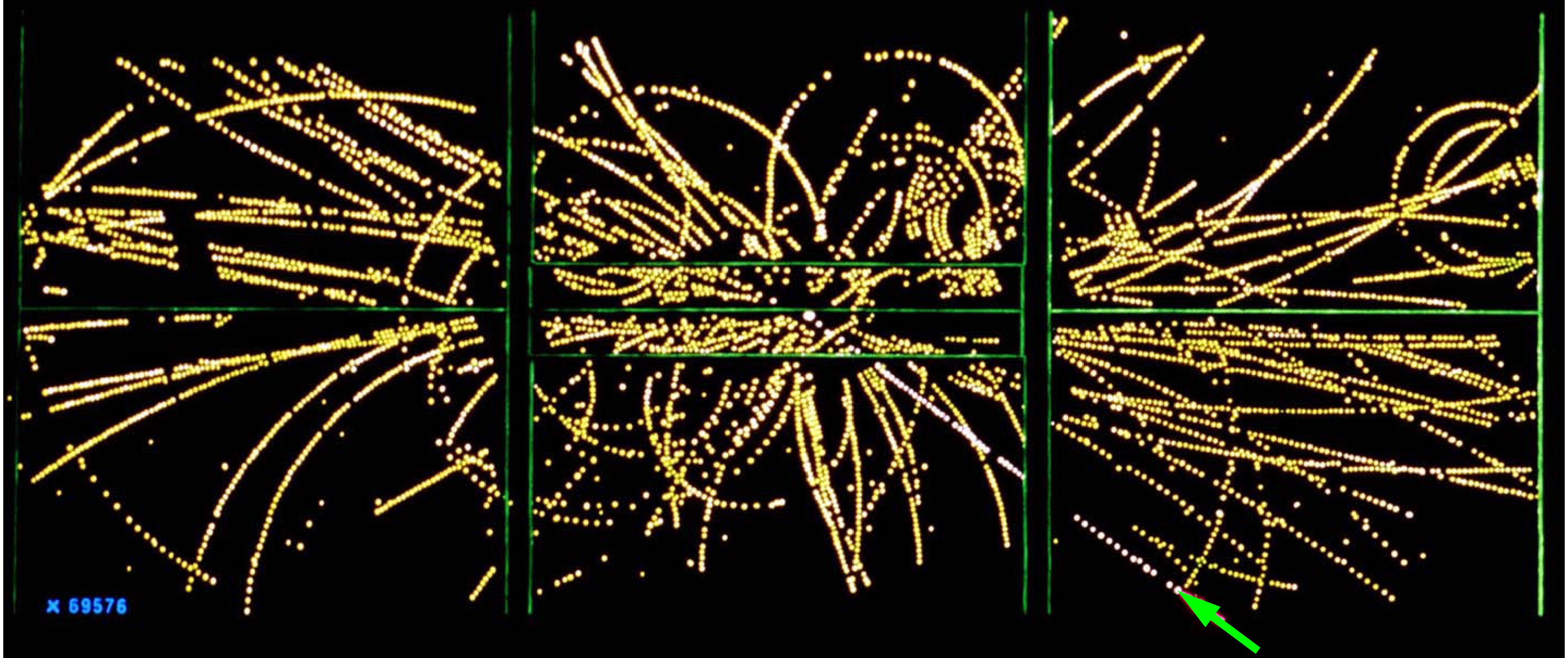
Cannot be found among all other hadrons produced.

The decays to leptons are easy to identify

The discovery of the W and Z bosons

➡ Analysis of W-events in UA1

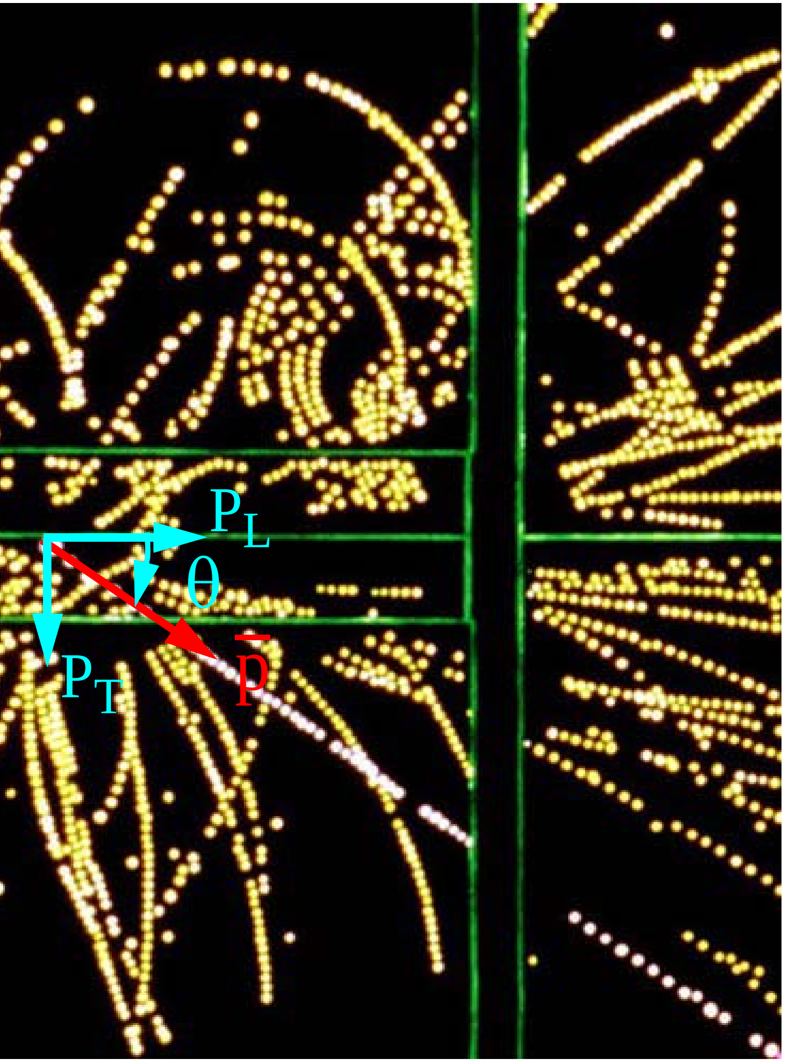
The analysis searched for events with a charged lepton + neutrino



A high momentum electron is indicated by the arrow.

The discovery of the W and Z bosons

➡ Transverse momentum



- The **transverse momentum** and energy of a particle:

$$P_T = P\sin(\theta)$$
$$E_T = E\sin(\theta)$$

The angle to the beam

- $E_T = P_T$ if the mass of the particle is small since $E^2=P^2+m^2$

- The **total momentum is zero** of all the particles in a collision.

- Neutrinos are not detectable ⇨ if the total momentum ∓ 0 ⇨ the event has **missing momentum** (or missing energy).

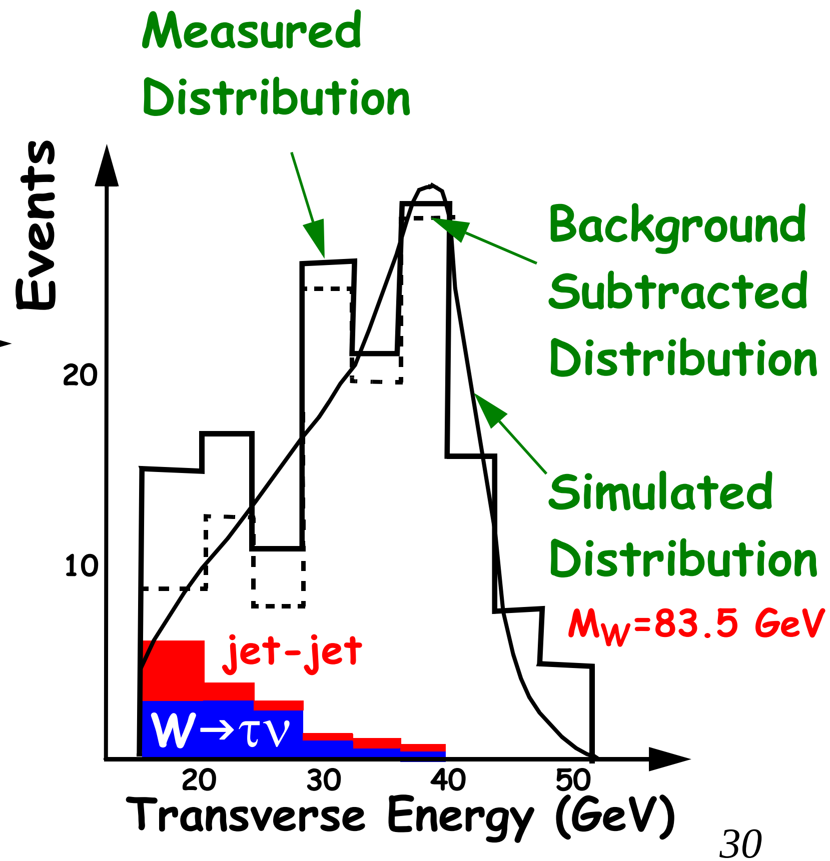
The discovery of the W and Z bosons

➡ Analysis of W-events in UA1

- The main selection criteria in the UA1 W-analysis was:

i) A charged electron or muon with a large momentum (>10 GeV/c)
ii) This lepton should be emitted at a wide angle to the beam (>5°)
iii) There should be large missing transverse momentum in the event

The expected distribution of the transverse energy of the selected electrons was compared with the measured distribution.



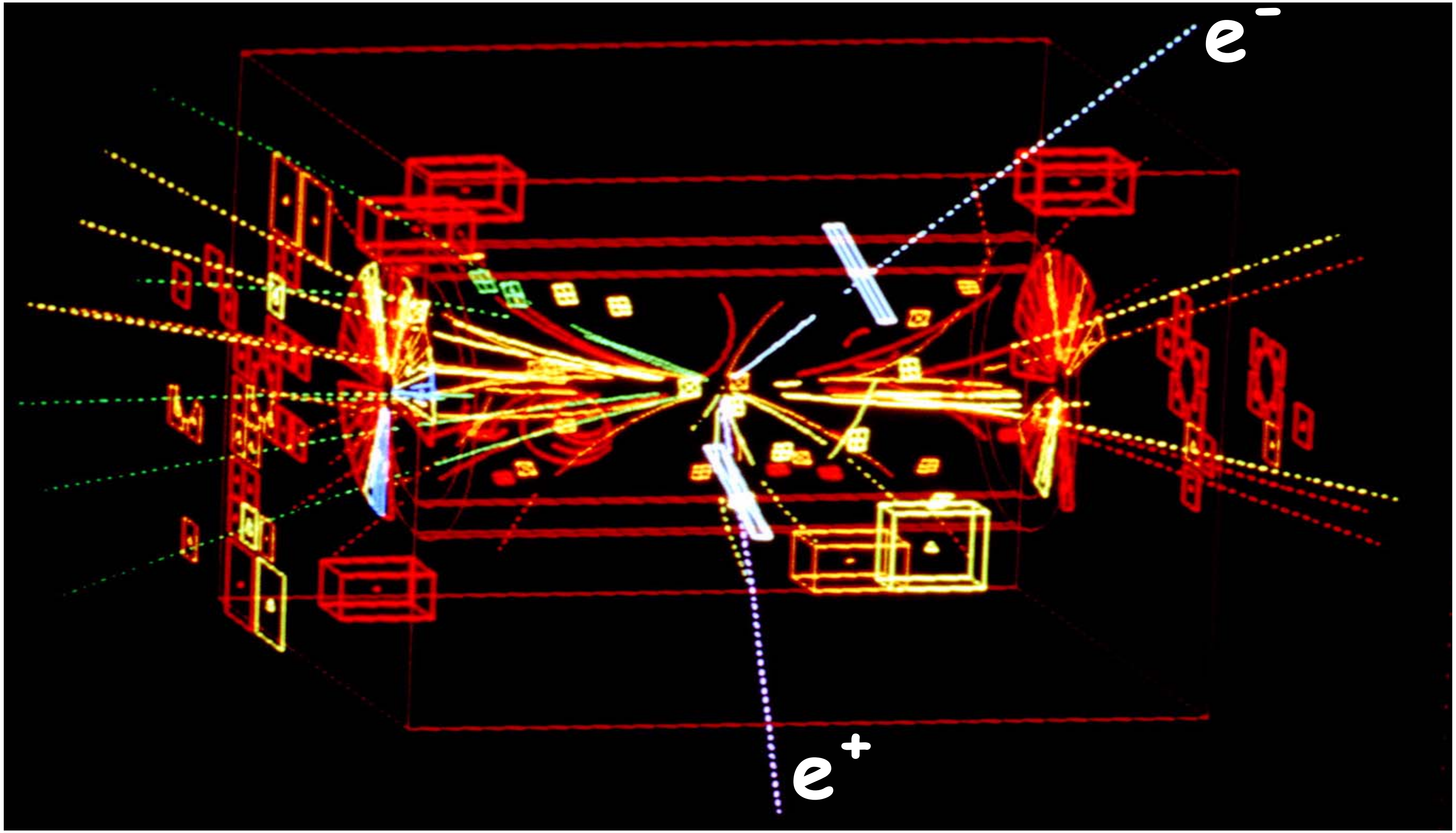
From the first 148 electron and 47 muon events it was estimated that:

$$M_W = 83{,}5 \pm 2{,}8\ GeV \qquad \Gamma_W \le 6{,}5\ \text{GeV}$$

The discovery of the W and Z bosons

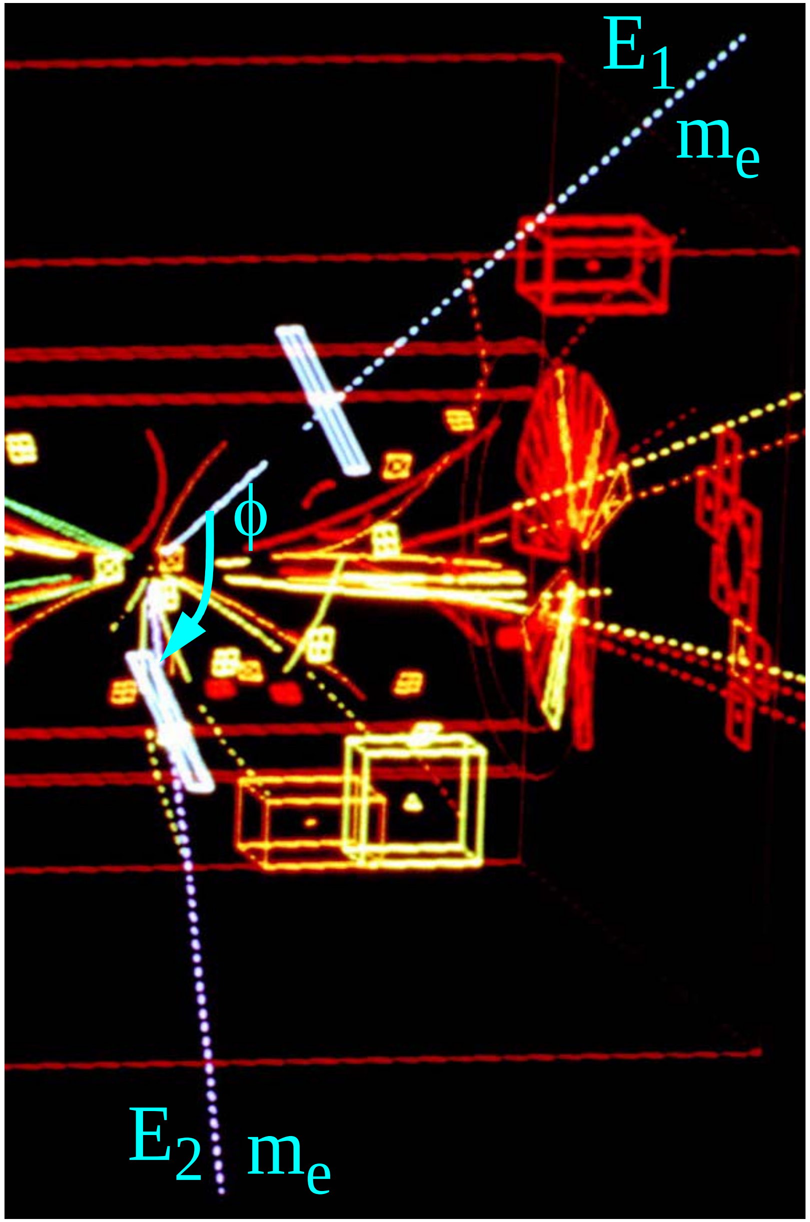
➡ Analysis of Z-events in UA1

- The analysis was looking for a pair of charged leptons.

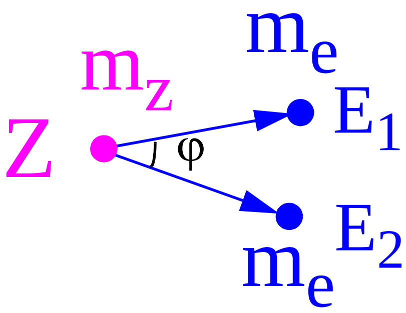


The discovery of the W and Z bosons

➡ Invariant mass



- The **invariant mass** of a particle that decays to two other particles:



$$m_Z^2 = (\vec{P}_1 + \vec{P}_2)^2$$ (4-vectors)

if m=0

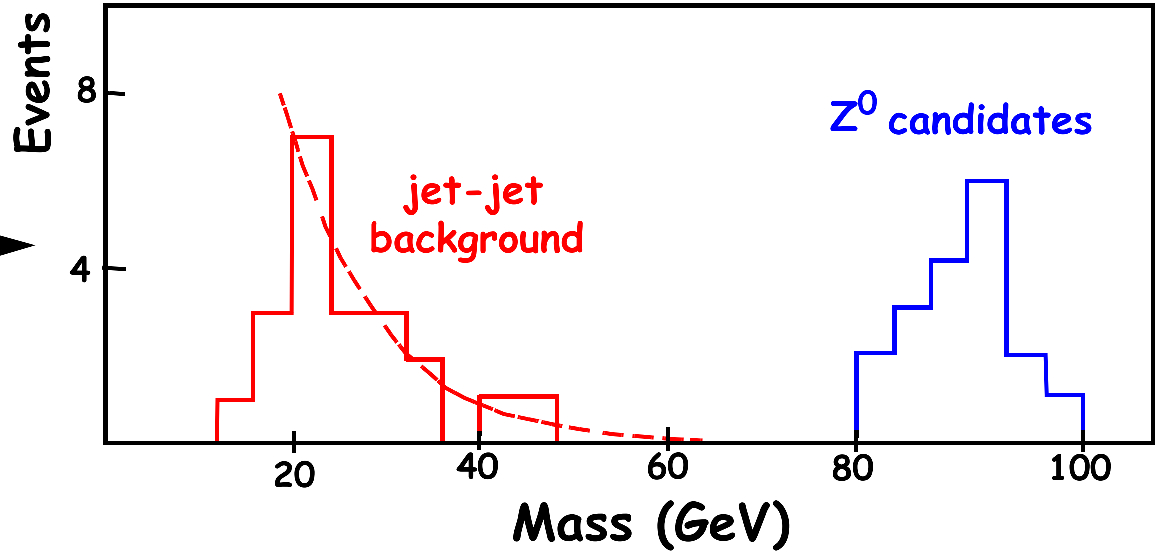
$$m_Z^2 = 2\,E_1 E_2\,(\,1 - \cos\varphi)$$

The discovery of the W and Z bosons

➡ Analysis of Z-events in UA1

- **Main search criteria** ⇨ **require pair of electrons or muons with a large transverse energy.**

The mass distribution of pairs of electrons where each electron has E_T>8 GeV.



The first 18 electron pairs and 10 muon pairs in UA1

$$M_Z = 93{,}0 \pm 1{,}4\ GeV$$

$$\Gamma_Z \leq 8{,}1 \text{ GeV}$$

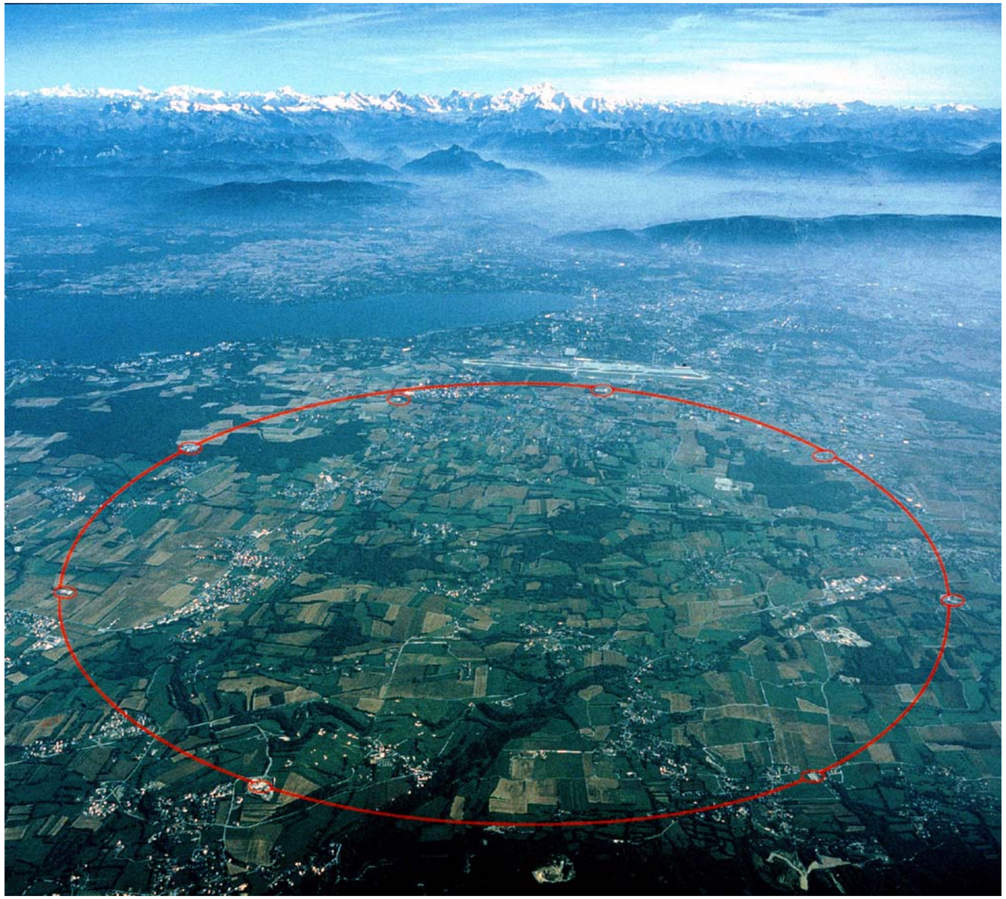
Precision studies of the W and Z bosons

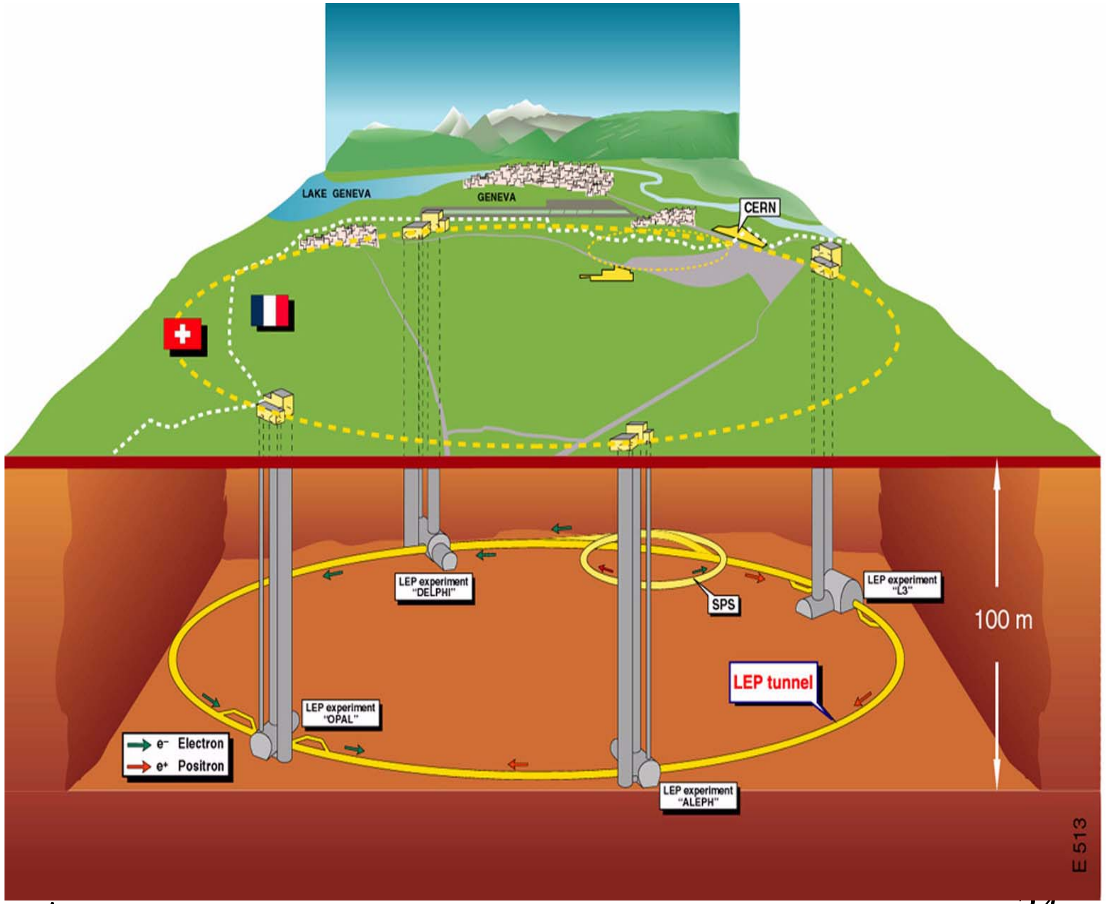
➡ The accelerator: The Large Electron Positron Collider

- Electrons-positrons collisions in four experiments.
- Collision energy: 91 GeV $\longrightarrow$ 209 GeV

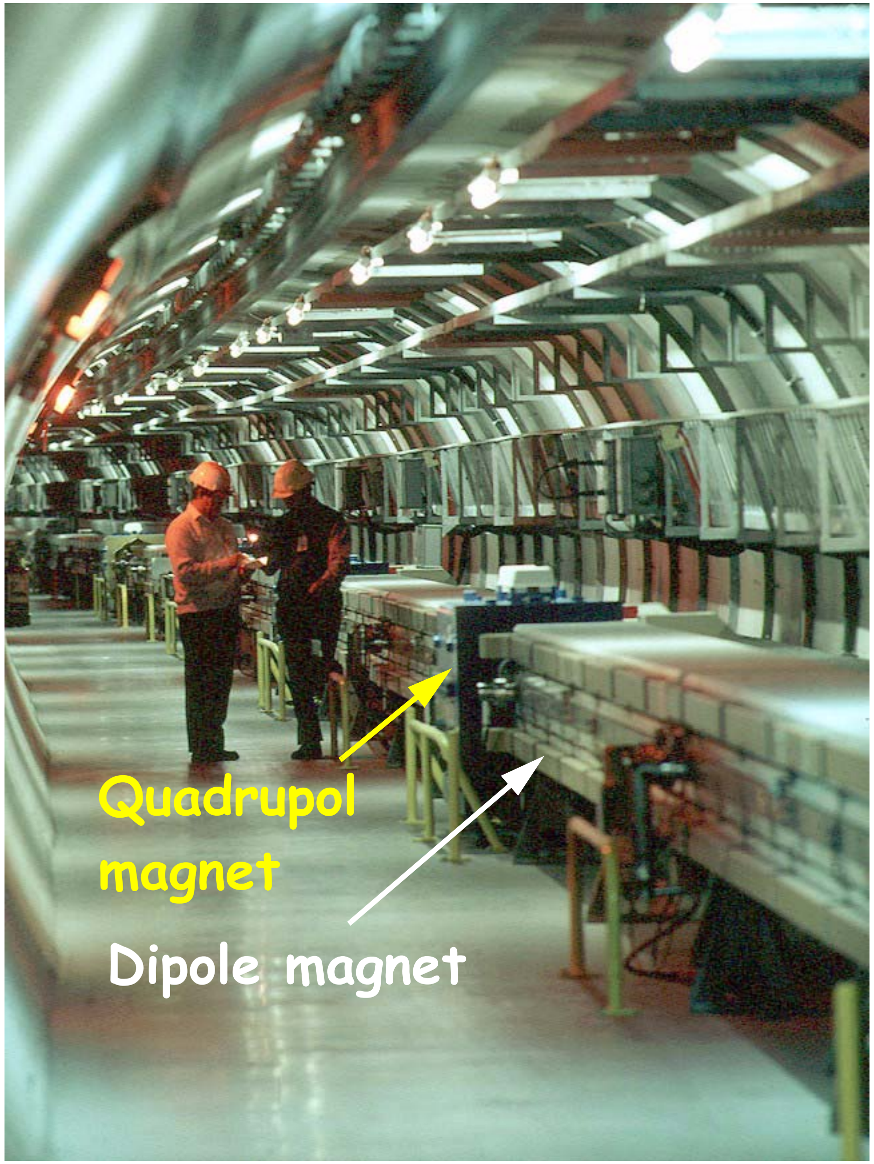
Z mass — maximum

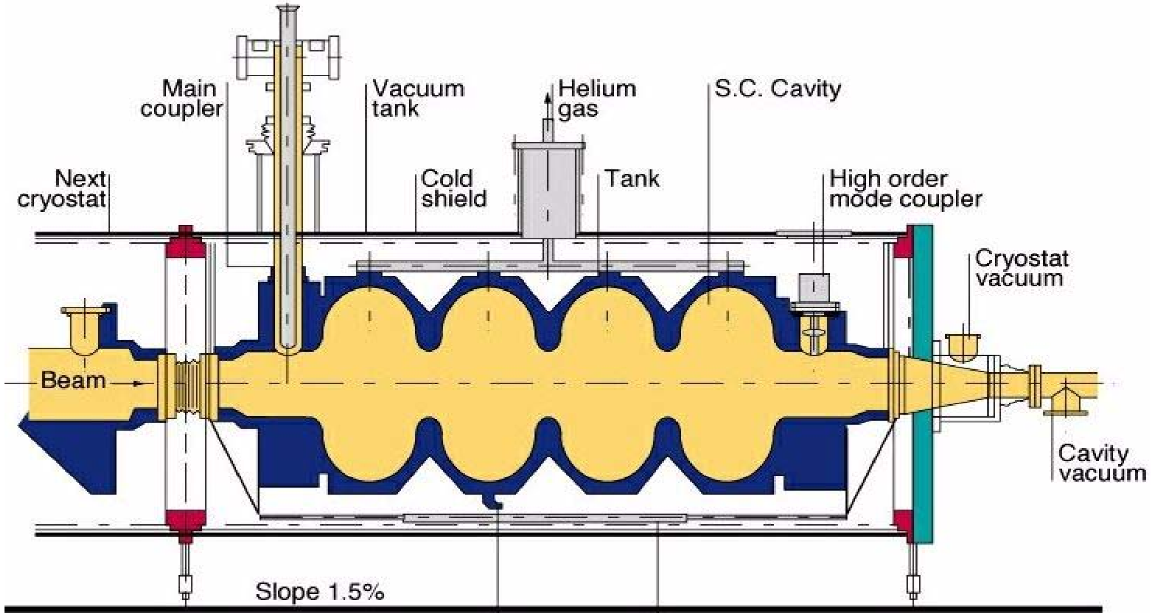
288 superconducting cavities.





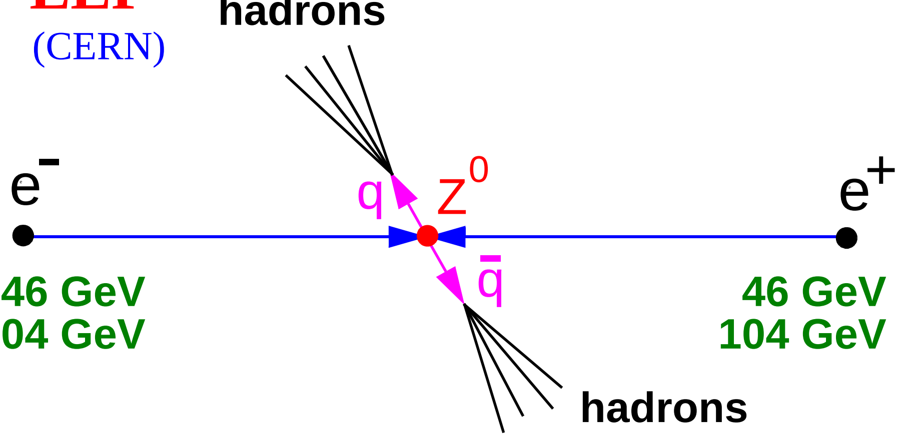
Precision studies of the W and Z bosons





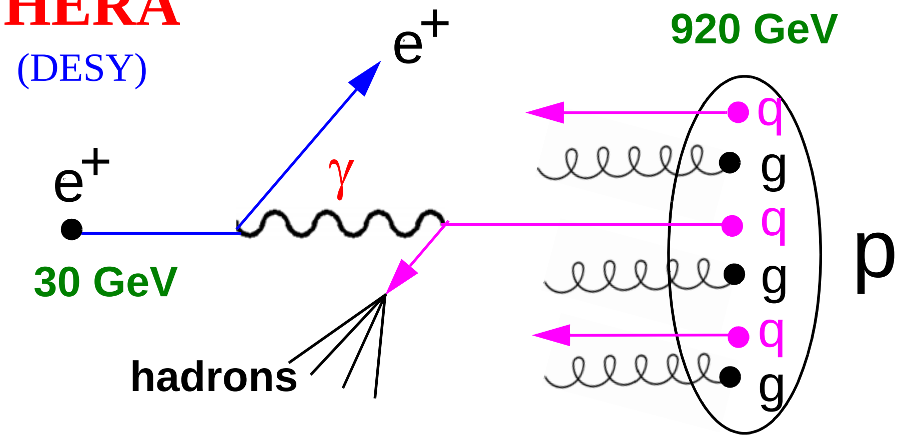
Precision studies of the W and Z bosons

LEP
(CERN)



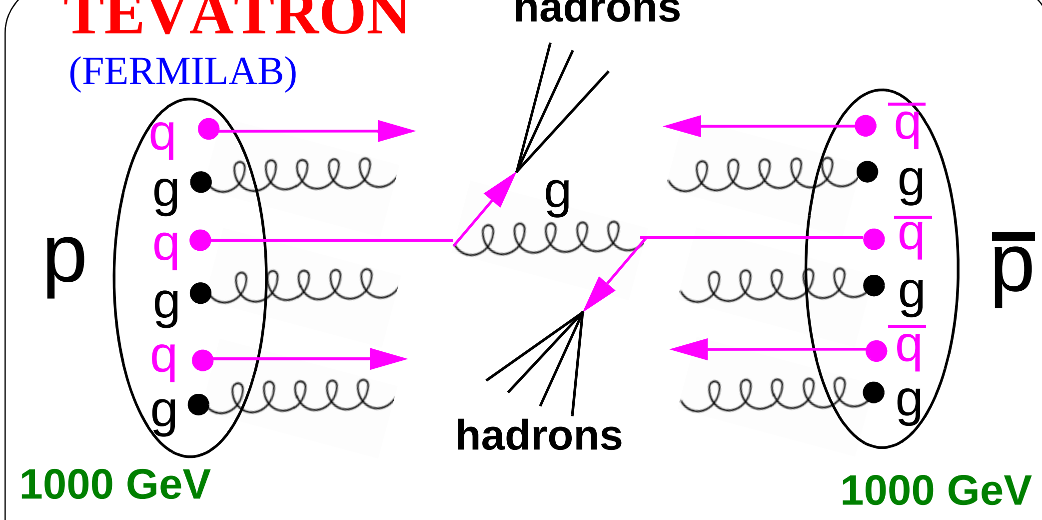
Length: 27 km (4184 magnets)
Experiments: DELPHI, OPAL, ALEPH, L3

HERA
(DESY)



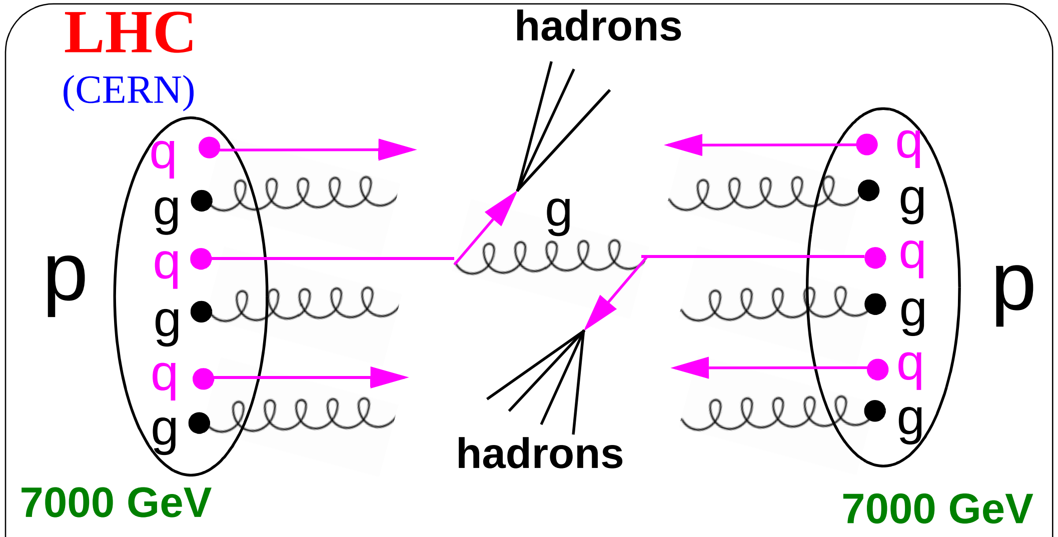
Length: 6 km (1650 magnets)
Experiments: H1, ZEUS

TEVATRON
(FERMILAB)



Length: 6 km (990 magnets)
Experiments: CDF, D0

LHC
(CERN)



Length: 27 km (1232 + 386 magnets)
Experiments: ATLAS, CMS, LHCb, ALICE

Precision studies of the W and Z bosons

➡ Differences between proton and electron colliders.

- **Synchrotron radiation:**

  Total energy loss is proportional to $1/\text{mass}^4$
  Total energy loss is proportional to E_{beam}^4
  Total energy loss is proportional to 1/Radius

- **Energy loss** in an electron accelerator is 10^{13} times larger.

- **LEP:** 344 cavities, accelerating voltage=**3630 MV**, energy=104 GeV
  LHC: 16 cavities, accelerating voltage = **16 MV**, energy =7000 GeV

- Magnetic bending field $\sim \dfrac{\text{Beam momentum}}{\text{Length of bending field}}$

- **LEP: 0.12 Tesla** bending field
  LHC: 8.38 Tesla bending field

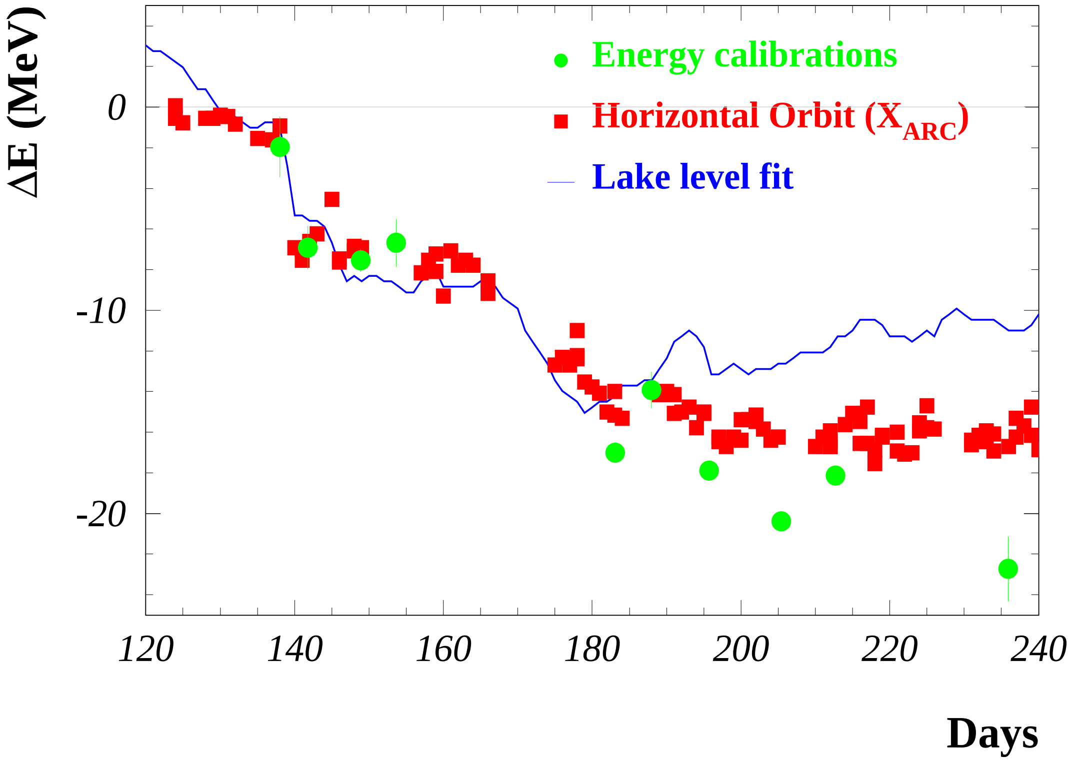
  $\Big\}$ factor 70

Precision studies of the W and Z bosons

The accelerator: The Large Electron Positron Collider

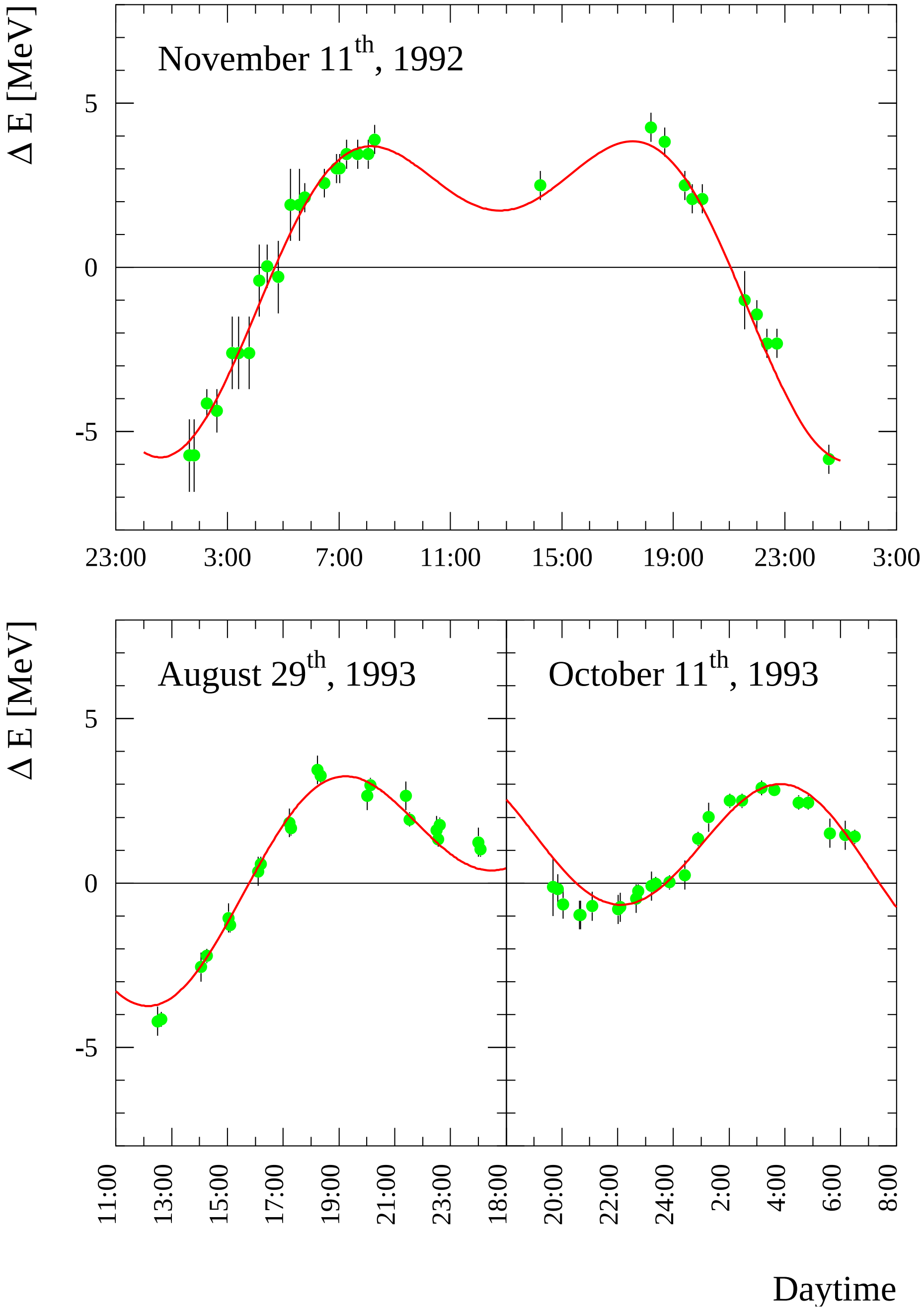
- Electron-positron collider: Precise measurement of $\sqrt{s}$ important !
- Problem: During 1993 the LEP energy was changing with time !

Geological shifts



- The water level in lake Geneva
- Rainfalls and the water table.

Precision studies of the W and Z bosons



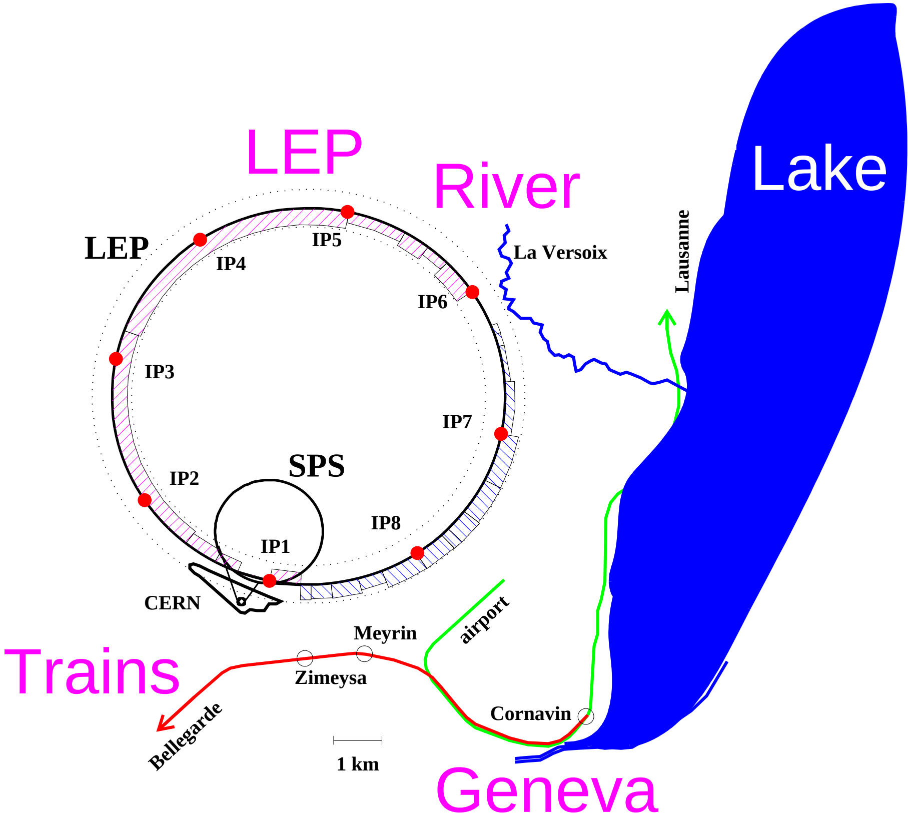
➡ Tides

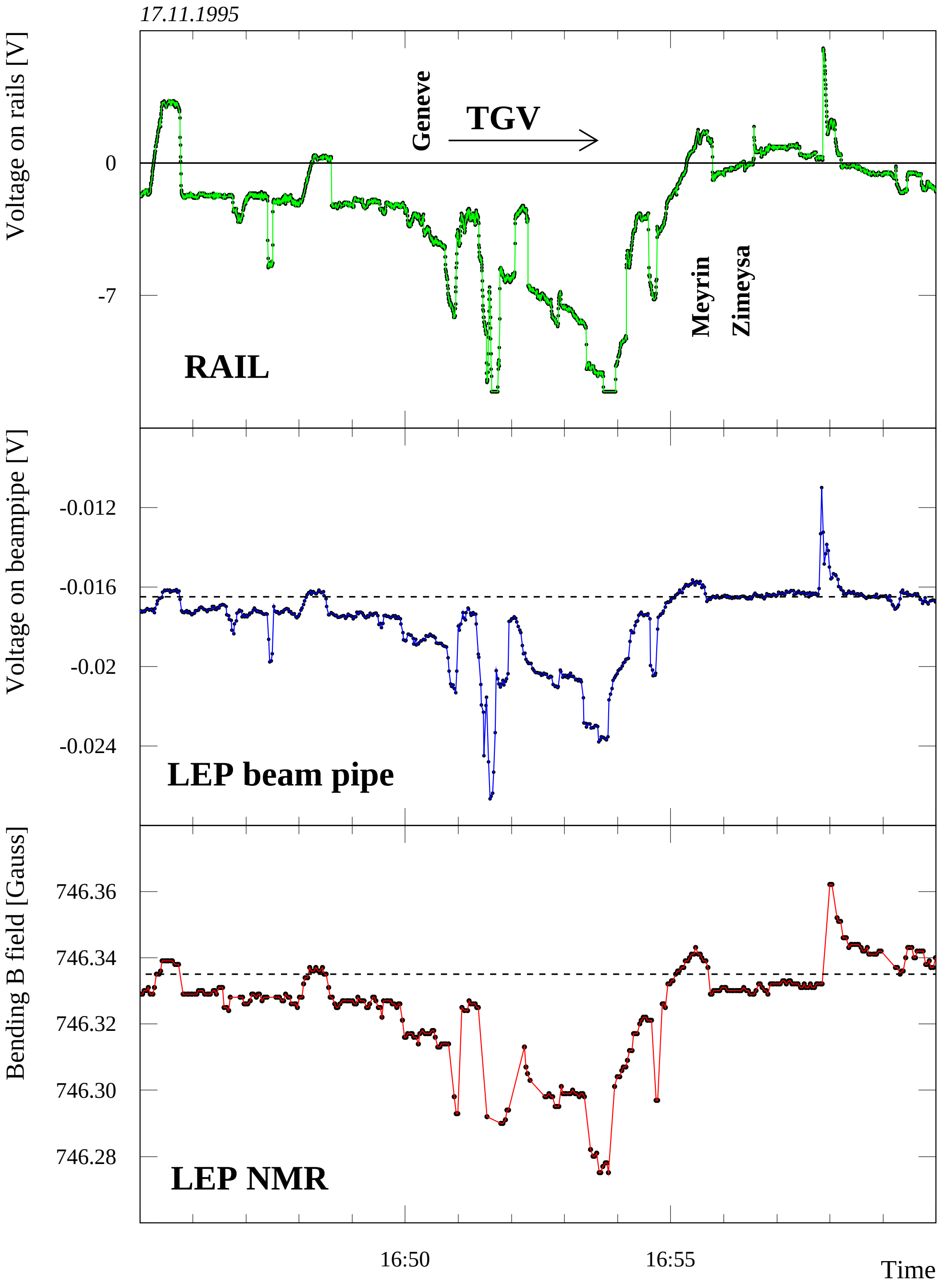
- **Earth tides** caused by the moon.
- The electron's **orbit changes.**
- **1 mm** ⇨ energy change of 10 MeV

Precision studies of the W and Z bosons

➡ Beampipe current

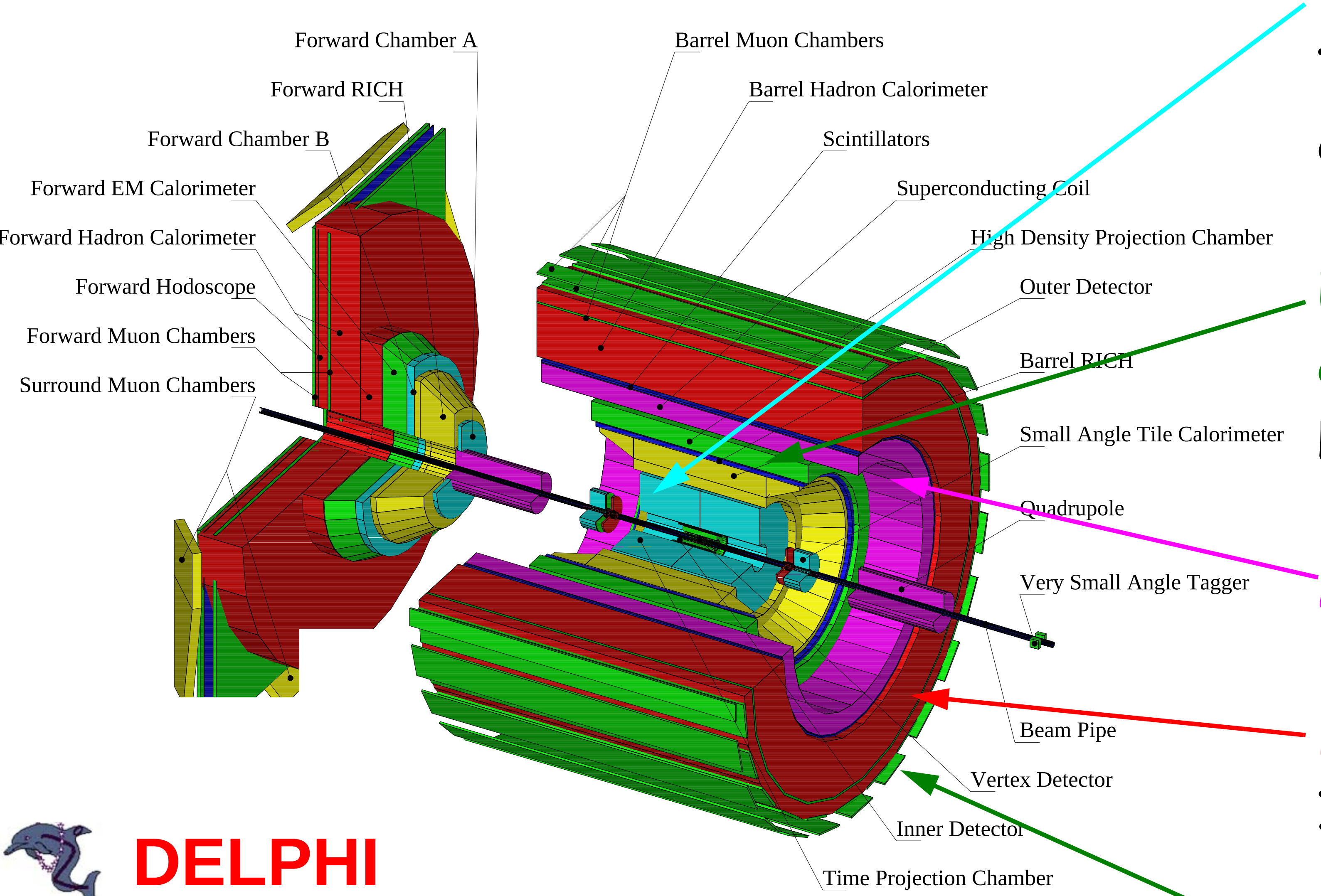
- Trains ⇨ parasitic currents
- Currents ⇨ the magnetic field
- Mag. field ⇨ electron's orbit
- Orbit ⇨ electron's energy





Precision studies of the W and Z bosons

➡ The DELPHI Experiment



Tracking detector:
Time projection chamber

Electromagnetic calorimeter:
Lead absorber

Magnet

Hadronic calorimeter
Iron absorber

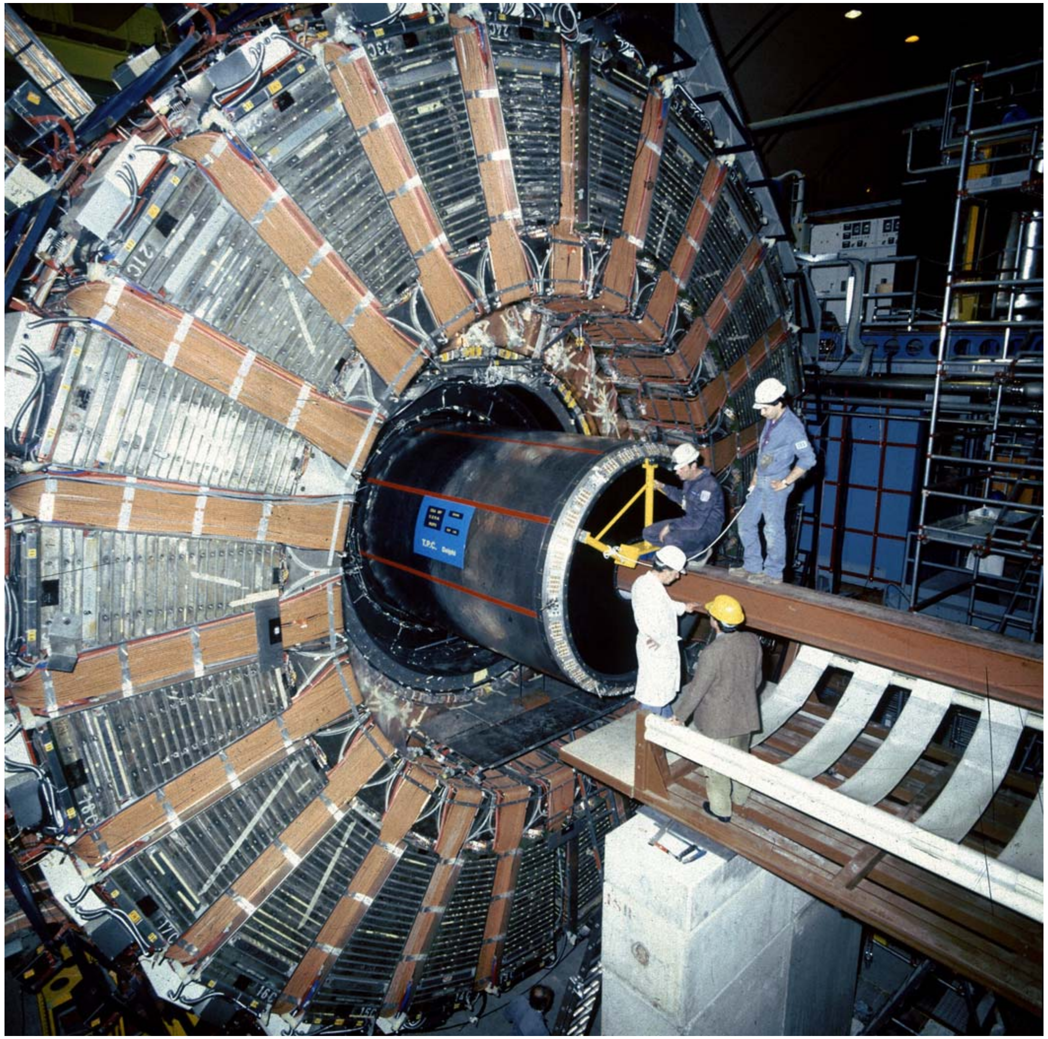
Muon detectors

DELPHI

Precision studies of the W and Z bosons

➡ The DELPHI Experiment

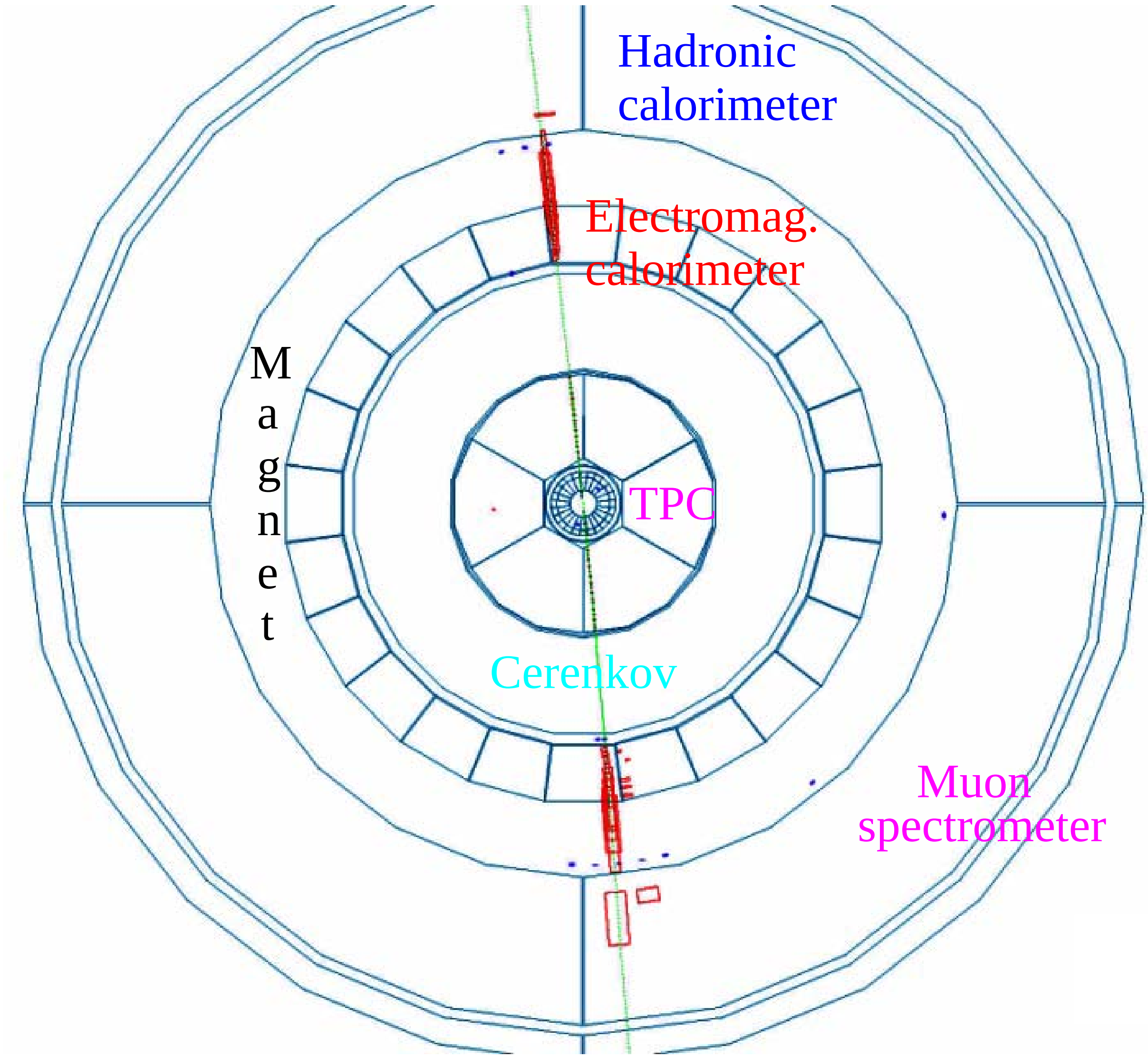
The DELPHI cavern

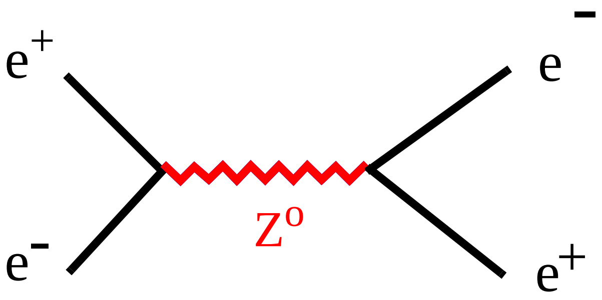


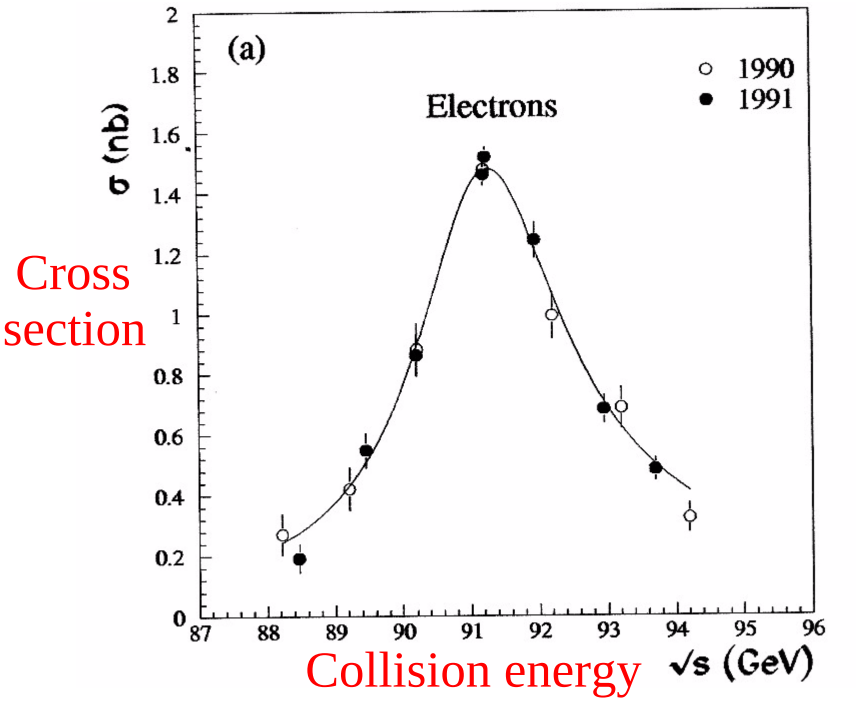
The Time Projection chamber

Precision studies of the W and Z bosons

➡ Studies of the Z-boson

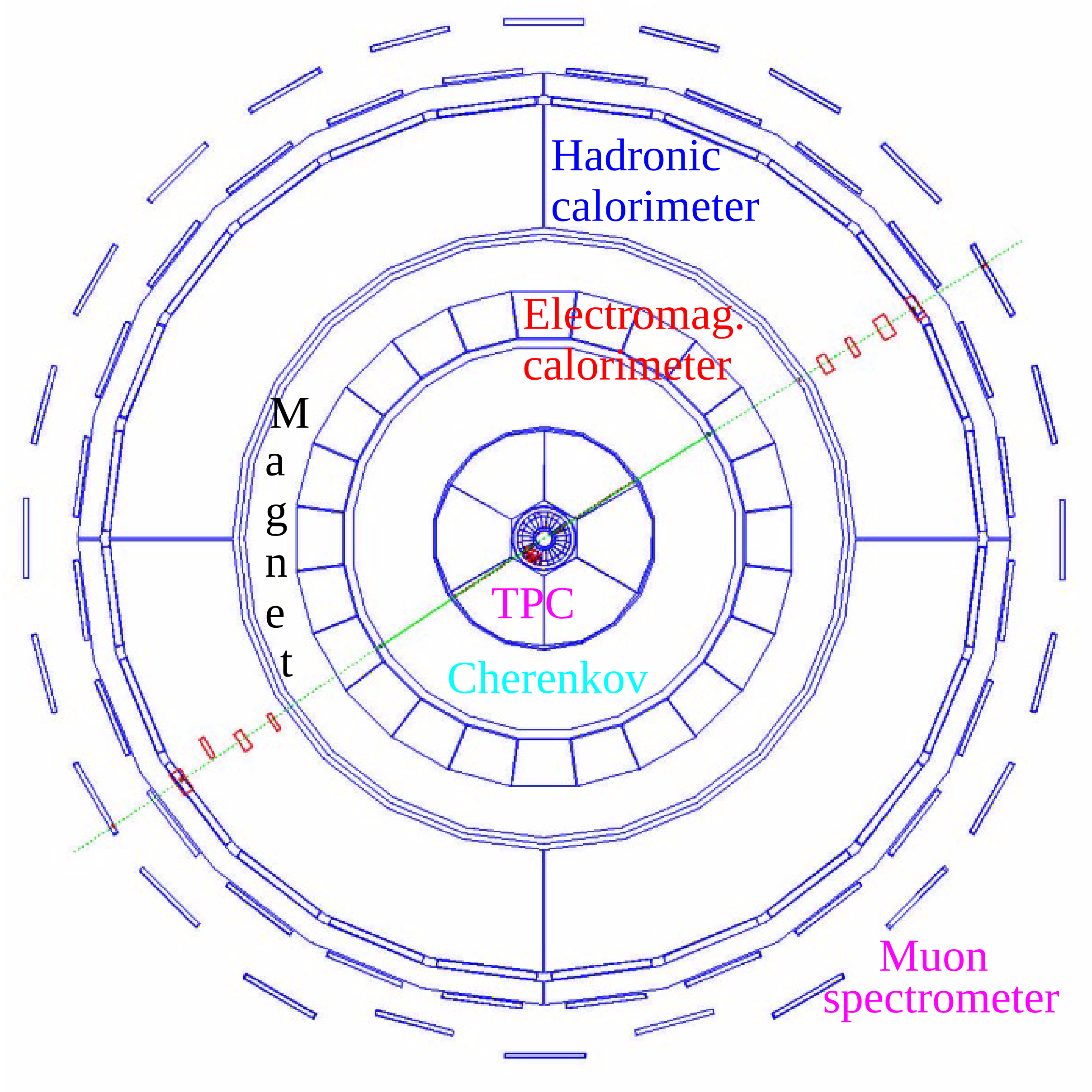


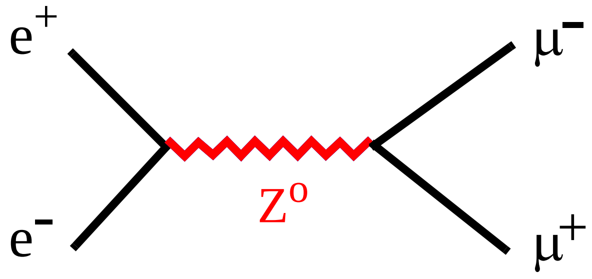


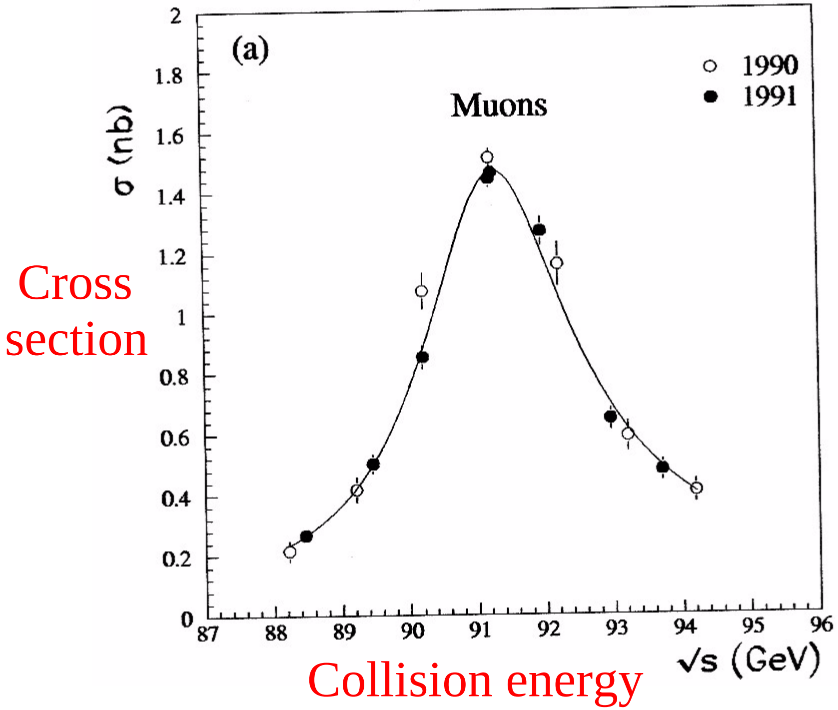


Precision studies of the W and Z bosons

➡ Studies of the Z-boson

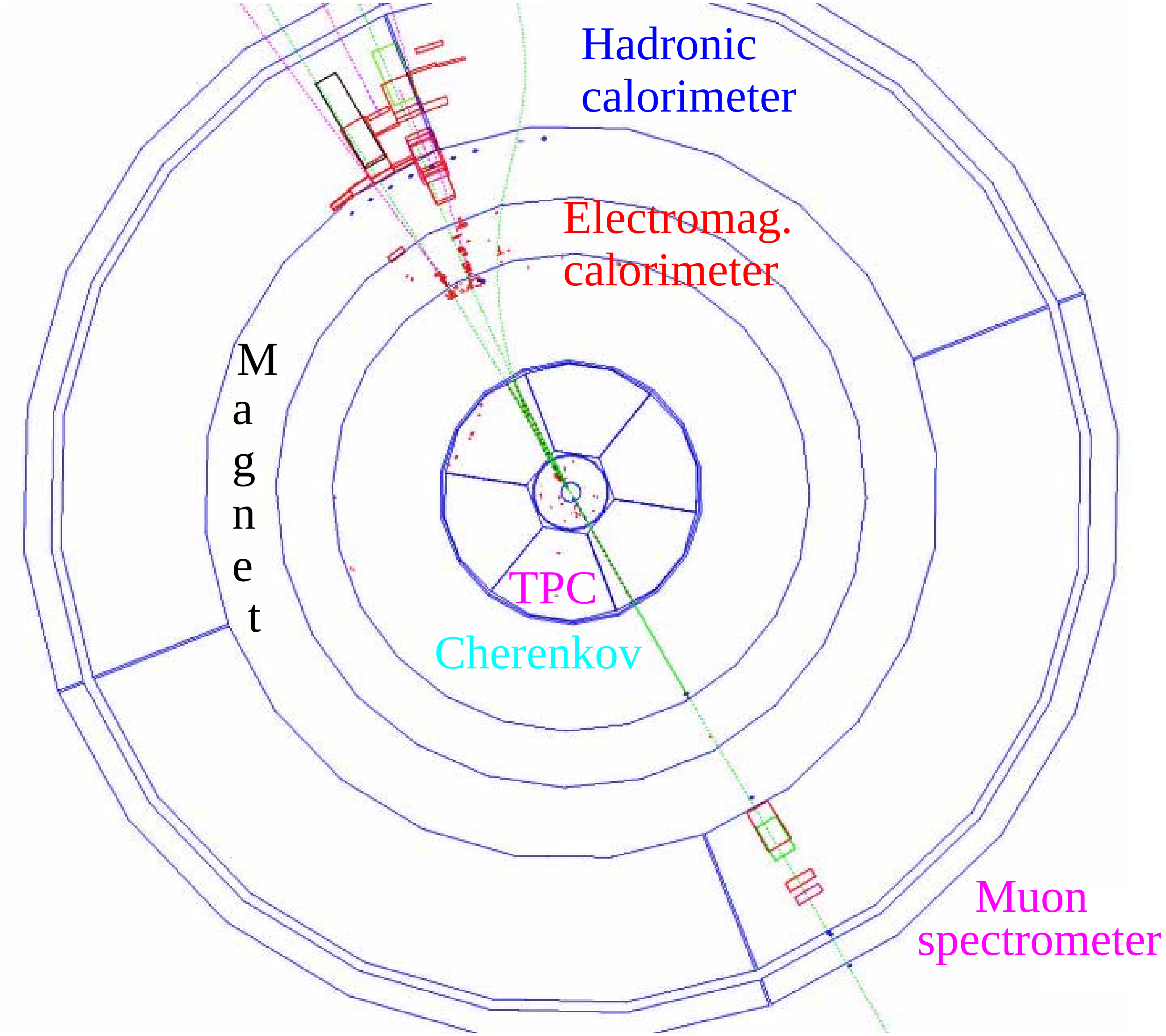


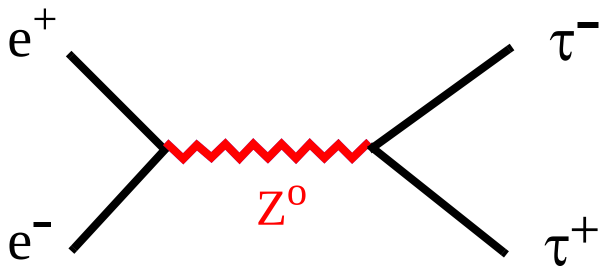


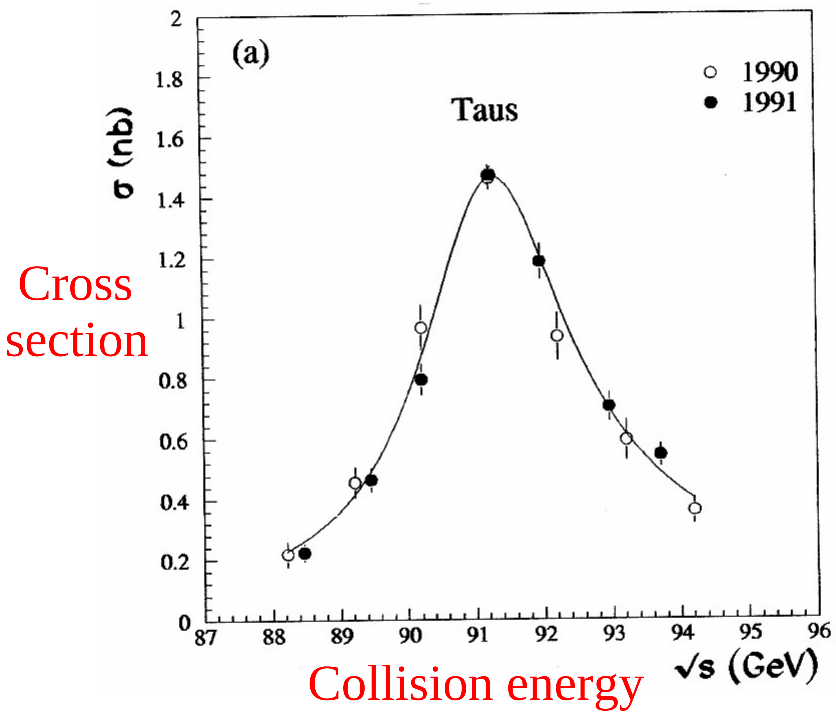


Precision studies of the W and Z bosons

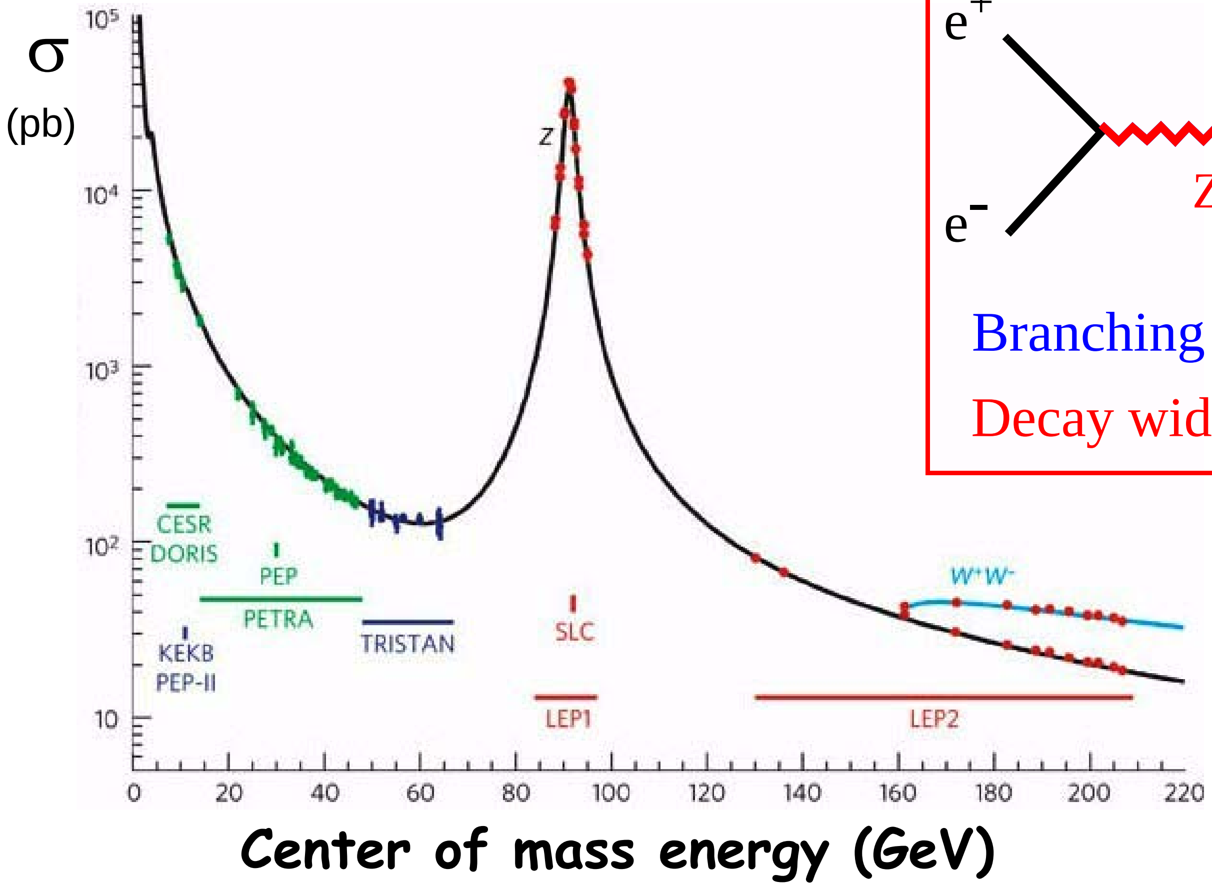
➡ Studies of the Z-boson

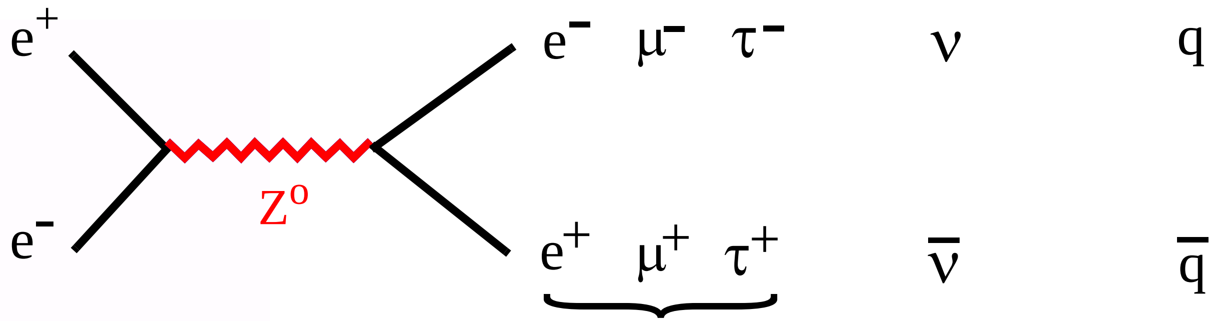






The number of neutrino families





Branching ratio (B):	0.10	0.20	0.70
Decay width (Γ):	**0.25 GeV**	**0.50 GeV**	**1.74 GeV**

Partial decay width:

$$\Gamma = \frac{B}{\tau}$$

Branching ratio

Decay time

$$\tau = \frac{B_{had}}{\Gamma_{had}} = \frac{B_{ll}}{\Gamma_{ll}} = \frac{B_{\nu\nu}}{\Gamma_{\nu\nu}} = 3\times10^{-25}s$$

1

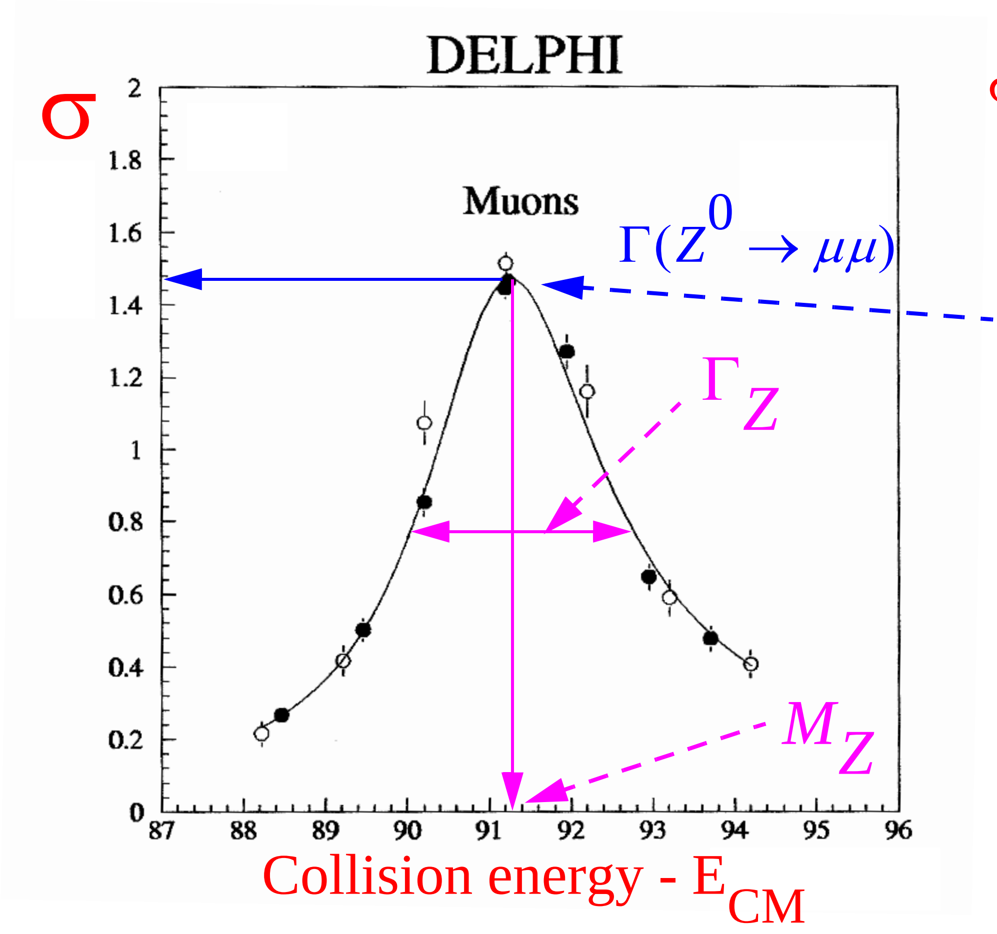
$$\tau = \frac{B_Z}{\Gamma_Z} = \frac{B_{had}+B_{ll}+B_{\nu\nu}}{\Gamma_{had}+\Gamma_{ll}+\Gamma_{\nu\nu}}$$

The total decay width

The partial decay widths

The number of neutrino families

- **Breit-Wigner ⇨ the muon partial decay widths of the Z^0**



$$\sigma(e^+e^- \to \mu\mu) = \frac{12\pi M_Z^2}{E_{CM}^2}\left[\frac{\Gamma(Z \to ee)\Gamma(Z \to \mu\mu)}{\left(E_{CM}^2 - M_Z^2\right)^2 + M_Z^2\Gamma_Z^2}\right]$$

For $E_{cm}=M_Z$:

The decay rate to ee

The decay rate to μμ

$$\sigma(e^+e^- \to \mu\mu) = \frac{12\pi}{M_Z^2}\left[\frac{\Gamma(Z \to ee)\Gamma(Z \to \mu\mu)}{\Gamma_Z^2}\right]$$

The mass of the Z^0

The total Z^0 decay rate

The number of neutrino families

- **The fitted parameters:**

$$M_Z = 91.187 \pm 0.007 \text{ GeV/c}^2 \qquad \Gamma(Z^0 \to \text{hadrons}) = 1.741 \pm 0.006 \text{ GeV}$$

$$\Gamma_Z = 2.490 \pm 0.007 \text{ GeV} \qquad \Gamma(Z^0 \to l^+ l^-) = 0.0838 \pm 0.0003 \text{ GeV}$$

- **Neutrinos cannot be measured in the experiments.**

- $$\Gamma_Z = \Gamma(Z^0 \to \text{hadrons}) + 3\Gamma(Z^0 \to l^+ l^-) + N_\nu \Gamma\left(Z^0 \to \nu_l \overline{\nu}_l\right)$$

- $$N_\nu \Gamma\left(Z^0 \to \nu_l \overline{\nu}_l\right) = 0.498 \pm 0.009 \text{ GeV}$$

Decay width to neutrinos

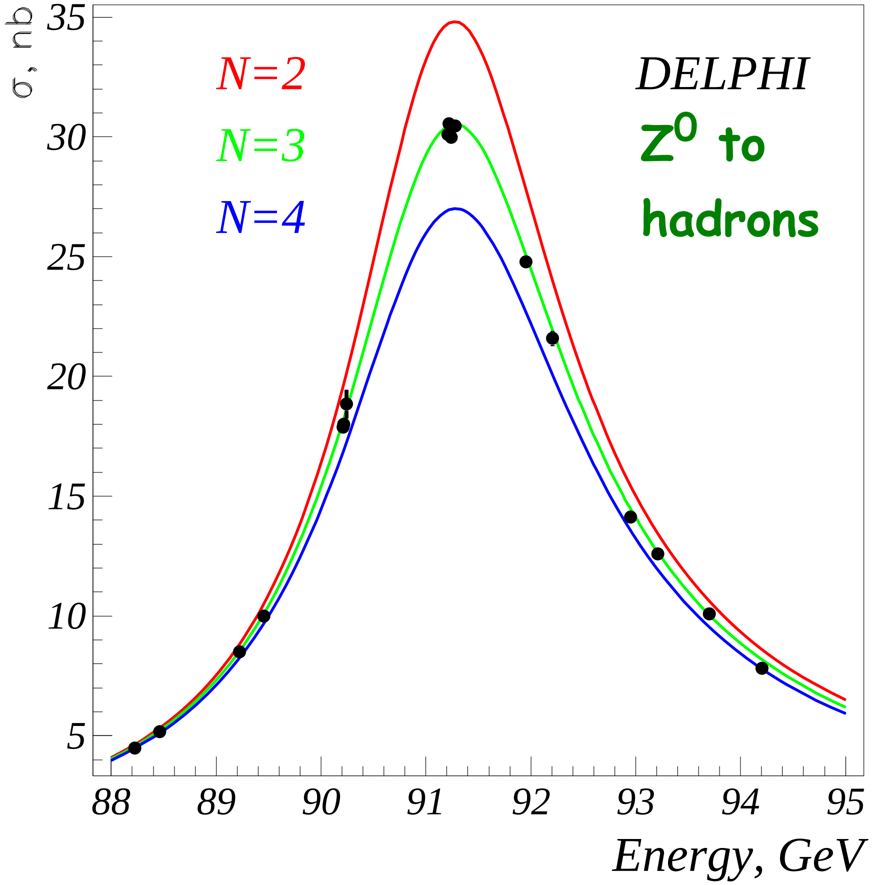
Number of neutrinos

The number of neutrino families

- **Measurement:** $N_\nu \Gamma\left(Z^0 \rightarrow \nu_l \bar{\nu}_l\right) = 0.498 \text{ GeV}$

  Calculation: $\Gamma\left(Z^0 \rightarrow \nu_l \bar{\nu}_l\right) = 0.166 \text{ GeV}$

⇨ $N_\nu = 2.994 \pm 0.011$

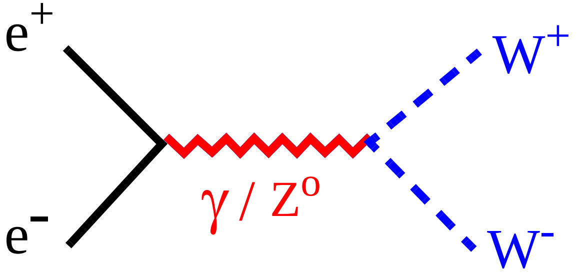


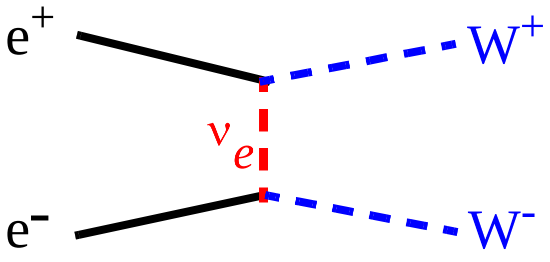
- **No restrictions on N_ν in SM.**
- **LEP ⇨ three types of light neutrinos**

Precision studies of the W and Z bosons

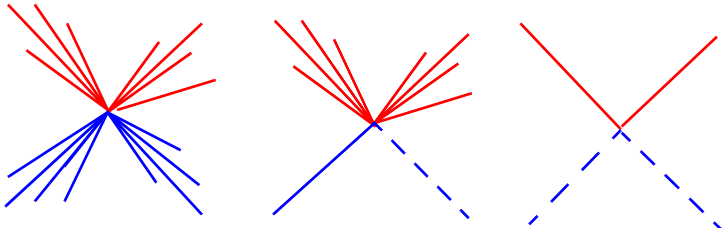
➡ Studies of the W-boson

- **W bosons were produced in pairs.**

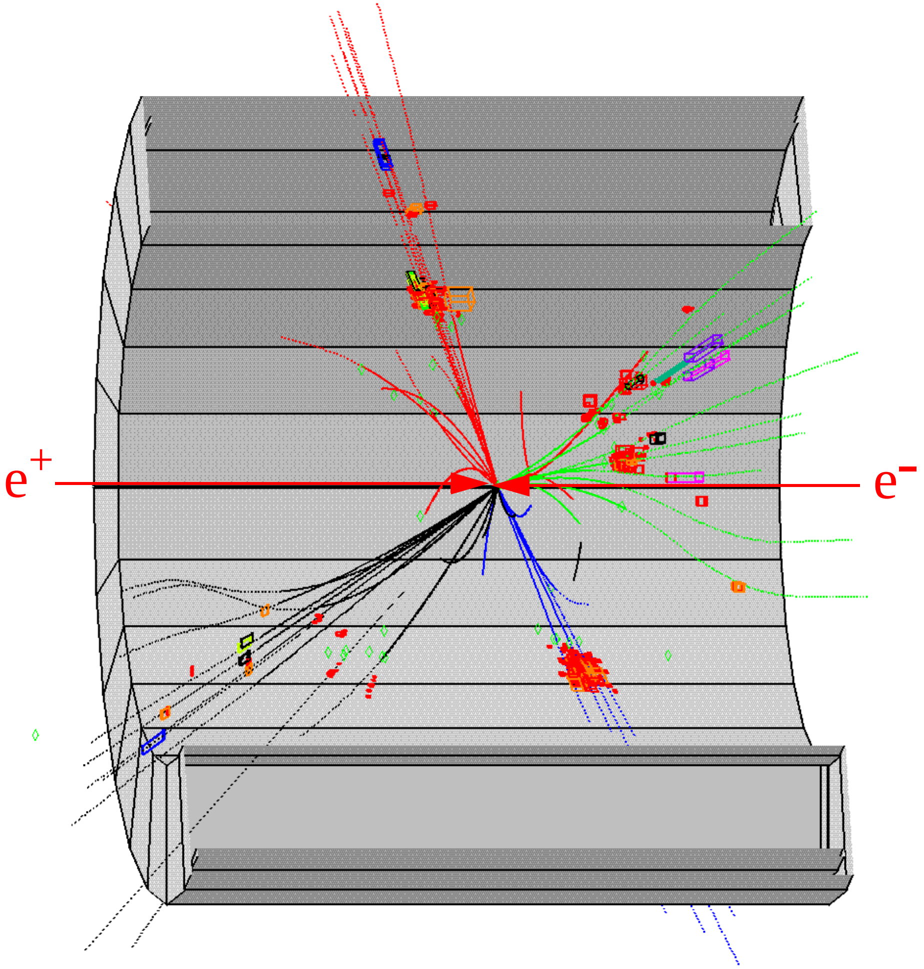




- **The signature:**



$q\bar{q}\,q\bar{q}$	$q\bar{q}\,l\nu$	$l\nu\,l\nu$
46%	44%	11%



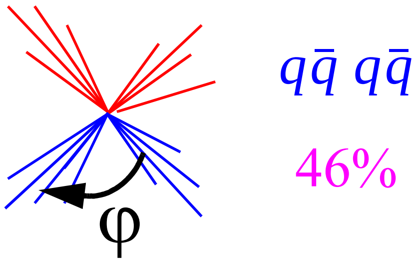
A WW-event with 4 jets

Precision studies of the W and Z bosons

➡ Studies of the W-boson

- **Step 1. Select WW-pair events**

Step 2. Calculate the W-mass



$$M_W^2 = (\vec{P}_q + \vec{P}_{\bar{q}})^2$$

(4-vectors)

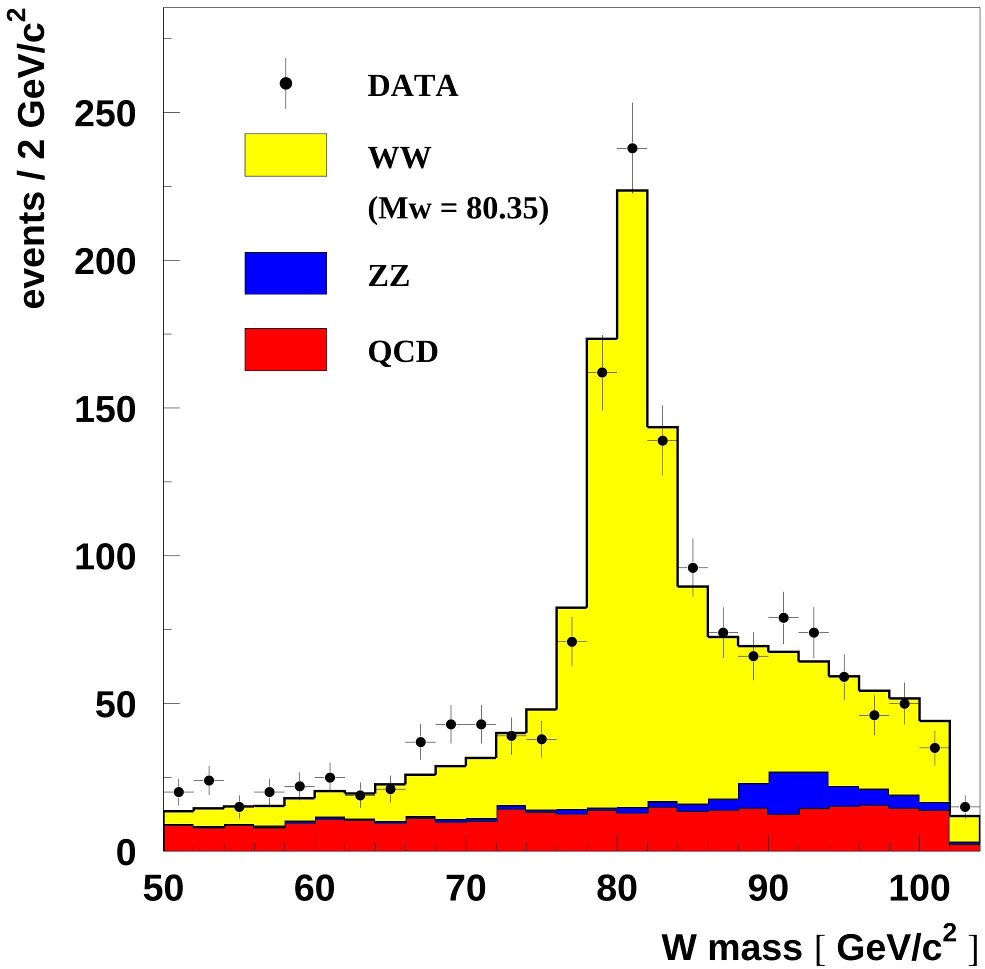
$$M_W^2 = 2\,E_q E_{\bar{q}}\,(\,1 - \cos\ \varphi\,)$$

if $m_q = 0$

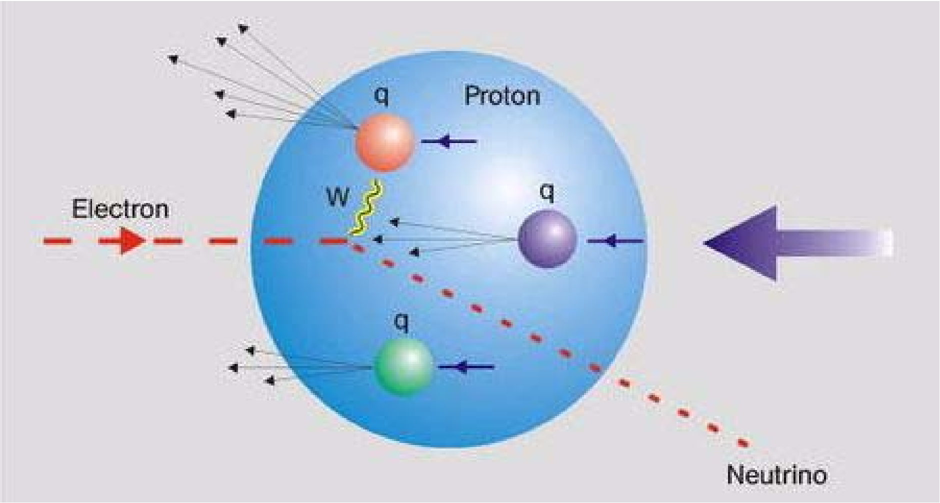
$$M_W = 80.376 \pm 0.033 \text{ GeV}$$ (LEP)

$$M_W = 83.5 \pm 2.8 \text{ GeV}$$ (UA1)

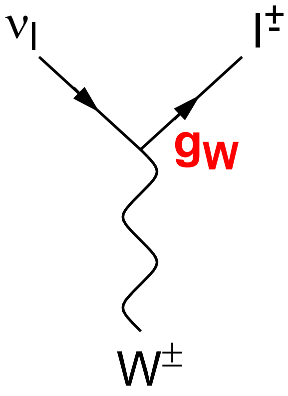
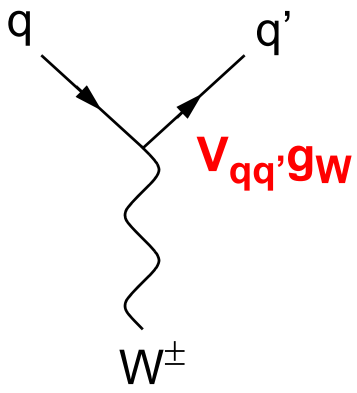
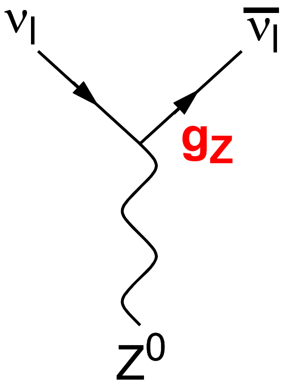
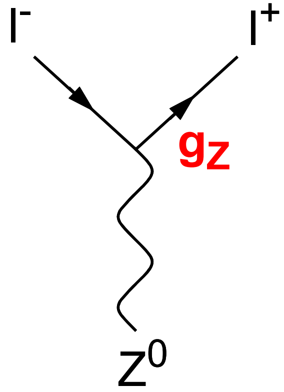
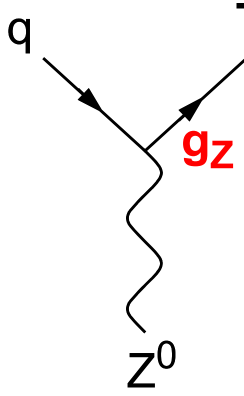
The mass distribution of jet-pairs



Charged current reactions



W and Z reactions

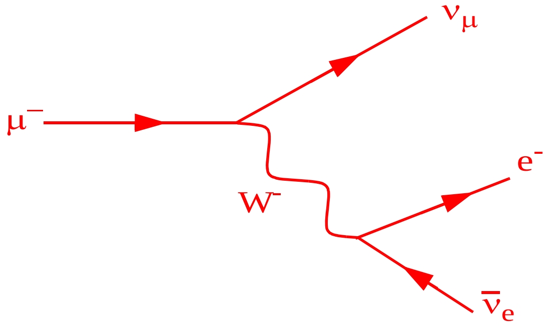
	Leptonic reactions	Hadronic reactions
Charge current reactions	 	
Neutral current reactions	  	

Charged current reactions

➡ Charged current reactions are mediated by a W.

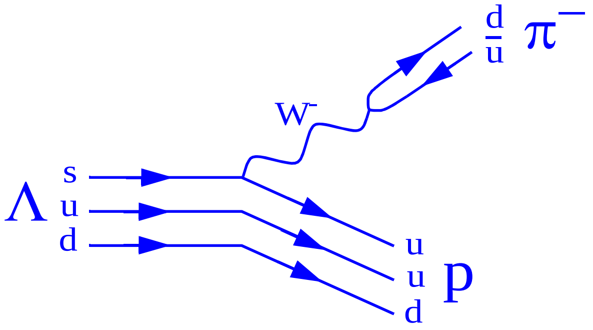
- **Purely leptonic processes:**

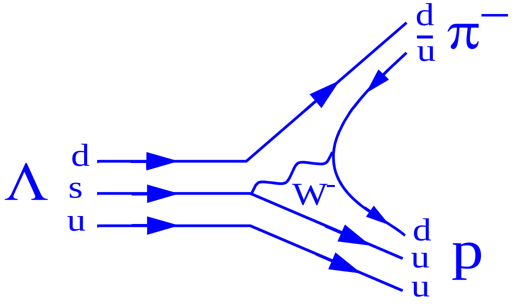
  $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$



- **Purely hadronic processes:**

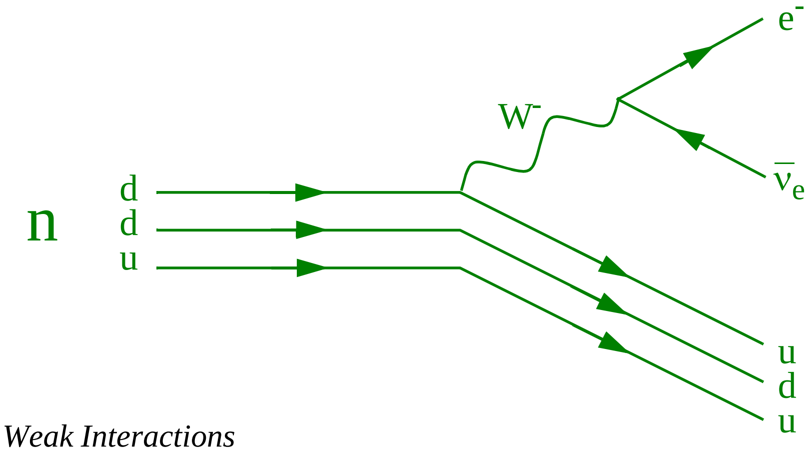
  $\Lambda \rightarrow \pi^- + p$





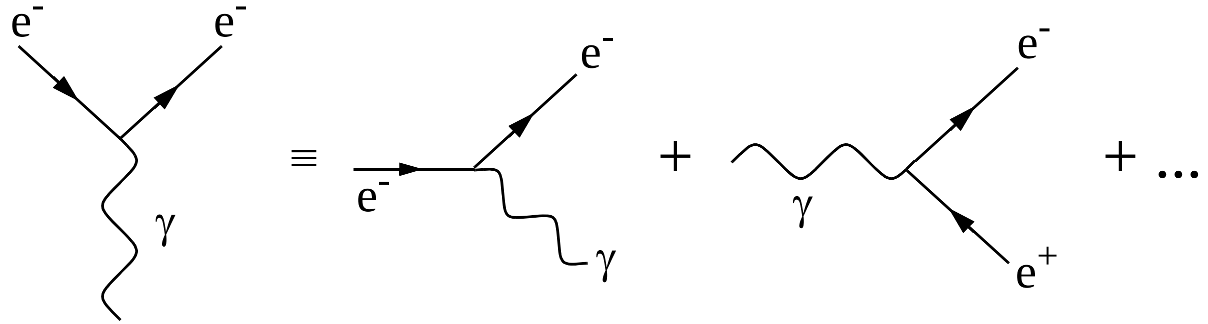
- **Semileptonic reactions:**

  $n \rightarrow p + e^- + \bar{\nu}_e$



Leptonic charged current reactions

- **All the electromagnetic interactions ⇨ eight basic interactions:**



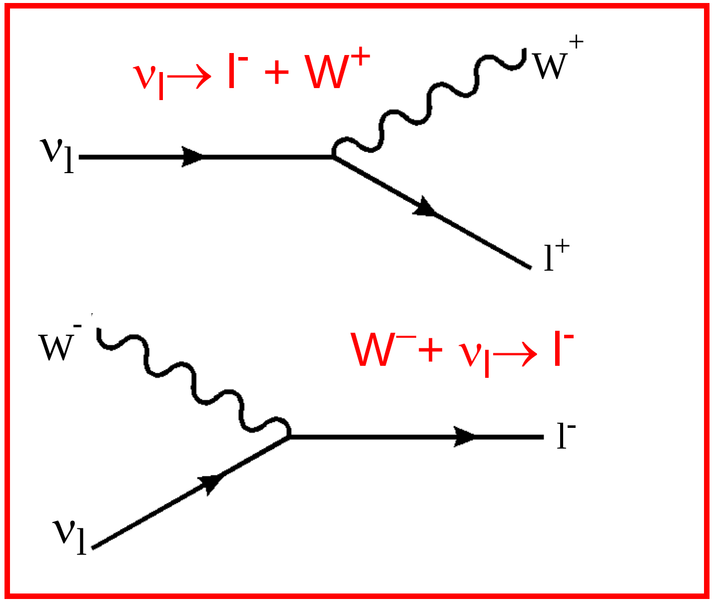
The basic vertex for electron-photon interactions.

- **Leptonic weak interaction ⇨ described by basic vertices:**

The two basic vertices for W-lepton interactions..

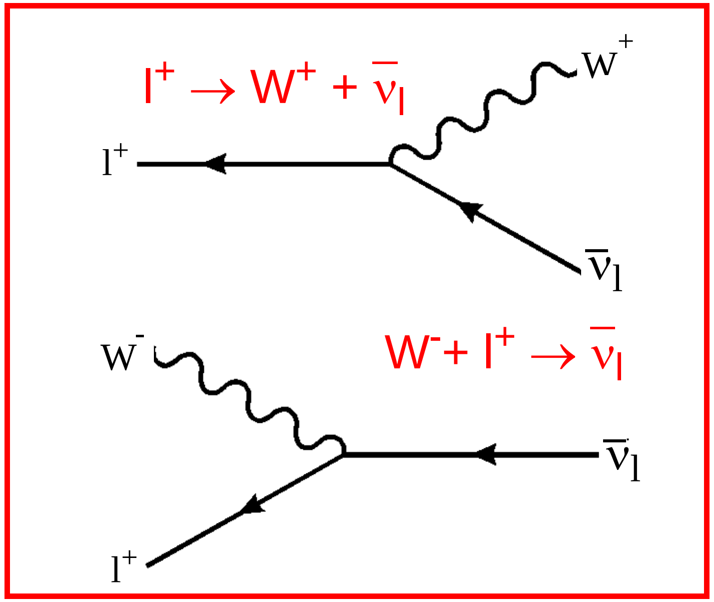
Leptonic charged current reactions

- **Eight basic charged current reactions** from two basic W vertices:



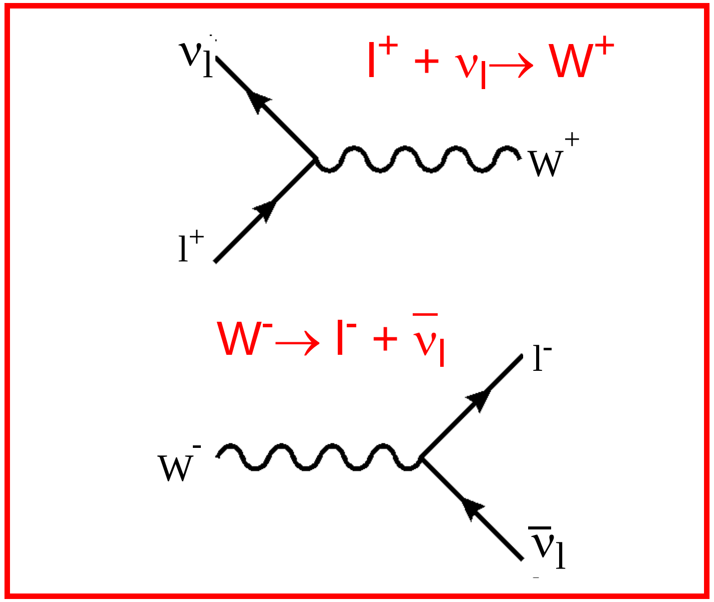
$\nu_l \rightarrow l^- + W^+$

$W^- + \nu_l \rightarrow l^-$



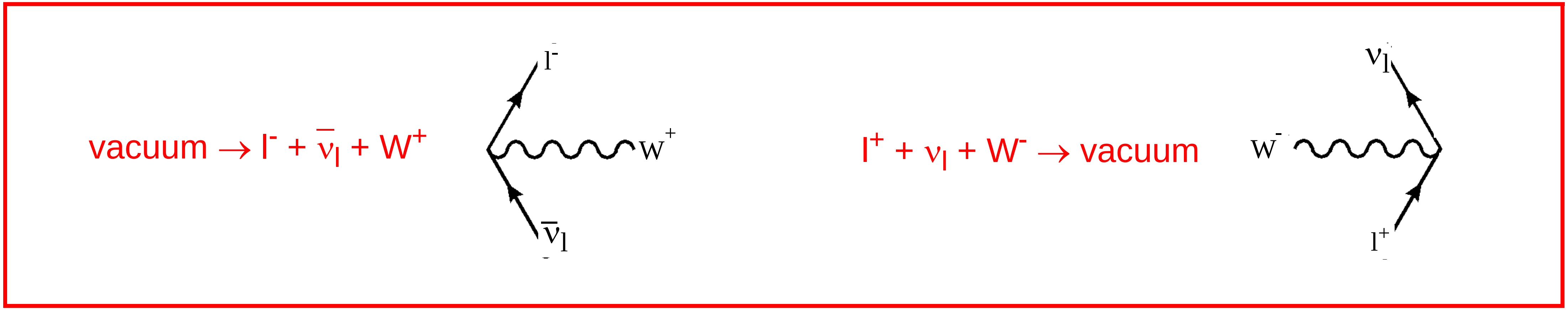
$l^+ \rightarrow W^+ + \bar{\nu}_l$

$W^- + l^+ \rightarrow \bar{\nu}_l$



$l^+ + \nu_l \rightarrow W^+$

$W^- \rightarrow l^- + \bar{\nu}_l$



vacuum $\rightarrow l^- + \bar{\nu}_l + W^+$

$l^+ + \nu_l + W^- \rightarrow$ vacuum

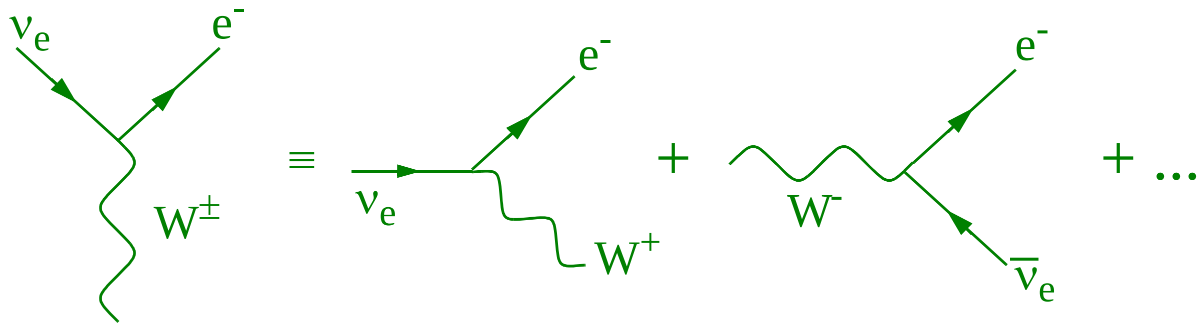
Leptonic charged current reactions

- Weak interactions conserve lepton numbers: L_e , L_μ , L_τ

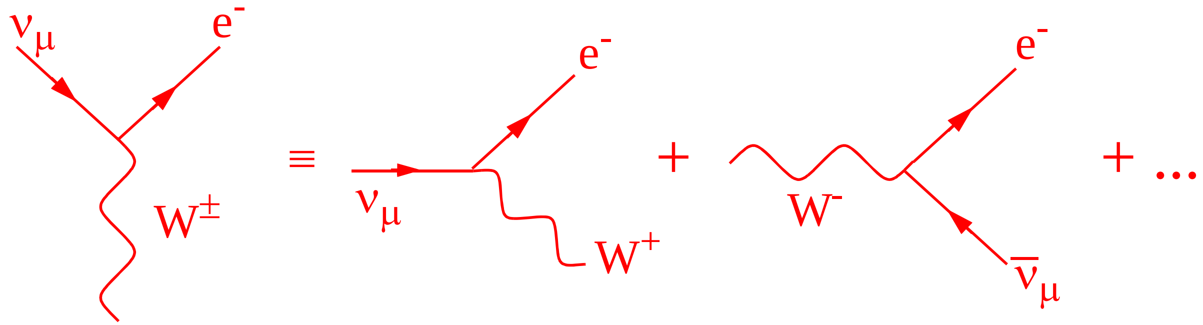
Feynman diagrams:

1) at each vertex, there is one arrow pointing in and one out
2) the lepton indices “l” are the same on both lines.

Allowed:



Forbidden:



Leptonic charged current reactions

The weak strength parameter: α_W

- α_W is the same at all leptonic vertices
  (it does not depend on lepton type)

Example: The decay rate of W to e+ν

Estimation from theory

$$\Gamma(W \to e\nu) \approx \alpha_W M_W \approx \alpha_W \, \mathbf{80\ GeV}$$

Measurement

$$\Gamma(W \to e\nu) \approx \mathbf{0.2\ GeV}$$

α_W=0.003

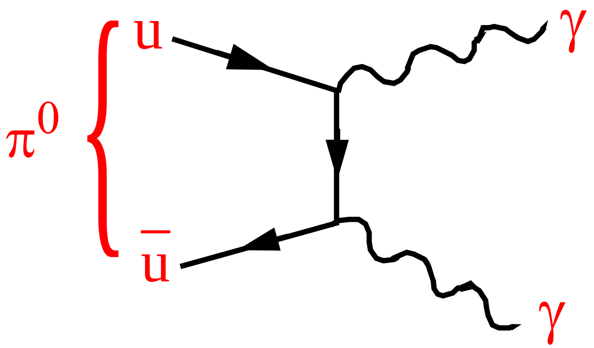
- Compare with the electromagnetic strength parameter: α_{em}=0.007

Leptonic charged current reactions

➡ Why is the weak interaction so weak if α_W and α_{em} is of a similar size ?

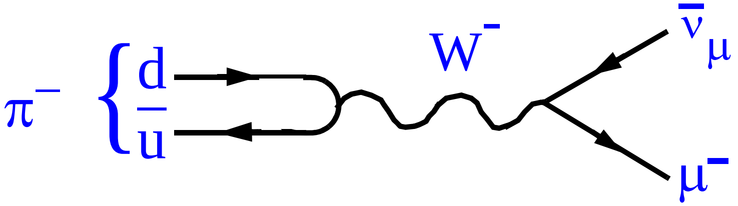
- **Compare the decay of charged and neutral pions:**

Electromagnetic decay



Lifetime = 8×10^{-17} s

Weak decay

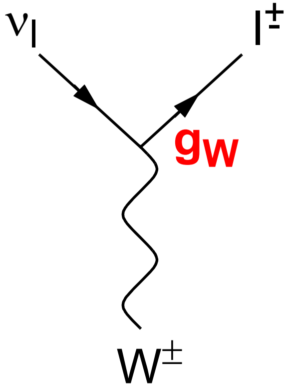
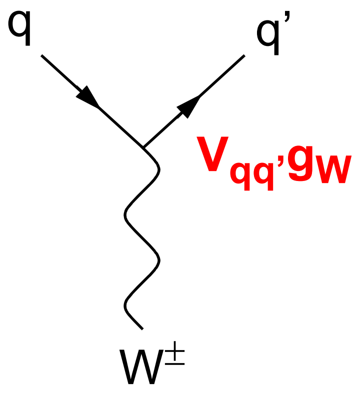
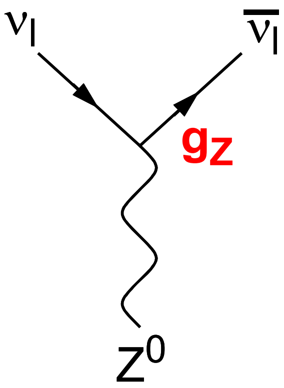
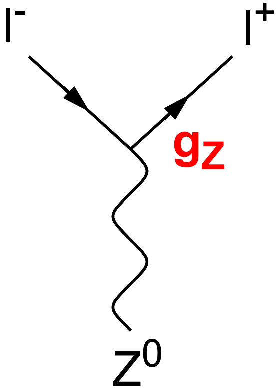
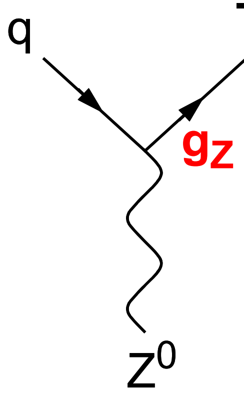


Lifetime = $3000000000 \times 10^{-17}$ s

(Lifetime of a real W = $0.00000003 \times 10^{-17}$ s)

- **CONCLUSION:** **Apparent weakness** ⇨ **large W and Z masses**

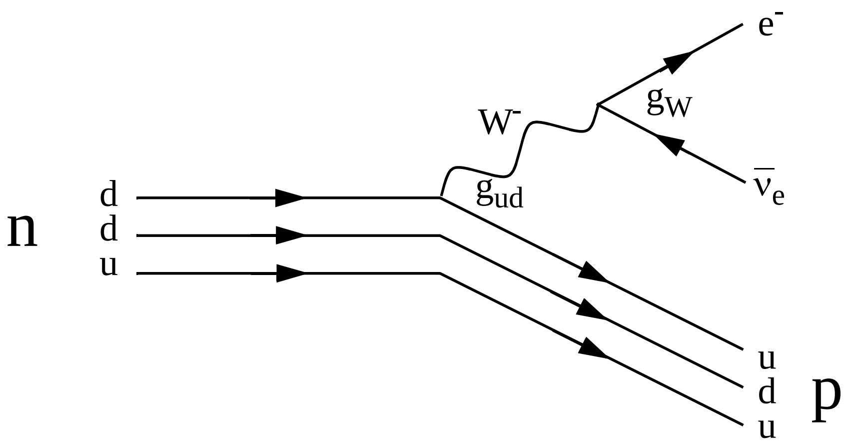
W and Z reactions

	Leptonic reactions	Hadronic reactions
Charge current reactions		
Neutral current reactions	 	

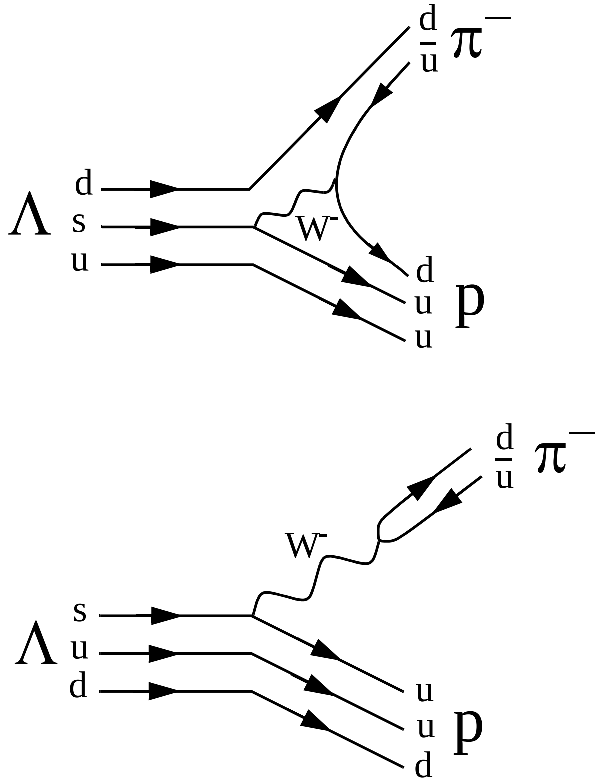
Hadronic charged current reactions

- In weak hadronic interactions, constituent quarks emit or absorb W or Z bosons.

Examples:



Neutron β-decay.



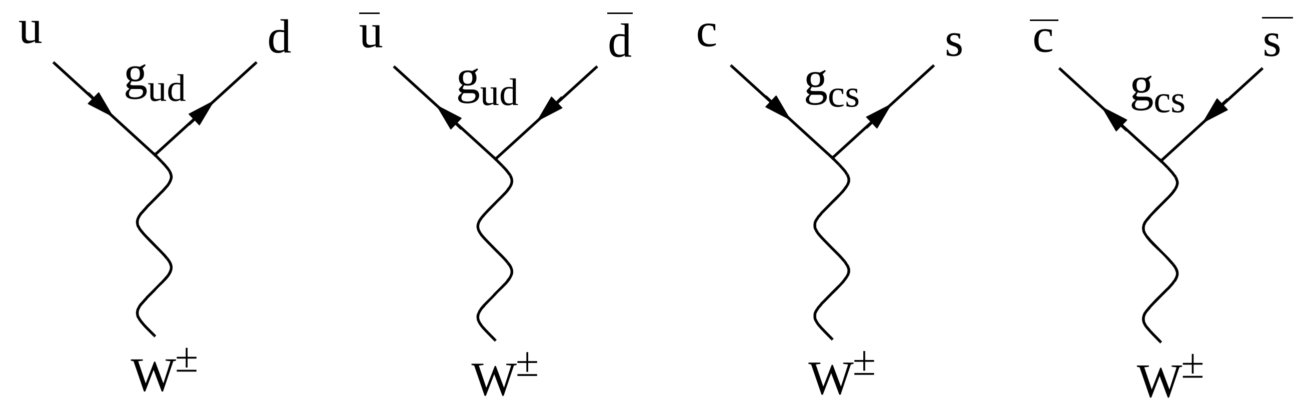
The dominant quark diagrams for Λ decay

Hadronic charged current reactions

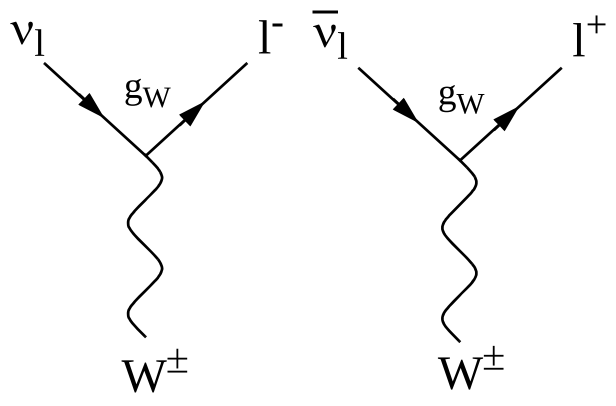
ASSUMPTION: Lepton-quark symmetry i.e. corresponding generations of quarks and leptons have identical weak interactions.

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}\leftrightarrow\begin{pmatrix}u\\ d\end{pmatrix}\quad\begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}\leftrightarrow\begin{pmatrix}c\\ s\end{pmatrix}\quad\begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}\leftrightarrow\begin{pmatrix}t\\ b\end{pmatrix}$$

- **Interactions will then only take place within a family !**



The W-quark vertices for the first two generations, assuming lepton-quark symmetry.



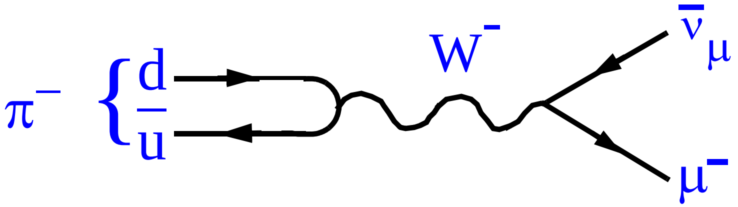
The W-lepton vertices.

Hadronic charged current reactions

Experimental tests of the assumption of lepton-quark symmetry.

- **Some weak reactions should be allowed and some should be forbidden if lepton-quark symmetry is true:**

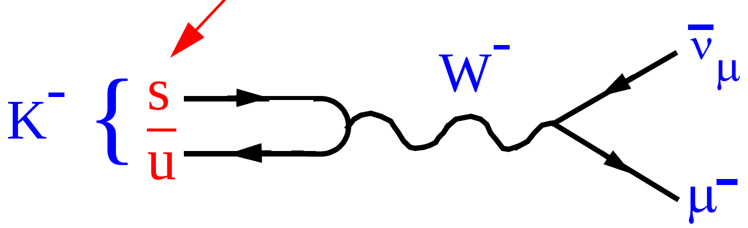
$\pi^- \to \mu^- + \bar{\nu}_\mu$ $d\bar{u} \to \mu^- + \bar{\nu}_\mu$



Allowed

Different families

$K^- \to \mu^- + \bar{\nu}_\mu$ $s\bar{u} \to \mu^- + \bar{\nu}_\mu$



Not Allowed

- **Measurements of these decays give:**

$\pi^- \to \mu^- + \bar{\nu}_\mu$ Branching ratio = 0.9999 $\tau = 2.6 \times 10^{-8}$ s

$K^- \to \mu^- + \bar{\nu}_\mu$ Branching ratio = 0.6343 $\tau = 1.2 \times 10^{-8}$ s

- **CONCLUSION: Quarks from different generations can participate in charged current interactions !**

Hadronic charged current reactions

- Cabibbo ⇨ quark mixing in order to explain kaon decays.

- **Quark mixing scheme** ⇨ d- and s-quarks participate in weak interactions via the linear combinations:

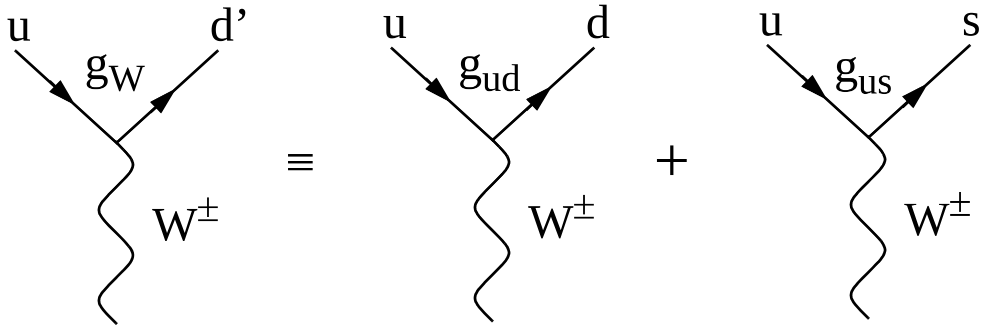
$$d' = d\cos\theta_C + s\sin\theta_C$$
$$s' = -d\sin\theta_C + s\cos\theta_C$$

where θ_C is called the **Cabibbo angle**.

- The quark-lepton symmetry applies to doublets like

$$\begin{pmatrix} u \\ d' \end{pmatrix} \text{ and } \begin{pmatrix} c \\ s' \end{pmatrix}$$

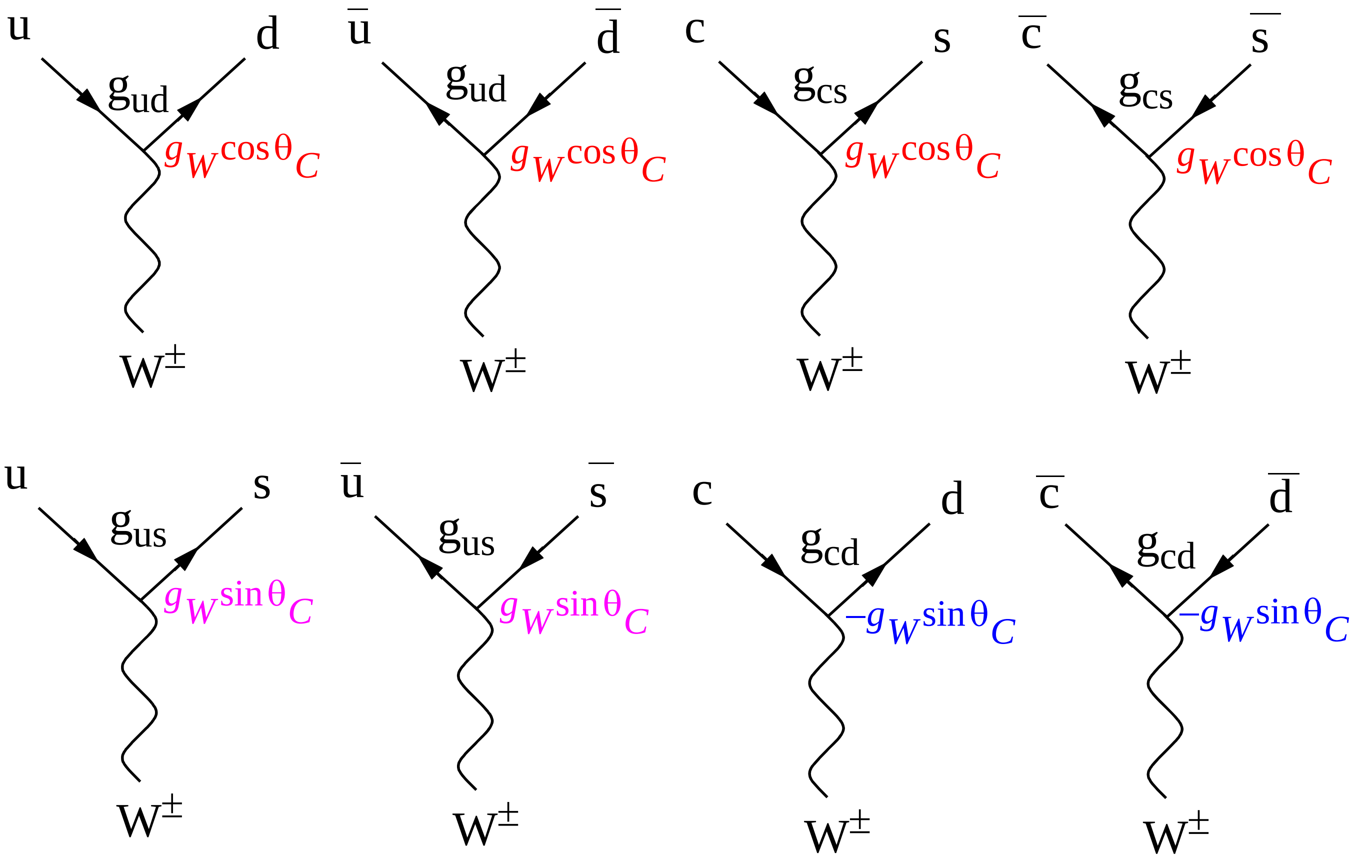
- The **ud'W vertex** ⇨ interpreted as a sum of the udW and usW vertices:



$g_{ud}=g_W\cos\theta_C$ $g_{us}=g_W\sin\theta_C$

Hadronic charged current reactions

- The quark mixing hypothesis ⇨ more W-quark vertices:



Within a generation: $g_{ud} = g_{cs} = g_W \cos\theta_C$

Between generations: $g_{us} = -g_{cd} = g_W \sin\theta_C$

Hadronic charged current reactions

➡ Measurements of the Cabibbo angle.

- The **Cabibbo angle** has to be measured.

- Comparing the decay rates of $\pi^- \to \mu^- + \bar{\nu}_\mu$ with $K^- \to \mu^- + \bar{\nu}_\mu$

$$\frac{\Gamma(K^- \to \mu^- \bar{\nu}_\mu)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \propto \frac{g_{us}^2}{g_{ud}^2} = \tan^2\theta_C \quad \Rightarrow \quad \theta_C = 12.7° \pm 0.1°$$

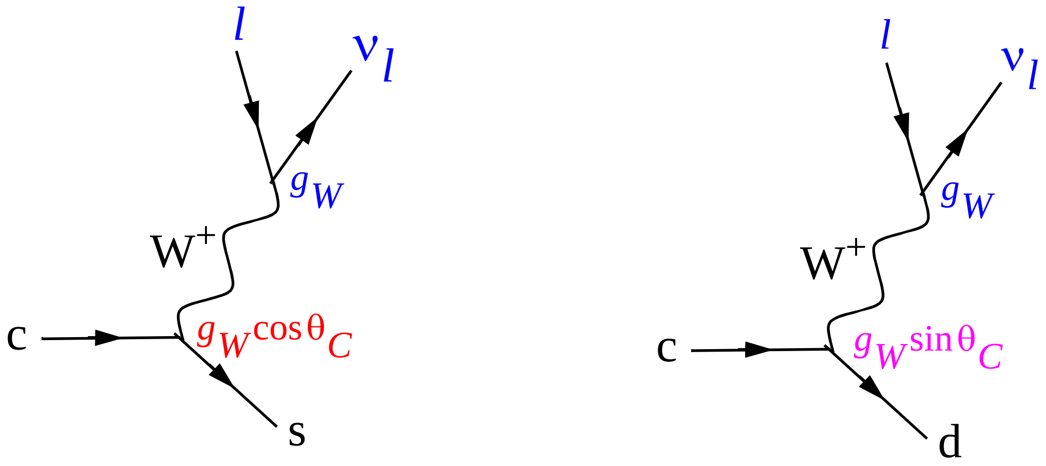
- The **coupling constants** within and between generations:

$$\begin{aligned} g_W \cos\theta_C &= 0,98 g_W \\ g_W \sin\theta_C &= 0,22 g_W \end{aligned}$$

Hadronic charged current reactions

➡ Charmed particle decays.

- Particles with charm quarks almost always give a strange particle in the final state because other decays are Cabibbo supressed:



$\theta_C = 12,6°$

- The supression factor is: $\frac{g_{cd}^2}{g_{cs}^2} = \tan^2\theta_C = \frac{1}{20}$

- Neutrino scattering experiments ⇨
  The charmed quark couplings g_{cd} and g_{cs}

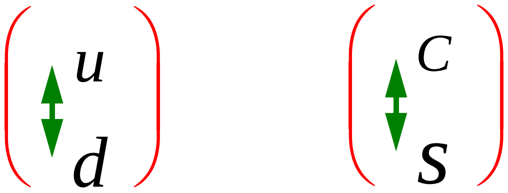
⇨ $\boxed{\theta_C = 12° \pm 1°}$

Weak interactions and the third generation

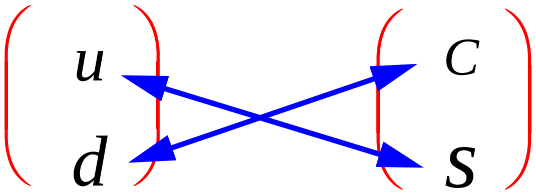
➡ Two generation quark mixing can be written in matrix form:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$ with transitions within $\begin{pmatrix} u \\ d' \end{pmatrix}$ and $\begin{pmatrix} c \\ s' \end{pmatrix}$

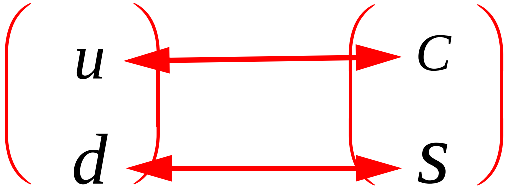
- This means that the following weak transitions are **favoured**:



- And that the following weak transitions are **suppressed**:



- Charge conservation **forbids** the following charged current transitions:



Charge = 2/3

Charge = -1/3

Weak interactions and the third generation

- The **c-quark** was **predicted** from lepton-quark symmetry ⇨ Discovered in experiments in 1974.

- Discovery of the τ lepton and the b-quark ⇨ **The sixth quark** was **predicted** to complete the symmetry ⇨ **Top quark** was **discovered in 1994**

- The third generation gives rise to the **Cabibbo-Kobayashi-Maskawa (CKM) matrix** $V_{\alpha\beta}$:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- $g_{\alpha\beta} = g_W V_{\alpha\beta}$

$\alpha = u, c, t$

$\beta = d, s, b$

Weak interactions and the third generation

- Weak transitions can now take place between:

$$u \leftrightarrow d' = V_{ud}d+V_{us}s+V_{ub}b$$
$$c \leftrightarrow s' = V_{cd}d+V_{cs}s+V_{cb}b$$
$$t \leftrightarrow b' = V_{td}d+V_{ts}s+V_{tb}b$$

- If the mixing between the b and t quarks with lighter quarks can be neglected the CKM-matrix is reduced to:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0 \\ -\sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- If V_{ub}, V_{cb}, V_{td} and V_{ts} are not small $\Rightarrow$
  The two-generation mixing model would not work

Weak interactions and the third generation

➡ b - quarks

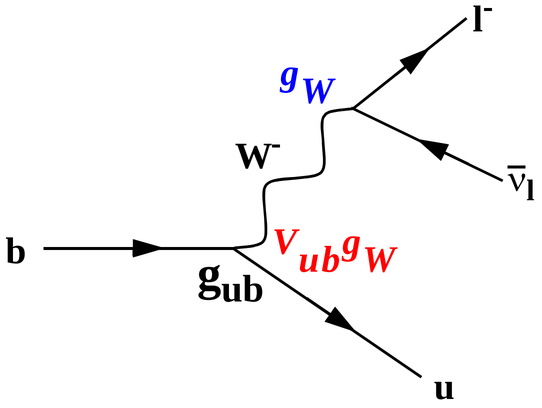
$V_{ub}=V_{cb}=V_{td}=V_{ts}=0$ →

- t-quark decay only to b-quarks
- b-quark is stable
  (since it cannot decay to u- or c-quarks)

↑ Not true

- **Semileptonic decays** of b-quarks to u- and c-quarks observed !

- The observed decay rate is proportional to the squared couplings: $|g_{ub}|^2 = |V_{ub}|^2 g_W^2$



Weak interactions and the third generation

➡ b - quarks

- **The most precise measurements at present**

$$|V_{ub}| \approx 0,004 \quad \text{and} \quad |V_{cb}| \approx 0,04$$

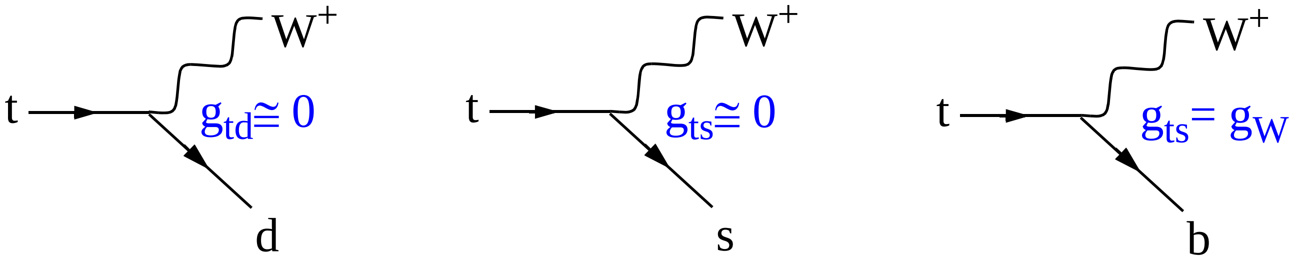
- **The CKM-matrix becomes**

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0{,}004 \\ -\sin\theta_C & \cos\theta_C & 0{,}04 \\ \sim 0 & \sim 0 & \sim 1 \end{pmatrix} \approx \begin{pmatrix} 0{,}98 & 0{,}22 & 0{,}004 \\ -0{,}22 & 0{,}98 & 0{,}04 \\ \sim 0 & \sim 0 & \sim 1 \end{pmatrix}$$

Weak interactions and the third generation

➡ The top quark

- The **top quark** is much heavier than even the W-bosons and it can **decay** by



- g_{td} and g_{ts} are close to zero ⇨ the only **significant decay mode:**

$$\boxed{t \to W + b}$$

rate proportional to $\alpha_W = g_W^2 / 4\pi \approx 0,0042$

- Estimation of the decay width of the top ($\Gamma \sim \alpha_W m_t \sim 1\ \text{GeV}$)

⇨ **very short lifetime** ⇨ $\tau_t \approx 4 \times 10^{-25} s$

Weak interactions and the third generation

➡ Compare particles decay length

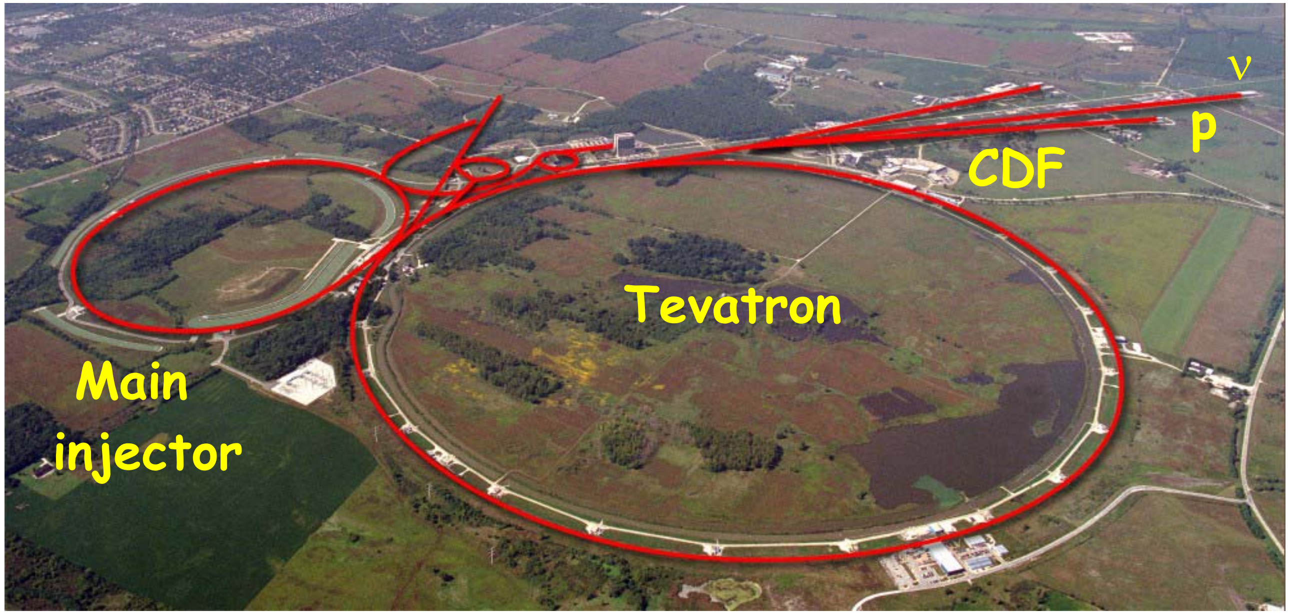
Particle	Lifetime	Decay Length	Decay Place	Decay Measurement
W, Z, top	3-4x10^{-13}ps	0	Beampipe	Not possible
π^0 ($\rightarrow\gamma\gamma$)	0.0008 ps	0.025μm	Beampipe	Not possible
τ	0.3 ps	90μm	Beampipe	Microvertex
Charm: $D^0/D^\pm/D_s$	0.4-1 ps	150-350μm	Beampipe	Microvertex
Bottom: $B^0/B^\pm/B_s/$	1.5 ps	450μm	Beampipe	Microvertex
K_s ($\rightarrow\pi\pi$)	80 ps	2.7cm	Tracker	Tracker
$K^\pm$	10,000 ps	3.7m	Tracker	Not possible
$\pi^\pm$	30,000 ps	7.8m	No decay	Not possible
K_L ($\rightarrow\pi\pi\pi$)	50,000 ps	16 m	No decay	Not possible
μ ($\rightarrow e\bar{\nu}_e\nu_\mu$)	2,000,000 ps	659 m	No decay	Not possible

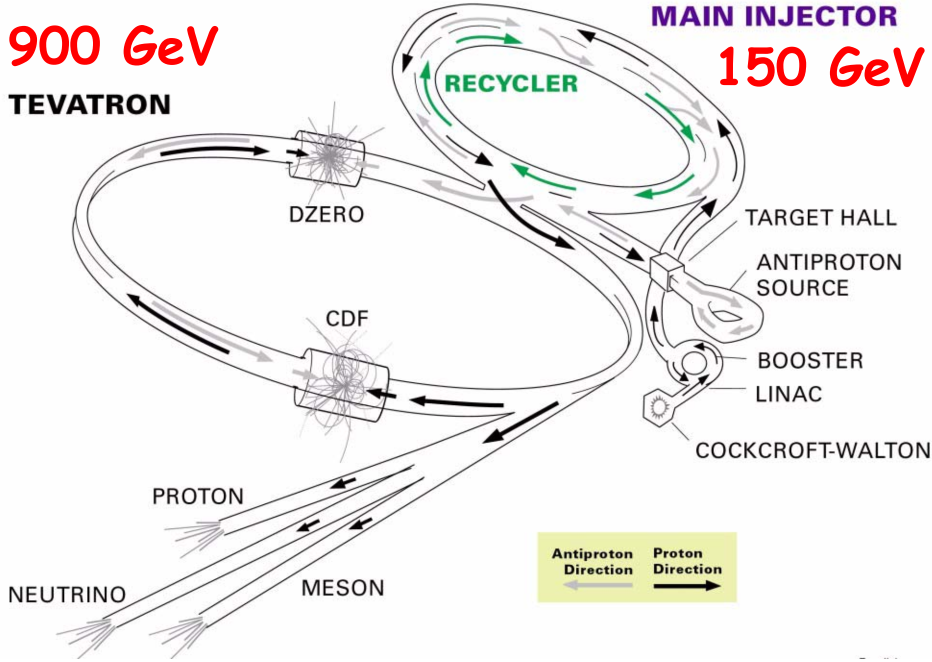
The discovery of the top quark

➡ The accelerator: The Tevatron

	Type	Bending field	Length	Collision energy
Tevatron:	$p\bar{p}$-collider	4.5 T	6.3 km	1800 GeV
SPS:	$p\bar{p}$-collider	1.8 T	6.9 km	900 GeV
LHC:	pp-collider	8.4 T	27 km	14000 GeV

Superconducting magnets

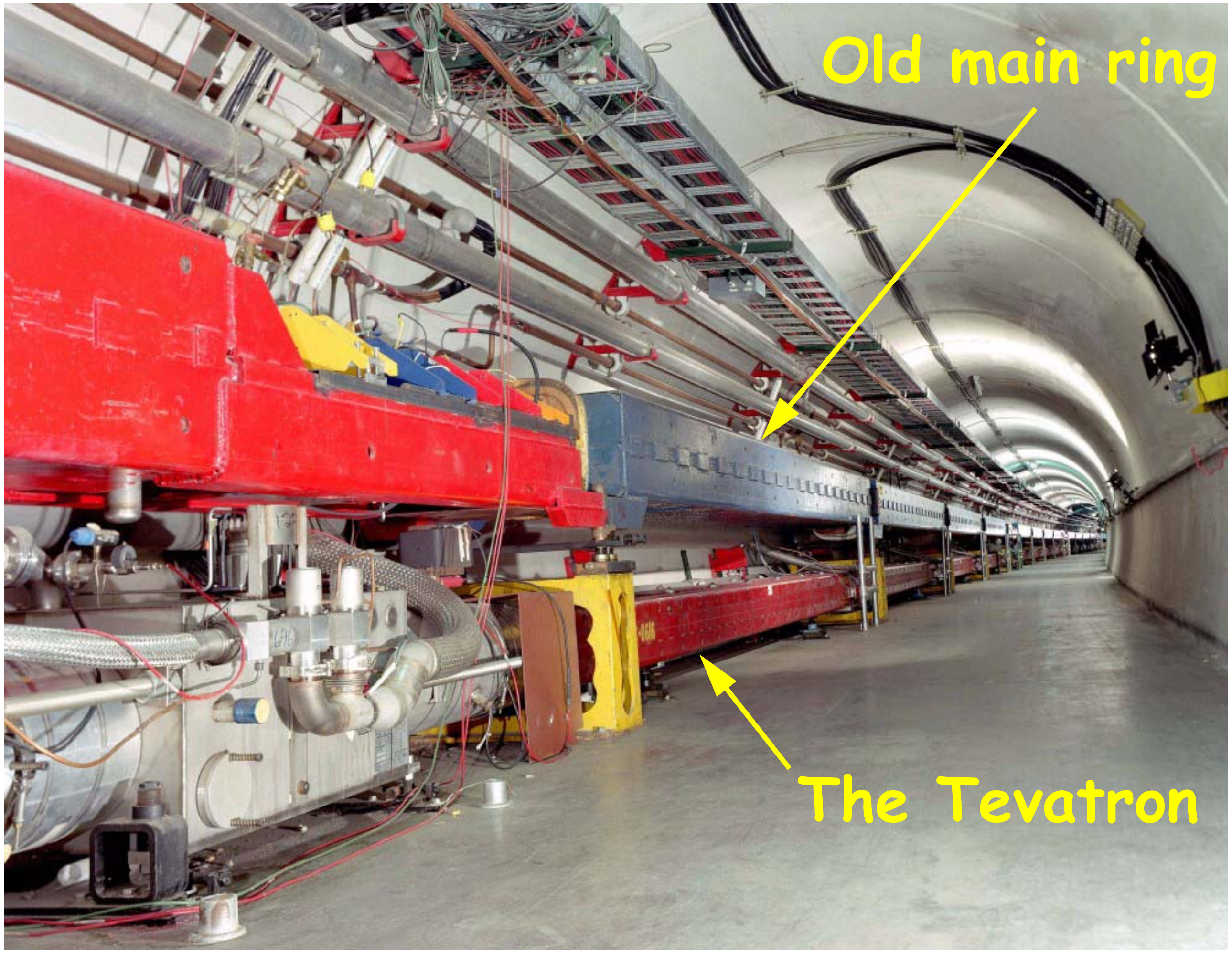




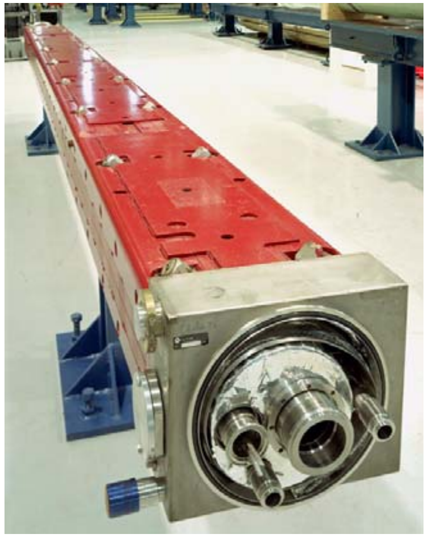
The discovery of the top quark

➡ The accelerator: The Tevatron

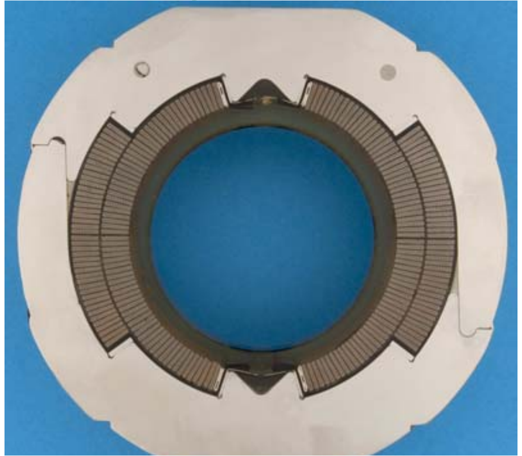
- The Tevatron accelerator was put under the old main ring which was used as a pre-accelerator.



Dipole magnet

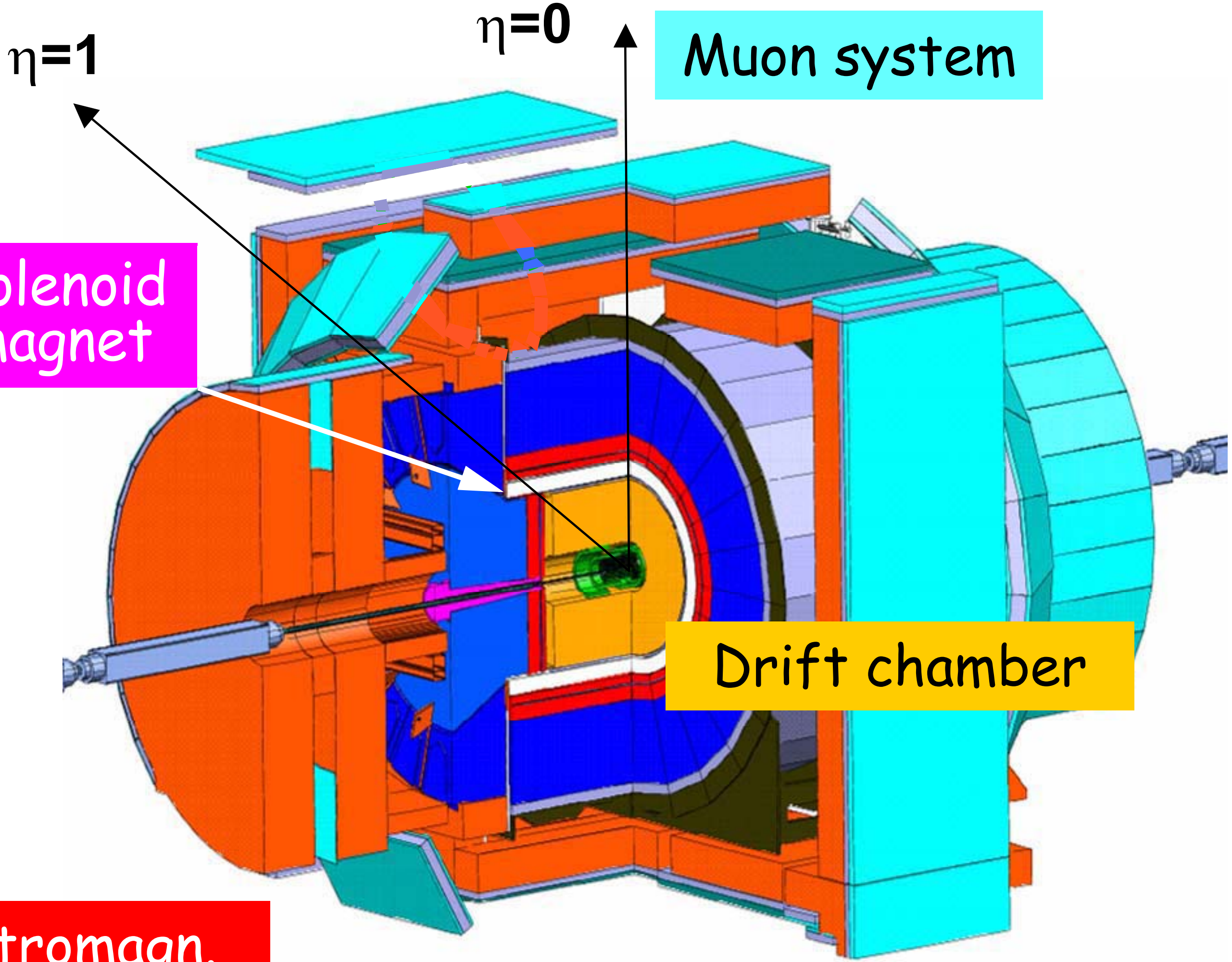


Magnet coil



The discovery of the top quark

➡ The experiment: The Collider Detector at Fermilab



η=1

η=0

Muon system

Solenoid magnet

Silicon Vertex Detector

Drift chamber

Electromagn. Calorimeter

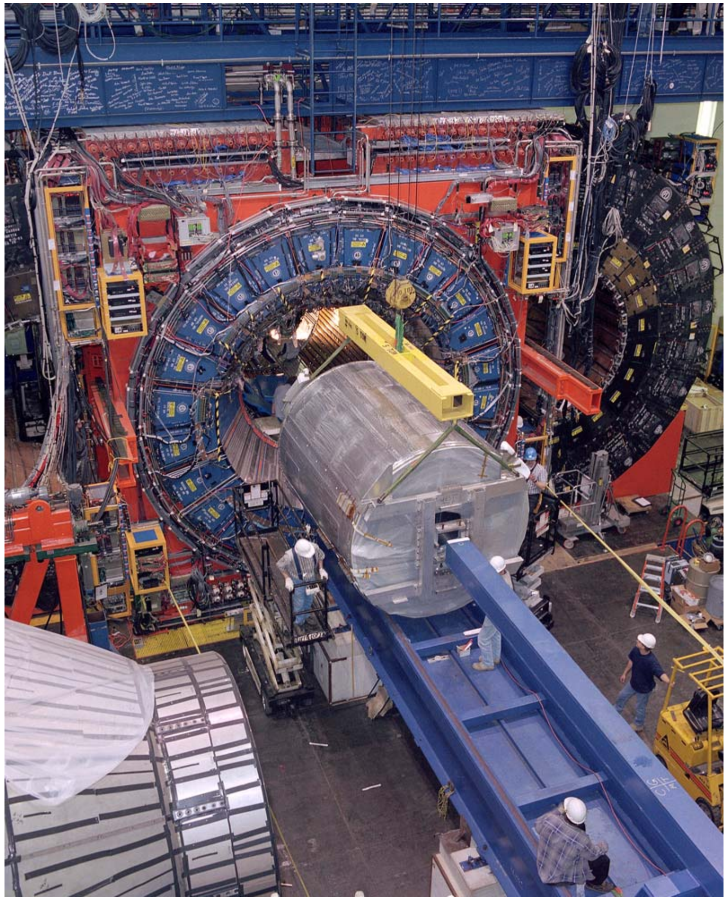
Lead/Scintillator

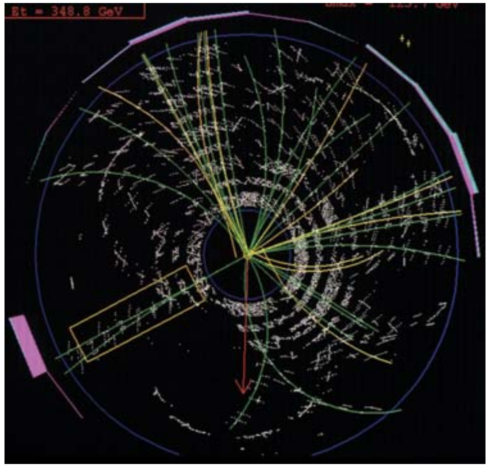
Had. Calorimeter

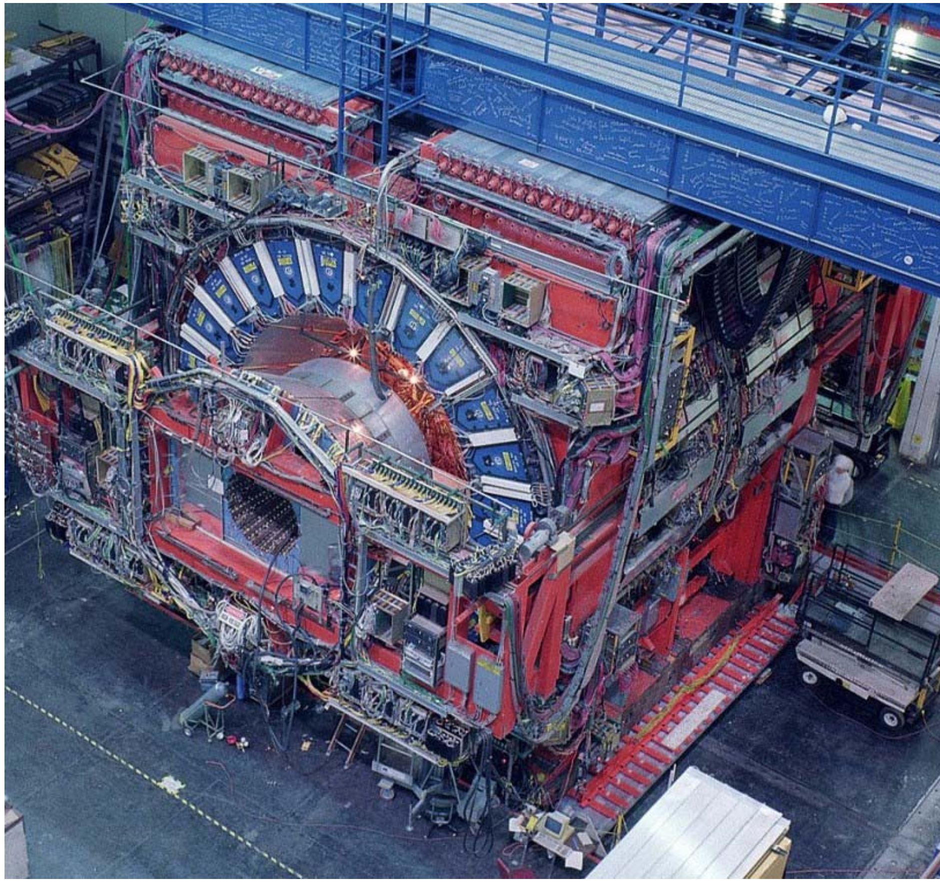
Iron/Scintillator

The discovery of the top quark

➡ The experiment: CDF





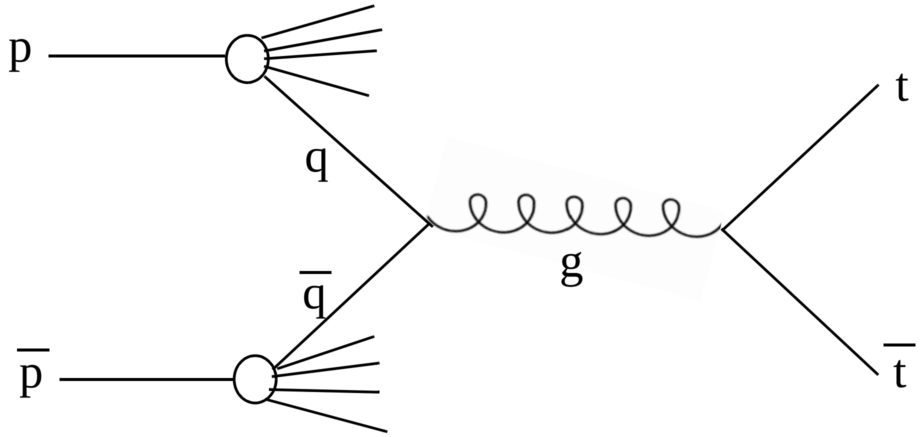


The discovery of the top quark

➡ Production of top-quarks

- **In proton-antiproton colliders, pairs of top quarks are mostly produced by quark-antiquark annihilation:**

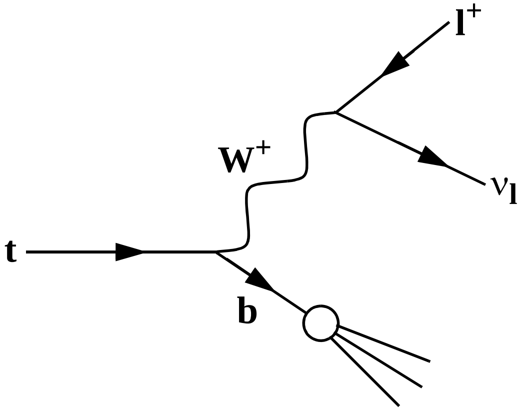
$$q+\overline{q}\rightarrow g\rightarrow t+\bar{t}$$

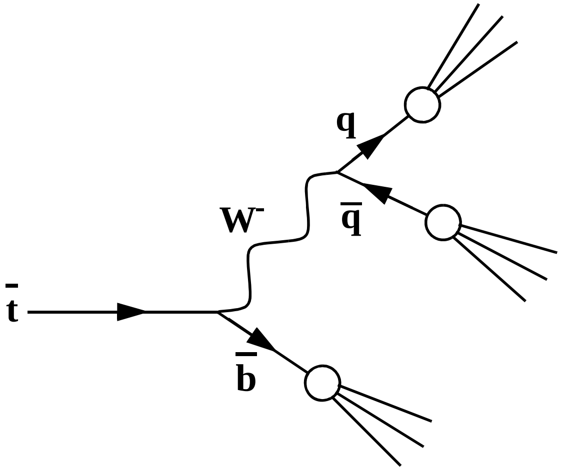


The discovery of the top quark

➡ The decay of top-quarks

- The most likely decay of a top quark is to a b quark and to a W.

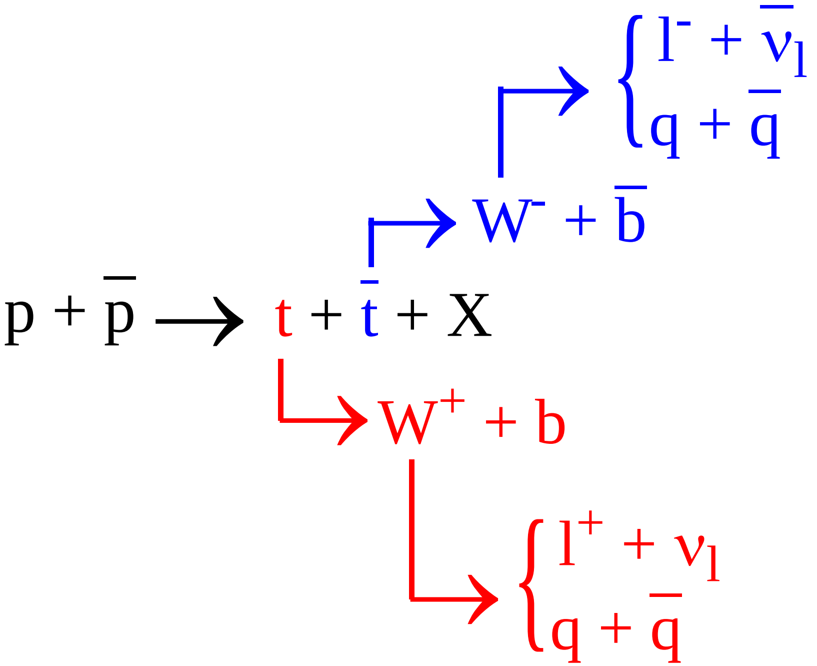




- The W can decay to leptons or hadrons

⇨

The final state is a complex mix of jets and leptons.



$$\left\{\begin{array}{l} l^- + \bar{\nu}_l \\ q + \bar{q} \end{array}\right.$$

$$W^- + \bar{b}$$

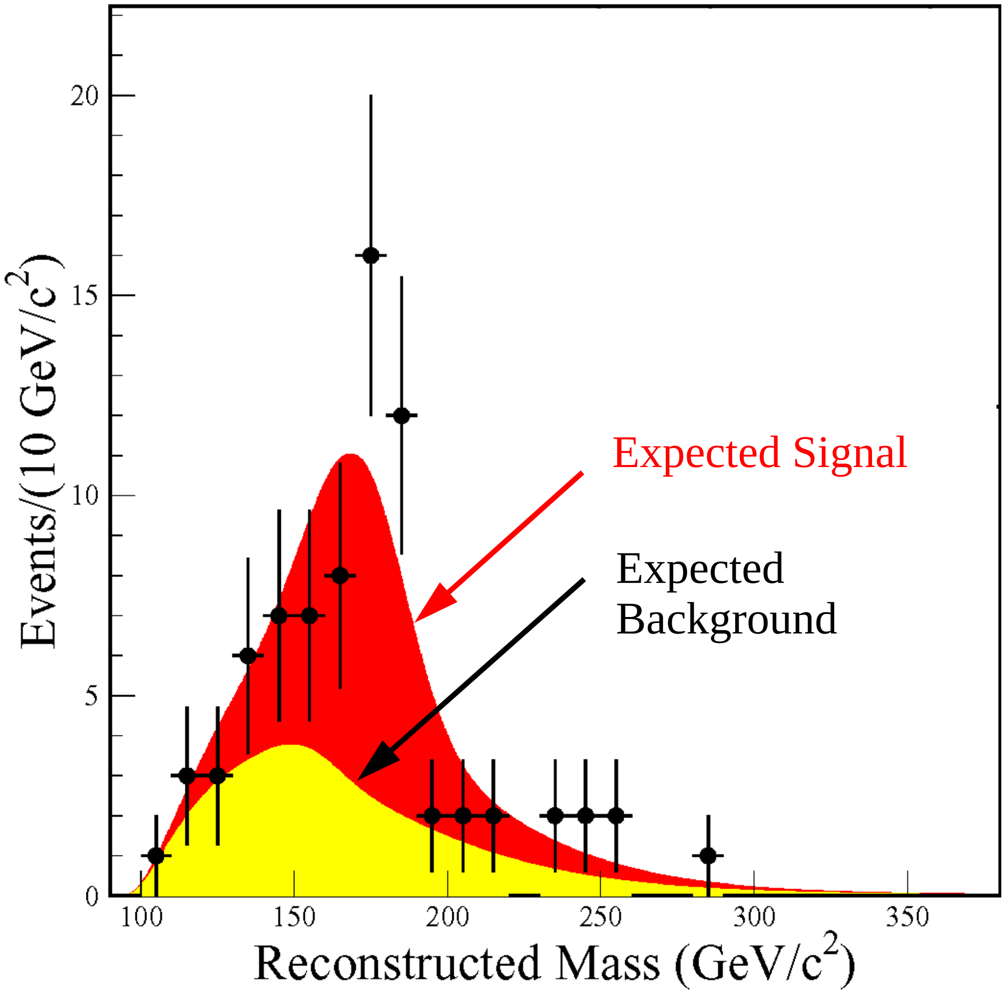
$$p + \bar{p} \longrightarrow t + \bar{t} + X$$

$$W^+ + b$$

$$\left\{\begin{array}{l} l^+ + \nu_l \\ q + \bar{q} \end{array}\right.$$

The discovery of the top quark

- After a selection of likely top event ⇨ **plot mass distribution** of the top-candidates.



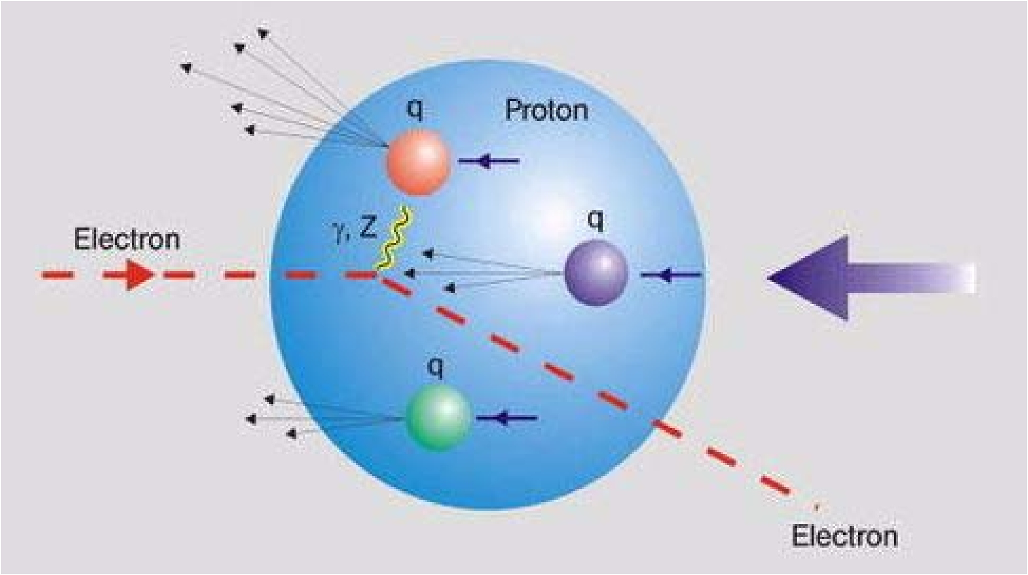
- A large background component
- Extract the **top mass**:

$$M_t = 176 \pm 5\ GeV$$

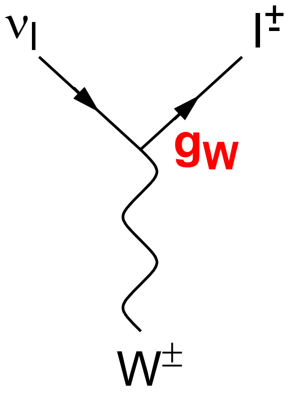
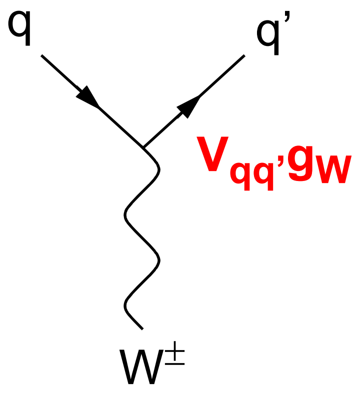
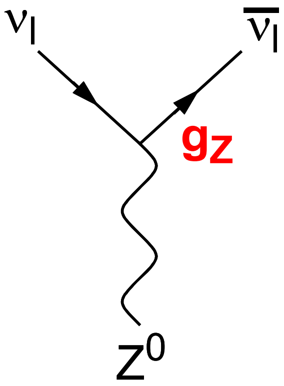
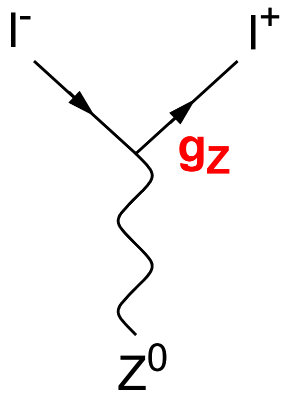
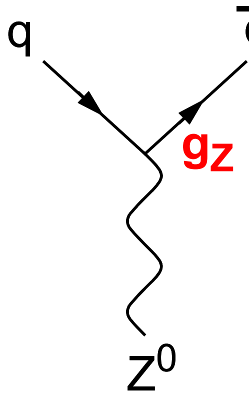
- Latest results:

$$M_t = 173 \pm 1 GeV$$

Neutral current reactions

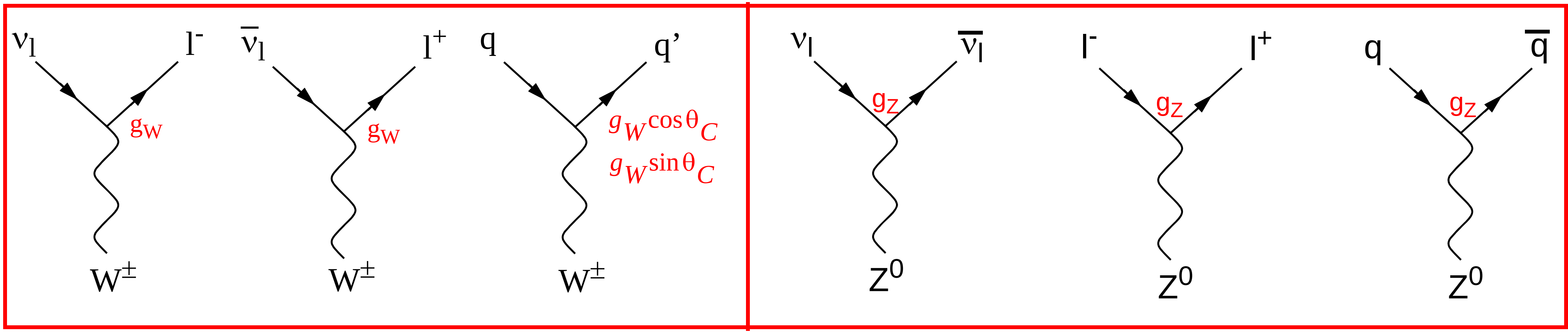


W and Z reactions

	Leptonic reactions	Hadronic reactions
Charge current reactions		
Neutral current reactions	 	

Neutral current reactions

- The basic vertices with W bosons:
  - Conserved lepton numbers
  - Not conserved quark flavour (quark mixing)
- The basic vertices with Z bosons:
  - Conserved lepton numbers
  - Conserved quark flavour (no quark mixing)

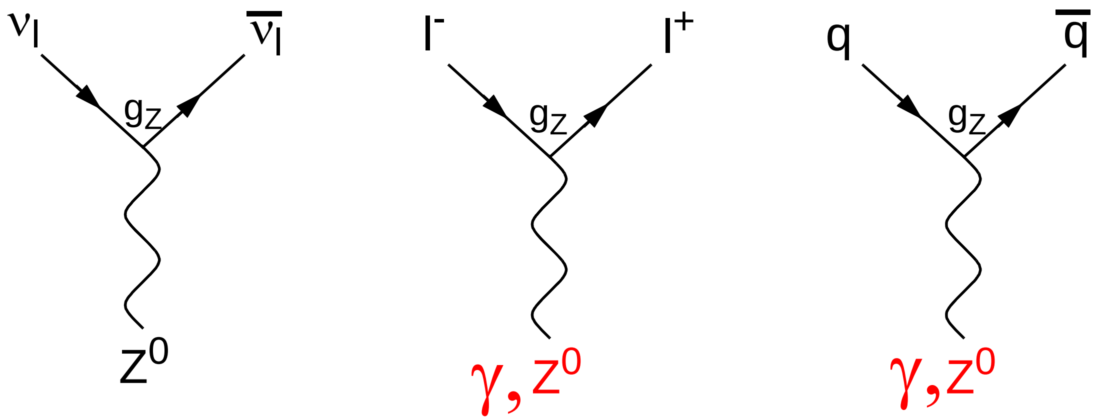


The basic W vertices

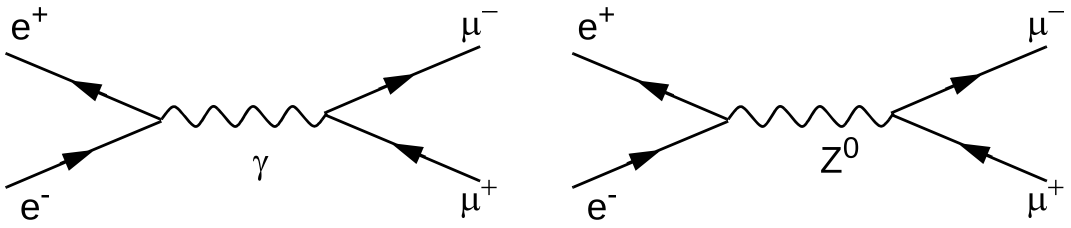
The basic Z vertices

Neutral current reactions

- In processes in which a photon can be exchanged, a Z^0 boson can be exchanged as well:



- Example: $e^+e^- \rightarrow \mu^+\mu^-$ has two dominant contributions



Neutral current reactions

- Estimation of the cross-section for the photon- and Z-exchange process in e^+e^- collisions at low energy:

$$\sigma_\gamma \approx \frac{\alpha^2}{E_{CM}^2} \qquad \sigma_Z \approx G_Z^2 E_{CM}^2$$

where E_{cm} is the collision energy.

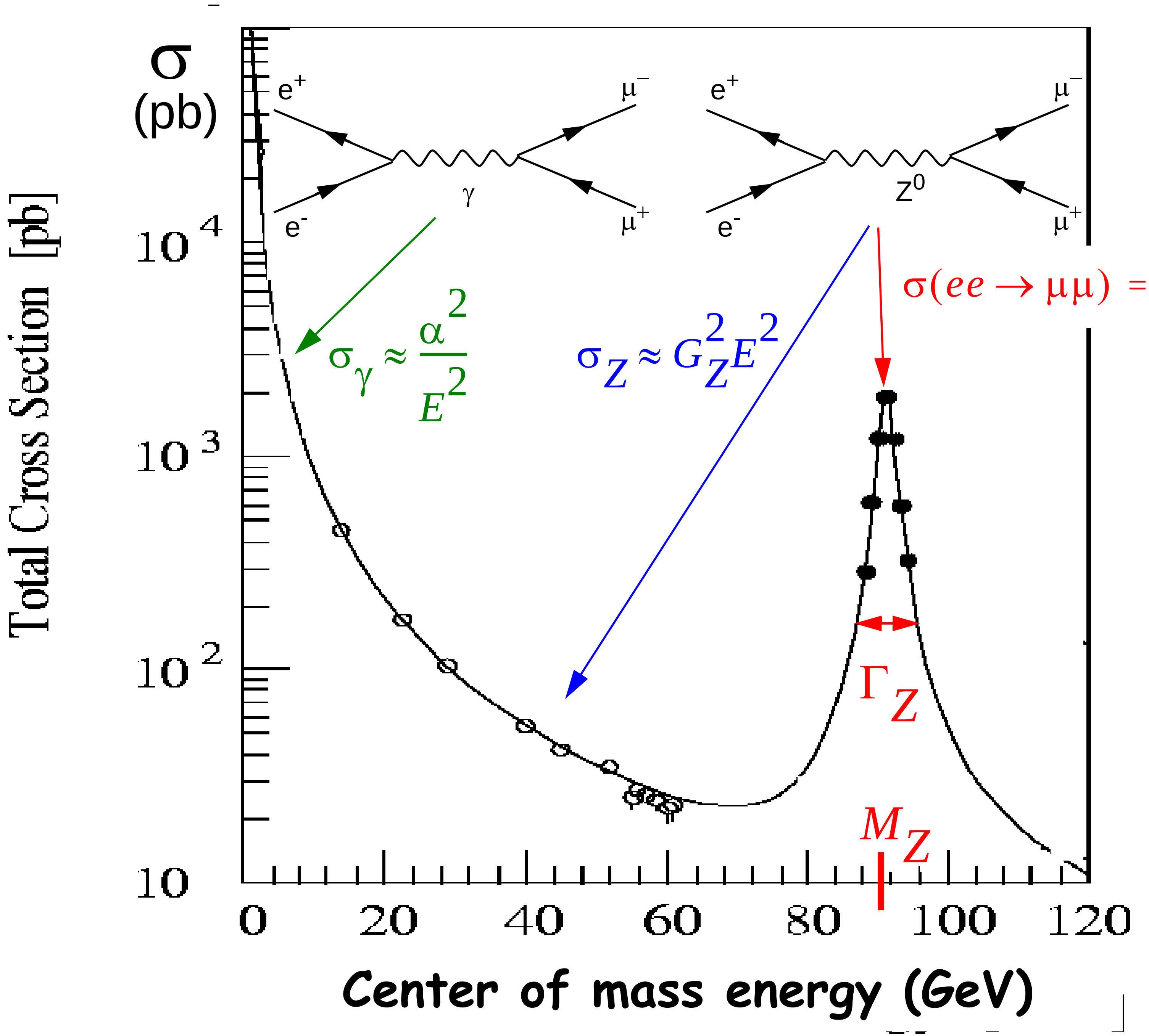
- The photon exchange process will dominate at low energies.

- At $E_{cm}=M_Z$ this low-energy approximation fails and the Z^0 peak is described by the Breit-Wigner formula:

$$\sigma(E_{CM}) = \frac{M^2}{E_{CM}^2}\left[\frac{C}{\left(E_{CM}^2 - M^2\right)^2 + M^2\Gamma^2}\right]$$

where M is the mass of the resonance and Γ its decay width.

Neutral current reactions



$$\sigma_\gamma \approx \frac{\alpha^2}{E^2}$$

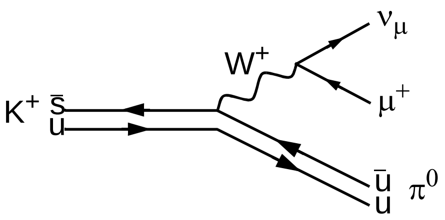
$$\sigma_Z \approx G_Z^2 E^2$$

$$\sigma(ee \rightarrow \mu\mu) = \frac{12\pi M_Z^2}{E_{CM}^2}\left[\frac{\Gamma(Z^0 \rightarrow ee)\Gamma(Z^0 \rightarrow \mu\mu)}{\left(E_{CM}^2 - M_Z^2\right)^2 + M_Z^2\Gamma_Z^2}\right]$$

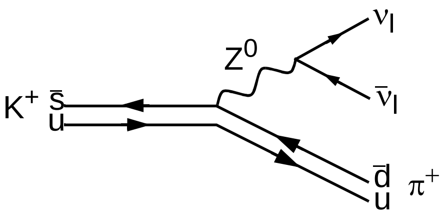
Test of flavour conservation

- That **flavour is conserved** at a **Z^0 vertex** can be verified by experiments.

- Example: measurement of the **decay rate of charged kaons**



$$K^+ \to \pi^0 + \mu^+ + \nu_\mu$$



$$K^+ \to \pi^+ + \nu_l + \bar{\nu}_l$$

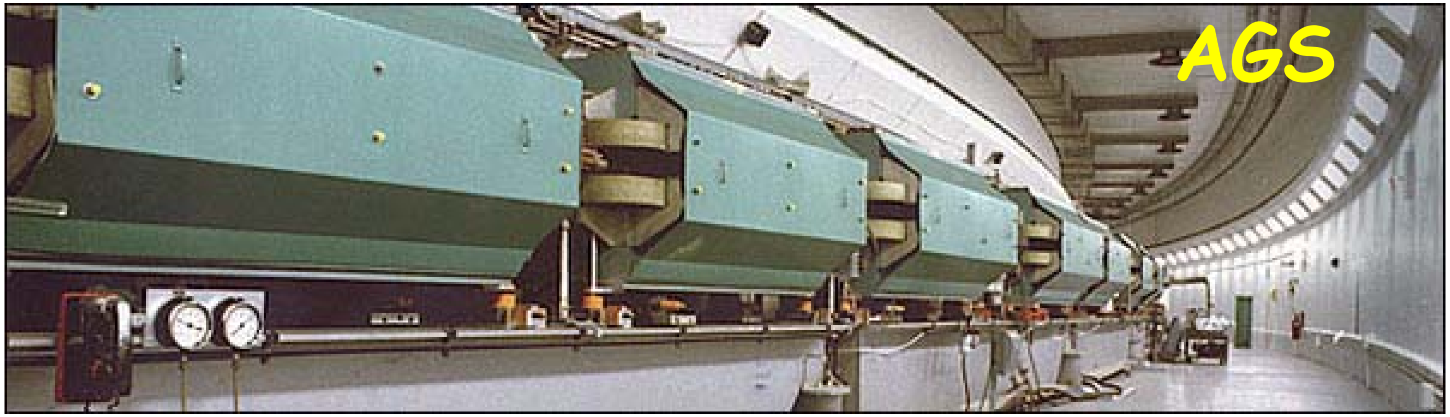
- The measured upper limit on the ratio of the decay rate of these two processes was

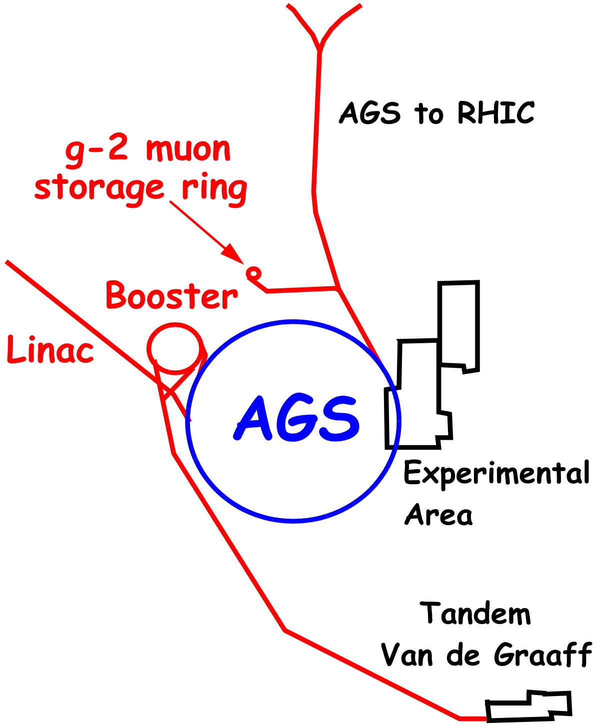
$$\frac{\sum_l \Gamma(K^+ \to \pi^+ + \nu_l + \bar{\nu}_l)}{\Gamma(K^+ \to \pi^0 + \mu^+ + \nu_\mu)} < 10^{-7}$$

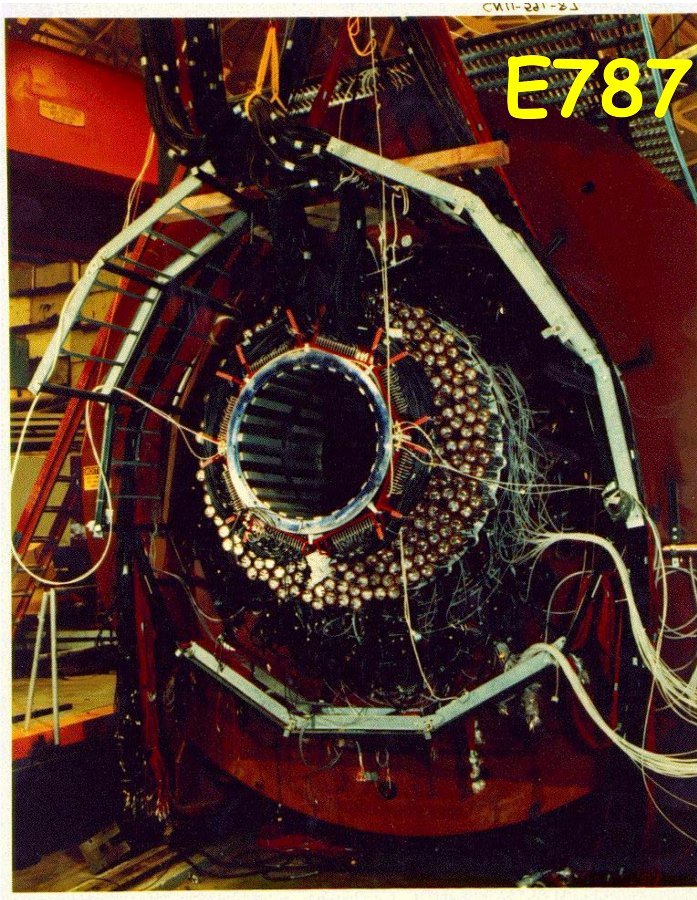
until experiment E787 came along .....

Test of flavour conservation

- The BNL experiment E787 was a **fixed target experiment.**
- It used a K^+ beam created by 24 GeV protons from the AGS.



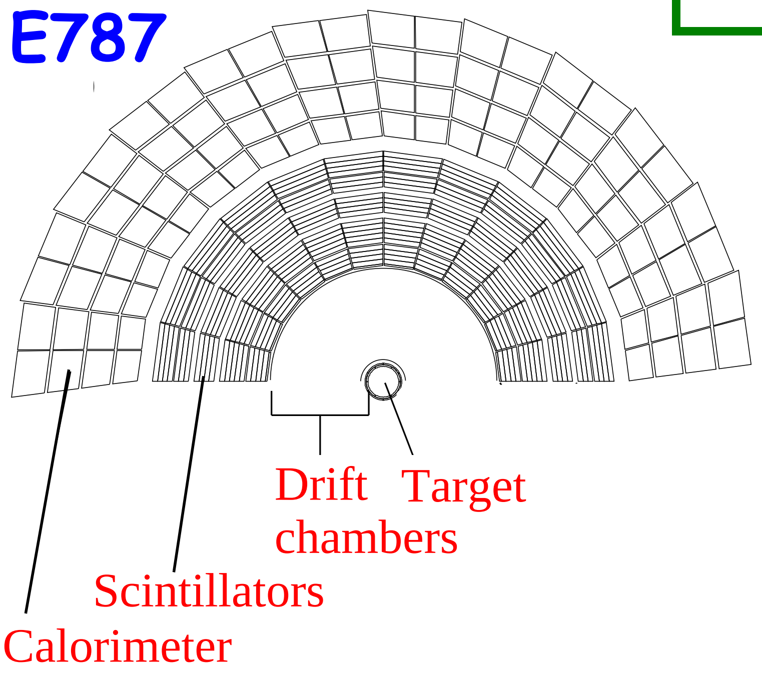


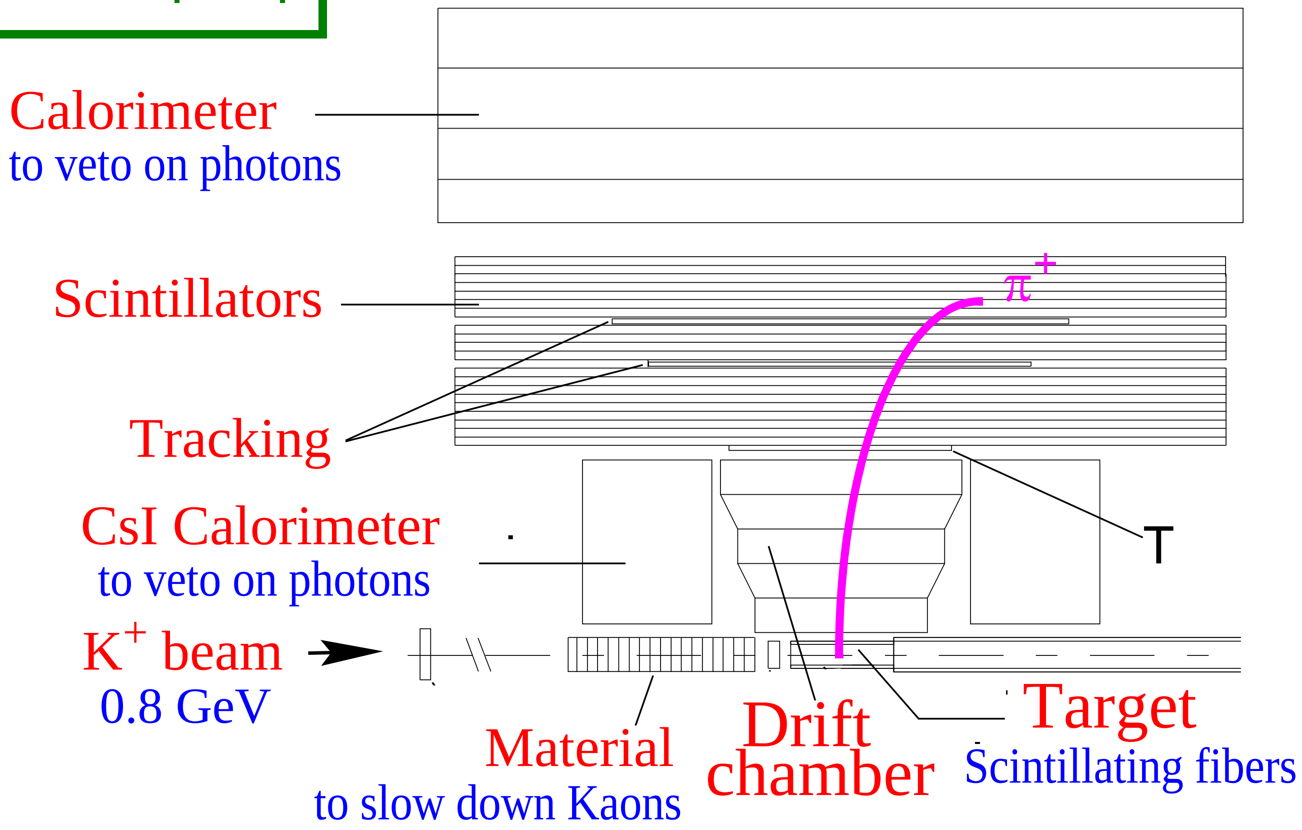


The 800 m long AGS has 240 magnets.

Test of flavour conservation

$$\boxed{K^+ \rightarrow \pi^+ + \nu_l + \bar{\nu}_l}$$

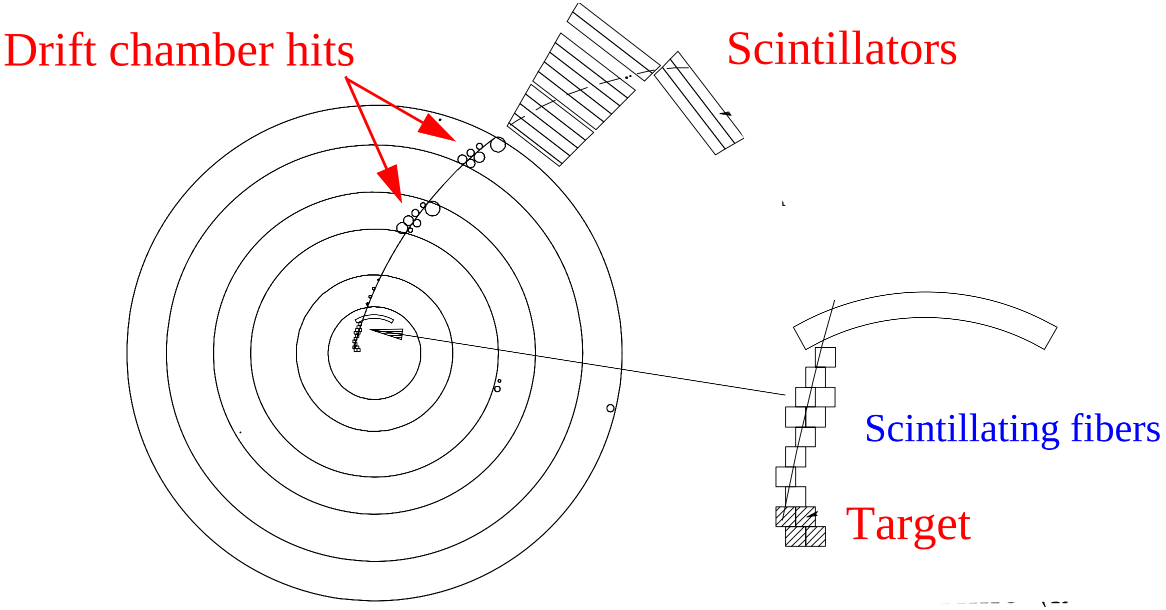




- Kaons stopped in a **target** made of **scintillating fibers**
- The decay of the K+ at rest was then studied.
- The **momentum, energy and range** of the particle from the decay was **measured**.

Test of flavour conservation

- After many years of running two candidate events for $K^+ \to \pi^+ + \nu_l + \bar{\nu}_l$ were found.

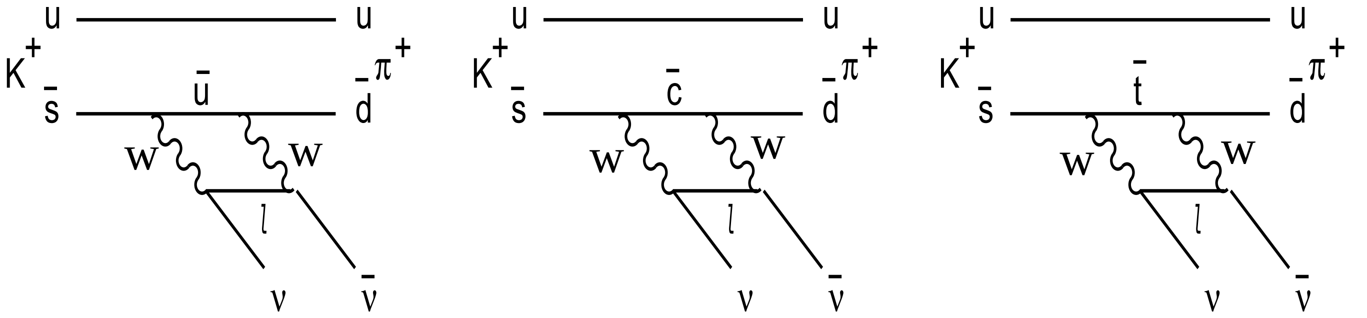


- The result from these two events were:

$$\frac{\sum_l \Gamma(K^+ \to \pi^+ + \nu_l + \bar{\nu}_l)}{\Gamma(K^+ \to \pi^0 + \mu^+ + \nu_\mu)} = \frac{1{,}6 \times 10^{-10}}{0{,}033} = 5\times 10^{-9}$$

Test of flavour conservation

- Explaination: **second-order charged current reactions** and not neutral current processes:



- The t-d vertex in the third diagram ⇨ **set limits** on the V_{td} element in the CKM matrix

$$0{,}007 < |V_{td}| < 0{,}030$$

The Higgs boson

- Experimental data agrees extremly well with predictions of the gauge invariant electroweak theory.

- Gauge invariance ⇨ the gauge bosons have zero mass.

  True for photons in QED and gluons in QCD but not for W and Z.

- A new scalar field called the Higgs field is introduced to generate mass to the W and Z bosons as well as fermion masses.

- Associated with the field is a new particle called the Higgs boson.

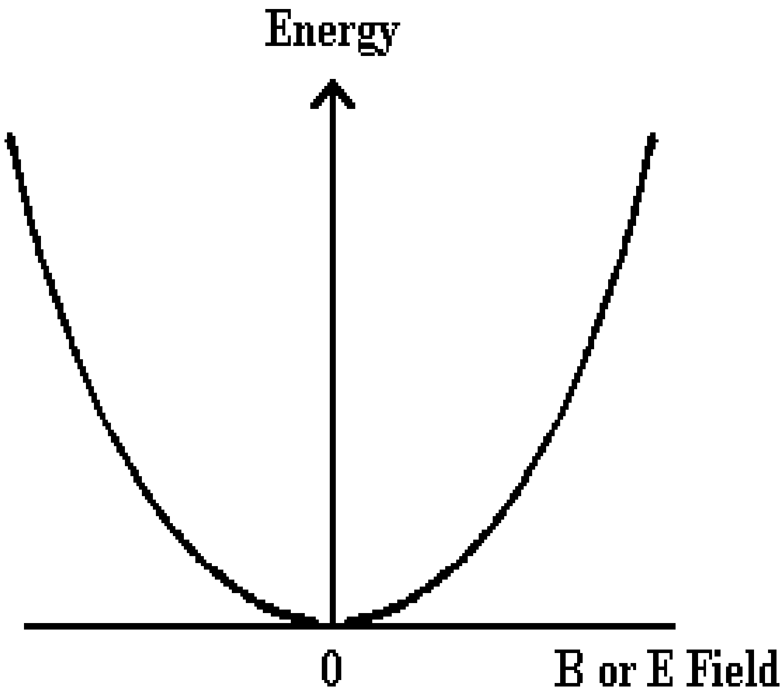
- The theory predicts how the Higgs boson couples to other particles but do not predict its mass.

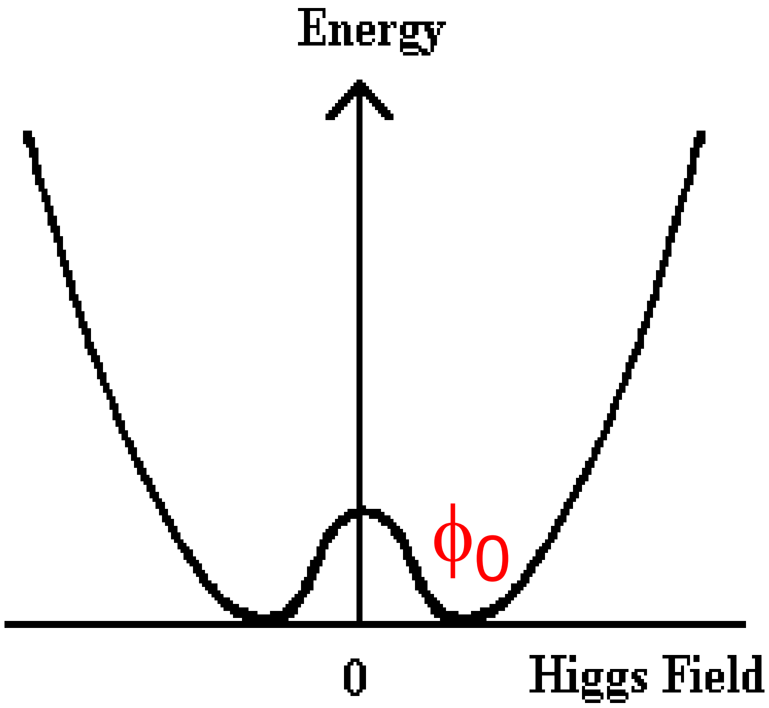
The Higgs boson

- Unusual characteristic of the Higgs field ⇨

  a non-zero value ϕ_0 in vacuum

  (i.e. the field is not zero in its groundstate).





- Non-zero vacuum expectation value ⇨

  The vacuum is populated with massive Higgs bosons ⇨

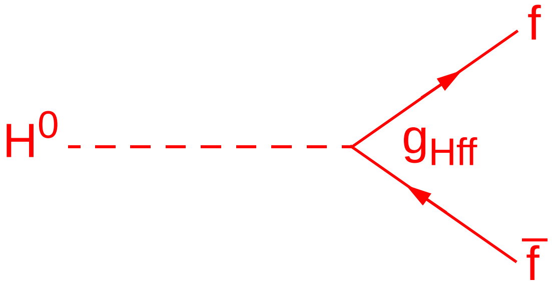
When a gauge field interacts with the Higgs field it acquires mass.

The Higgs boson

- The interaction with the Higgs field ⇨ W and Z bosons obtain masses with the ratio given by

$$\cos\theta_W = \frac{M_W}{M_Z}$$

- Fermions acquire mass by interacting with the Higgs boson:



- The coupling constant depends on the fermion mass.

$$g_{Hff}^2 = \sqrt{2}G_F m_f^2$$

- The Higgs boson has not been experimentally verified !

Search for the Higgs boson at LEP

- **The LEP project had two phases:**

LEP 1: The collision **energy** was equal to the **mass of the Z^0**.

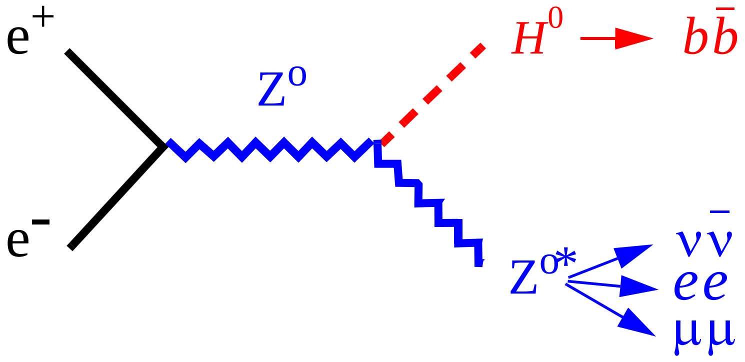
LEP 2: The collision **energy** was increased gradually by adding superconducting cavities ⇨ Maximum energy = **209 GeV**

➡ Higgs search at LEP 1

- If the **Higgs** particle was **lighter than the Z^0**

$$Z^0 \rightarrow H^0 + l^+ + l^-$$

$$Z^0 \rightarrow H^0 + \nu_l + \bar{\nu}_l$$



Search for the Higgs boson at LEP

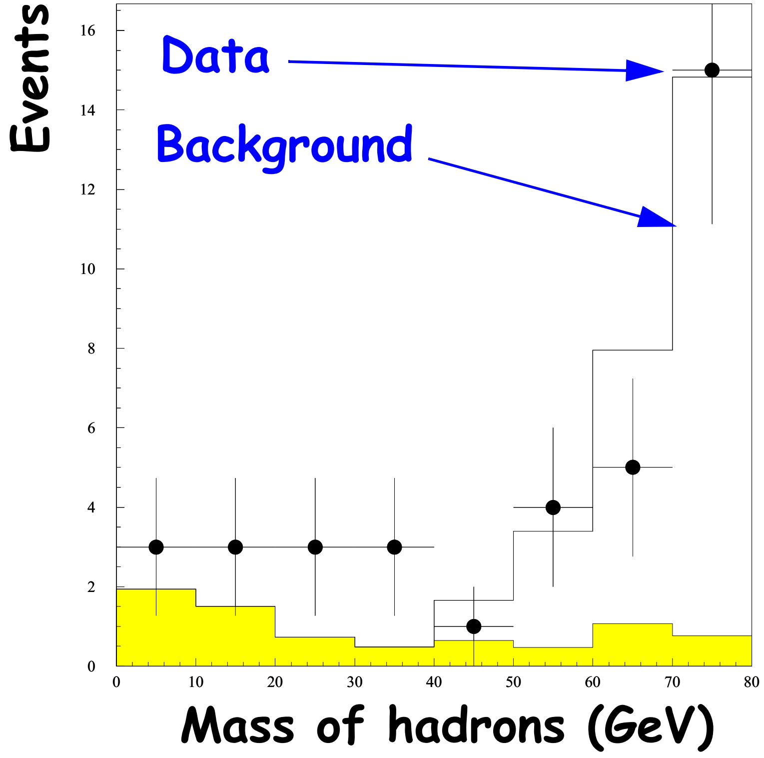
- The predicted branching ratio for Higgs production was low

$$3\times10^{-6} \le \frac{\Gamma(Z^0 \to H^0 l^+ l^-)}{\Gamma_{tot}} \le 10^{-4}$$

- The DELPHI experiment looked at one million Z^0 events

- Selection of events that contained both leptons and hadrons.

- No signal was observed and one concluded that $m_H > 56$ GeV.

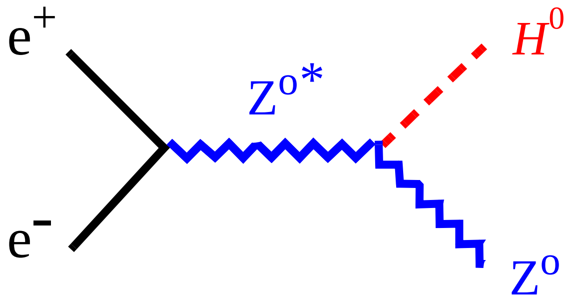


Search for the Higgs boson at LEP

➡ Higgs search at LEP 2

- LEP 2: Higgs production by **Higgs strahlung:**

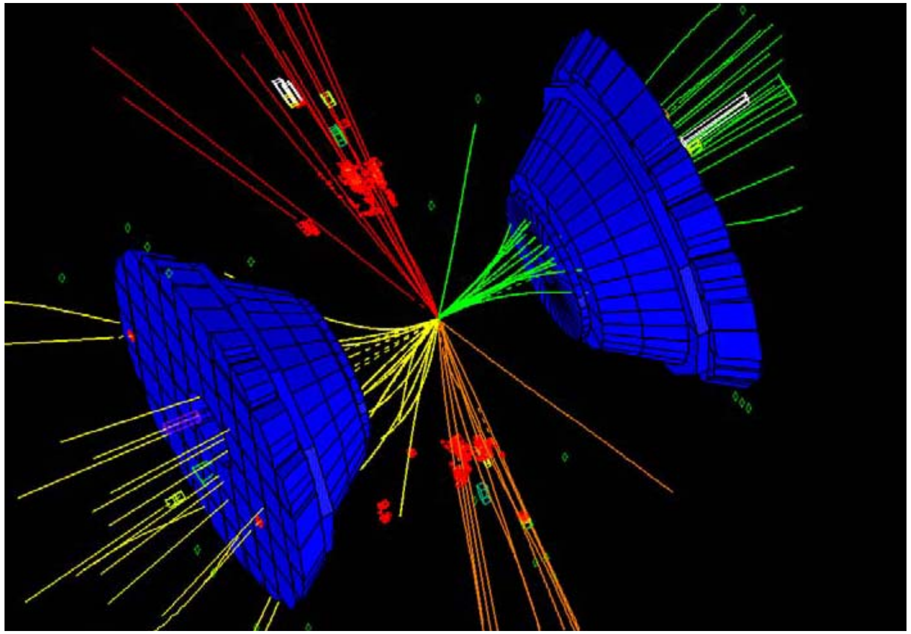
$$e^+ + e^- \to H^0 + Z^0$$



$H^0 \to$	$b\bar{b}$ 74%	$\tau\bar{\tau}$ 8%	WW 8%	gg 6%	$c\bar{c}$ 4%
$Z^o \to$	$q\bar{q}$ 70%	$\nu\bar{\nu}$ 20%	ee 3%	$\mu\mu$ 3%	$\tau\tau$ 3%

- Most of the Higgs events would have **4 jets** in the final state.

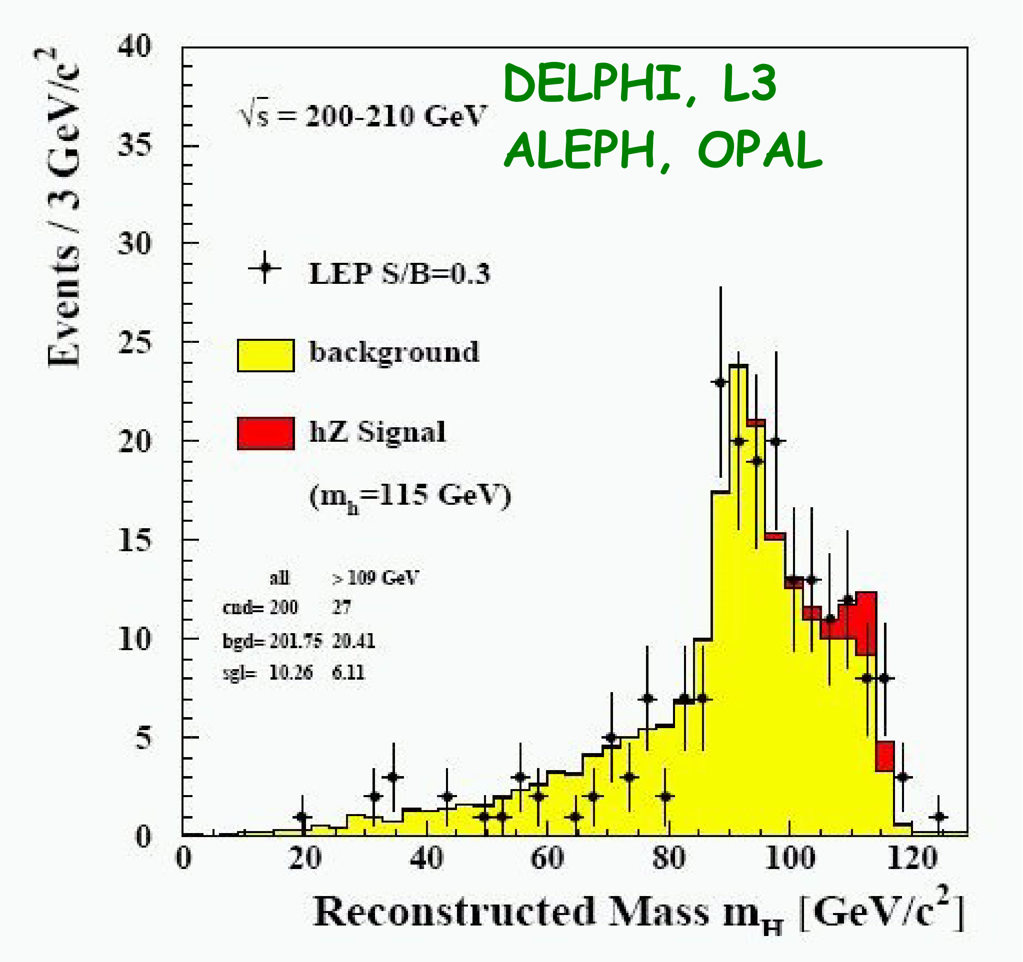
- Two of these should be coming from **b-quarks.**



Search for the Higgs boson at LEP

- ALEPH reported a couple of Higgs candidates at 115 GeV
- DELPHI, L3 and OPAL had no signal.
- All data added together ⇨ no discovery.
- LEP was turned off in 2000.
- The DELPHI experiment put a limit on the Higgs mass of:

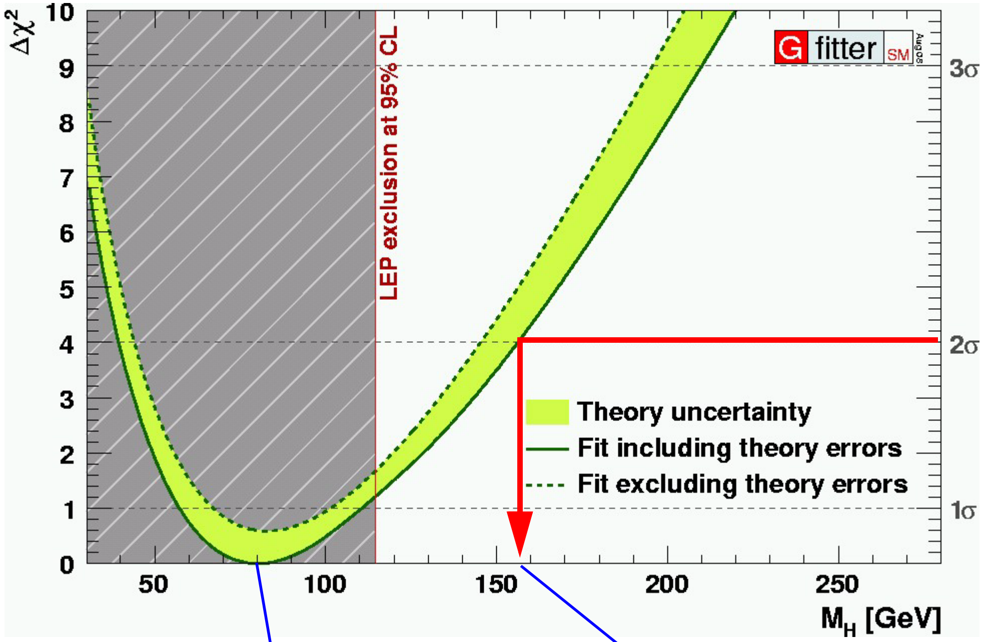
$$M_H > 114 \text{ GeV/c}^2$$



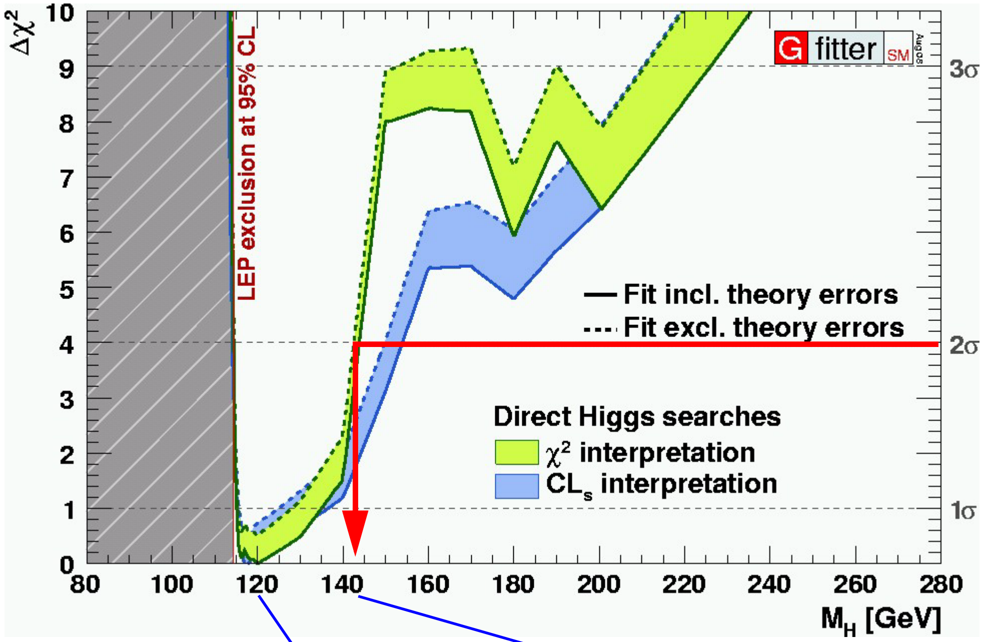
Search for the Higgs boson at LEP

- **The measurement of many electroweak parameters at LEP (and other places) makes it possible to make a global fit with the Higgs mass as a free parameter.**

Fit without using the result of direct searches:



Fit using the limits from direct searches;



$m_H = 80^{+30}_{-23}$ GeV < 154 GeV

$m_H = 120^{+15}_{-5}$ GeV < 144 GeV