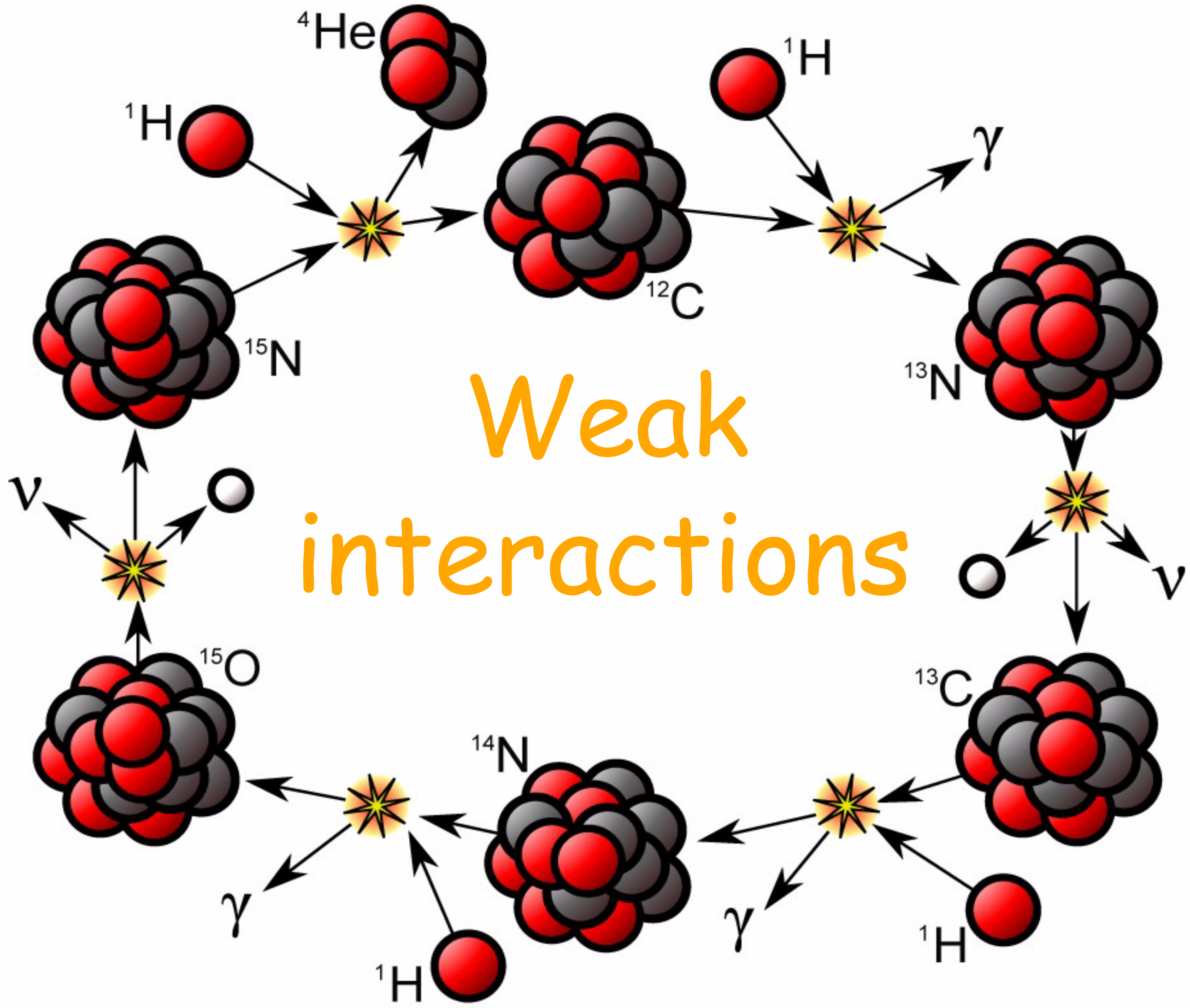
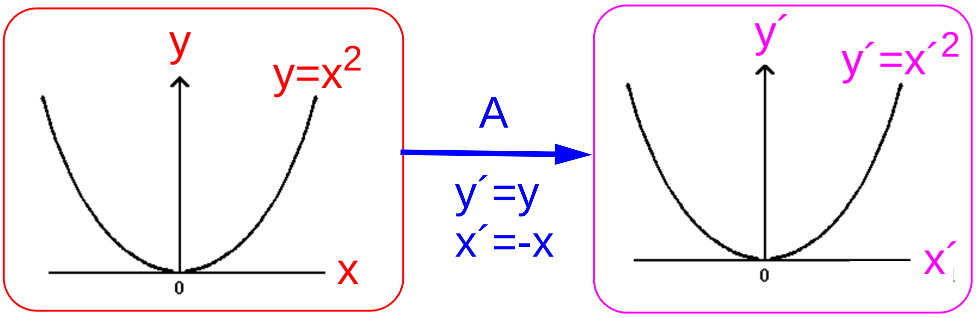
Weak interactions



The electroweak theory

➡ Gauge invariant theories.



The equation $y=x^2$ is symmetric or invariant under the transformation *A*, i.e. it looks the same before and after the transformation.

- Modern quantum field theories are gauge invariant theories i.e. they are theories were the main equations do not change when a gauge transformation is performed.

- By requiring that the theories are gauge invariant one can in fact deduce the various interactions.

The electroweak theory

➡ What is a gauge transformation ?

- There are several forms of gauge transformations corresponding to different interactions.

- As an example we can look at non-relativistic electromagnetism and start by assuming that the equation of motion for a free non-relativist particle is:

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\nabla^2\psi$$

where $\Psi(\mathbf{r},\mathbf{t})$ is the wavefunction of the particle and m is its mass.

The free particle Schrödinger equation

- Assume that we want to modify this equation so that it also describes particles that interact electromagnetically.

The electroweak theory

- Assume further that we suspect that the new equation has to be invariant under a so-called U(1) phase transformation:

$$\psi \rightarrow \psi' = e^{iq\alpha}\psi$$

were $\alpha(r,t)$ is an arbirary continous function and q is the charge of the particle.

- However, if we try to put the transformed wavefunction into the Schrödinger equation we discover that it is not a solution.

$$i\frac{\partial\psi'}{\partial t} = \left[\frac{1}{2m}(\nabla + iq\alpha) - 2mq\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right]\Psi'$$

new terms

The electroweak theory

- In electromagnetism the field is described by a vector potential **A** and a scalar potential **V** that are defined by

$$\bar{E} = -\nabla V - \frac{d}{dt}\bar{A} \qquad \bar{B} = \nabla x \bar{A}$$

where E is the electrical field and B the magnetic field.

- Question: How can we add these potentials to the Shrödinger equation while keeping it invariant under a U(1) phase transformation of the wavefunction ?

- The interaction is introduced by requiring that the Schroedinger equation is also invariant under a gauge transformation of type:

$$\bar{A} \rightarrow \bar{A}' = \bar{A} + \nabla\alpha$$
$$V \rightarrow V' = V - \frac{\partial\alpha}{\partial t}$$

The electroweak theory

- In order for the free-particle Schroedinger equation to be invariant under both the U(1) phase transformation and the gauge transformation, the equation has to be changed to:

$$i\frac{\partial\psi}{\partial t} = \left[\frac{1}{2m}(\nabla - q\bar{A}) + qV\right]\Psi$$

The equation for a non-relativistic particle with charge q moving in an electromagnetic field.

The Gauge principle

- **Gauge principle:** to keep the invariance condition satisfied it is necessary to add a minimal field to the Schroedinger equation

  ⇨ an interaction will have to be introduced.

The electroweak theory

➡ Glashow, Weinberg and Salam formulated in the sixties a unified theory for the weak and electromagnetic interactions - The electro weak theory.

- This is a quantum field theory and the details is beyond the scope of this course. We will, however, study some of the predictions of the theory and how these have been tested experimentally.
- The theory introduces weak isospin charge (I_3^W) and weak hypercharge (Y^W) that are related to electric charge (Q) by

$$Q = I_3^W + Y^W/2$$

- It also introduces massless gauge particles (W^+, W^-, W^0, B^0) that interacts with massless fermions in order to make the theory gauge-invariant.

The electroweak theory

- A new field called the Higgs field is introduced in the theory to generate mass to the gauge bosons and the fermions.
- While W^+ and W^- are the well-known bosons responsible for weak radioactive decay, the W^0 and B^0 bosons are not observed experimentally. Instead the gauge boson for the electromagnetic interaction (the photon) and the gauge boson for the weak neutral current interaction (the Z^0) are linear combinations of W^0 and B^0:

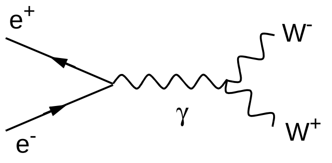
$$\gamma = B^0 \cos\theta_W + W^0 \sin\theta_W$$

$$Z^0 = -B^0 \sin\theta_W + W^0 \cos\theta_W$$

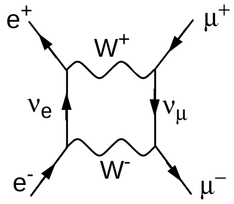
- The weak mixing angle (θ_W) is a parameter that is not predicted by the theory but has to be determined experimentally.

The electroweak theory

- The theory would lead to infinities if weak interactions could only take place by W-exchange due to divergent processes:

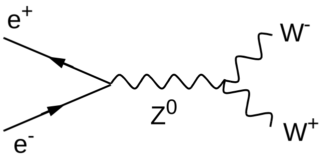


Non-divergent integrals

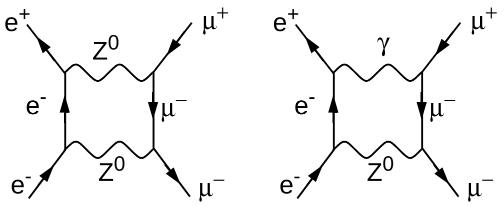


Divergent integrals

- The Z^0-boson fixes this problem beacuse the addition of its diagrams cancel out the divergencies:



Non-divergent integrals



Divergent integrals

The electroweak theory

- Two new coupling constants (g_W and g_Z) are introduced in addition to the electric charge (e) that is used in QED and g_s in QCD.

- The coupling constants at the W-, Z- and γ-vertices cannot be independent from each other in order for all the infinities to cancel out in the electroweak theory:

$$e/\sqrt{8} = g_W \sin\theta_W = g_Z \cos\theta_W$$ The unification condition.

(alternatively $e = g\sin\theta_W = g'\cos\theta_W$ with $g=\sqrt{8}\, g_W$ and $g'=\sqrt{8}\, g_Z$)

- The weak mixing angle, or Weinberg angle, is given by:

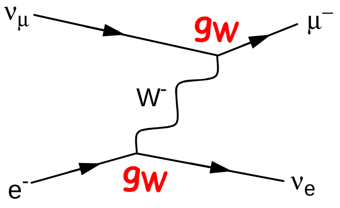
$$\cos\theta_W = \frac{M_W}{M_Z}$$

- Strength parameters can also be introduced for all interactions:

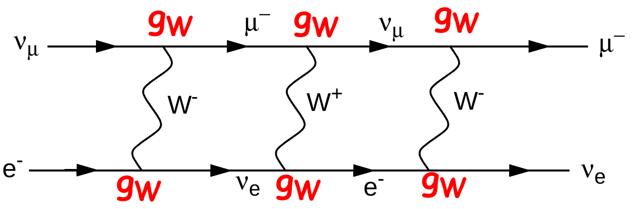
QED: $\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$ QCD: $\alpha_s = \frac{g_s^2}{4\pi} \approx \frac{1}{9}$ EW: $\alpha_W = \frac{g_W^2}{4\pi} \approx \frac{1}{250}$ $\alpha_Z = \frac{g_Z^2}{4\pi} \approx \frac{1}{850}$

The electroweak theory

- The **strength parameters** can be used to estimate the contribution from different processes (diagrams) to the **cross-section.**
- As an example one can take muon-scattering on electrons $\nu_\mu + e^- \rightarrow \mu^- + \nu_e$ for which diagrams with increasing complexity can contribute:



Second order diagram



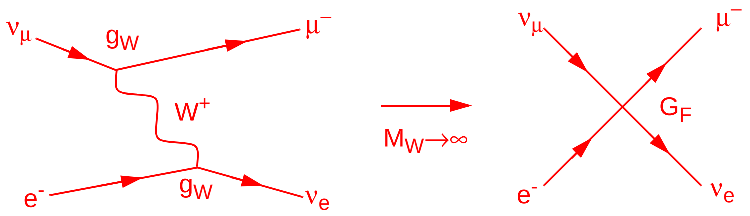
Sixth order diagram

- The **second order diagram** give a contribution to the cross section that is proportional to a_W^2 and the **sixth order diagram** a contribution proportional to a_W^6.

The electroweak theory

➡ Point-like interactions

- Before the electroweak theory, there was **Fermi's theory** for weak interactions that assumed **four-fermion point-like interactions** without W and Z exchange.
- Since W bosons are heavy, charged current interactions can be **approximated** by a **zero-range interaction** at low energy e.g.



- The strength of the zero-range interaction was given by the **Fermi coupling constant (G_F)** which is related to g_W by

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2}} = \frac{4\pi\alpha_W}{M_W^2}$$

where α_W is a weak strength parameter analogous to α in electromagnetic interactions.

The electroweak theory

- **A bit of algebra:**

The unification condition

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2}} \quad \text{and} \quad \sqrt{\frac{\pi\cdot\alpha_{em}}{2}} = g_W\sin\theta_W \quad \text{gives} \quad M_W^2 = \frac{g_W^2\sqrt{2}}{G_F} = \frac{\pi\alpha_{em}}{\sqrt{2}G_F\sin^2\theta_W}$$

$$\cos\theta_W = \frac{M_W}{M_Z} \quad \text{and} \quad M_W^2 = \frac{\pi\alpha_{em}}{\sqrt{2}G_F\sin^2\theta_W} \quad \text{gives} \quad M_Z^2 = \frac{\pi\alpha_{em}}{\sqrt{2}G_F\cos^2\theta_W\sin^2\theta_W}$$

Definition Weinberg angle

- **If one then introduces a neutral current coupling constant (G_Z) in the low energy zero-range approximation also, one has:**

$$\frac{G_Z}{\sqrt{2}} = \frac{g_Z^2}{M_Z^2} \quad \text{which gives} \quad \frac{G_Z}{G_F} = \frac{\sqrt{2}g_Z^2}{\sqrt{2}g_W^2}\frac{M_W^2}{M_Z^2} = \frac{\frac{\pi\alpha}{2\cos^2\theta_W}}{\frac{\pi\alpha}{2\sin^2\theta_W}}\cos^2\theta_W = \sin^2\theta_W$$

The electroweak theory

- **One could use the relationship $G_Z/G_F=\sin^2(\theta_W)$ together with measurements of weak interaction rates at low energy, i.e. G_Z and G_F, to determine that**

$$\sin^2\theta_W = 0,277 \pm 0.014$$

- **This measurement of the weak mixing angle made it possible to predict the masses of the W and Z:**

$$M_W = 78.3 \pm 2.4\ \mathrm{GeV/c^2}$$
$$M_Z = 89.0 \pm 2.0\ \mathrm{GeV/c^2}$$

- **It was a strong confirmation that the electroweak theory was correct when the W and Z bosons were later discovered at CERN with the masses predicted from these low energy experiments.**

The electroweak theory

- Today the most precise estimation of the Weinberg angle using many experiments is: $\sin^2\theta_W = 0.2255 \pm 0.0021$

- Putting these values into the previous formulas give:

$M_W = 78.5\ \mathrm{GeV/c^2}$

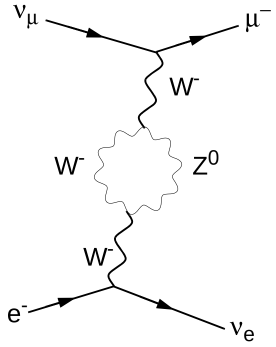
$M_Z = 89.0\ \mathrm{GeV/c^2}$

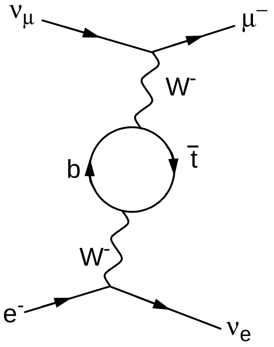
while direct measurements give

$M_W = 80.4\ \mathrm{GeV/c^2}$

$M_Z = 91.2\ \mathrm{GeV/c^2}$

- The reason for the **difference** is that higher-order diagrams, such as those below, were not taken into account in the low-energy formulas:





The electroweak theory

- Since the top-quark is involved in these higher-order corrections, the measurement of electroweak processes could again be used to **predict the top-quark mass** before it had been discovered:

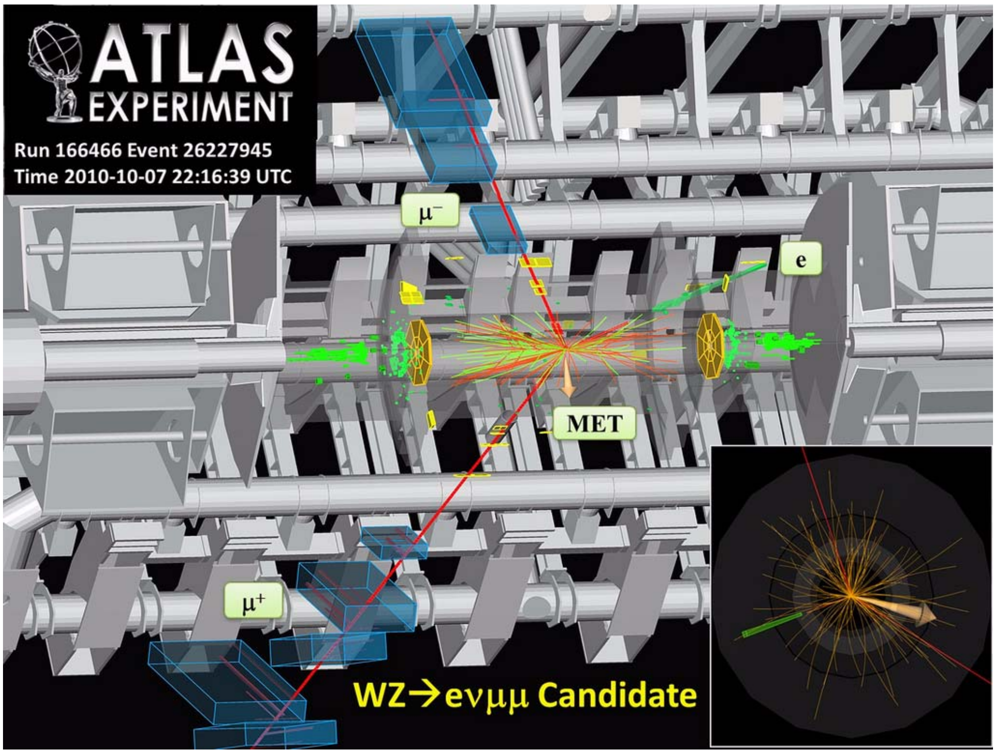
$$m_t = 170 \pm 30\ \mathrm{GeV/c^2}$$

- The **directly measured mass** of the top quark at Fermilab by the CDF experiment gave a value

$$m_t = 176 \pm 5\ \mathrm{GeV/c^2}$$

in perfect agreement with the prediction !

The W and Z bosons



The W and Z bosons

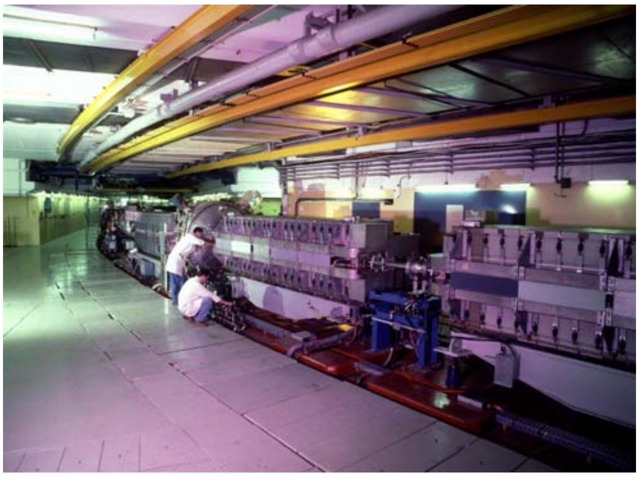
➡ The force carriers in weak interactions are spin-1 bosons (as in QED and QCD) that couple to quarks and leptons.

- The force carriers of weak interactions are three intermediate vector bosons: W^+ , W^- and Z^0.
- Since the W^+, W^- and Z^0 bosons are very massive particles (m_W=80.4 GeV and M_Z=91.2 GeV) weak interactions have a very short range (order of 2×10^{-3} fm).
- Before the Electroweak Theory was developed, all observed weak processes were charged current reactions (e.g. β-decay) mediated by W^+ or W^- bosons.
- The Electroweak Theory predicted that neutral current reactions caused by the Z^0 boson should exist.

Discovery of the neutral current

Accelerator: The Proton Synchrotron (PS) at CERN

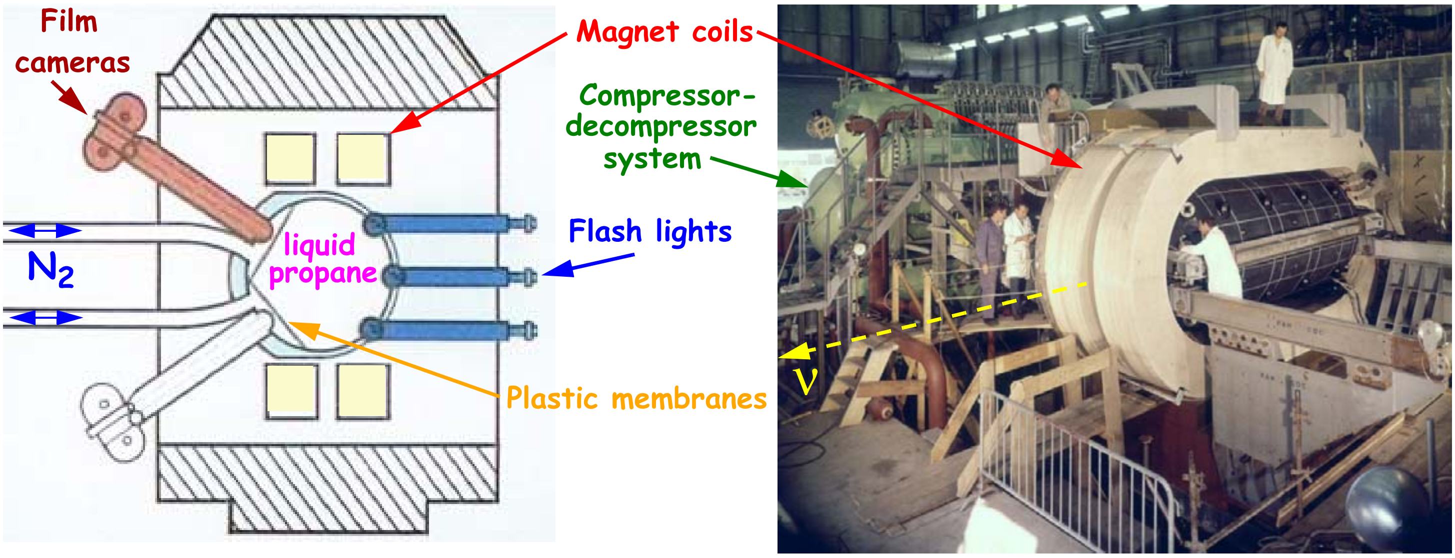
- The neutrino beams were created by letting an intense proton beam hit a target. Charged pions and kaons are created in the collision. These decay to neutrinos e.g. $\pi^+ \to \mu^+ + \nu_\mu \to e^+ + \nu_e + \bar{\nu}_\mu + \nu_\mu$.



The PS accelerator is 628 m long and has 277 magnets (including 100 dipole bending magnets). It can accelerate protons to 28 GeV.

Discovery of the neutral current

Neutral current events was first observed by the Gargamelle experiment at CERN in 1973.

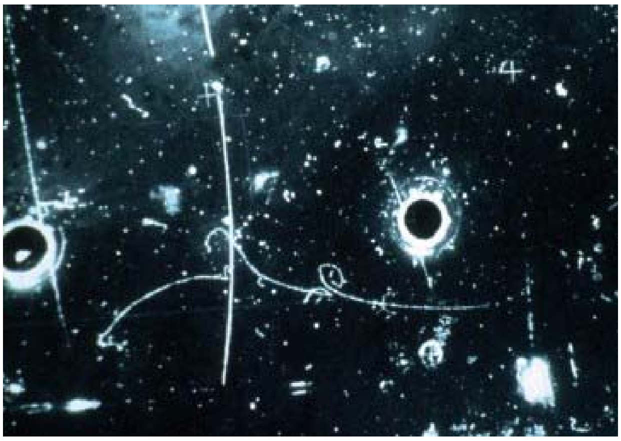


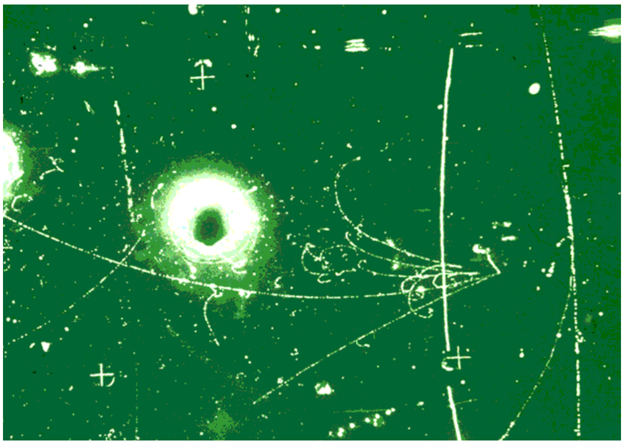
Step 1. The liquid propane is at a temperature below its boiling point.
Step 2. When the ν enters the propane, its pressure is lowered to make it superheated.
Step 3. Charged tracks ionize the propane and these ions create bubbles in the liquid.
Step 4. The bubble tracks are photographed by film cameras.

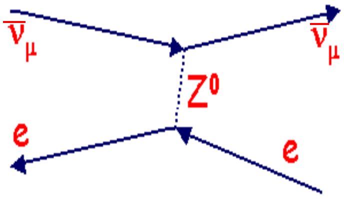
For more details watch the film at: http://cdsweb.cern.ch/record/43141

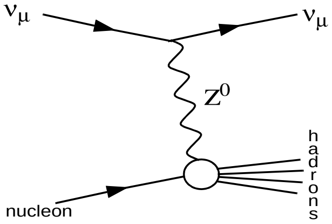
Discovery of the neutral current

➡ One looked for elastic and inelastic neutral current reactions.



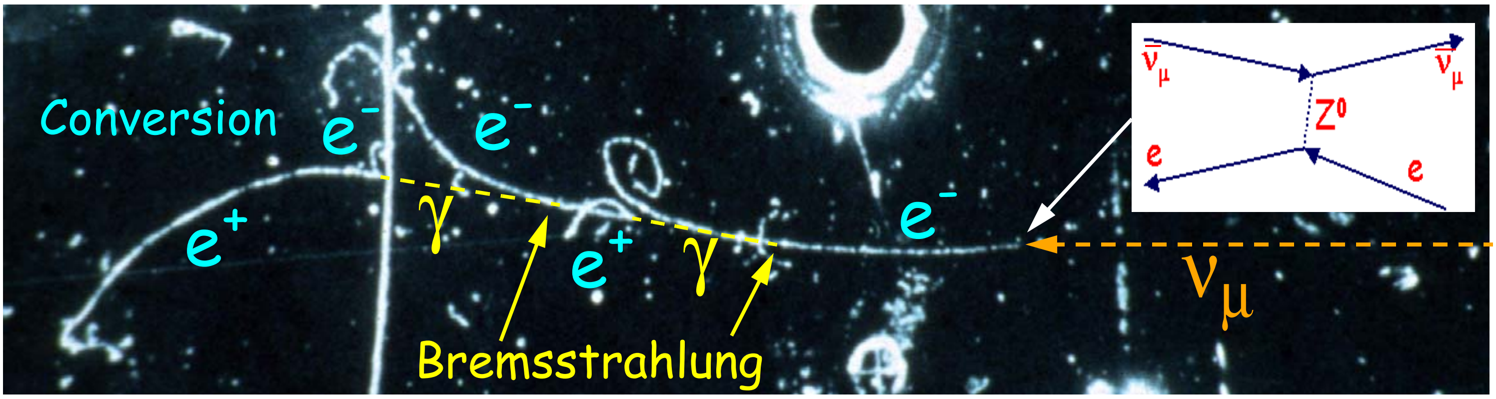






- The main background was from neutron - nucleon interactions.

Discovery of the neutral current

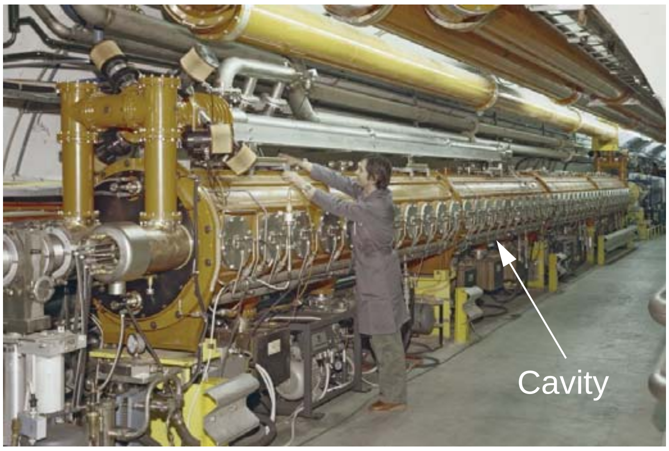


Interpretation of one of the first neutral current reactions seen by the Gargamelle experiment.

The discovery of the W and Z bosons

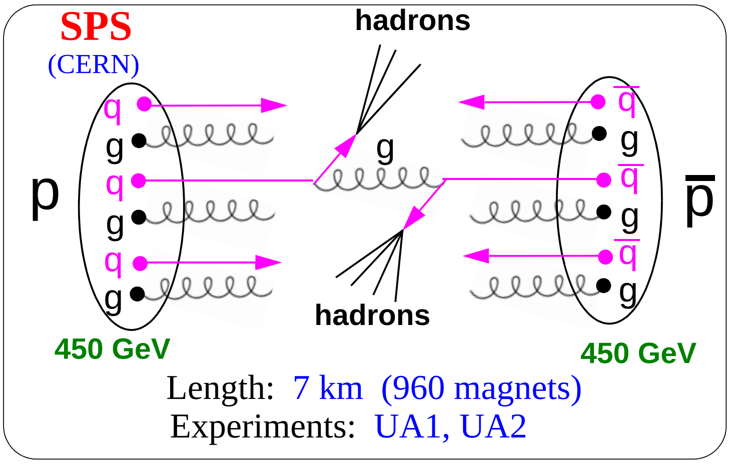
➡ The accelerator: The Super Proton Synchrotron

- The first study of direct production and decay of the W and Z vector bosons were made by the UA1 and UA2 experiments at the SPS proton-antiproton collider at CERN.

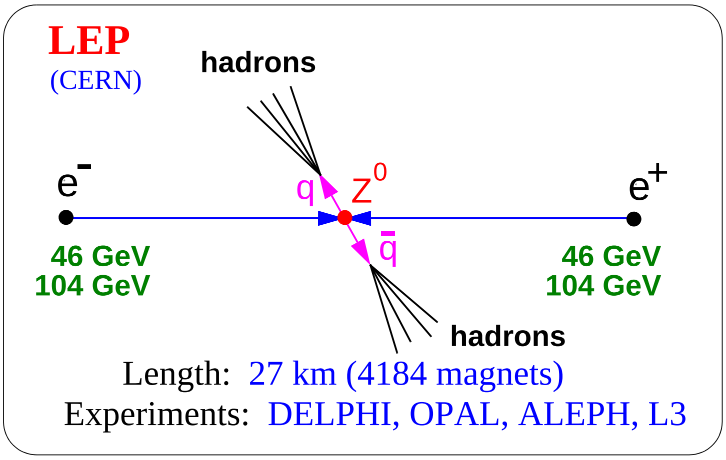


The SPS war originally a 450 GeV fixed target machine but was later converted to a collider with protons colliding with anti-protons. The accelerator is 6.9 km long and has 744 dipole magnets and 216 quadrupol magnets. The acceleration is given by 4 cavities operating at 200 MHz.

The discovery of the W and Z bosons



Length: 7 km (960 magnets)
Experiments: UA1, UA2



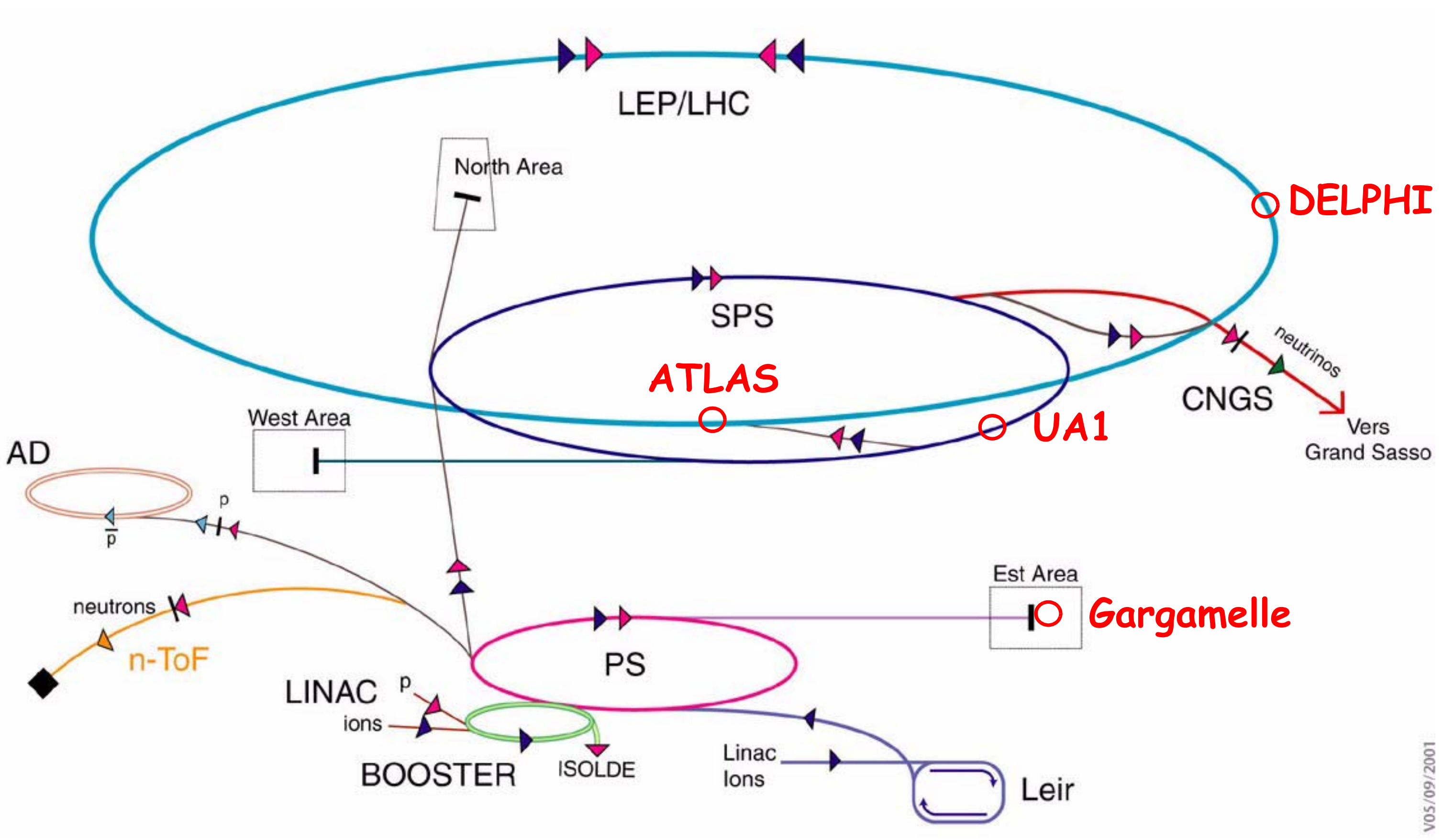
Length: 27 km (4184 magnets)
Experiments: DELPHI, OPAL, ALEPH, L3

- In hadron colliders, the energy of the quarks and gluons that participate in the interaction is only a fraction of the energy of the colliding protons.

- To produce W- and Z-bosons with a mass of 80-90 GeV it was therefore necessary to build an accelerator with a beam energy of 270 GeV.

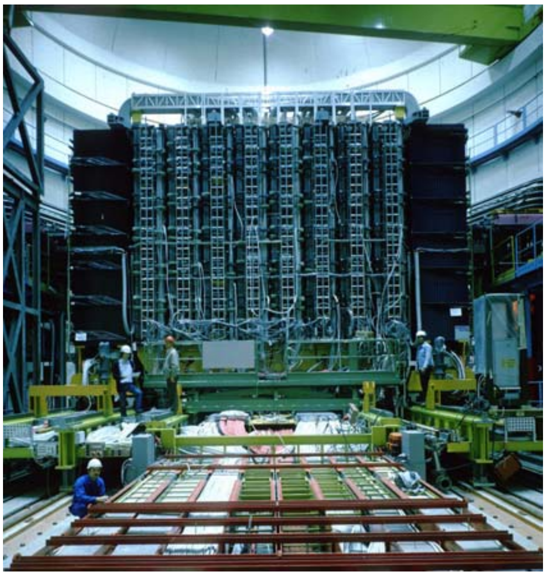
- The beam energy was later increased to 450 GeV.

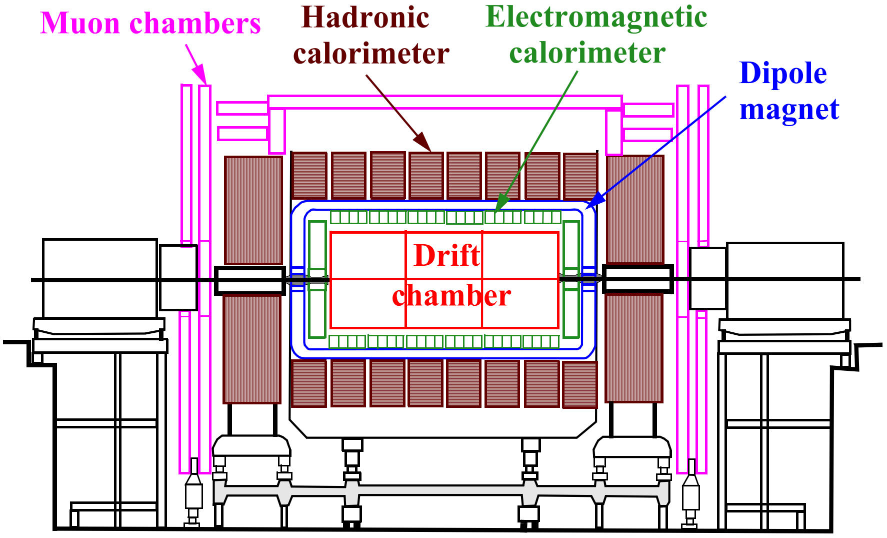
The discovery of the W and Z bosons



The discovery of the W and Z bosons

➡ The experiment: UA1





The UA1 detector had a central wire chamber that could measure tracks. A dipole magnet produced a 0.7 T field perpendicular to the beam direction. The electromagnetic calorimeter was of lead/scintillator type and the hadronic was an iron/scintillator sandwich. The muon detector consisted of 8 planes of drift chambers.

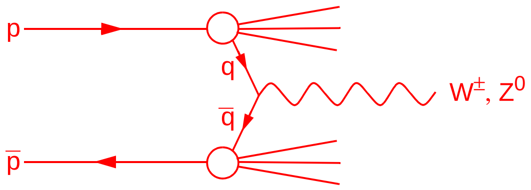
The discovery of the W and Z bosons

➡ Production of W and Z bosons

- **The W and Z bosons are in proton colliders produced by quark-antiquark annihilations:**

$u+\bar{d}\to W^+$ $d+\bar{u}\to W^-$

$u+\bar{u}\to Z^0$ $d+\bar{d}\to Z^0$



- **The lifetime of both the W and the Z is about 3×10^{-25} s and particles with such short lifetime are never seen directly in the experiments.**

The discovery of the W and Z bosons

➡ The decay of W and Z bosons

- **The W and Z bosons decay in most cases to hadrons but these decays cannot be identified among all the other hadrons created in $p\bar{p}$-collisions.Instead one looks for decays to leptons.**

hadronic decays	**Branching ratio**
$\bar{p}+p\to W^{+}+X$ $\hookrightarrow q' + \bar{q}$	**68%**
$\bar{p}+p\to W^{-}+X$ $\hookrightarrow q' + \bar{q}$	**68%**
$\bar{p}+p\to Z^{0}+X$ $\hookrightarrow q + \bar{q}$	**70%**

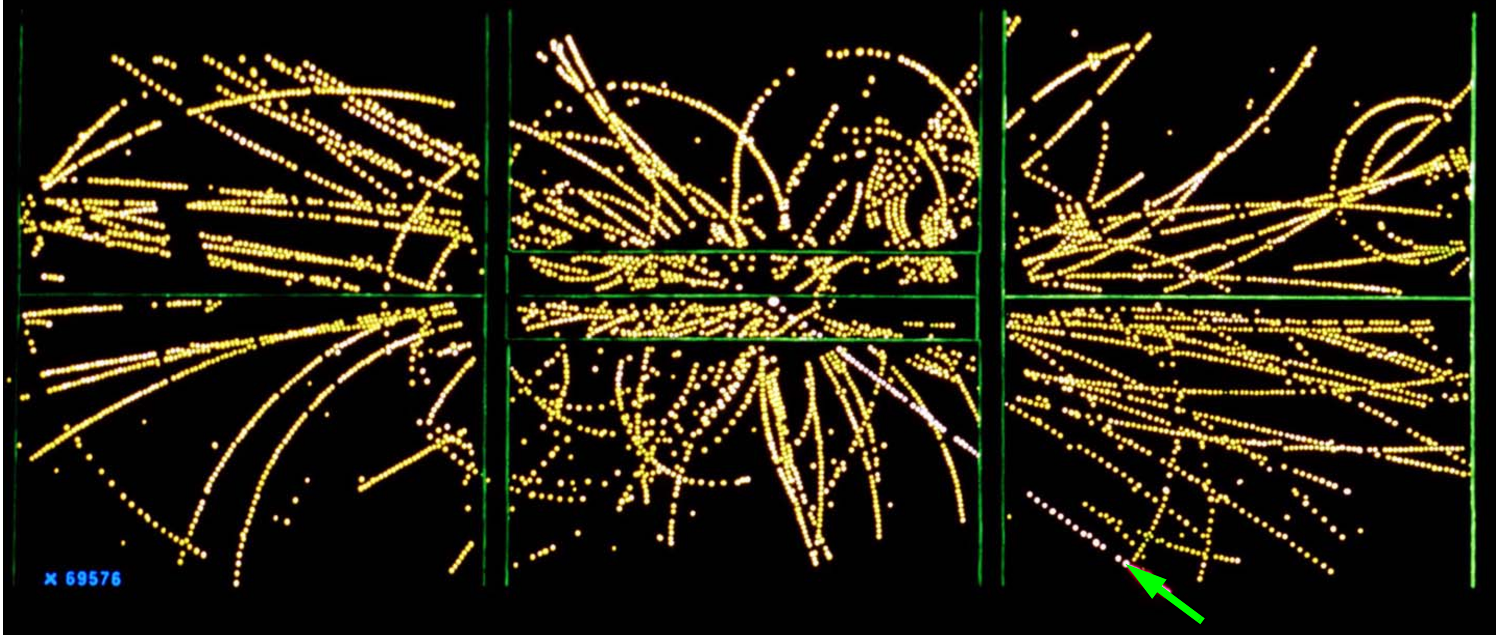
leptonic decays	**Branching ratio**
$\bar{p}+p\to W^{+}+X$ $\hookrightarrow l^{+} + \nu_l$	**32%** **e & μ**
$\bar{p}+p\to W^{-}+X$ $\hookrightarrow l^{-} \mp \bar{\nu}_l$	**32%** **e & μ**
$\bar{p}+p\to Z^{0}+X$ $\hookrightarrow l^{+} + l^{-}$	**6.7%** **e & μ**

The discovery of the W and Z bosons

EVENT 2958. 1279.

Analysis of W-events in UA1

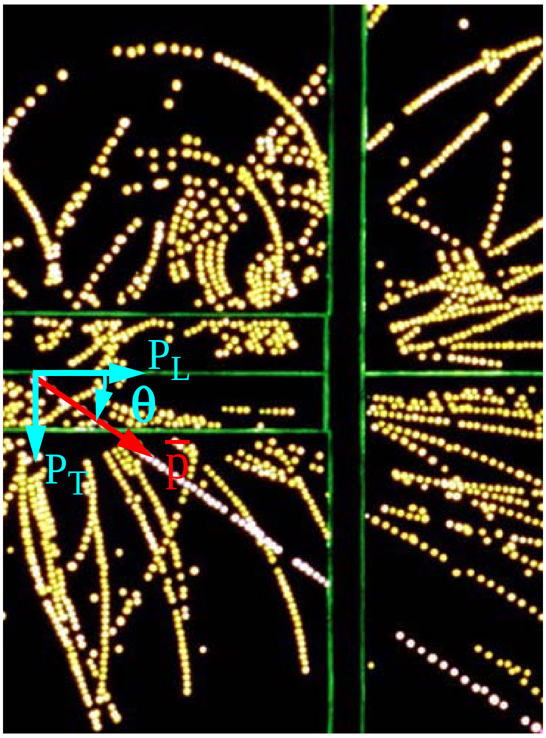
- The analysis was looking for events with a charged lepton and a neutrino.



A high momentum electron is indicated by the arrow.

The discovery of the W and Z bosons

Transverse momentum



- The **transverse momentum** and energy of a particle is defined in the following way:

$$P_T = P \sin(\theta)$$
$$E_T = E \sin(\theta)$$

where θ is the angle to the beam.

- $E_T = P_T$ if the mass of the particle is small since $E^2=P^2+m^2$
- The **total momentum is zero** if the momentum of all the particles in a collision is added up (momentum conservation).
- Neutrinos are, however, not detectable and if the total momentum is different from zero the event is said to have **missing momentum** (or missing energy).

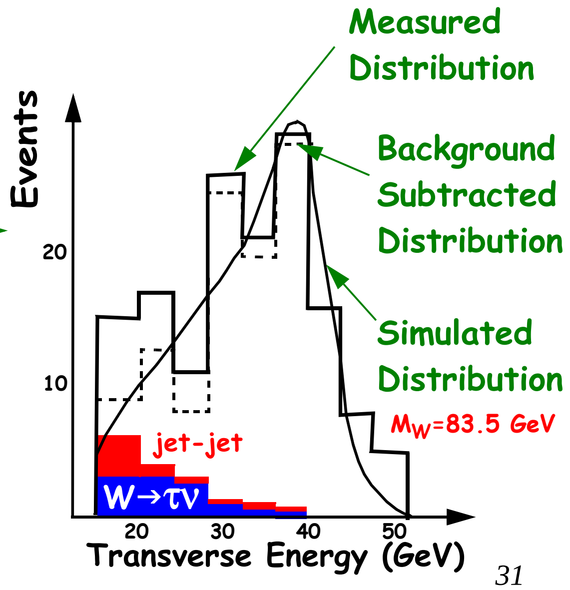
The discovery of the W and Z bosons

Analysis of W-events in UA1

- The main selection criteria in the UA1 W-analysis was:

i) A charged electron or muon with a large momentum (>10 GeV/c);
ii) This lepton should be emitted at a wide angle to the beam (>5°);
iii) There should be large missing transverse momentum in the event to indicate neutrino production.

The expected distribution of the transverse energy of the selected electrons was compared with the measured distribution.



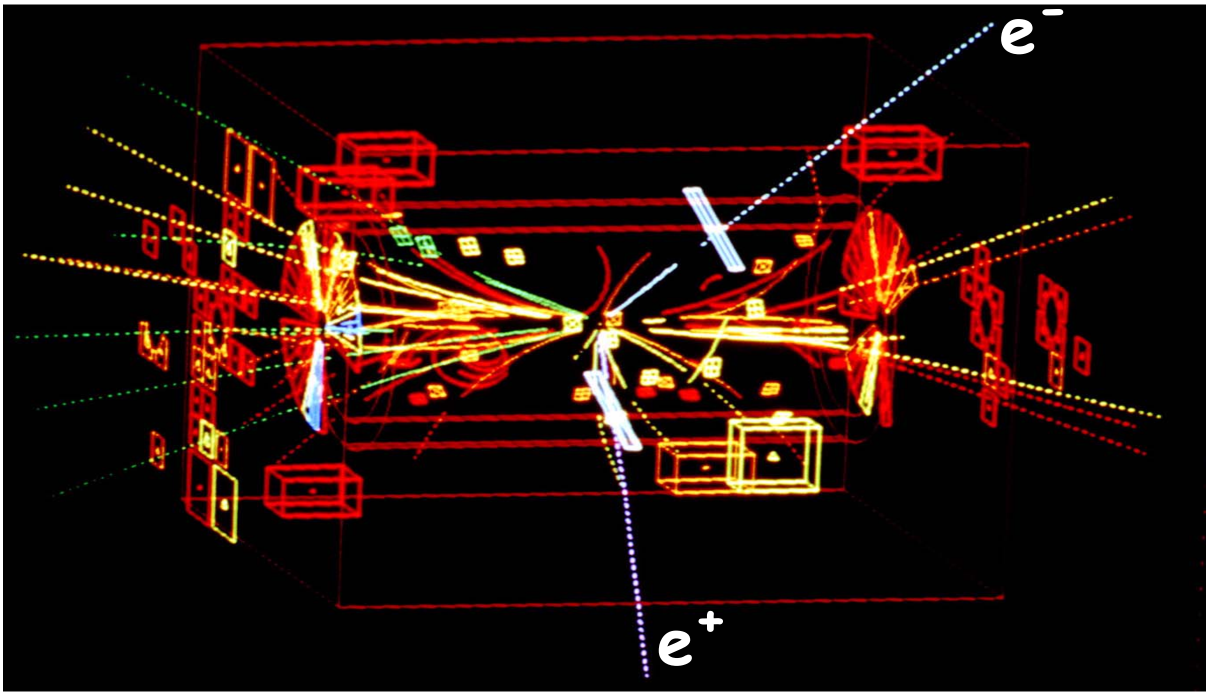
From the first 148 electron and 47 muon events it was estimated that:

$$M_W = 83{,}5 \pm 2{,}8\ GeV \qquad \Gamma_W \le 6{,}5 \text{ GeV}$$

The discovery of the W and Z bosons

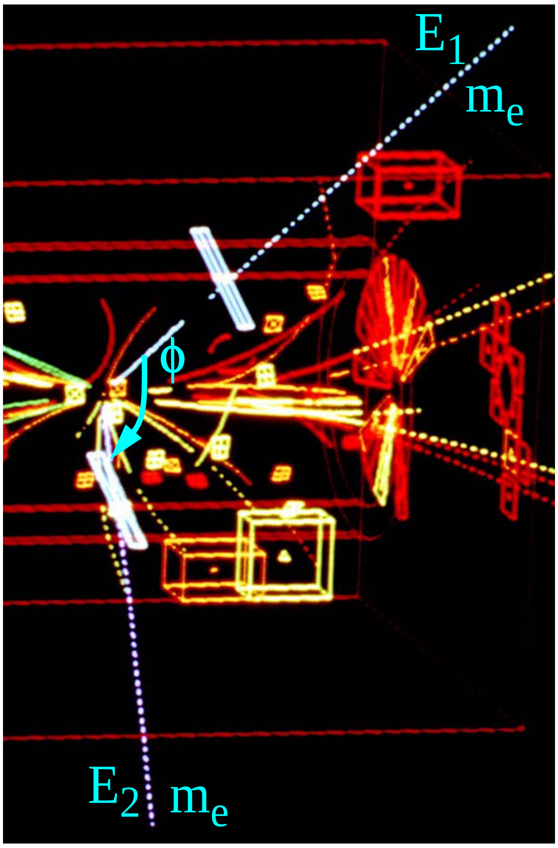
Analysis of Z-events in UA1

- The analysis was looking for a pair of charged leptons.

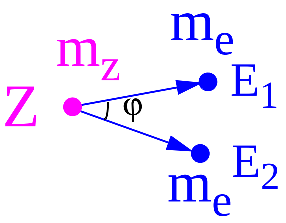


The discovery of the W and Z bosons

➡ Invariant mass



- The **invariant mass** of a particle that decays to two other particles can be calculated from the new particles energies and the angle between the two particles:



$$m_Z^2 = (\vec{P}_1 + \vec{P}_2)^2$$ (4-vectors)

if m=0

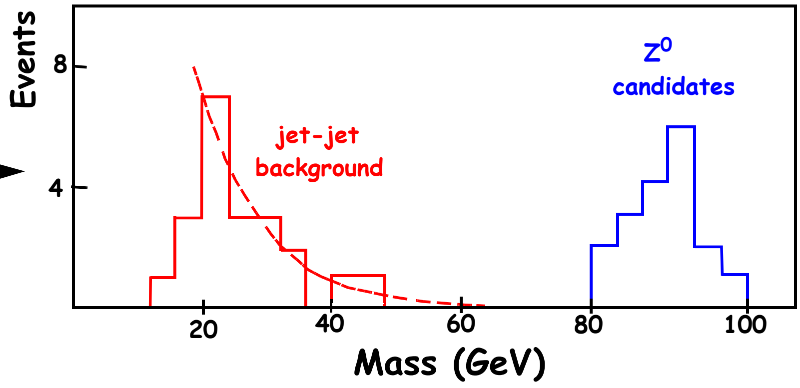
$$m_Z^2 = 2\,E_1E_2\,(1 - \cos\varphi)$$

The discovery of the W and Z bosons

➡ Analysis of Z-events in UA1

- The main search criteria in the UA1 Z-analysis was to require a pair of charged electrons or muons with a large transverse energy.

The mass distribution of pairs of electrons where each electron has E_T>8 GeV.



The first 18 electron pairs and 10 muon pairs in UA1

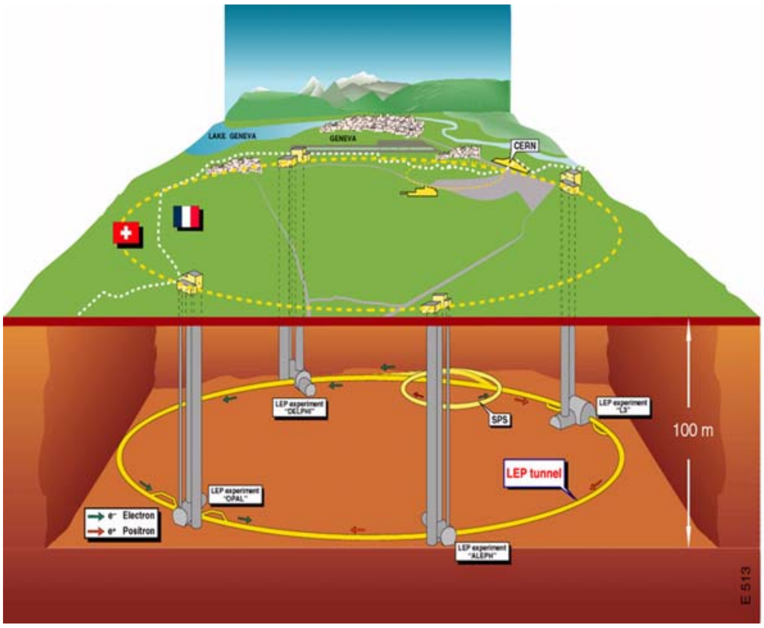
$$M_Z = 93{,}0 \pm 1{,}4\ GeV$$

$$\Gamma_Z \le 8{,}1\ \mathrm{GeV}$$

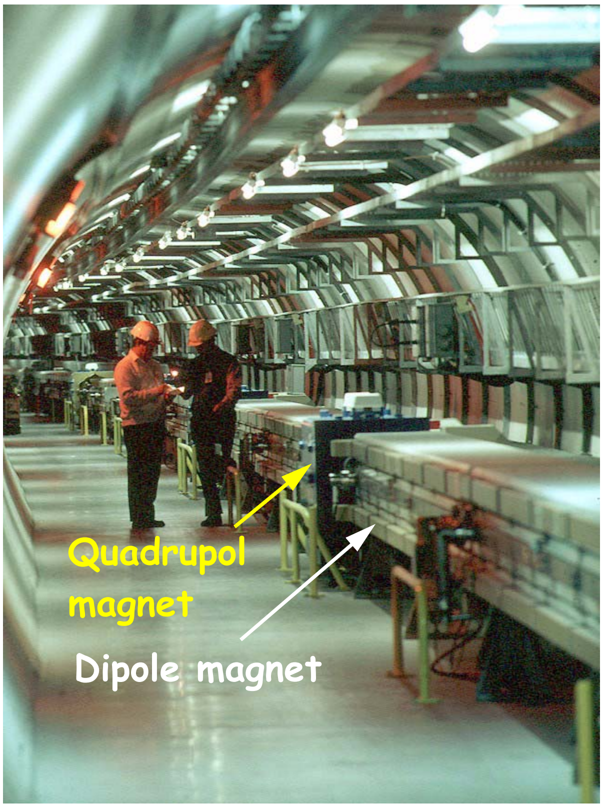
Precision studies of the W and Z bosons

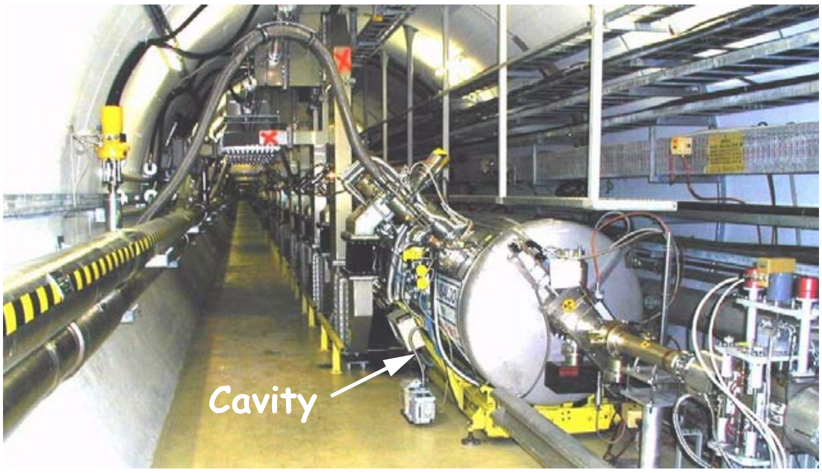
➡ The accelerator: The Large Electron Positron Collider

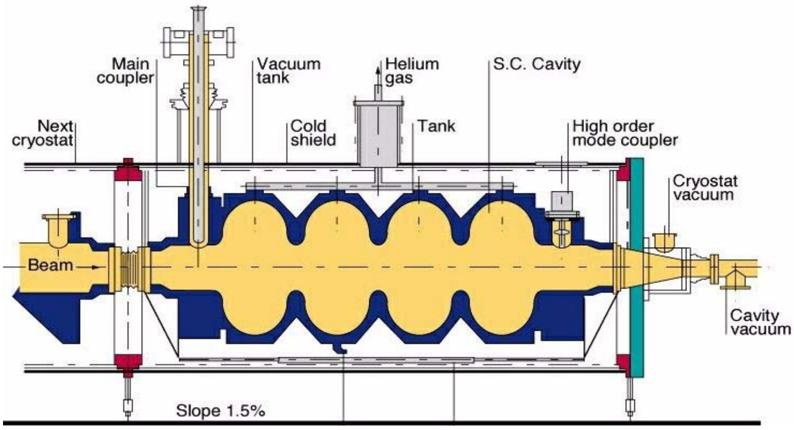
The LEP accelerator was the largest accelerator ever built. It collided electrons with positrons at four places along a 27 km long tunnel.



Precision studies of the W and Z bosons

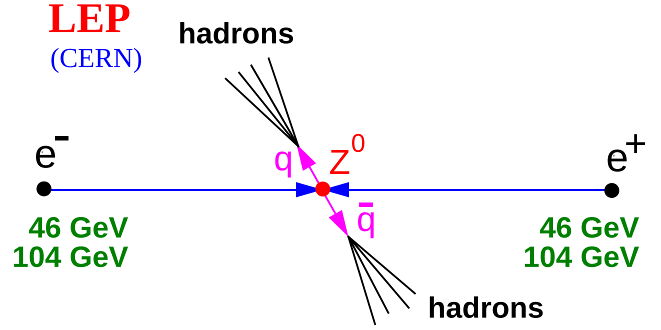






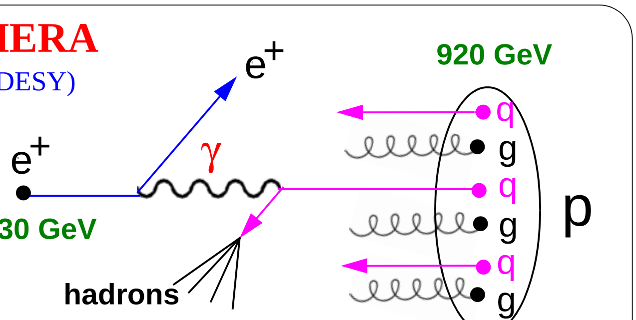
Precision studies of the W and Z bosons

LEP
(CERN)



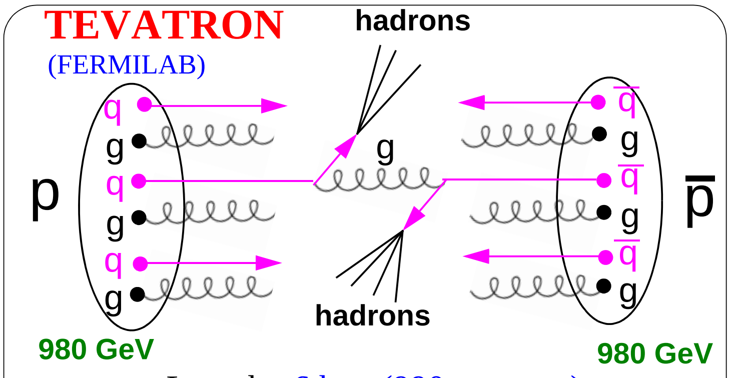
Length: 27 km (4184 magnets)
Experiments: DELPHI, OPAL, ALEPH, L3

HERA
(DESY)



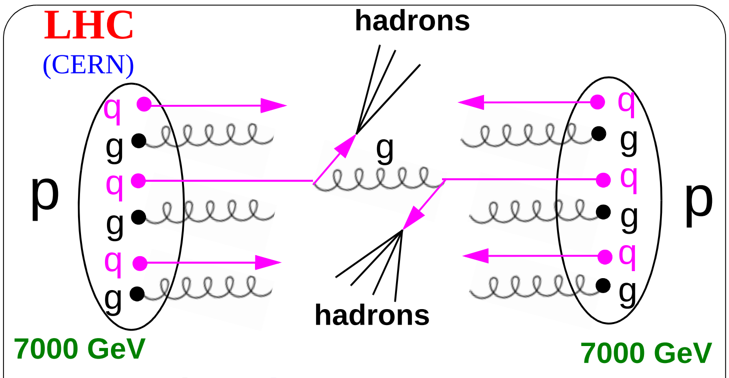
Length: 6 km (1650 magnets)
Experiments: H1, ZEUS

TEVATRON
(FERMILAB)



Length: 6 km (990 magnets)
Experiments: CDF, D0

LHC
(CERN)



Length: 27 km (1232 + 386 magnets)
Experiments: ATLAS, CMS, LHCb, ALICE

Precision studies of the W and Z bosons

➡ Basic differences between proton and electron colliders.

- The limiting factor for an **electron collider** is the **synchrotron radiation**. Total energy loss per turn is proportional to **1/mass^4** (and E^4_{beam} and 1/radius) i.e the energy loss in an electron machine is 10^{13} times higher than in a proton accelerator.
- **288** superconducting **and 56** warm **cavities** were used at **LEP** but only **16 cavities** are needed at **LHC**. Accelerating voltage = 3630 MV (LEP) and = 16 MV (LHC)
- The limiting factor for a **proton collider** is the **magnetic field** in the dipole bending magnets. The maximum field needed is proportional to

$$\frac{\text{Beam momentum}}{\text{Length of bending field}}$$

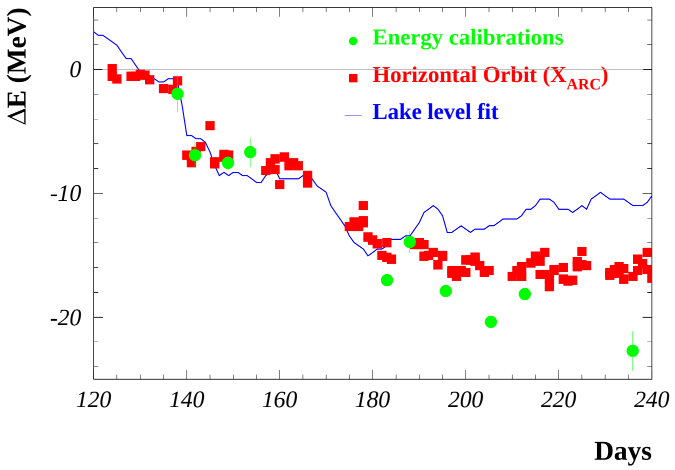
- The bending field at **LHC (8.38 T)** therefore has to be a factor 70 larger than the bending field at **LEP (0.12 T)**.

Precision studies of the W and Z bosons

The accelerator: The Large Electron Positron Collider

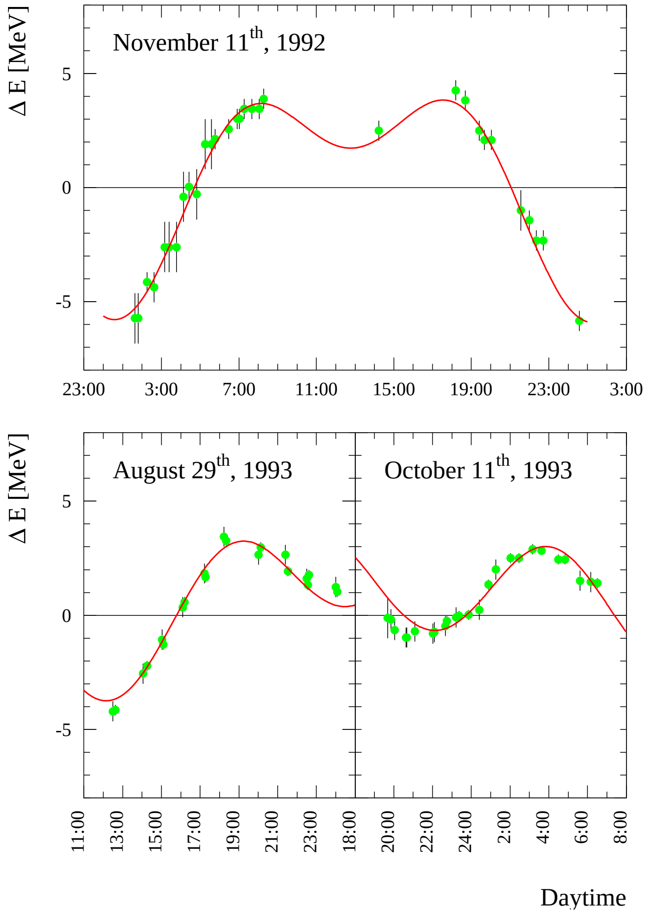
- **In an electron-positron collider it is important to know the collision energy with a high accuracy. It took many years to understand the factors that influenced the collision energy in LEP.**

Geological shifts



- **During 1993 the LEP energy was observed to change with time.**
- **Part of the change was due to the water level in lake Geneva which caused small geological shifts of the accelerator.**
- **Rainfalls and the water table in the Jura mountains also affected the LEP energy.**

Precision studies of the W and Z bosons



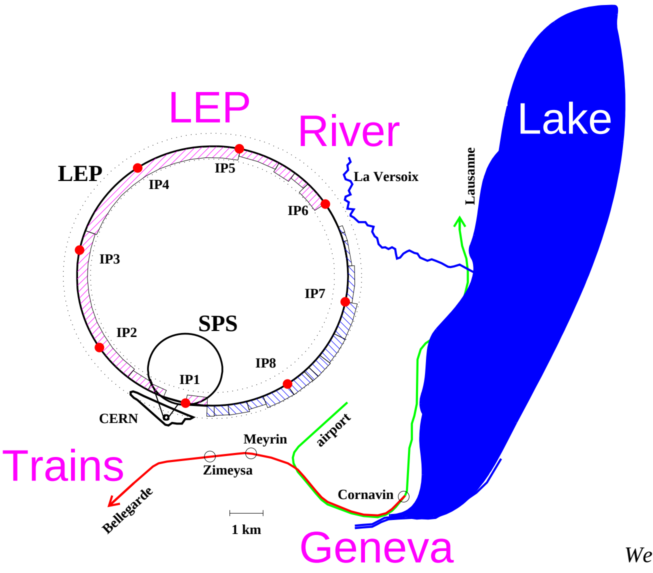
Tides

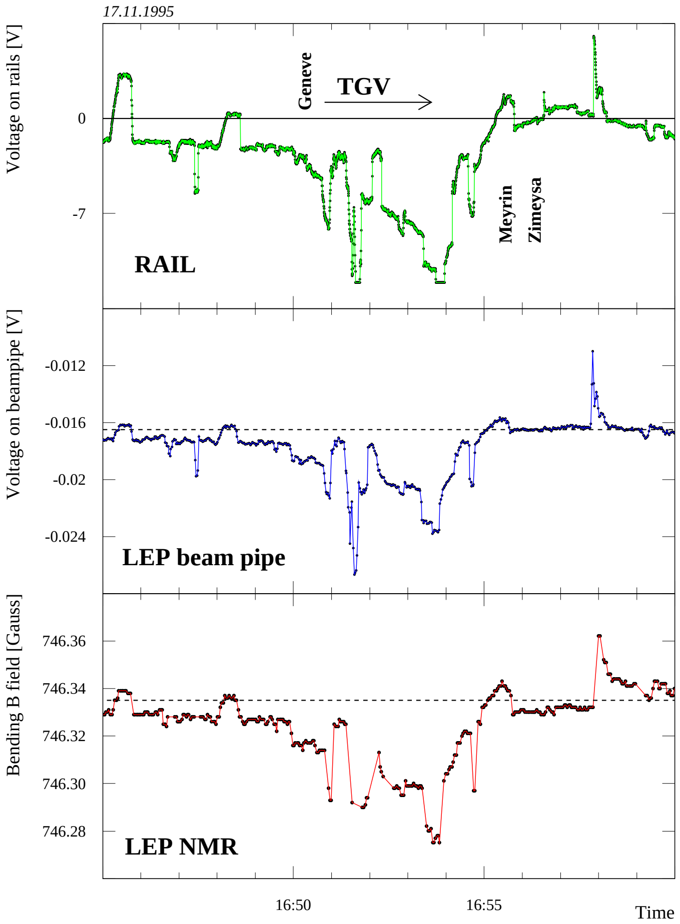
- **Earth tides caused by the moon will produce small distortions of the earth's crust.**
- **This can affect the accelerator so that the electrons orbit change.**
- **An orbit change of 1 mm will change the energy with about 10 MeV.**

Precision studies of the W and Z bosons

Beampipe current

- The **trains** from Geneva to France caused **parasitic currents** on the LEP beampipe.
- These currents (1 A) affected the **magnetic field** in the LEP magnets and this changed the energy.





Precision studies of the W and Z bosons

The DELPHI Experiment

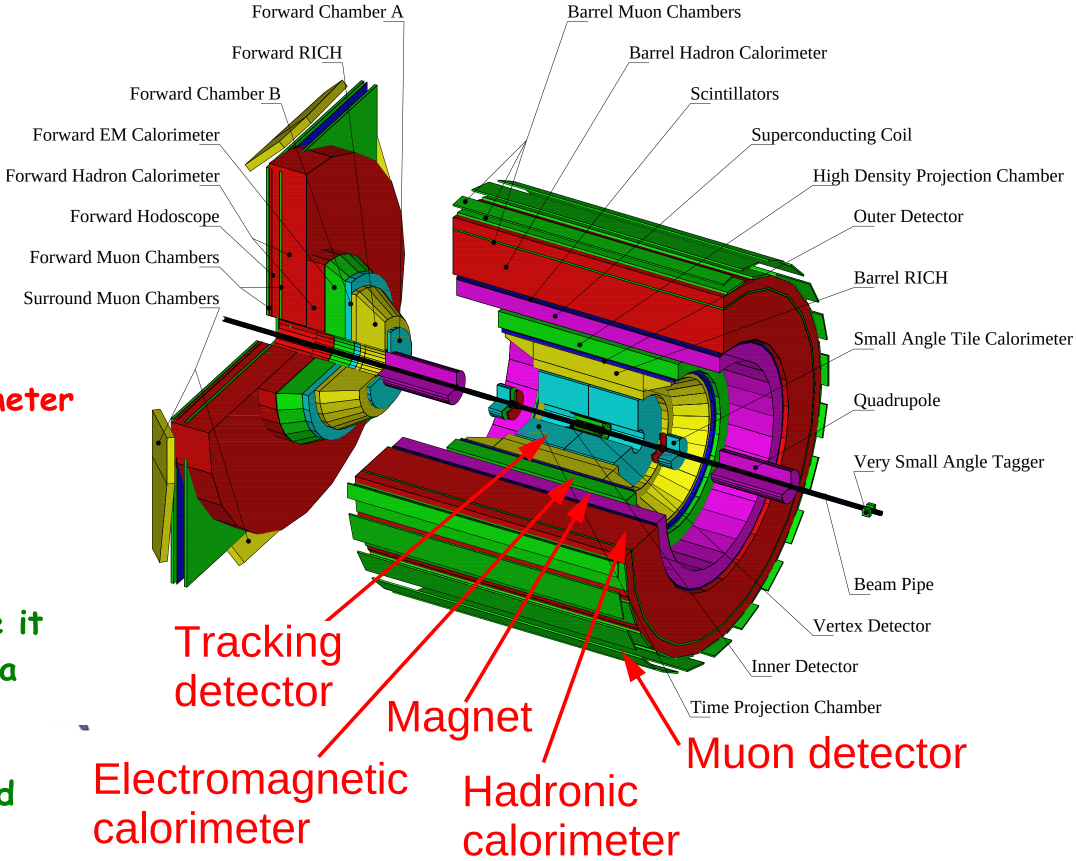
A Time Projection Chamber was used as the main **tracking detector**.

It was surrounded by an electromagnetic calorimeter sitting inside a solenoid **magnet**.

The **electromagnetic calorimeter** had a lead absorbers and a gas detector readout.

The iron return yoke of the magnet had detectors inside it so that it could be used as a **hadronic calorimeter**.

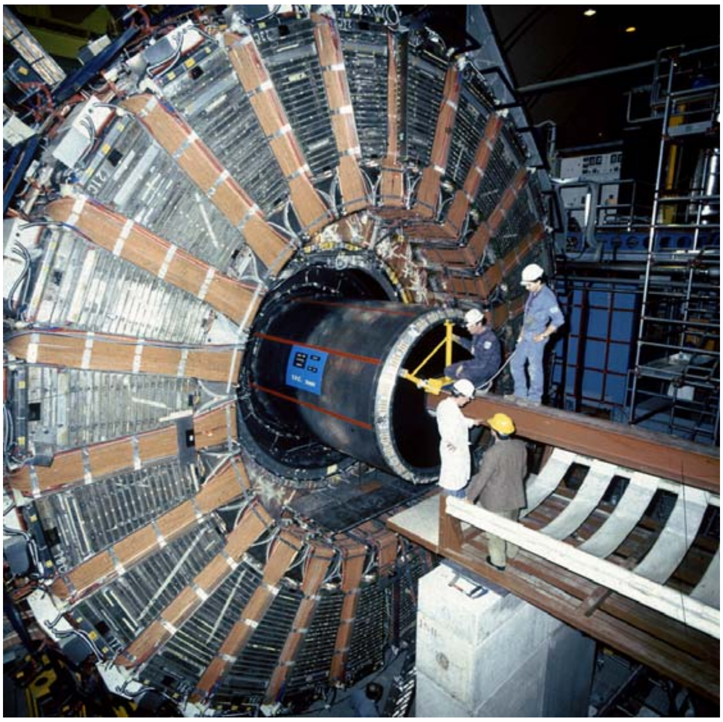
The **muon detector** consisted of two driftchamber layers.



Precision studies of the W and Z bosons

The DELPHI Experiment

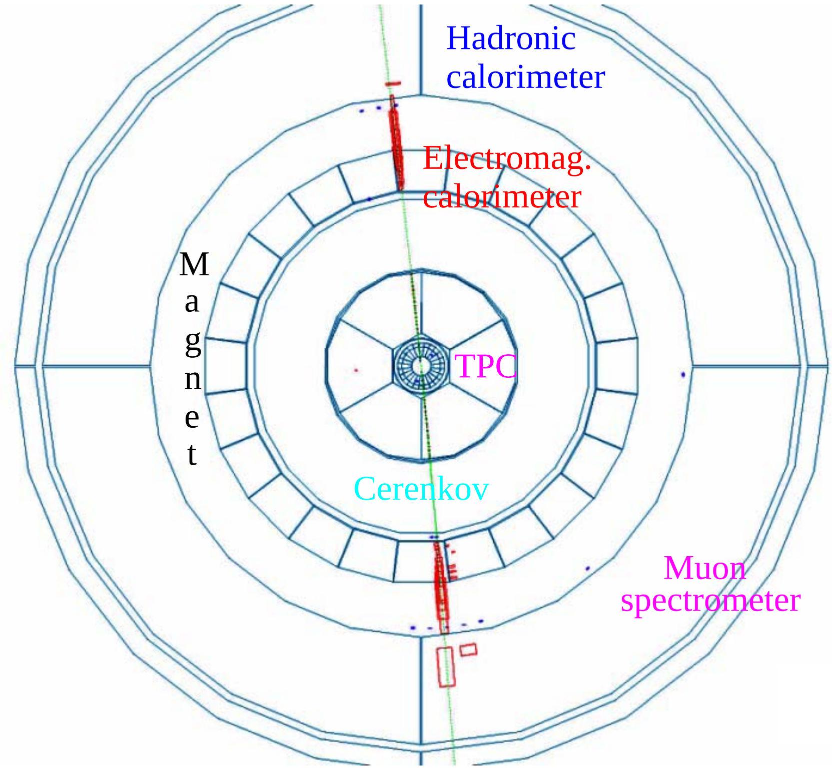
The photo shows the DELPHI cavern with the buildings for the electronics hiding the experiment.



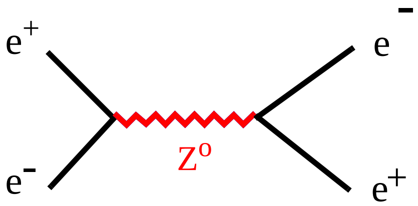
Installation of the large Time Projection Chamber in DELPHI.

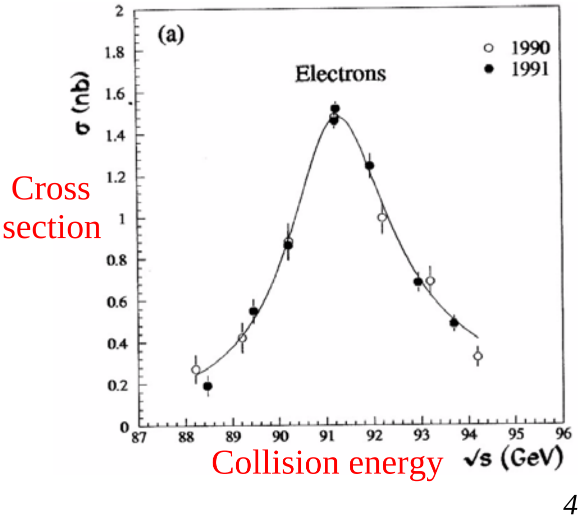
Precision studies of the W and Z bosons

Studies of the Z-boson



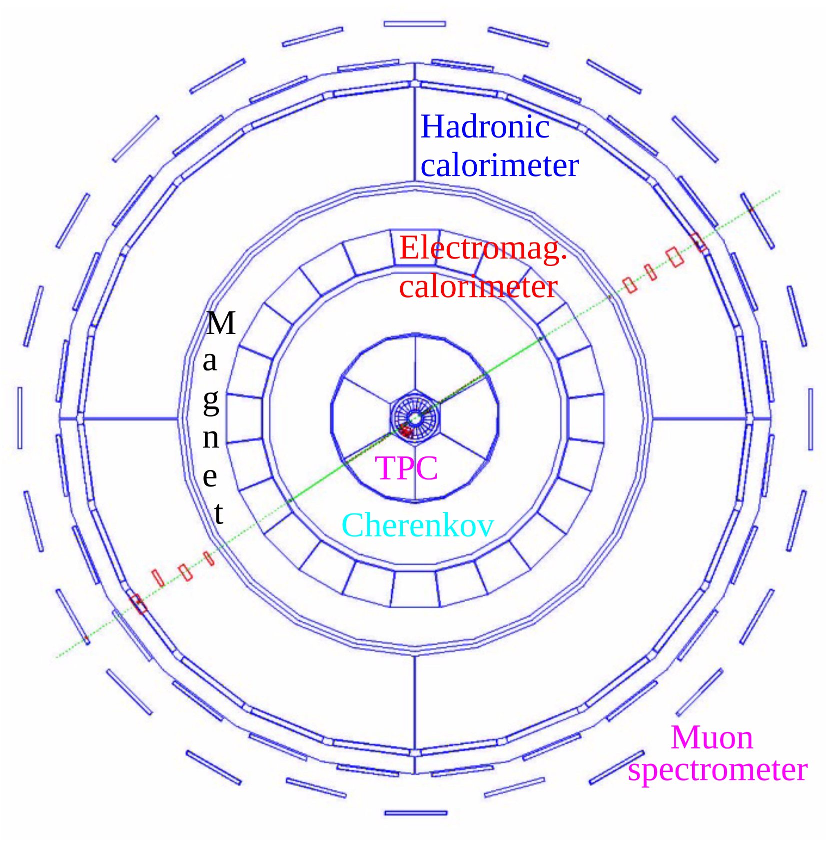
From the number of produced electron pair events one can calculate the cross section for the following process:



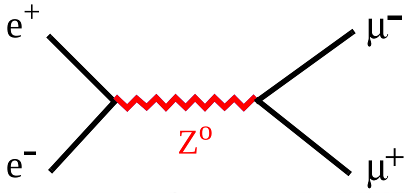


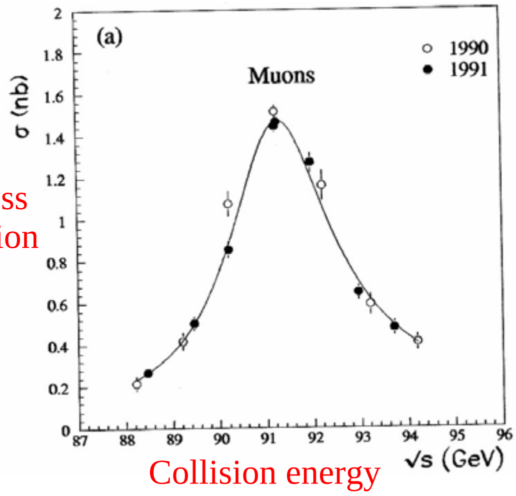
Precision studies of the W and Z bosons

➡ Studies of the Z-boson



From the number of produced muon pair events one can calculate the cross section for the following process:



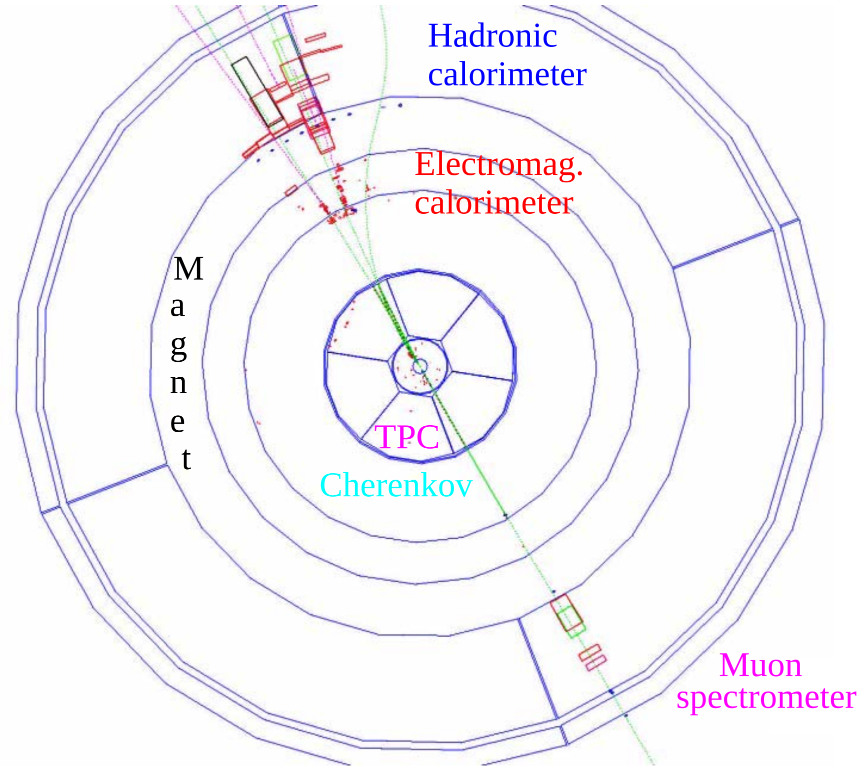


Cross section

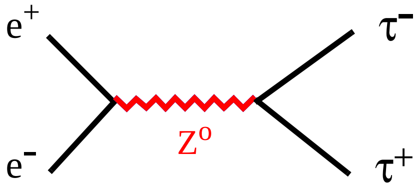
Collision energy

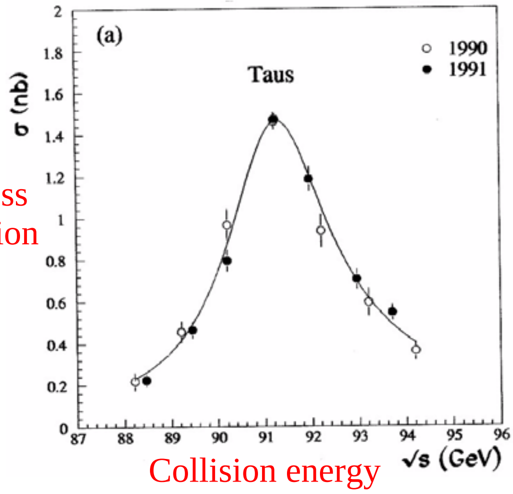
Precision studies of the W and Z bosons

➡ Studies of the Z-boson



From the number of produced tau pair events one can calculate the cross section for the following process:

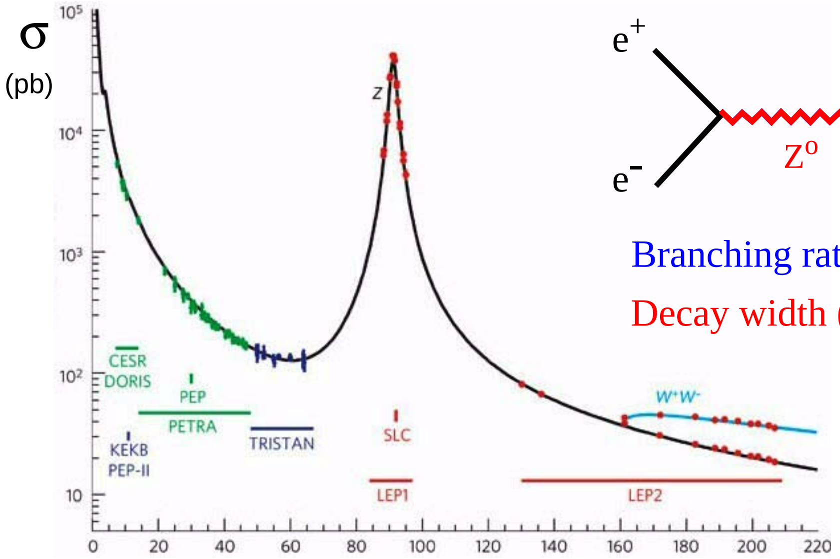




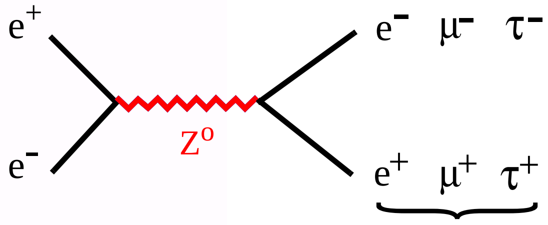
Cross section

Collision energy

The number of neutrino families



Center of mass energy (GeV)



Branching ratio (B):	0.10	0.20	0.70
Decay width (Γ):	0.25 GeV	0.50 GeV	1.74 GeV

where the lifetime is given by

$$\boxed{\tau = \frac{B}{\Gamma}} \qquad B_{xx} = \frac{\Gamma_{xx}}{\Gamma_Z}$$

$$\tau = \frac{B_{had}}{\Gamma_{had}} = \frac{B_{ll}}{\Gamma_{ll}} = \frac{B_{\nu\nu}}{\Gamma_{\nu\nu}} = 3\times10^{-25}s$$

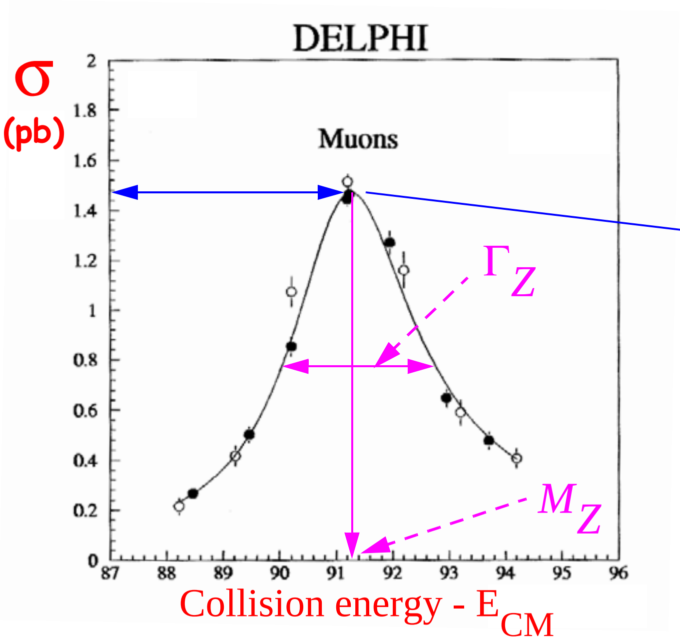
Note: 1 $\text{GeV}^{-1} = 6.58 \times 10^{-25}$ s

The total decay width Γ_Z is the sum of the partial decay widths.

$$\tau = \frac{B_Z}{\Gamma_Z} = \frac{B_{had}+B_{ll}+B_{\nu\nu}}{\Gamma_{had}+\Gamma_{ll}+\Gamma_{\nu\nu}}$$

The number of neutrino families

- It was possible to determine the partial decay widths of the Z^0 by fitting Breit-Wigners to the measured distributions at LEP.



$$\sigma(e^+e^-\to\mu\mu) = \frac{12\pi M_Z^2}{E_{CM}^2}\left[\frac{\Gamma(Z\to ee)\Gamma(Z\to\mu\mu)}{\left(E_{CM}^2-M_Z^2\right)^2+M_Z^2\Gamma_Z^2}\right]$$

σ at $E_{cm}= M_Z$ give $\Gamma(Z^0\to\mu\mu)$ because then

The decay rate to ee

The decay rate to μμ

$$\sigma(e^+e^-\to\mu\mu) = \frac{12\pi}{M_Z^2}\left[\frac{\Gamma(Z\to ee)\Gamma(Z\to\mu\mu)}{\Gamma_Z^2}\right]$$

The mass of the Z^0

The total Z^0 decay rate

In reality the measurements are a bit more complicated because the measured peaks have to be corrected for initial state radiation (photons radiated off incoming electrons).

The number of neutrino families

- The fitted parameters of the Z^0 peak gave the following result:

$$M_Z = 91.187 \pm 0.007 \text{ GeV/c}^2 \qquad \Gamma(Z^0 \to \text{hadrons}) = 1.741 \pm 0.006 \text{ GeV}$$

$$\Gamma_Z = 2.490 \pm 0.007 \text{ GeV} \qquad \Gamma(Z^0 \to l^+ l^-) = 0.0838 \pm 0.0003 \text{ GeV}$$

- However, the leptonic and hadronic decays account for only 80% of all the Z^0 decays.

- The remaining **decays** are those **to neutrinos** that cannot be measured directly by the experiment since

$$\Gamma_Z = \Gamma(Z^0 \to \text{hadrons}) + 3\Gamma(Z^0 \to l^+ l^-) + N_\nu \Gamma\left(Z^0 \to \nu_l \bar{\nu}_l\right)$$

- From the measured values of the decay widths one gets

$$N_\nu \Gamma\left(Z^0 \to \nu_l \bar{\nu}_l\right) = 0.498 \pm 0.009 \text{ GeV}$$

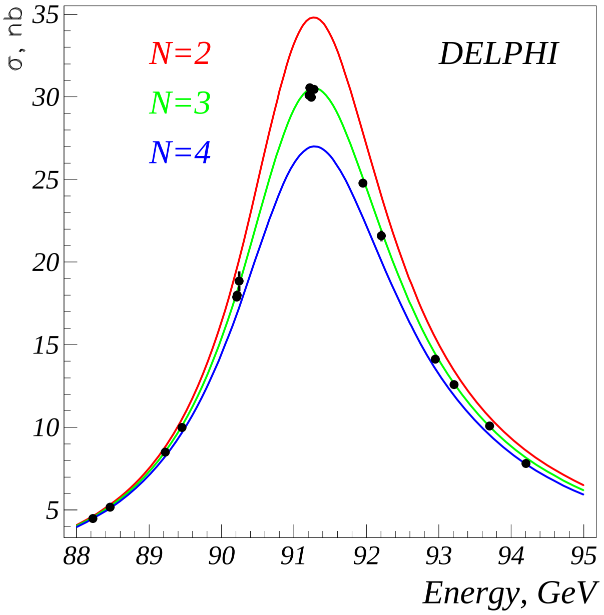
The number of neutrino families

- The decay rate of the Z^0 to neutrinos can be calculated:

$\Gamma\left(Z^0 \to \nu_l \bar{\nu}_l\right) = 0.166 \text{ GeV}$ which together with $N_\nu \Gamma\left(Z^0 \to \nu_l \bar{\nu}_l\right) = 0.498 \text{ GeV}$ gives $N_\nu = 2.994 \pm 0.011$

- There are no explicit restrictions on the **number of generations** in the standard model.

- However, the study of the Z^0 peak at LEP shows that there are only **three types of light neutrinos**, i.e., with a mass less than M_Z.

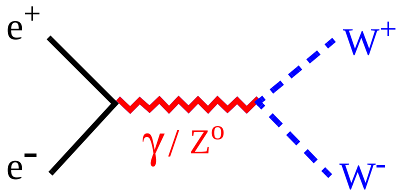


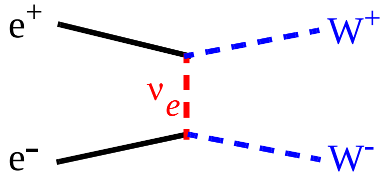
The decay of the Z^0 to hadrons and the predictions for different number of neutrino families (N).

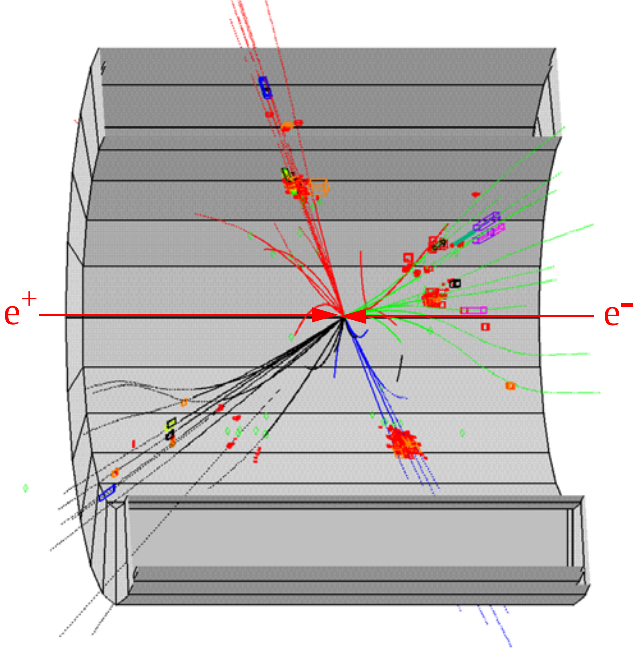
Precision studies of the W and Z bosons

➡ Studies of the W-boson

- The W bosons were produced in pairs at LEP.

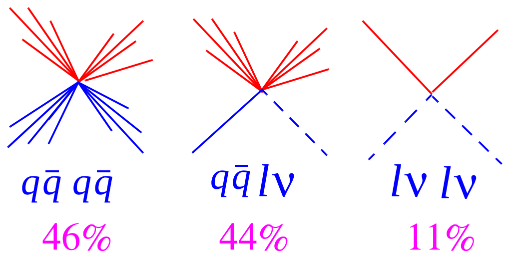






A WW-event with 4 jets in the DELPHI experiment.

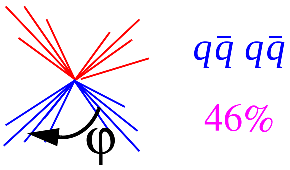
- The signature for WW-events was:



Precision studies of the W and Z bosons

➡ Studies of the W-boson

- When the events with WW-pairs had been selected it was possible to calculate the W-mass from the energy and directions of the jets:

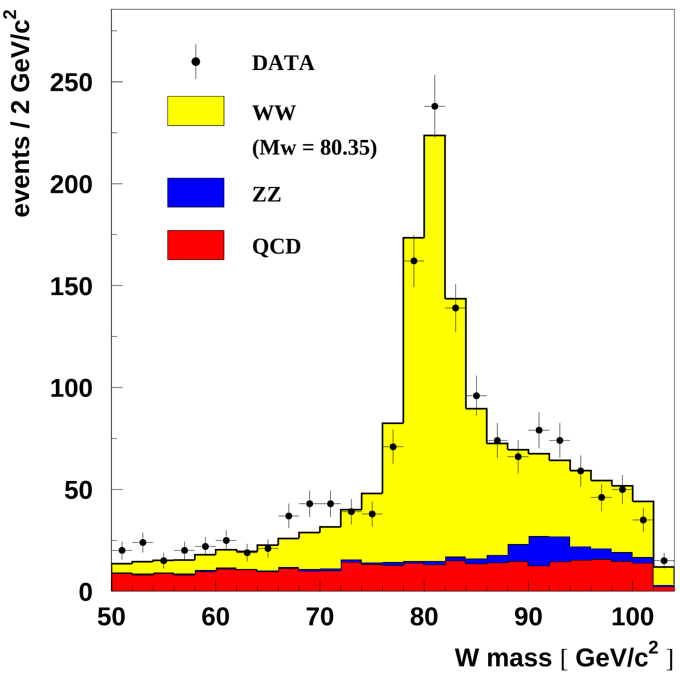


$$M_W^2 = (\vec{P}_q + \vec{P}_{\bar{q}})^2 \quad \text{(4-vectors)}$$

$$M_W^2 = 2\,E_q E_{\bar{q}}\,(1 - \cos\varphi)$$

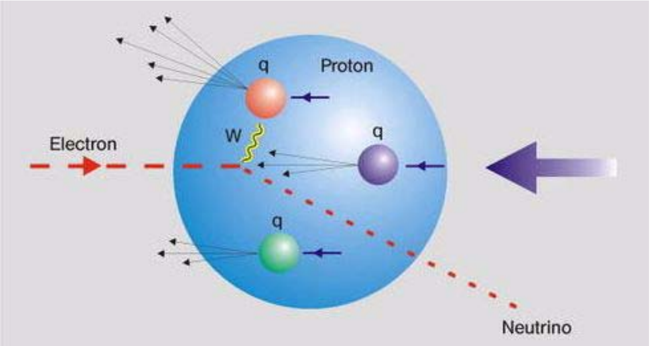
if mq= 0

$$\boxed{\begin{array}{l} M_W = 80.376 \pm 0.033 \text{ GeV} \ \text{(LEP)} \\ M_W = 83.5 \pm 2.8 \text{ GeV} \ \text{(UA1)} \end{array}}$$

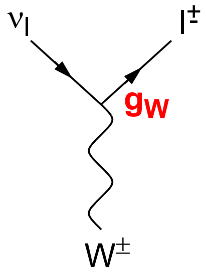
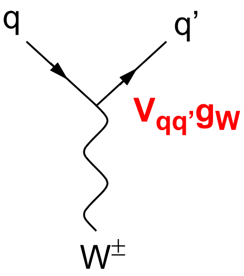
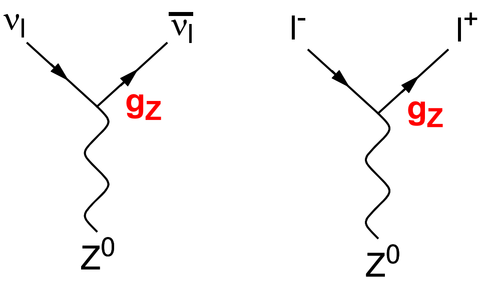
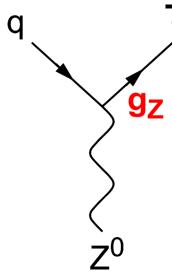


The mass distribution from jet-pairs in WW events selected by the DELPHI experiment.

Charged current reactions



W and Z reactions

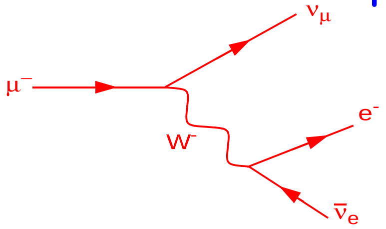
	Leptonic reactions	Hadronic reactions
Charge current reactions		
Neutral current reactions		

Charged current reactions

➡ Charged current reactions are reactions mediated by the charged W-bosons. They can be divided up into:

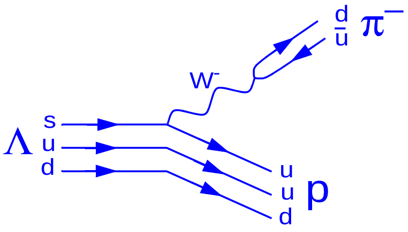
- **Purely leptonic processes:**

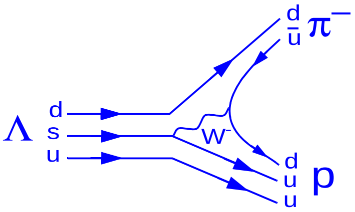
  $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$



- **Purely hadronic processes:**

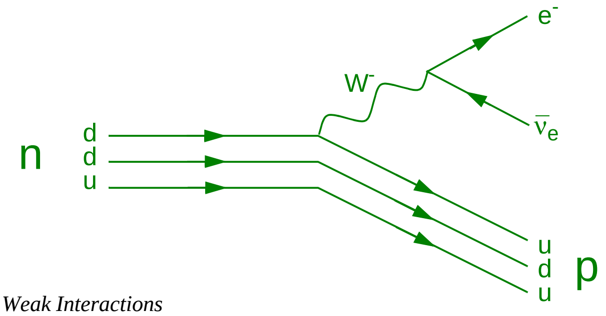
  $\Lambda \to \pi^- + p$





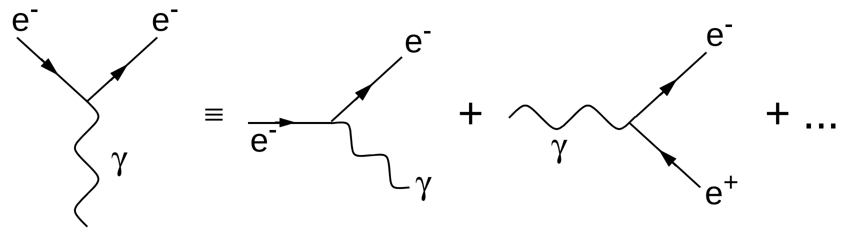
- **Semileptonic reactions:**

  $n \to p + e^- + \bar{\nu}_e$



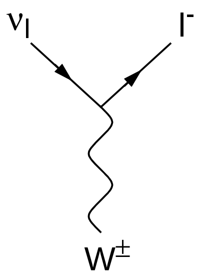
Leptonic charged current reactions

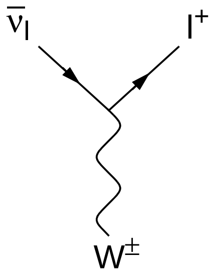
- **All the electromagnetic interactions can be built from eight basic interactions:**



The basic vertex for electron-photon interactions.

- **Leptonic weak interaction processes can in a similar way be built from a certain number of reactions corresponding to basic vertices:**

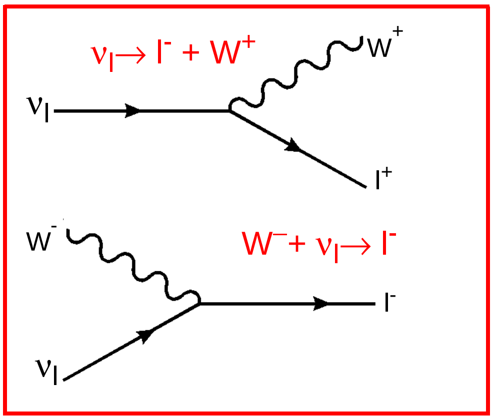




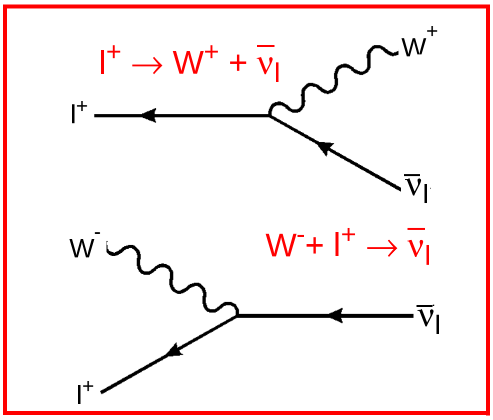
The two basic vertices for W-lepton interactions..

Leptonic charged current reactions

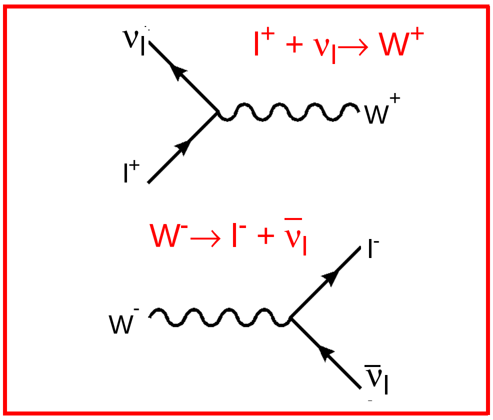
- It is possible to derive **eight basic charged current reactions** from the two basic W vertices:


$\nu_l \to l^- + W^+$

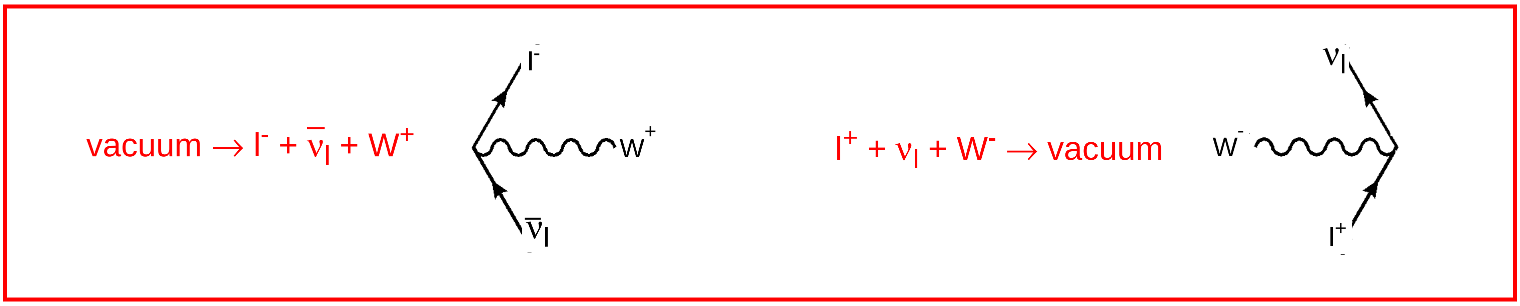
$W^- + \nu_l \to l^-$


$l^+ \to W^+ + \bar{\nu}_l$

$W^- + l^+ \to \bar{\nu}_l$


$l^+ + \nu_l \to W^+$

$W^- \to l^- + \bar{\nu}_l$


$\text{vacuum} \to l^- + \bar{\nu}_l + W^+$

$l^+ + \nu_l + W^- \to \text{vacuum}$

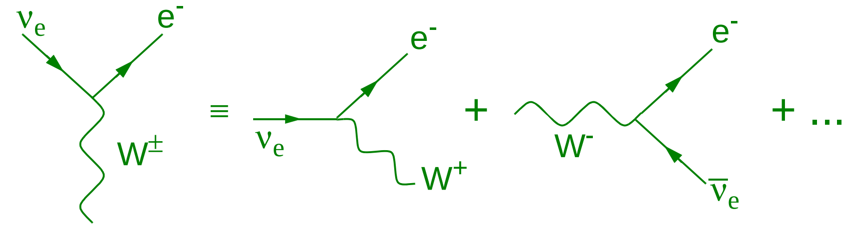
Leptonic charged current reactions

Weak interactions always conserve the lepton numbers: L_e , L_μ , L_τ .

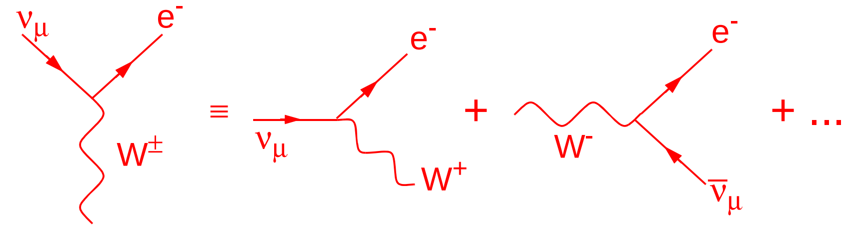
This conservation is guaranteed in Feynman diagrams by:

- at each vertex, there is one arrow pointing in and one out;
- the lepton indices "l" are the same on both lines.

Allowed:



Forbidden:



Leptonic charged current reactions

➡ Leptonic vertices are characterized by the corresponding weak strength parameter, α_W , which do not depend on lepton type.

- The decay rate of **W to e+ν**, can be **estimated** to first order as

$$\Gamma(W \to e\nu) \approx \alpha_W M_W \approx 80\alpha_W \text{ GeV}$$

since the process only involves one vertex and the lepton masses are negligible.

- A **measurement** of the decay rate of **W to e+ν** gives

$$\Gamma(W \to e\nu) \approx 0.2 \text{ GeV}$$

which translates into an approximative value of α_W=0.003 for the weak strength parameter.

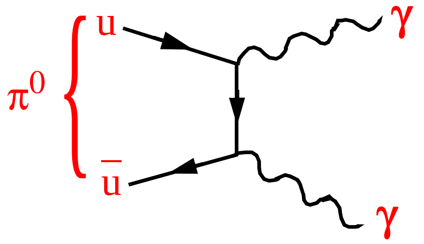
- This is comparable with the value for the **electromagnetic** strength parameter: α_{em}=0.007 .

Leptonic charged current reactions

➡ Why is the weak interaction so weak if α_W and α_{em} is of a similar size ?

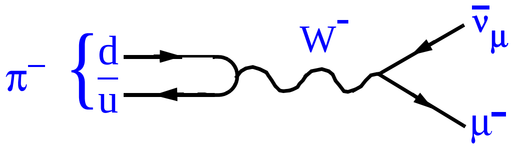
- Compare the decay of charged and neutral pions:

Electromagnetic decay



Lifetime = 8 x 10^{-17} s

Weak decay

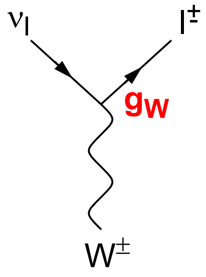
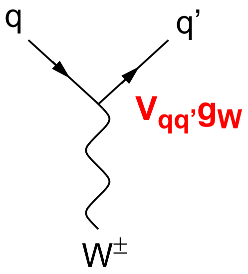
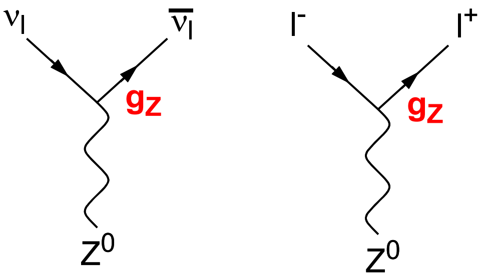
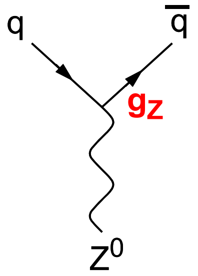


Lifetime = 3000000000 x 10^{-17} s

(Lifetime of a real W = 0.00000003 x 10^{-17}s)

- **CONCLUSION:** The apparent weakness of the weak interactions is due to the very **large W and Z masses**.

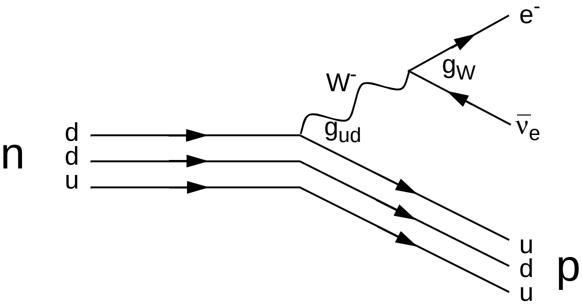
W and Z reactions

	Leptonic reactions	Hadronic reactions
Charge current reactions	 	
Neutral current reactions	 	

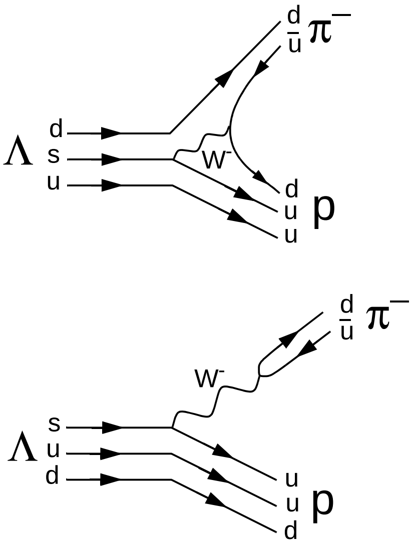
Hadronic charged current reactions

- In weak hadronic interactions, constituent quarks emit or absorb W or Z bosons.

Examples:



Neutron β-decay.



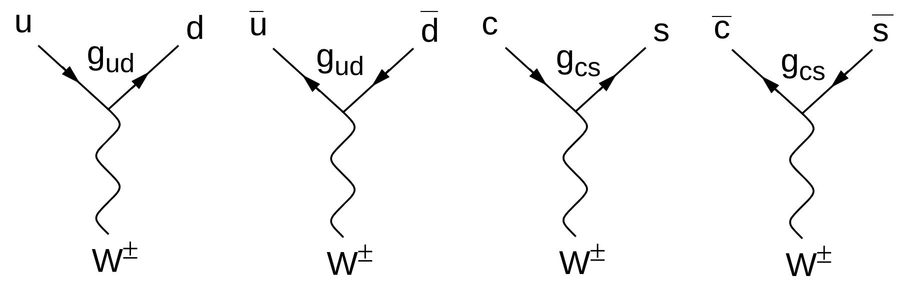
The dominant quark diagrams for Λ decay

Hadronic charged current reactions

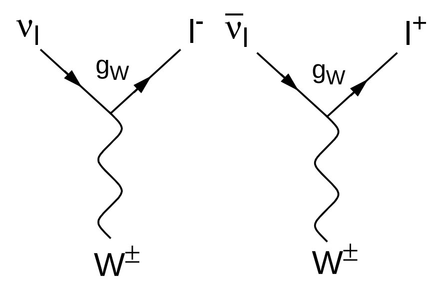
ASSUMPTION: Lepton-quark symmetry i.e. corresponding generations of quarks and leptons have identical weak interactions.

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}\leftrightarrow\begin{pmatrix}u\\ d\end{pmatrix}\quad\begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}\leftrightarrow\begin{pmatrix}c\\ s\end{pmatrix}\quad\begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}\leftrightarrow\begin{pmatrix}t\\ b\end{pmatrix}$$

- **Interactions will then only take place within a family !**



The W-quark vertices for the first two generations, assuming lepton-quark symmetry.



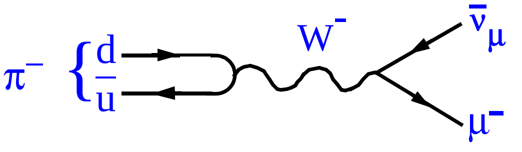
The W-lepton vertices.

Hadronic charged current reactions

Experimental tests of the assumption of lepton-quark symmetry.

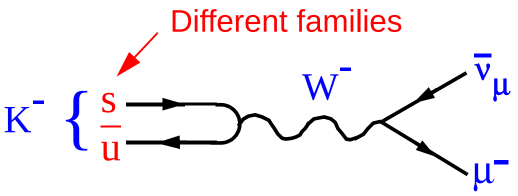
- **Some weak reactions should be allowed and some should be forbidden if lepton-quark symmetry is true:**

$\pi^- \to \mu^- + \bar{\nu}_\mu$ $\quad d\bar{u} \to \mu^- + \bar{\nu}_\mu$



Allowed

$K^- \to \mu^- + \bar{\nu}_\mu$ $\quad s\bar{u} \to \mu^- + \bar{\nu}_\mu$


Different families

Not Allowed

- **Measurements of these decays give:**

$\pi^- \to \mu^- + \bar{\nu}_\mu$ Branching ratio = 0.9999 $\tau = 2.6 \times 10^{-8}$ s

$K^- \to \mu^- + \bar{\nu}_\mu$ Branching ratio = 0.6343 $\tau = 1.2 \times 10^{-8}$ s

- **CONCLUSION: Quarks from different generations can participate in charged current interactions !**

Hadronic charged current reactions

- Cabibbo introduced the concept of quark mixing in order to explain the kaon decays.

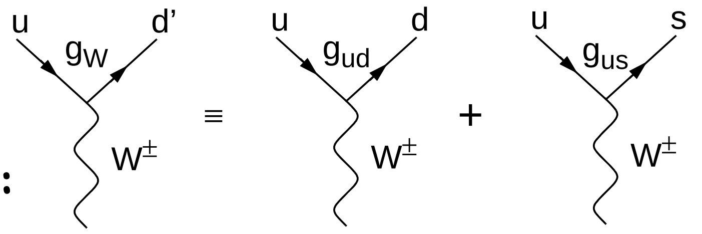
- **According to the quark mixing scheme, d- and s-quarks participate in the weak interactions via the linear combinations:**

$$d' = d\cos\theta_C + s\sin\theta_C$$
$$s' = -d\sin\theta_C + s\cos\theta_C$$

where θ_C is called the Cabibbo angle.

- **With quark mixing, the quark-lepton symmetry applies to doublets like** $\begin{pmatrix} u \\ d' \end{pmatrix}$ and $\begin{pmatrix} c \\ s' \end{pmatrix}$

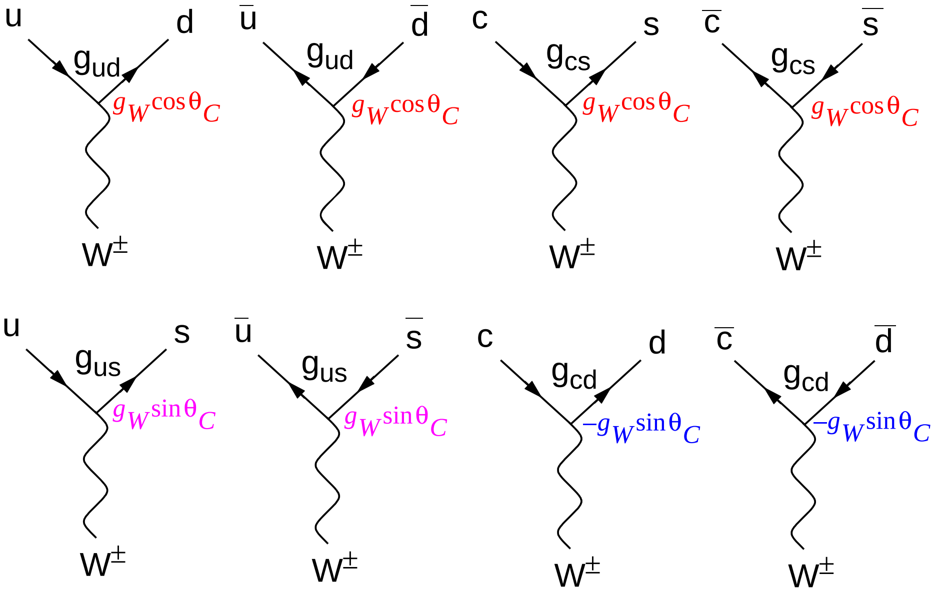
- **The ud'W vertex can now be interpreted as a sum of the udW and usW vertices:**



$g_{ud}=g_W\cos\theta_C$ $g_{us}=g_W\sin\theta_C$

Hadronic charged current reactions

- With the quark mixing hypothesis, some more W-quark vertices are allowed:



where the vertices with quarks within a generation have the coupling constants $g_{ud} = g_{cs} = g_W\cos\theta_C$ and the vertices with quarks from different generations have the coupling constants $g_{us} = -g_{cd} = g_W\sin\theta_C$

Hadronic charged current reactions

➡ Measurements of the Cabibbo angle.

- **The Cabibbo angle is not given by the theory but has to be measured e.g. by comparing the decay rates of $\pi^- \to \mu^- + \bar{\nu}_\mu$ with that of $K^- \to \mu^- + \bar{\nu}_\mu$**

$$\frac{\Gamma(K^- \to \mu^- \bar{\nu}_\mu)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \propto \frac{g_{us}^2}{g_{ud}^2} = \tan^2\theta_C$$

gives the result $\theta_C = 12.7° \pm 0.1°$

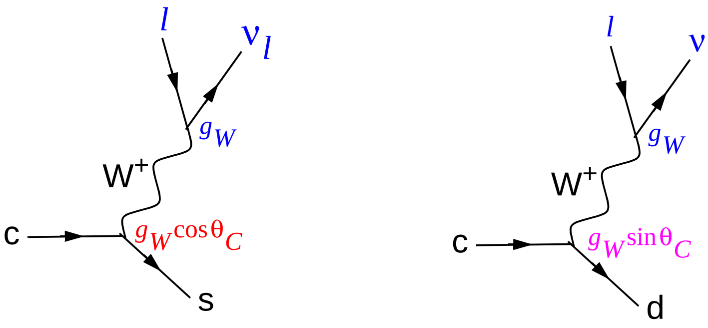
- **When the Cabibbo angle is measured it is also possible to compare the coupling constants within and between generations:**

$$g_W \cos\theta_C = 0,98 g_W$$
$$g_W \sin\theta_C = 0,22 g_W$$

Hadronic charged current reactions

➡ Charmed particle decays.

- **Particles with charm quarks almost always give a strange particle in the final state. The reason is that other decays are Cabibbo supressed.**



- **The suppression factor is given by:**

$$\frac{g_{cd}^2}{g_{cs}^2} = \tan^2\theta_C = \frac{1}{20} \qquad \text{if} \qquad \theta_C = 12,6°$$

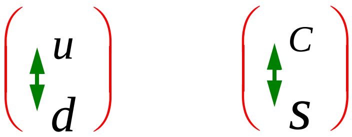
- **The charmed quark couplings g_{cd} and g_{cs} have been measured in neutrino scattering experiments with the result:** $\theta_C = 12° \pm 1°$

Weak interactions and the third generation

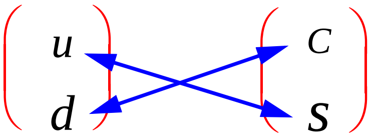
Two generation quark mixing can be written in matrix form:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$ with transitions within $\begin{pmatrix} u \\ d' \end{pmatrix}$ and $\begin{pmatrix} c \\ s' \end{pmatrix}$

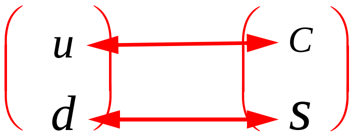
- This means that the following weak transitions are **favoured**:



- And that the following weak transitions are **suppressed**:



- While charge conservation **forbids** the following charged current transitions:



Charge = 2/3

Charge = -1/3

Weak interactions and the third generation

- The **existence** of the **c-quark** was **predicted** from lepton-quark symmetry before it was discovered in experiments in 1974.
- After the discovery of the τ lepton and the b-quark, the sixth quark was predicted to complete the symmetry and the **top quark** was **discovered in 1994** with a mass of about 180 GeV/c^2.
- The third generation gives rise to the **Cabibbo-Kobayashi-Maskawa (CKM) matrix** $V_{\alpha\beta}$:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- The relation between the matrix elements in the CKM-matrix and the coupling constants are given by $g_{\alpha\beta} = g_W V_{\alpha\beta}$ where α=u,c,t and β=d,s,b .

Weak interactions and the third generation

- Weak transitions can now take place between:

$$u \leftrightarrow d' = V_{ud}d+V_{us}s+V_{ub}b$$
$$c \leftrightarrow s' = V_{cd}d+V_{cs}s+V_{cb}b$$
$$t \leftrightarrow b' = V_{td}d+V_{ts}s+V_{tb}b$$

- If the mixing between the b and t quarks with lighter quarks can be neglected the CKM-matrix is reduced to:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0 \\ -\sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

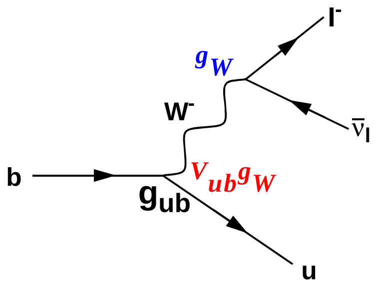
- In reality, this is not correct but V_{ub}, V_{cb}, V_{td} and V_{ts} must be small since otherwise the two-generation mixing model would not agree so well with the data.

Weak interactions and the third generation

➡ b - quarks

- If $V_{ub}=V_{cb}=V_{td}=V_{ts}=0$ then the t-quark decays to b-quarks 100% of the time but the b-quark cannot decay to lighter quarks i.e. it must be stable. However, experimentally we know that this is not the case.

- Semileptonic decays of the heavy b-quark (mass=4.5 GeV) to the lighter u and c quarks are observed with a decay rate that is proportional to the squared couplings:



$$|g_{ub}|^2 = |V_{ub}|^2 g_W^2$$

Weak interactions and the third generation

b - quarks

- The most precise measurements at present give

$$|V_{ub}| \approx 0,004 \quad \text{and} \quad |V_{cb}| \approx 0,04$$

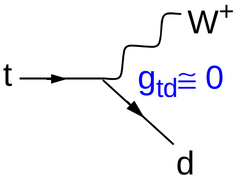
- This means that the CKM-matrix becomes

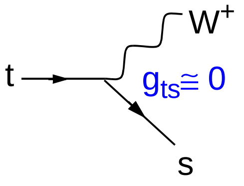
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0{,}004 \\ -\sin\theta_C & \cos\theta_C & 0{,}04 \\ \sim 0 & \sim 0 & \sim 1 \end{pmatrix} \approx \begin{pmatrix} 0{,}98 & 0{,}22 & 0{,}004 \\ -0{,}22 & 0{,}98 & 0{,}04 \\ \sim 0 & \sim 0 & \sim 1 \end{pmatrix}$$

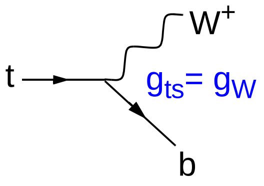
Weak interactions and the third generation

The top quark

- The top quark is much heavier than even the W-bosons and it can decay by







- Since g_{td} and g_{ts} are close to zero, the only significant decay mode of the t-quarks is

$$t \rightarrow W^+ + b$$

with a rate proportional to $\alpha_W = g_W^2/4\pi \approx 0,0042$

- Estimation of the decay width of the top ($\Gamma \sim \alpha_W m_t \sim 1\ \mathrm{GeV}$) suggests a very short lifetime for this quark:

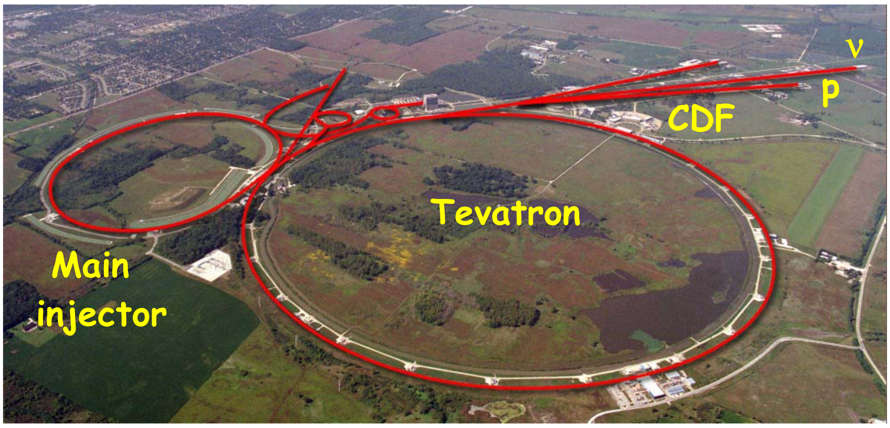
$$\tau_t \approx 4 \times 10^{-25} s$$

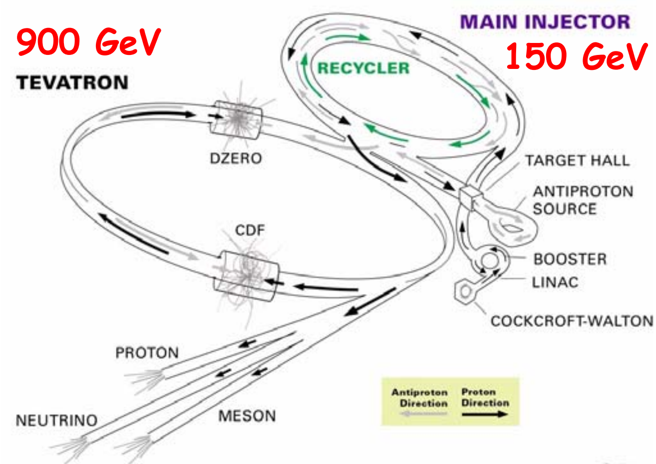
The discovery of the top quark

The accelerator: The Tevatron

	Type	Bending field	Length	Collision energy
Tevatron:	$p\bar{p}$-collider	4.5 T	6.3 km	1960 GeV
SPS:	$p\bar{p}$-collider	1.8 T	6.9 km	900 GeV
LHC:	pp-collider	8.4 T	27 km	14000 GeV

Superconducting magnets

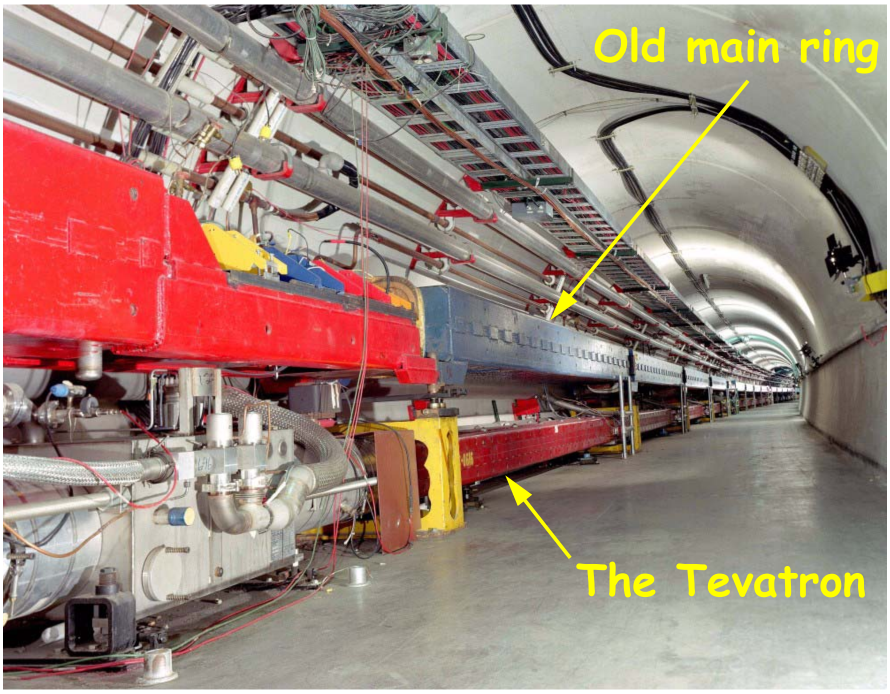


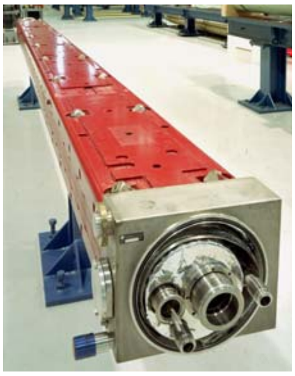


The discovery of the top quark

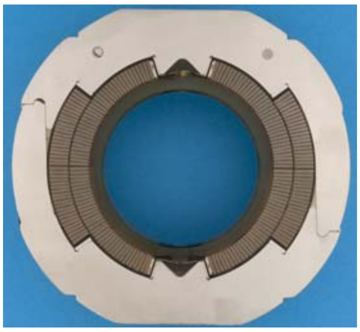
The accelerator: The Tevatron

- The Tevatron accelerator was put under the old main ring which was used as a pre-accelerator (it was later removed).





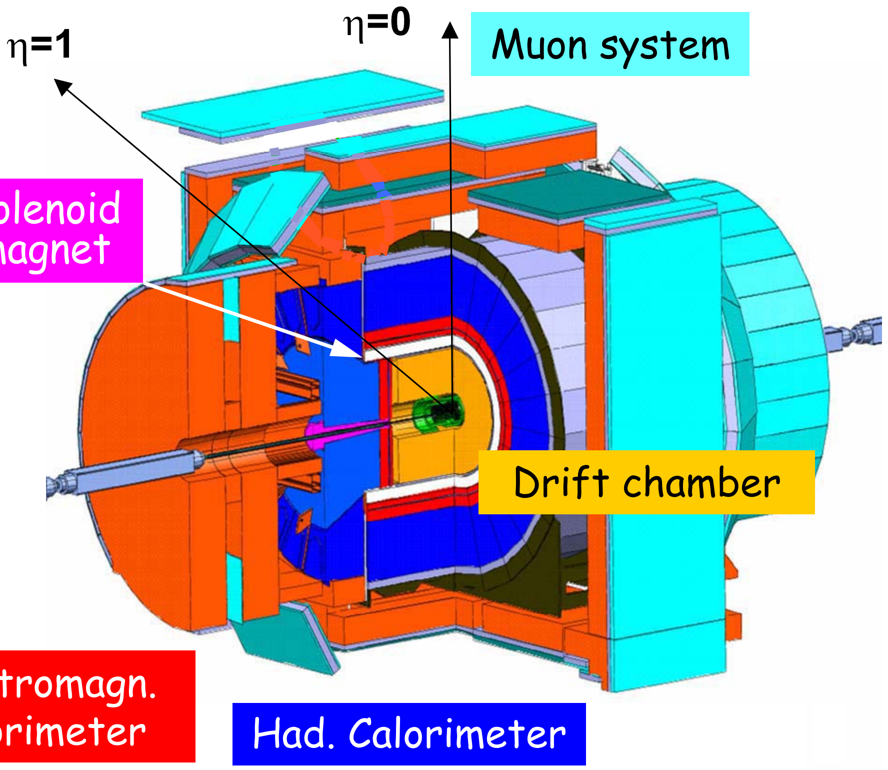
One of the superconducting dipole bending magnets.



A cross section of a dipole magnet showing the coil structure.

The discovery of the top quark

The experiment: The Collider Detector at Fermilab



Muon system

Solenoid magnet

Silicon Vertex Detector

Drift chamber

Electromagn. Calorimeter

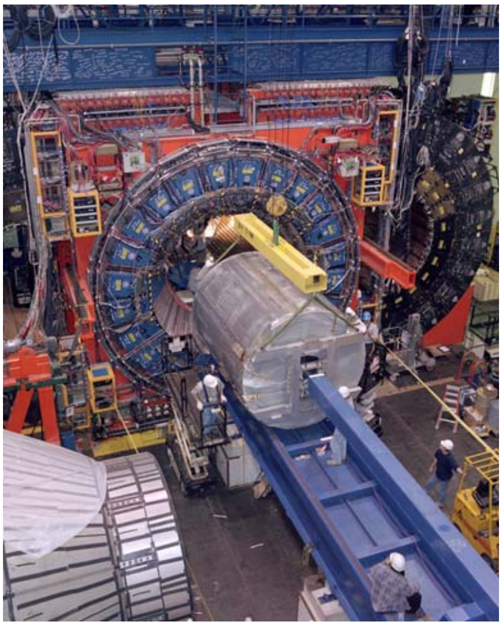
Had. Calorimeter

Lead/Scintillator

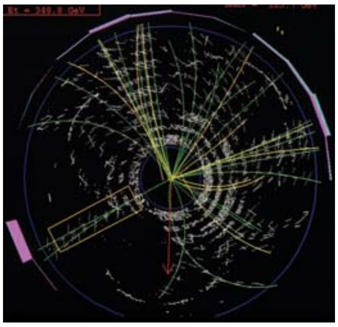
Iron/Scintillator

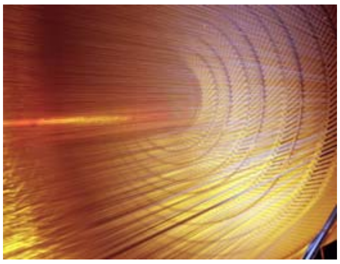
The discovery of the top quark

The experiment: CDF

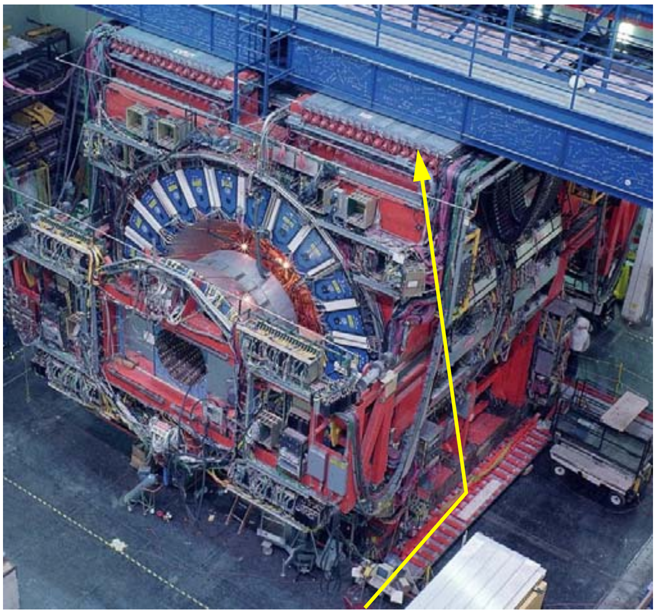


The photo shows how the large central tracker is installed in the experiment.





The tracker was a drift chamber with thousands of wires parallel to the beams.



The muon detectors were also drift chambers.

The discovery of the top quark

➡ Compare particles decay length

Particle	Lifetime	Decay Length	Decay Place	Decay Measurement
W, Z, top	$3\text{-}4\times10^{-13}$ps	0	Beampipe	Not possible
π^0 ($\rightarrow\gamma\gamma$)	0.0008 ps	0.025μm	Beampipe	Not possible
τ	0.3 ps	90μm	Beampipe	Microvertex
Charm: $D^0/D^{\pm}/D_s$	0.4-1 ps	150-350μm	Beampipe	Microvertex
Bottom: $B^0/B^{\pm}/B_s/$	1.5 ps	450μm	Beampipe	Microvertex
K_s ($\rightarrow\pi\pi$)	80 ps	2.7cm	Tracker	Tracker
$K^{\pm}$	10,000 ps	3.7m	Tracker	Not possible
$\pi^{\pm}$	30,000 ps	7.8m	No decay	Not possible
K_L ($\rightarrow\pi\pi\pi$)	50,000 ps	16 m	No decay	Not possible
μ ($\rightarrow e\bar{\nu}_e\nu_\mu$)	2,000,000 ps	659 m	No decay	Not possible

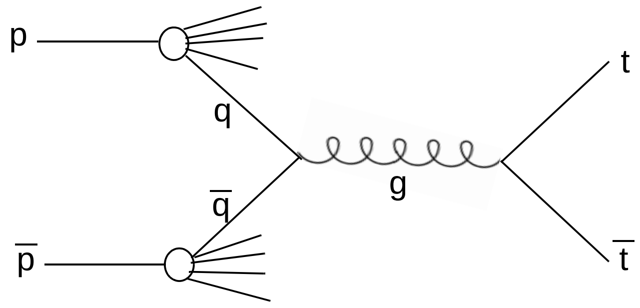
---

The discovery of the top quark

➡ Production of top-quarks

- **In proton-antiproton colliders, pairs of top quarks are mostly produced by the quark-antiquark annihilation process:**

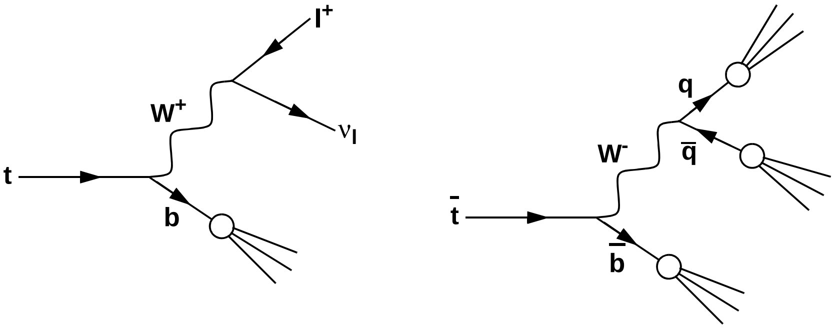
$$q+\bar{q}\rightarrow g\rightarrow t+\bar{t}$$



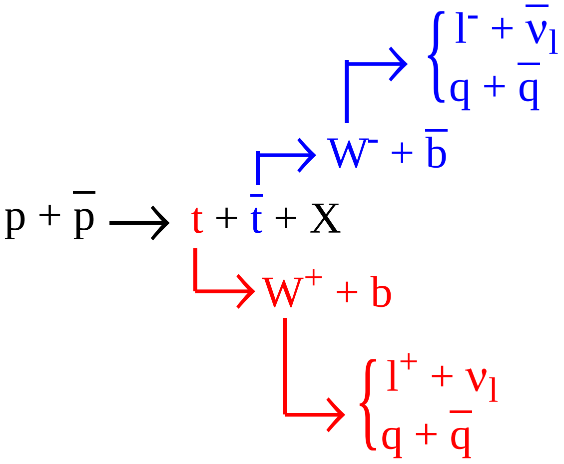
The discovery of the top quark

The decay of top-quarks

- The most likely **decay** of a top quark is to a **b quark and to a W**.



- The **W** can decay to **leptons or hadrons** and so the final state is a complex mix of jets and leptons.



$$\begin{cases} l^- + \bar{\nu}_l \\ q + \bar{q} \end{cases}$$

$$W^- + \bar{b}$$

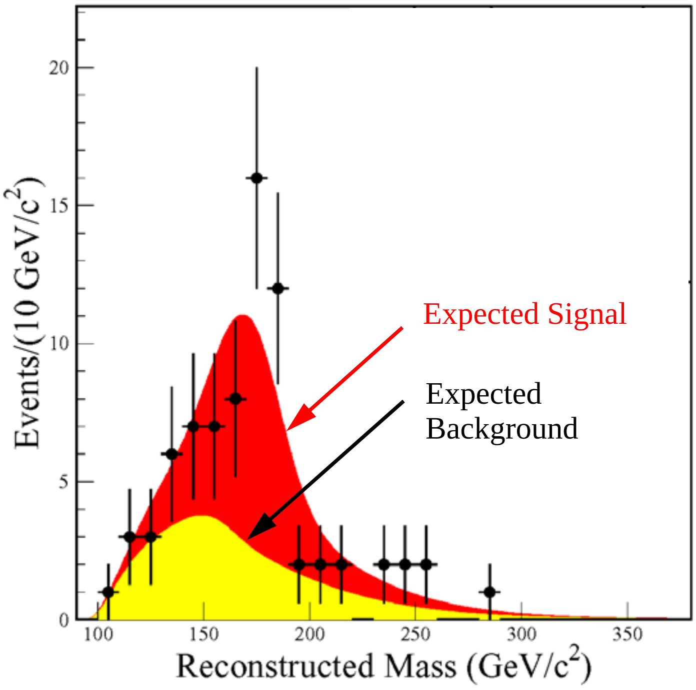
$$p + \bar{p} \longrightarrow t + \bar{t} + X$$

$$W^+ + b$$

$$\begin{cases} l^+ + \nu_l \\ q + \bar{q} \end{cases}$$

The discovery of the top quark

- After a selection of likely top event had been made, one could plot the **mass distribution** of the top-candidates.



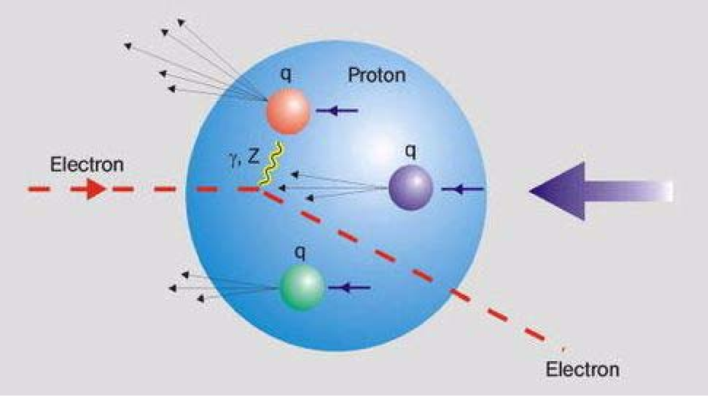
- The resulting distribution had a large background component but one could nevertheless extract a **top mass:**

$$M_t = 176 \pm 5 \ GeV$$

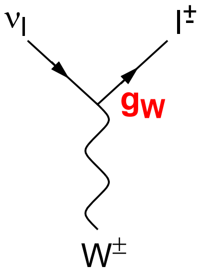
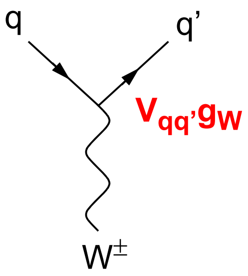
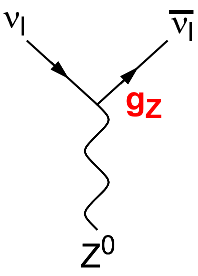
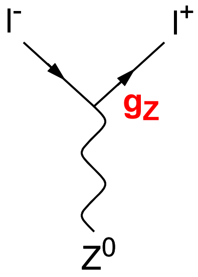
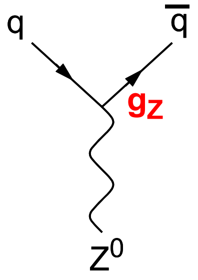
- The latest Tevatron result

$$M_t = 173 \pm 1 GeV$$

Neutral current reactions

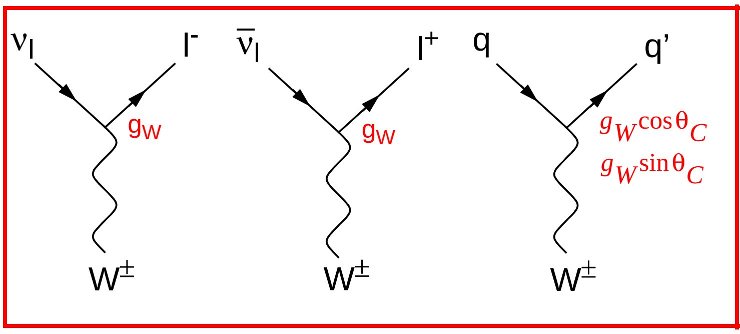


W and Z reactions

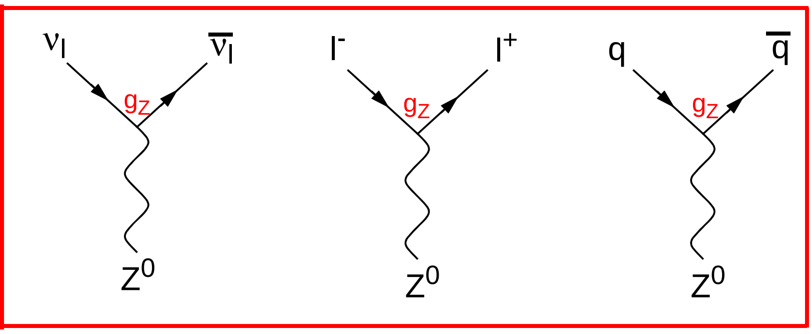
	Leptonic reactions	Hadronic reactions
Charge current reactions	 	
Neutral current reactions	  	

Neutral current reactions

- The basic vertices with W bosons have:
  - Conserved lepton numbers
  - Not conserved quark flavour (quark mixing)
- The basic vertices with Z bosons have:
  - Conserved lepton numbers
  - Conserved quark flavour (no quark mixing)



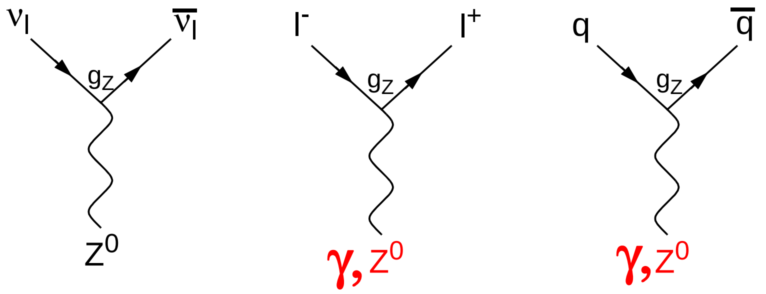
The basic W vertices



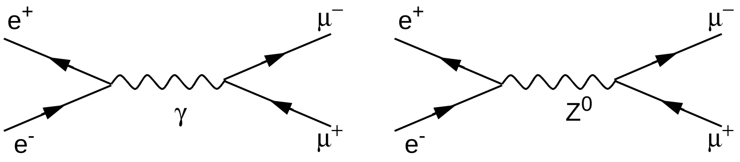
The basic Z vertices

Neutral current reactions

- In processes in which a photon can be exchanged, a Z^0 boson can be exchanged as well:



- The reaction $e^+e^- \to \mu^+\mu^-$ has as an example two dominant contributions:



Neutral current reactions

- With simple dimensional arguments one can estimate the cross-section for the photon- and Z-exchange process at low energy:

$$\sigma_\gamma \approx \frac{\alpha^2}{E_{CM}^2} \qquad \sigma_Z \approx G_Z^2 E_{CM}^2$$

where E_{cm} is the energy of the colliding electrons and positrons.

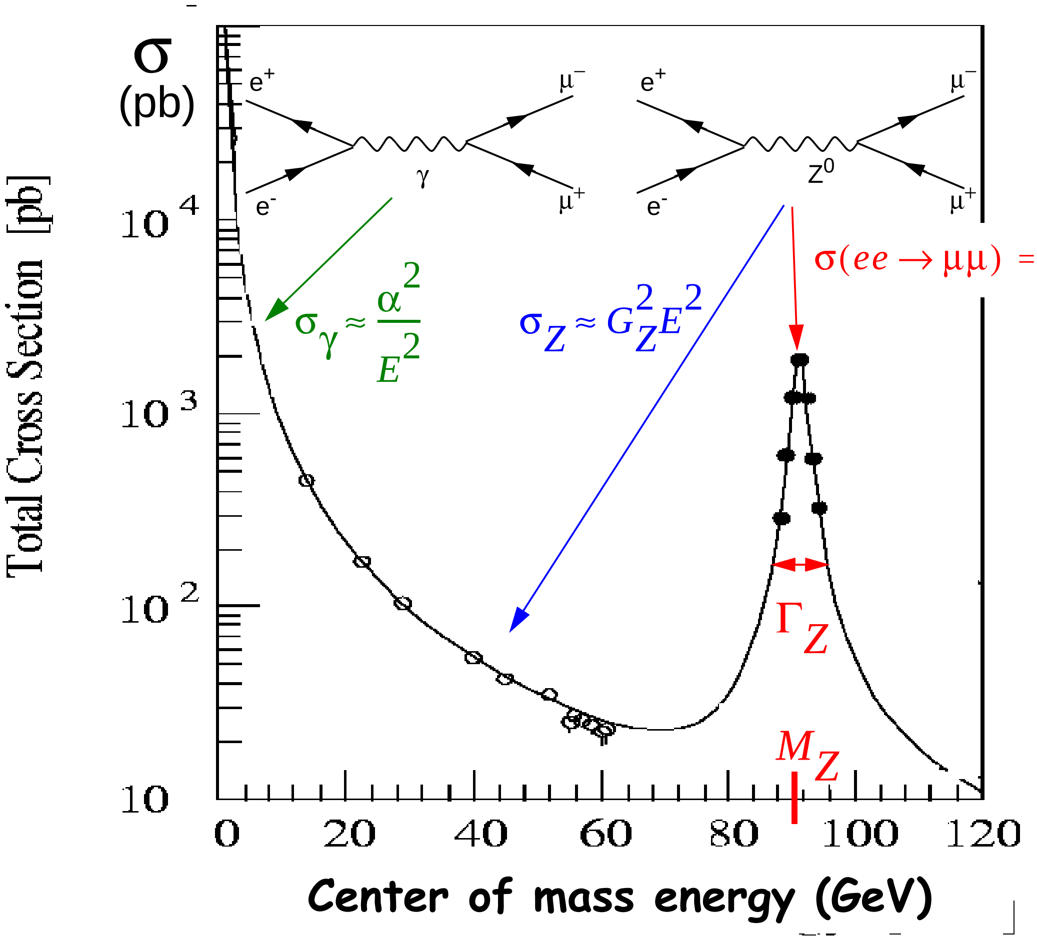
- From these formulas one can conclude that the **photon exchange** process will dominate **at low energies**.

- However, at $E_{cm}=M_Z$ this low-energy approximation fails and the **Z^0 peak** is described by the **Breit-Wigner formula:**

$$\sigma(E_{CM}) = \frac{M^2}{E_{CM}^2}\left[\frac{C}{\left(E_{CM}^2 - M^2\right)^2 + M^2\Gamma^2}\right]$$

where M is the mass of the resonance and Γ its decay width.

Neutral current reactions

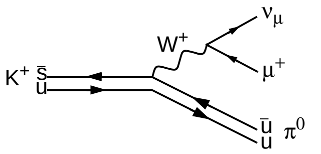


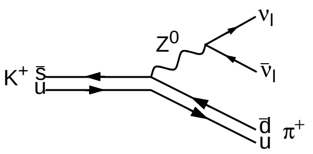
$$\sigma(ee \to \mu\mu) = \frac{12\pi M_Z^2}{E_{CM}^2}\left[\frac{\Gamma(Z^0 \to ee)\Gamma(Z^0 \to \mu\mu)}{\left(E_{CM}^2 - M_Z^2\right)^2 + M_Z^2\Gamma_Z^2}\right]$$

Test of flavour conservation

- That **flavour is conserved** at a **Z^0 vertex** can be verified by experiments.

- One way of doing this is to study the **decay of charged kaons** by measuring the decay rate of the following two processes:





$$K^+ \to \pi^0 + \mu^+ + \nu_\mu \qquad K^+ \to \pi^+ + \nu_l + \bar{\nu}_l$$

- The measured upper limit on the ratio of the decay rate of these two processes was

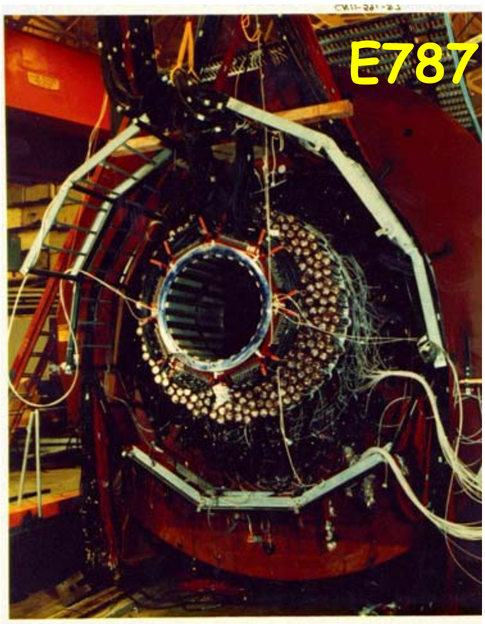
$$\frac{\sum_l \Gamma(K^+ \to \pi^+ + \nu_l + \bar{\nu}_l)}{\Gamma(K^+ \to \pi^0 + \mu^+ + \nu_\mu)} < 10^{-7}$$

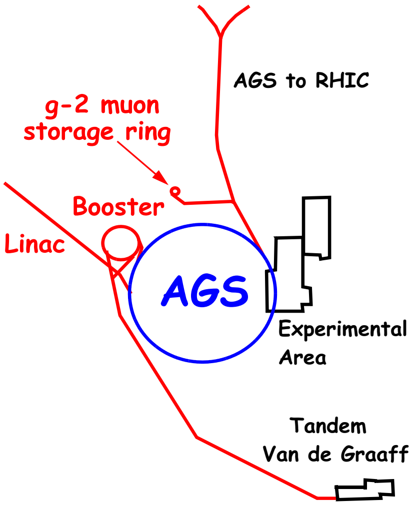
until experiment E787 came along .....

Test of flavour conservation

- The BNL experiment E787 was a **fixed target experiment** that used a K^+ beam created by 24 GeV protons from the AGS.

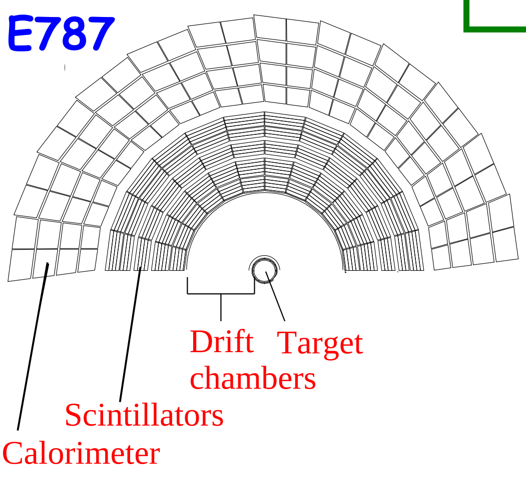
The 800 m long AGS has 240 magnets.





Test of flavour conservation

$$\boxed{K^+ \to \pi^+ + \nu_l + \bar{\nu}_l}$$



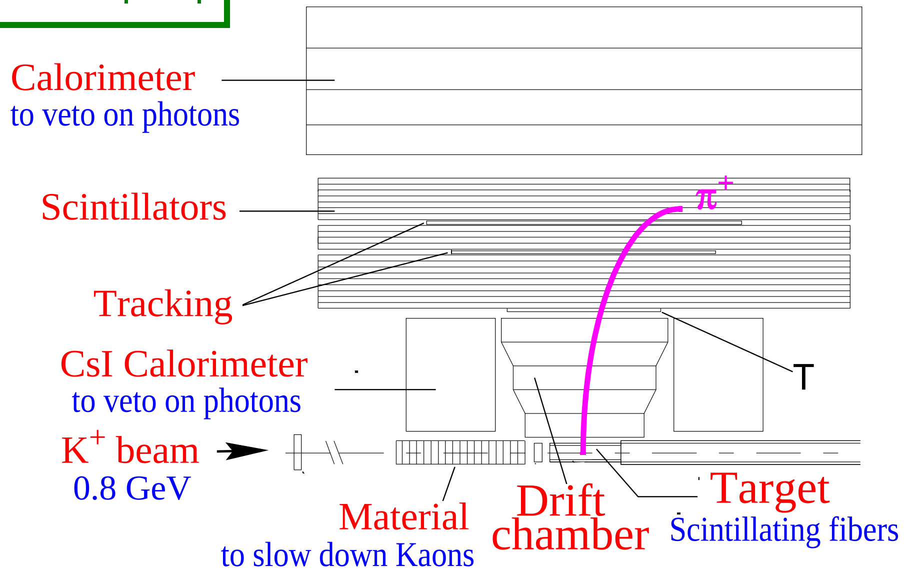
E787

Drift chambers

Target

Scintillators

Calorimeter



Calorimeter
to veto on photons

Scintillators

Tracking

CsI Calorimeter
to veto on photons

K^+ beam
0.8 GeV

Material
to slow down Kaons

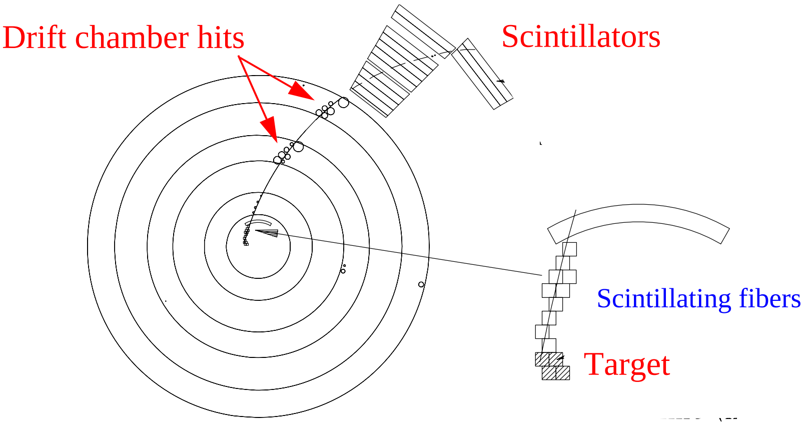
Drift chamber

Target
Scintillating fibers

- The Kaons were stopped in a **target** made of **scintillating fibers** and the decay of the K+ at rest was then studied.
- The **momentum, energy and range** of the particle from the decay was **measured**.

Test of flavour conservation

- After many years of running **two candidate events** for $K^+ \to \pi^+ + \nu_l + \bar{\nu}_l$ were found.



Drift chamber hits

Scintillators

Scintillating fibers

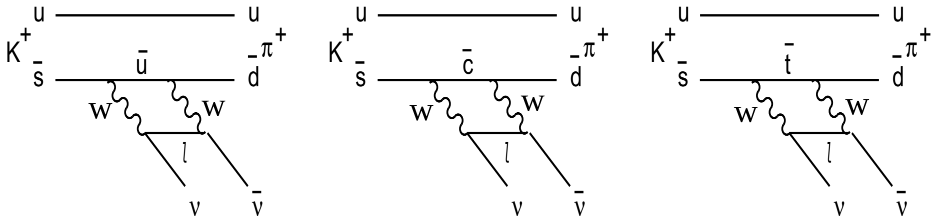
Target

- The result from these two events were:

$$\frac{\sum_l \Gamma(K^+ \to \pi^+ + \nu_l + \bar{\nu}_l)}{\Gamma(K^+ \to \pi^0 + \mu^+ + \nu_\mu)} = \frac{1{,}6 \times 10^{-10}}{0{,}033} = 5 \times 10^{-9}$$

Test of flavour conservation

- The events could, however, be explained by second-order charged current reactions rather than neutral current processes:



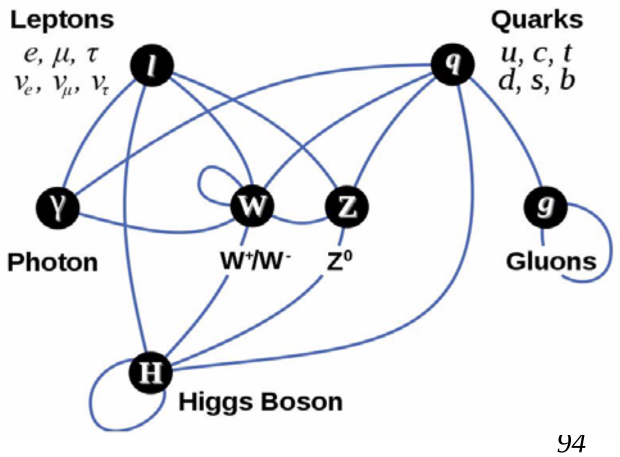
- Due to the t-d vertex in the third diagram above, it was also possible to set limits on the V_{td} element in the Cabibbo-Kobayashi-Maskawa matrix:

$$0{,}007 < |V_{td}| < 0{,}030$$

The Higgs boson

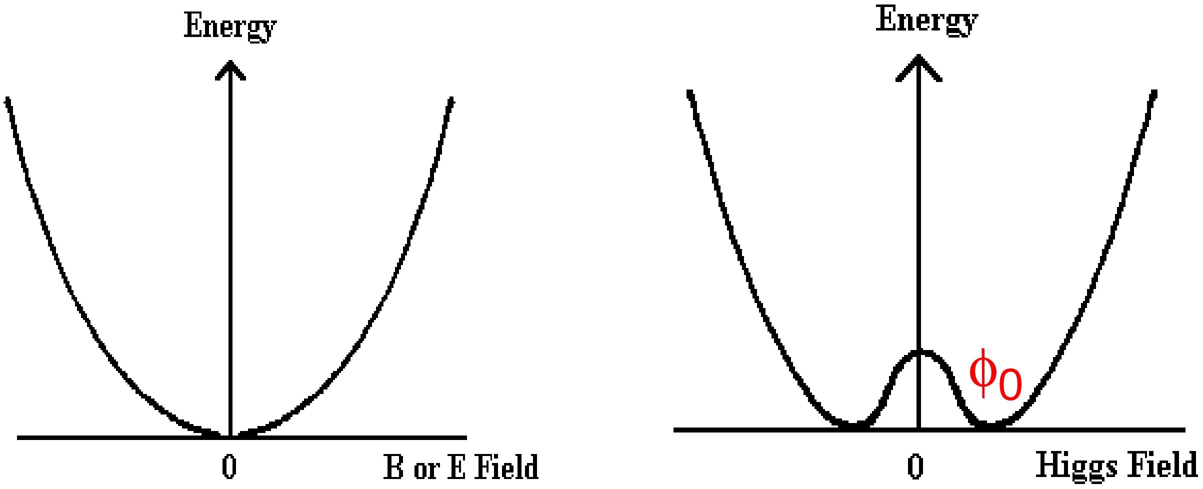
- Experimental data agrees extremely well with predictions of the gauge invariant electroweak theory.
- Gauge invariance ⇨ the gauge bosons have zero mass.

  True for photons in QED and gluons in QCD but not for W and Z.
- A new scalar field called the Higgs field is introduced to generate mass to the W and Z bosons as well as fermion masses.
- Associated with the field is a new particle called the Higgs boson with zero spin.
- The theory predicts how the Higgs boson couples to other particles but do not predict its mass.



The Higgs boson

- The Higgs field has the unusual characteristic of having a **non-zero value ϕ_0 in vacuum** (i.e. the field is not zero in its groundstate).



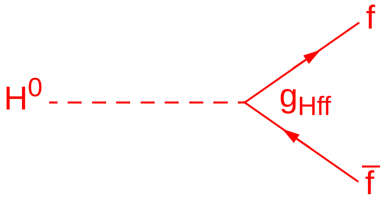
- Since the vacuum expectation value is not zero, the **vacuum** is supposed to be **populated with massive Higgs bosons** and when a gauge field interacts with the Higgs field it acquires mass.

The Higgs boson

- From the interaction with the Higgs field, the W and Z bosons require masses with the ratio given by

$$\cos\theta_W = \frac{M_W}{M_Z}$$

- In the same way, fermions acquire mass by interacting with the Higgs bosons.



- The coupling constant for this process depends on the fermion mass:

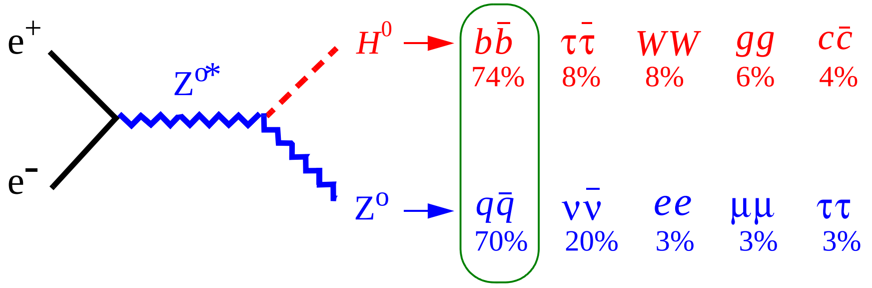
$$g_{Hff}^2 = \sqrt{2}G_F m_f^2$$

- Photons and gluons do not interact with the Higgs boson.

Search for the Higgs boson at LEP

- At LEP 2 one expects the main Higgs production to happen by so-called Higgs strahlung:

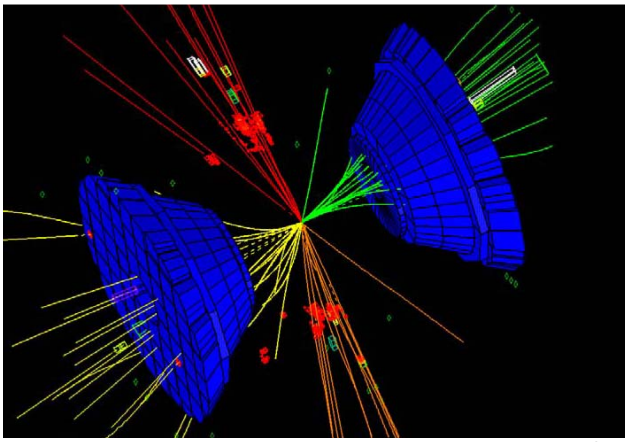
$e^+ + e^- \rightarrow H^0 + Z^0$



$H^0 \rightarrow$	$b\bar{b}$ 74%	$\tau\bar{\tau}$ 8%	WW 8%	gg 6%	$c\bar{c}$ 4%
$Z^o \rightarrow$	$q\bar{q}$ 70%	$\nu\bar{\nu}$ 20%	ee 3%	$\mu\mu$ 3%	$\tau\tau$ 3%

- Most of the Higgs events would have 4 jets in the final state. Two of these should be coming from b-quarks.

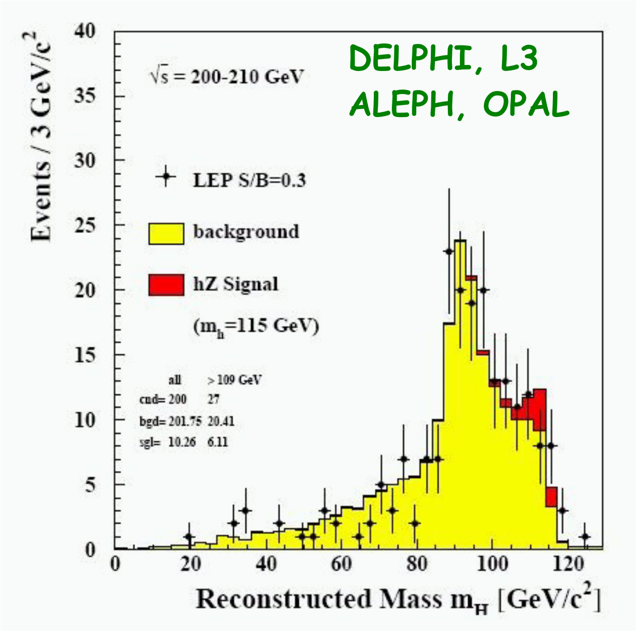
Example of a Higgs candidate in DELPHI →



Search for the Higgs boson at LEP

- During the last year of operation of LEP 2, the ALEPH experiment recorded a couple of events which could be due to the decays of a Higgs boson with a mass of about 115 GeV.

- The other experiments at LEP did not see a signal and when all data was added together there was no discovery.



- The DELPHI experiment put a limit on the Higgs mass of:

$$M_H > 114 \text{ GeV/c}^2$$

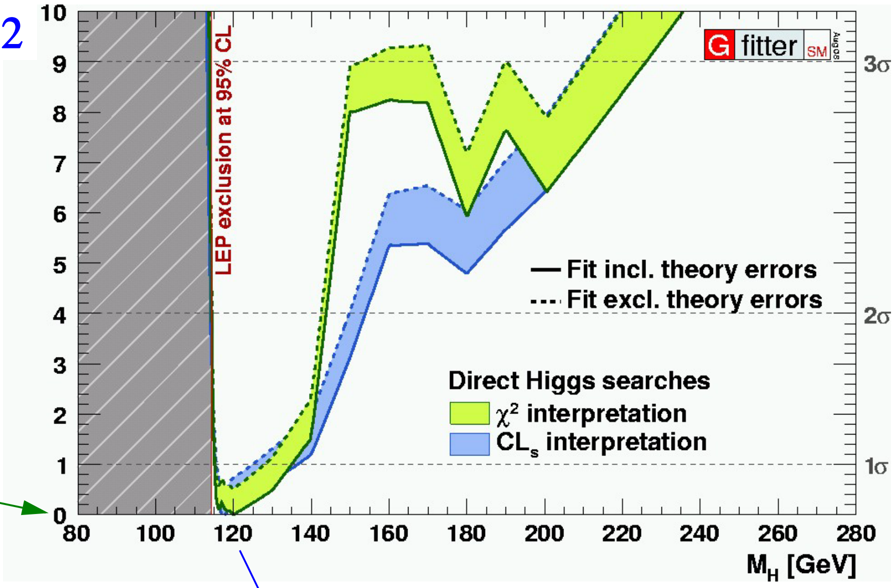
Search for the Higgs boson at LEP

- The measurement of many electroweak parameters at LEP (and other places) makes it possible to make a global fit with the Higgs mass as a free parameter.

χ^2

Quantifies how good the fit is

Perfect fit



$m_H = 120^{+15}_{-5}$ GeV