

Quantum Chromodynamics

The theory of strong interactions



Quantum Chromodynamics

➔ Interactions by massless spin-1 particles: Gauge bosons

- Quantum electrodynamics (QED): Photons
Quantum chromodynamics (QCD): Gluons
- QED: Photons couple to electric charges (Q)
QCD: Gluons couple to colour charges (Y^c and I_3^c).
- Y^c : colour hypercharge.
 I_3^c : colour isospin charge.
- The strong interaction is flavour-independent:
It acts the same on u, d, s, c, b and t

Quantum Chromodynamics

- The colour hypercharge (Y^c) + the colour isospin charge (I_3^c)
 ⇒ three colour and three anti-colour quark states

QCD



	Y^c	I_3^c		Y^c	I_3^c
r	1/3	1/2	\bar{r}	-1/3	-1/2
g	1/3	-1/2	\bar{g}	-1/3	1/2
b	-2/3	0	\bar{b}	2/3	0

- Colour confinement: Mesons and baryons total colour charge = 0.

- Colour wave-functions:

QCD



$$q\bar{q} = \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$$

$$q_1q_2q_3 = \frac{1}{\sqrt{6}}(r_1g_2b_3 - g_1r_2b_3 + b_1r_2g_3 - b_1g_2r_3 + g_1b_2r_3 - r_1b_2g_3)$$

Quantum Chromodynamics

- QCD: Colour hypercharge (Y^c) and colour isospin charge (I_3^c)
 Quark model: Flavour hypercharge (Y) and flavour isospin (I_3)

Quark Model



	Q	Y	I_3		Q	Y	I_3
d	-1/3	1/3	-1/2	d̄	1/3	-1/3	1/2
u	2/3	1/3	1/2	ū	-2/3	-1/3	-1/2
s	-1/3	-2/3	0	s̄	1/3	2/3	0
c	2/3	4/3	0	c̄	-2/3	-4/3	0
b	-1/3	-2/3	0	b̄	1/3	2/3	0
t	2/3	4/3	0	t̄	-2/3	-4/3	0

- The total wavefunction of hadrons:

$$\Psi_{\text{total}} = \Psi_{\text{space}} \times \Psi_{\text{spin}} \times \Psi_{\text{flavour}} \times \Psi_{\text{colour}}$$

Quantum Chromodynamics

➔ Photons electric charge = 0 !
 Gluons have colour charge !

- Gluons exist in 8 different colour states:

QCD



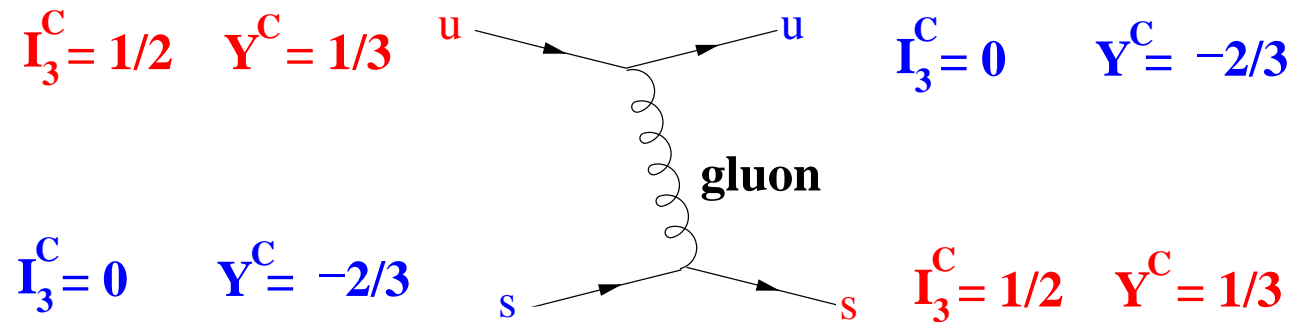
$\chi_{g1}^C = r \bar{g}$	$I_3^C = 1$	$Y^C = 0$
$\chi_{g2}^C = \bar{r} g$	$I_3^C = -1$	$Y^C = 0$
$\chi_{g3}^C = r \bar{b}$	$I_3^C = 1/2$	$Y^C = 1$
$\chi_{g4}^C = \bar{r} b$	$I_3^C = -1/2$	$Y^C = -1$
$\chi_{g5}^C = g \bar{b}$	$I_3^C = -1/2$	$Y^C = 1$
$\chi_{g6}^C = \bar{g} b$	$I_3^C = 1/2$	$Y^C = -1$
$\chi_{g7}^C = 1/\sqrt{2} (g \bar{g} - \bar{r} r)$	$I_3^C = 0$	$Y^C = 0$
$\chi_{g8}^C = 1/\sqrt{6} (g \bar{g} - r \bar{r} - 2 b \bar{b})$	$I_3^C = 0$	$Y^C = 0$

- Color confinement ➔ Gluons do **not** exist as **free particles**.

Quantum Chromodynamics

- Colour hypercharge + colour isospin charge are **additive quantum numbers** (like the electric charge).

Example:



Gluon:

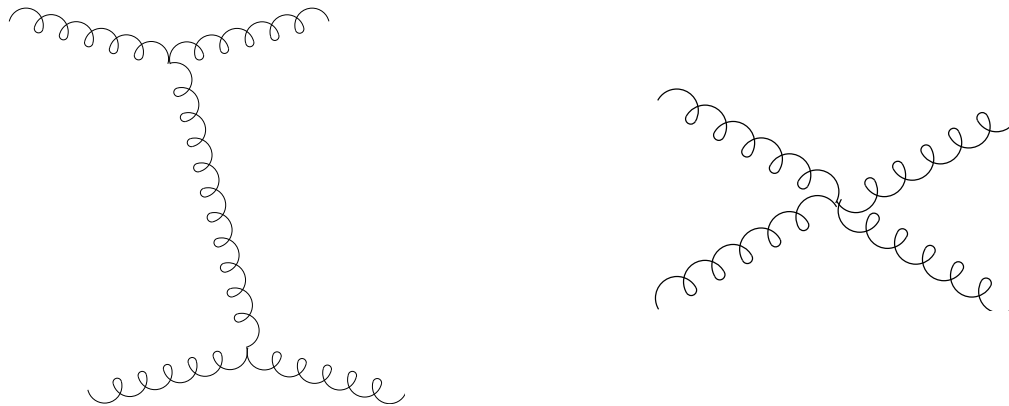
$$I_3^C = I_3^C(r) - I_3^C(b) = \frac{1}{2}$$

$$Y^C = Y^C(r) - Y^C(b) = 1$$

$$\chi_{g3}^c = \mathbf{r} \bar{\mathbf{b}}$$

Quantum Chromodynamics

➔ Gluons can couple to other gluons
(since they carry colour charge).



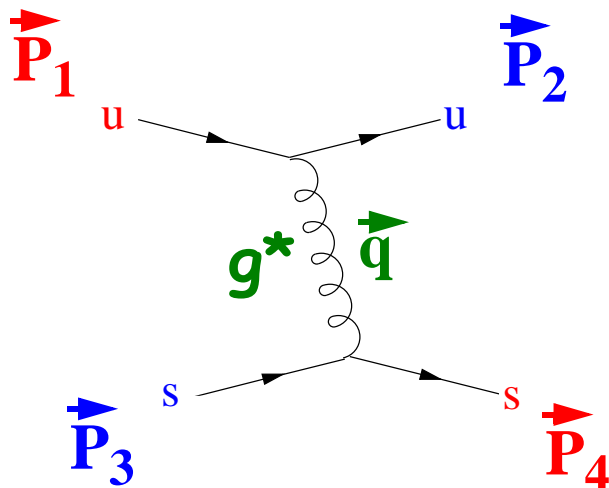
- QCD: Gluons can form colourless states.
- **Glueballs**
- Experiments: Difficult to prove existence of glueballs.

Quantum Chromodynamics

➔ The strong coupling constant

- QED: The electromagnetic coupling constant: α_{em}
- QCD: The **strong coupling constant**: α_s
- α_s : “running constant” ➔ Decreases with increasing Q^2

● What is Q^2 ?



The 4-vectors of the interacting quarks:

$$\vec{P} = (E, \vec{p}) = (E, p_x, p_y, p_z)$$

The 4-vectors of the quarks ➔ 4-vector of gluon

$$\vec{q} = (E_q, \vec{q}) = \vec{P}_1 - \vec{P}_2 = (E_1 - E_2, \vec{P}_1 - \vec{P}_2)$$

The squared 4-vector energy-momentum transfer:

$$Q^2 = -\vec{q} \cdot \vec{q} \quad (\text{i.e. } Q = \text{the "mass" of the gluon})$$

Quantum Chromodynamics

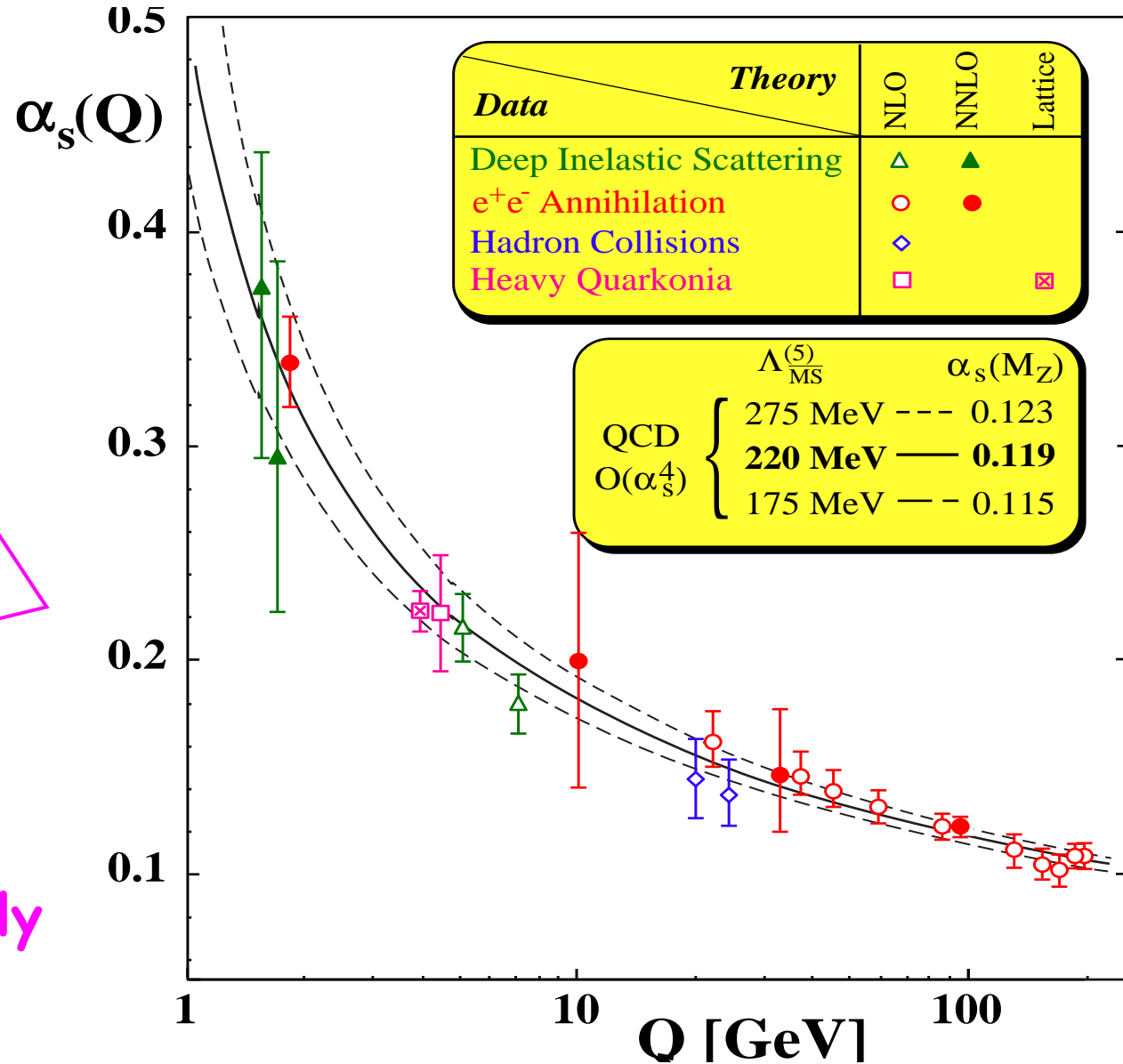
➔ The strong coupling constant

QCD prediction:

$$\alpha_s = \frac{12\pi}{(33 - 2N_f) \ln(Q^2 / \Lambda^2)}$$

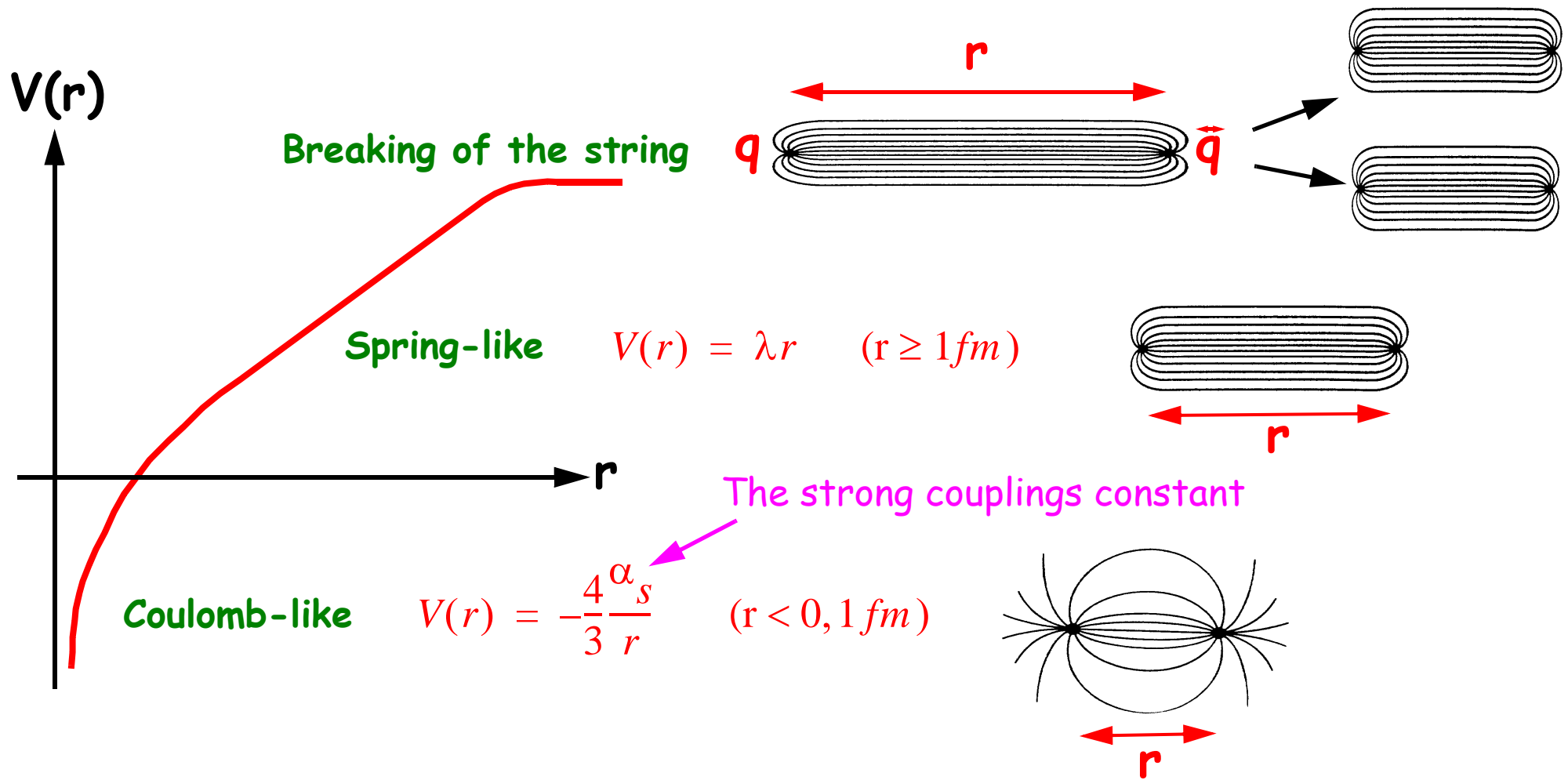
N_f : Number of allowed quark flavours

Λ : QCD scale parameter
Determined experimentally
($\Lambda \approx 0.2$ GeV)



Quantum Chromodynamics

➔ The quark-antiquark potential (mesons)



Quantum Chromodynamics

➔ The principle of asymptotic freedom.

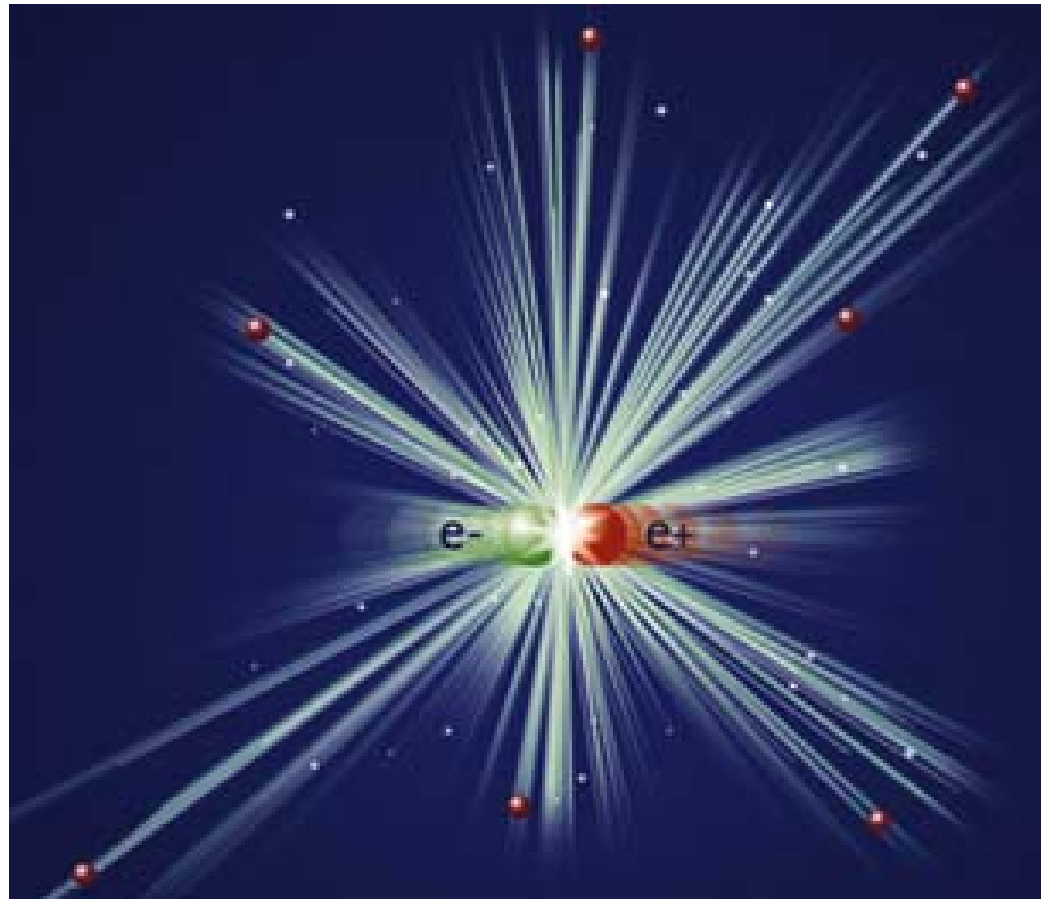
- QCD - **Short distances** - interaction is **weaker**
- QCD - **Long distances** - interaction is **stronger**

Small distance \Rightarrow Coulomb-like potential
Small distance \Rightarrow large $Q^2 \Rightarrow$ small α_s

quarks and gluons
free particles

- Small distances \Rightarrow first order diagrams.
Large distances \Rightarrow higher order diagrams.
- Higher-order diagrams \Rightarrow
Confinement cannot be calculated analytically

Electron-positron collisions



Electron-positron annihilation

➔ The R-value from e^+e^- experiments

- Measurement:

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

rate of collisions that give hadrons

rate of collisions that give muons

- Prediction from theory:

$$R = N_c \sum e_q^2$$

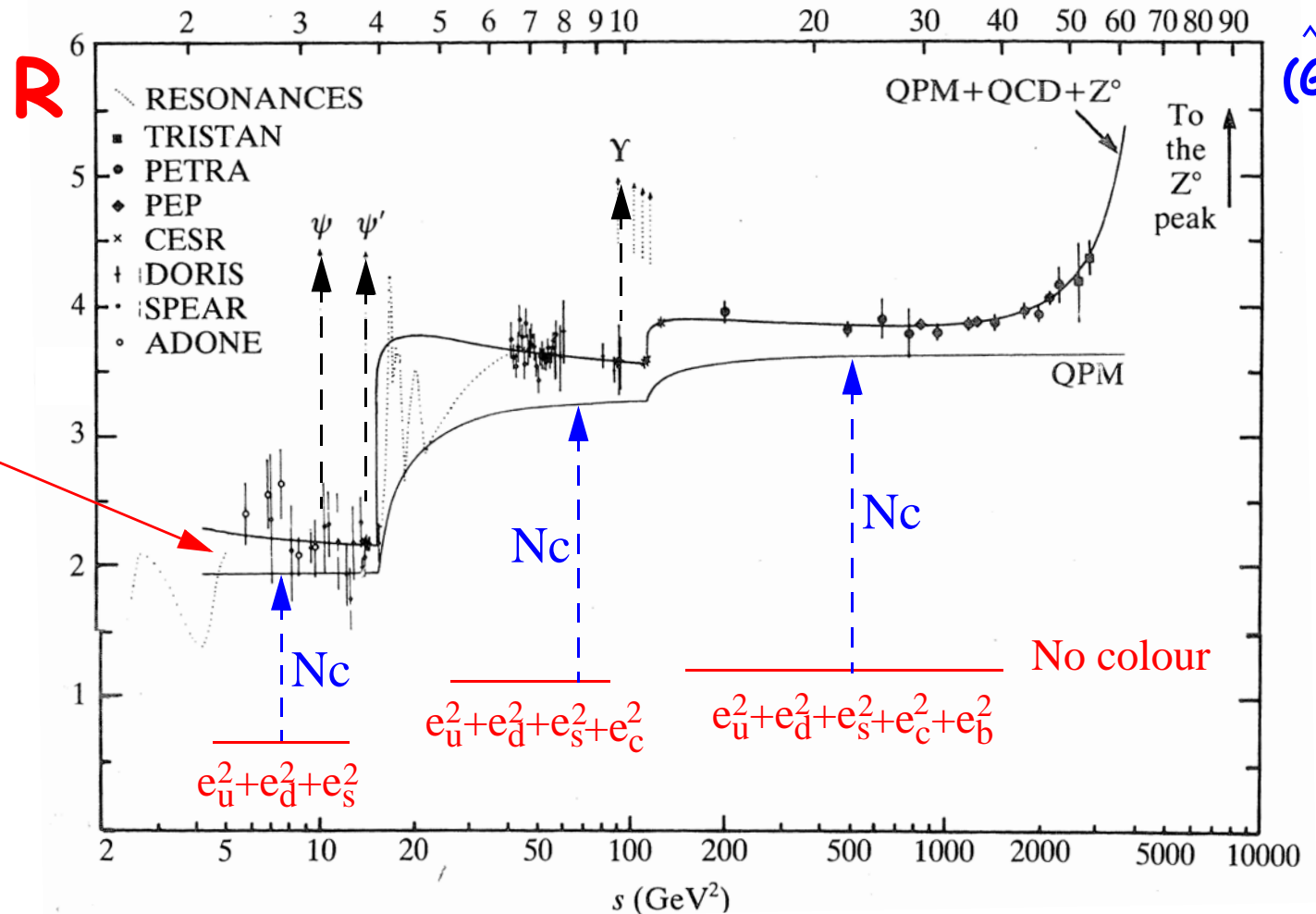
N_c is the number of colours (=3)

e_q the charge of the quarks.

Electron-positron annihilation

$$\begin{aligned}
 R &= N_c(e_u^2 + e_d^2 + e_s^2) = 3 \left((-1/3)^2 + (-1/3)^2 + (2/3)^2 \right) = 2 && \text{if } \sqrt{s} < m_\psi \\
 R &= N_c(e_u^2 + e_d^2 + e_s^2 + e_c^2) = 10/3 && \text{if } \sqrt{s} < m_\gamma \\
 R &= N_c(e_u^2 + e_d^2 + e_s^2 + e_c^2 + e_b^2) = 11/3 && \text{if } \sqrt{s} > m_\gamma
 \end{aligned}$$

\sqrt{s}
(GeV)



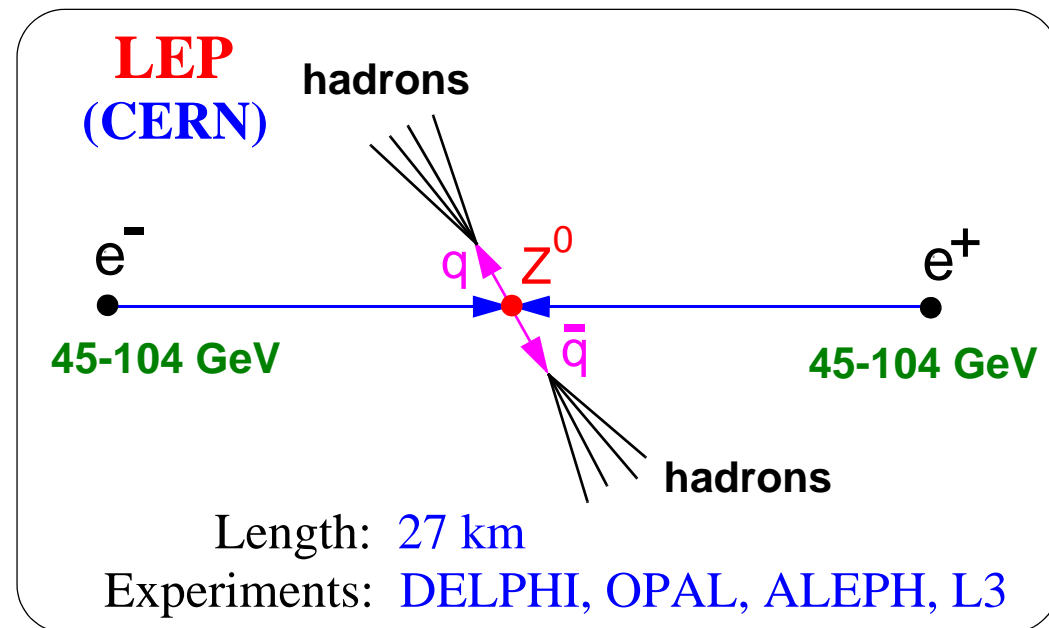
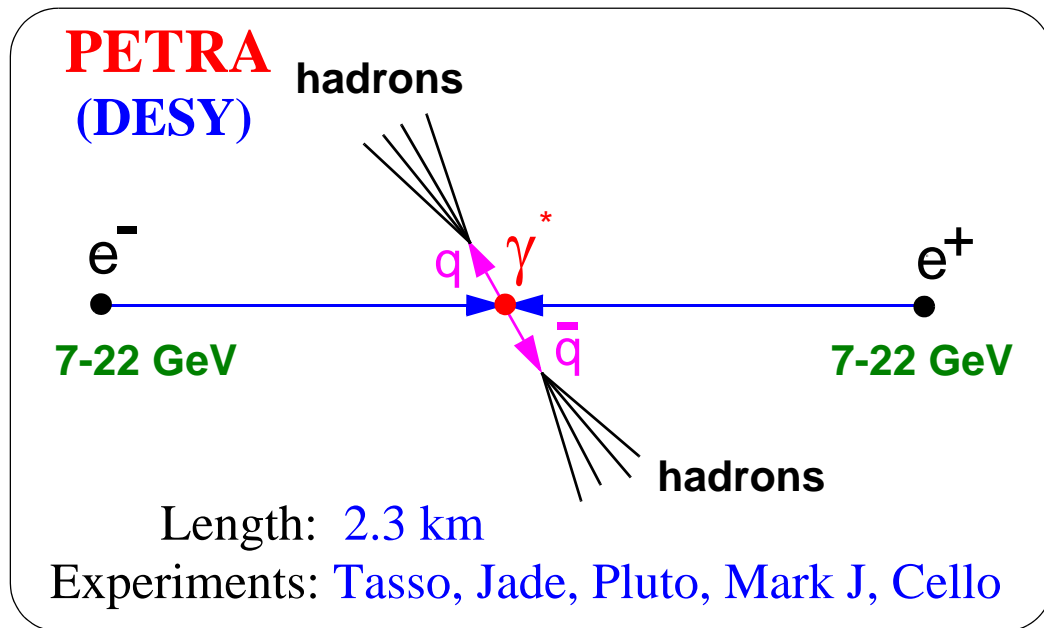
$$R = 3 \sum_q e_q^2 \left(1 + \frac{\alpha_s(Q^2)}{\pi} \right)$$

Correction for
gluon radiation

Electron-positron annihilation

➔ Jets of particles

- e^+e^- annihilation process:
A **photon** or a Z^0 is produced ➔ **quark-antiquark pair**.
- The quark and the antiquark **fragment** into observable **hadrons**.



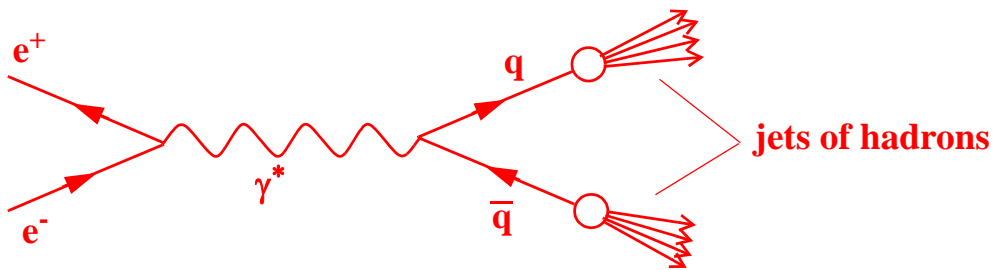
Electron-positron annihilation

➔ Two jets of particles

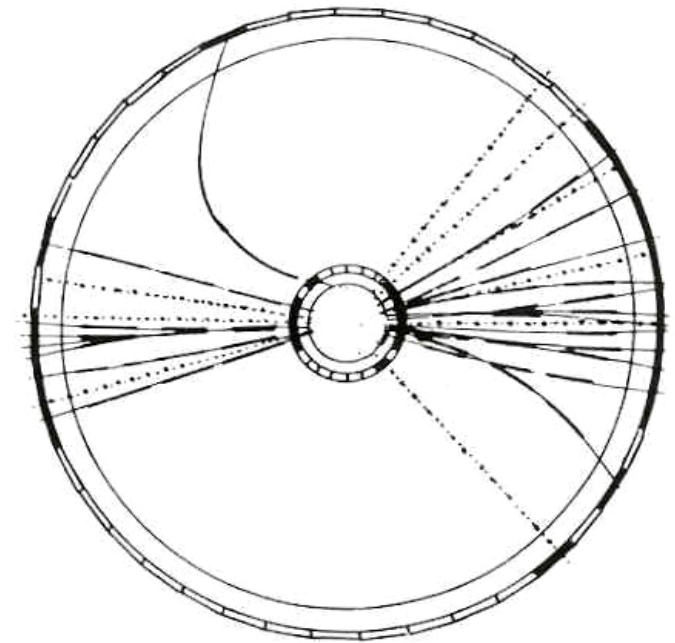
- The direction of the jet ➔ The direction of the quark

Two-jet events in the Jade experiment

Two-jet events in theory:



$$e^+ + e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$$



- Energy and momentum conservation ➔

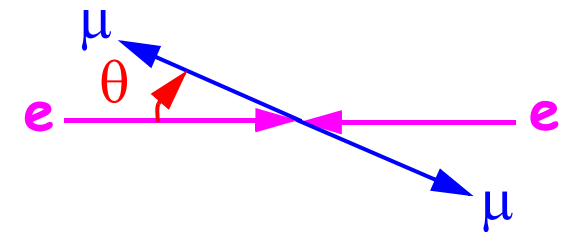
The quark and anti-quark have equal energy and opposite direction
Jets have the same energy and opposite direction

Electron-positron annihilation

➔ The angular distribution of 2 jets and the spin of the quarks.

- $e^+ + e^- \rightarrow \gamma^* \rightarrow \mu^+ + \mu^-$

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow \mu^+\mu^-) = \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta)$$



- $e^+ + e^- \rightarrow \gamma^* \rightarrow q + \bar{q}$

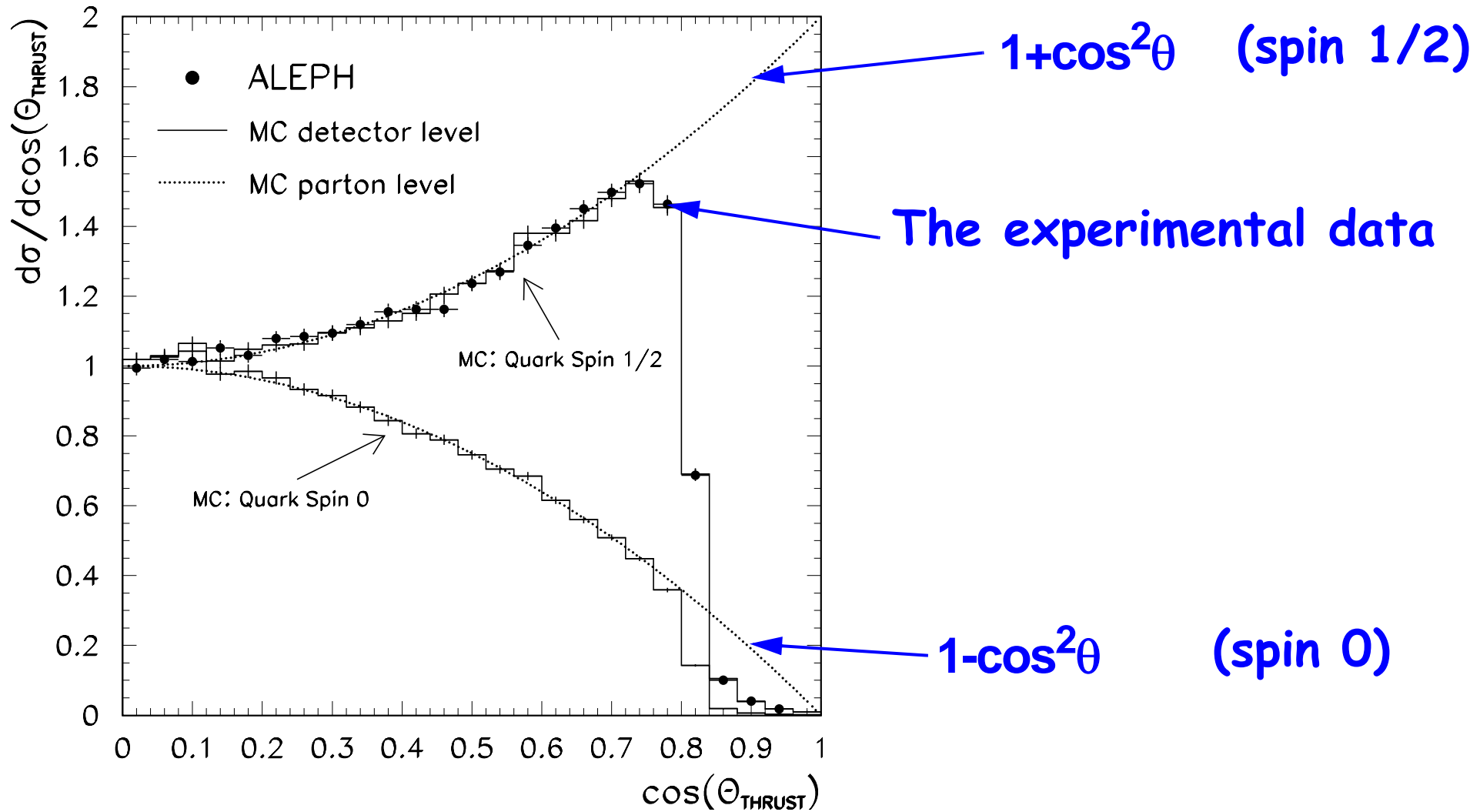
$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow q\bar{q}) = N_c e_q^2 \frac{2\pi\alpha^2}{2Q^2}(1 + \cos^2\theta) \Rightarrow \text{quark spin} = 1/2$$

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow q\bar{q}) = N_c e_q^2 \frac{2\pi\alpha^2}{2Q^2}(1 - \cos^2\theta) \Rightarrow \text{quark spin} = 0$$

e_q is the quark charge

N_c is the number of colours (=3)

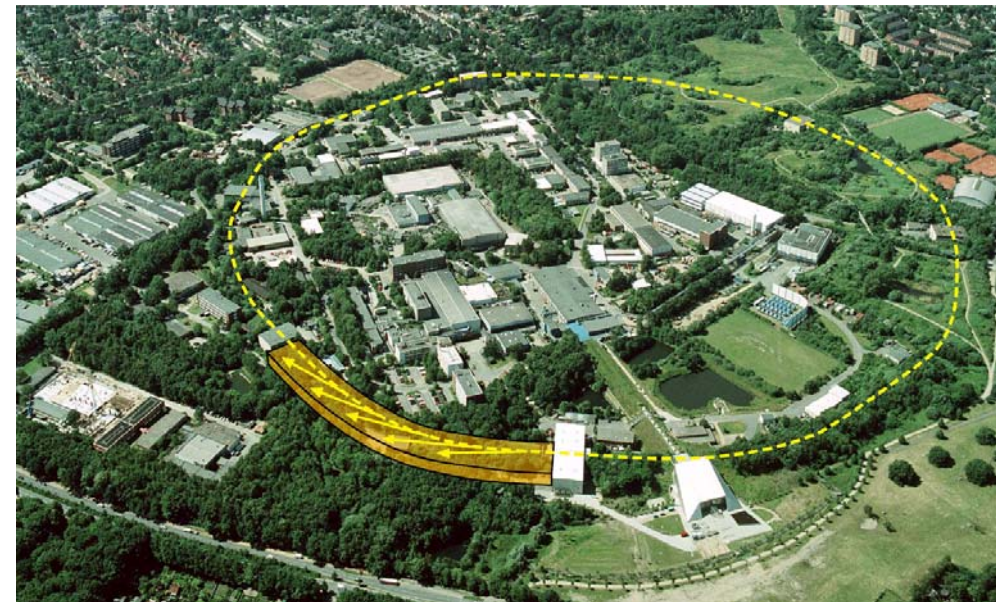
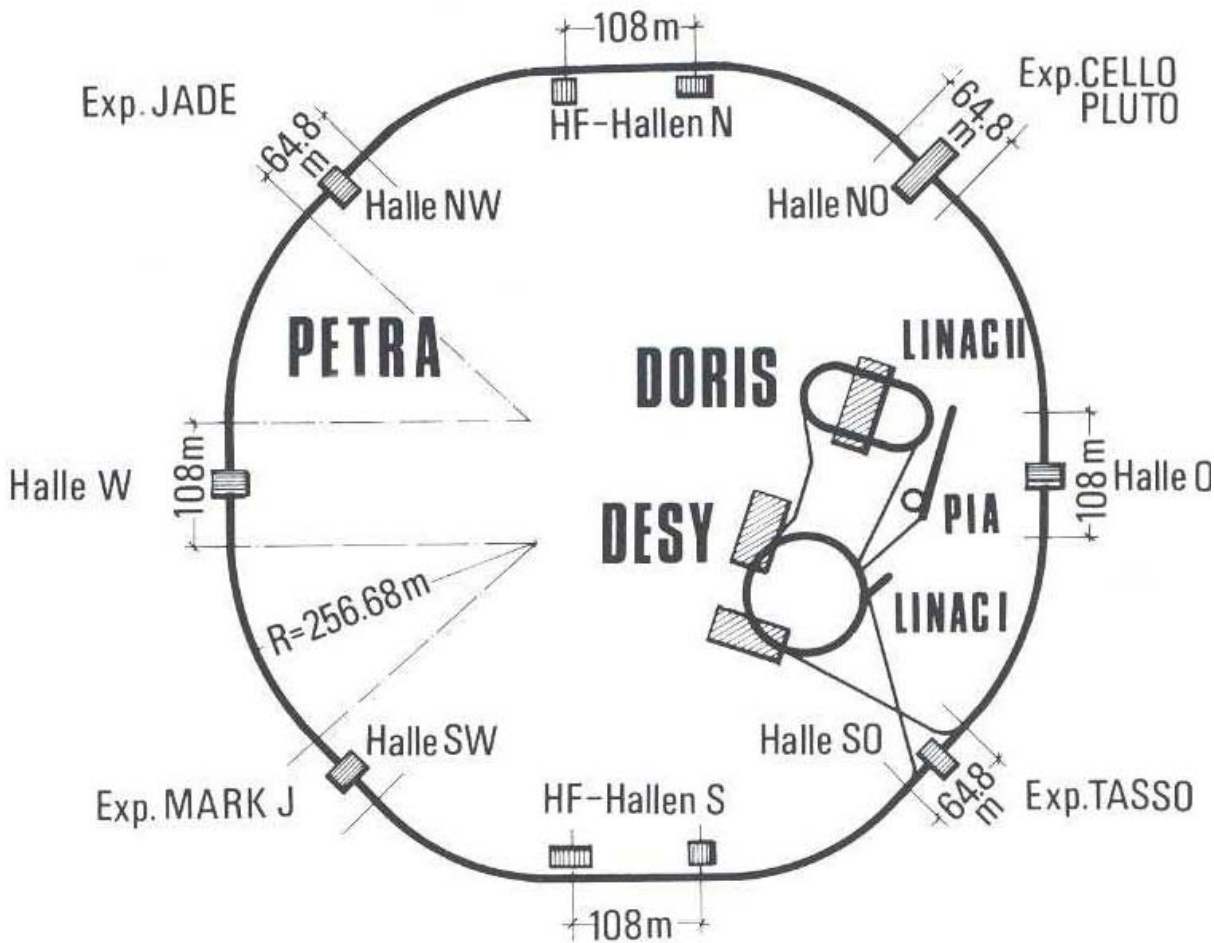
Electron-positron annihilation



Conclusion: Quarks have spin = 1/2 !

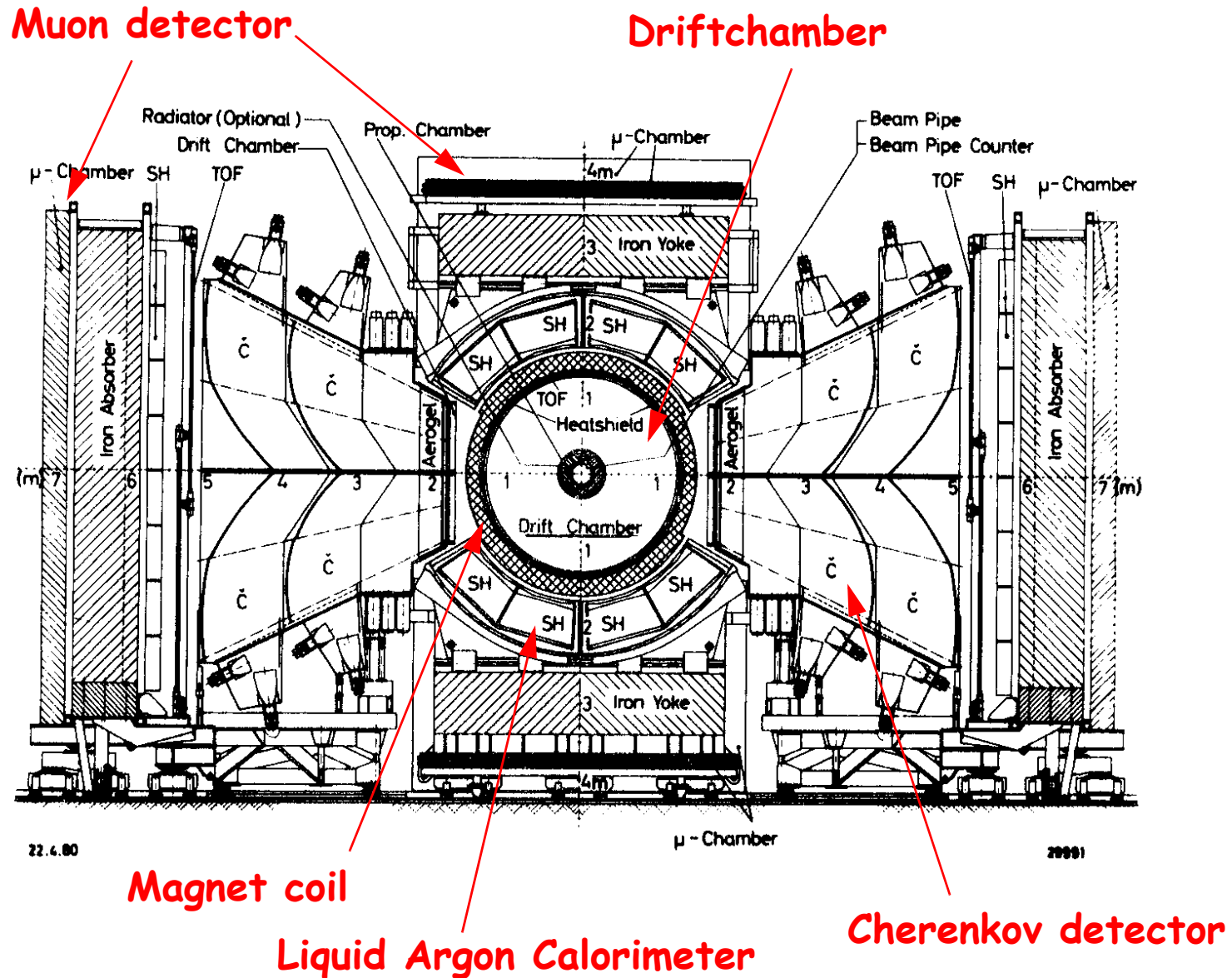
The discovery of the gluon

➔ The accelerator: PETRA at the German laboratory DESY.



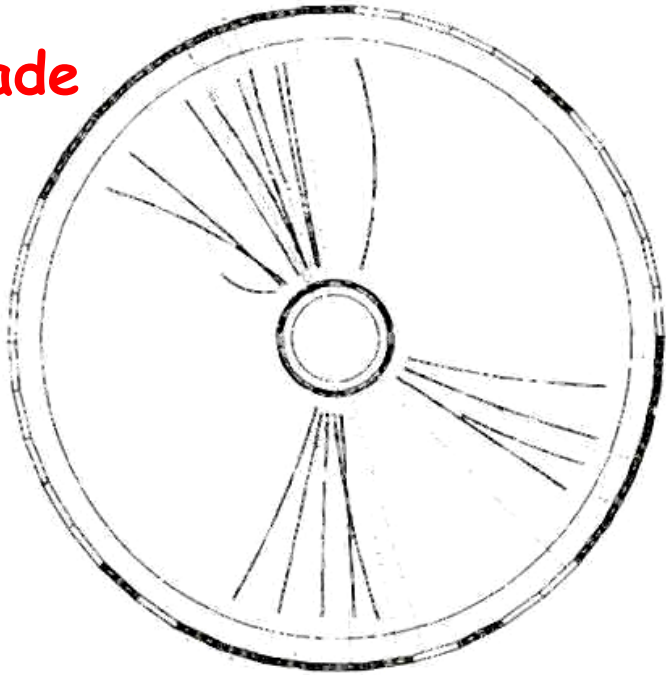
The discovery of the gluon

→ The experiment: TASSO

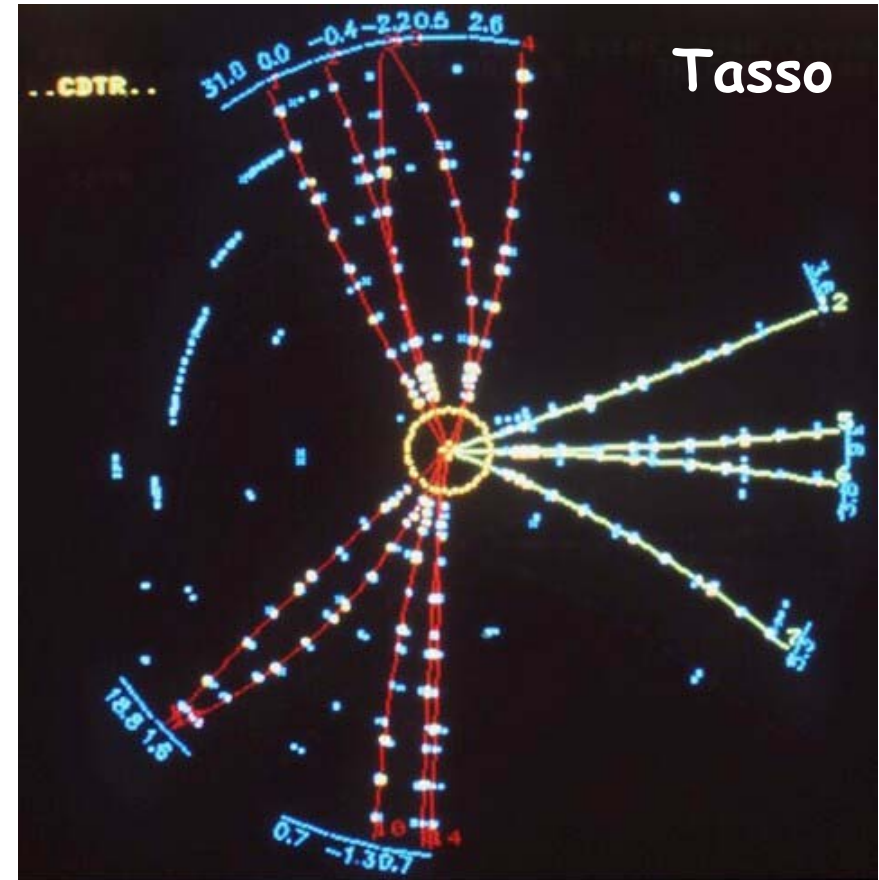
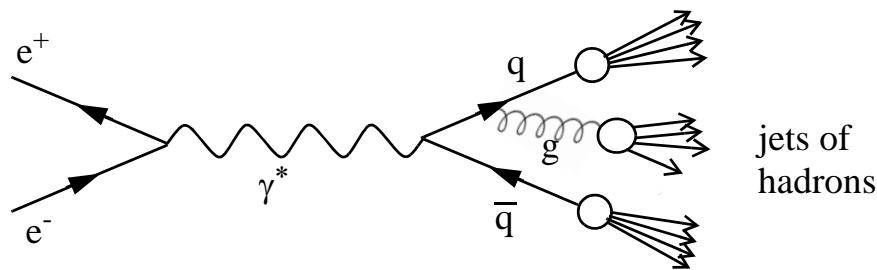


The discovery of the gluon

Jade



Three-jet events in theory:



Three-jet events \Rightarrow The third jet from a gluon !

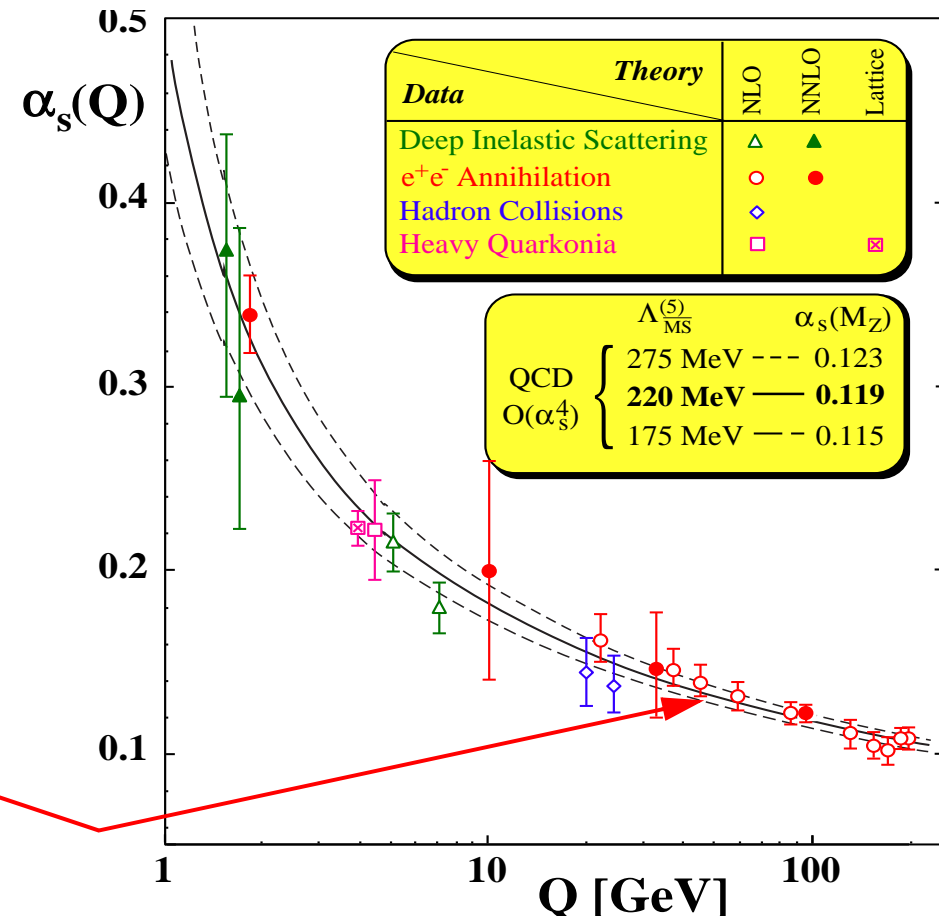
The discovery of the gluon

- The **probability** for **gluon** emission is proportional to α_s .

$$\alpha_s = \frac{\text{Number of three-jet events}}{\text{Number of two-jet events}}$$

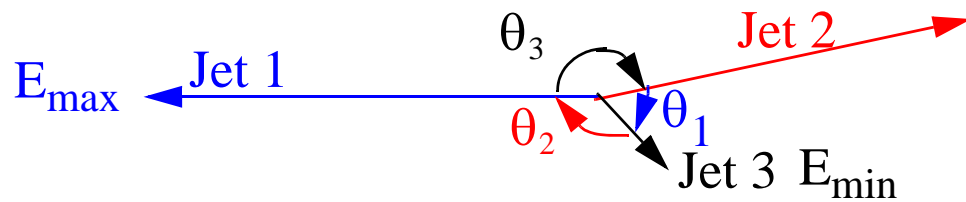
- **PETRA:**

$$\alpha_s = 0.15 \pm 0.03 \quad \text{for } \sqrt{s} = 30\text{-}40 \text{ GeV}$$

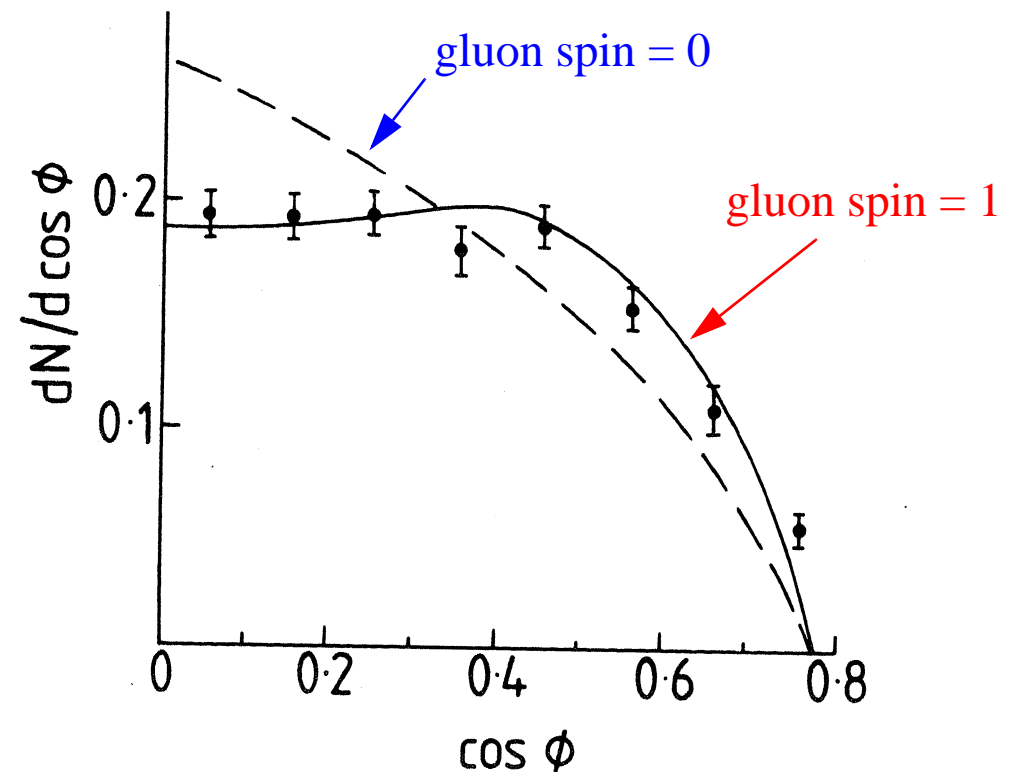


Electron-positron annihilation

➔ Angular distribution of 3 jets and the spin of the gluon.

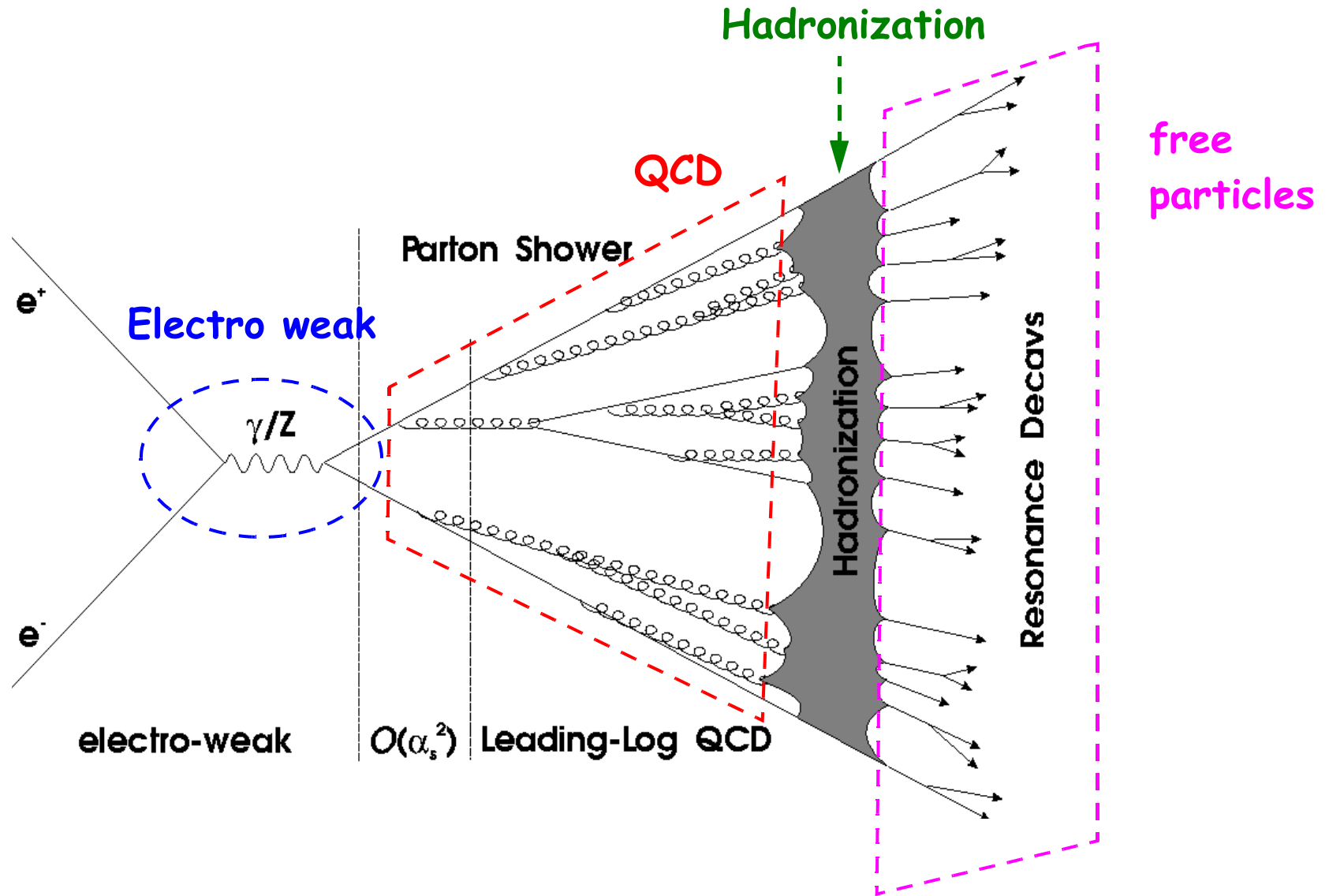


$$\cos \phi = \frac{\sin \theta_2 - \sin \theta_3}{\sin \theta_1}$$

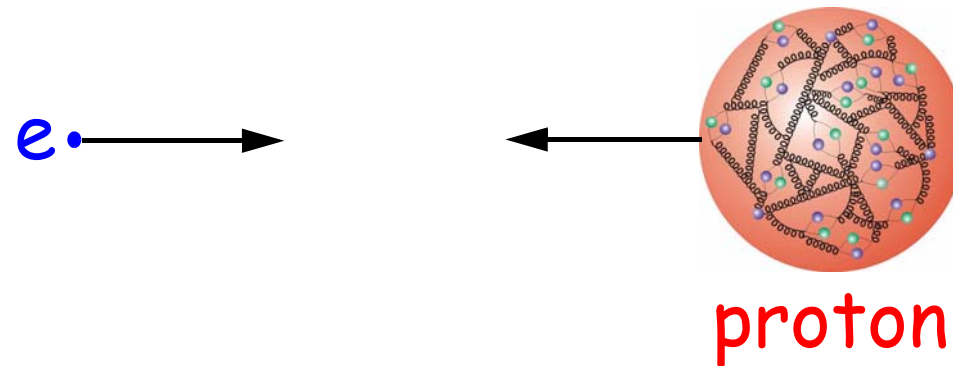


● Angular distribution ➔ **Gluons have spin = 1**

Electron-positron annihilation



Electron-proton collisions



Electron-proton scattering

$\lambda \gg r_p$ Very low electron energies

Scattering from a "point-like" spin-less object.

$\lambda = r_p$ Low electron energies

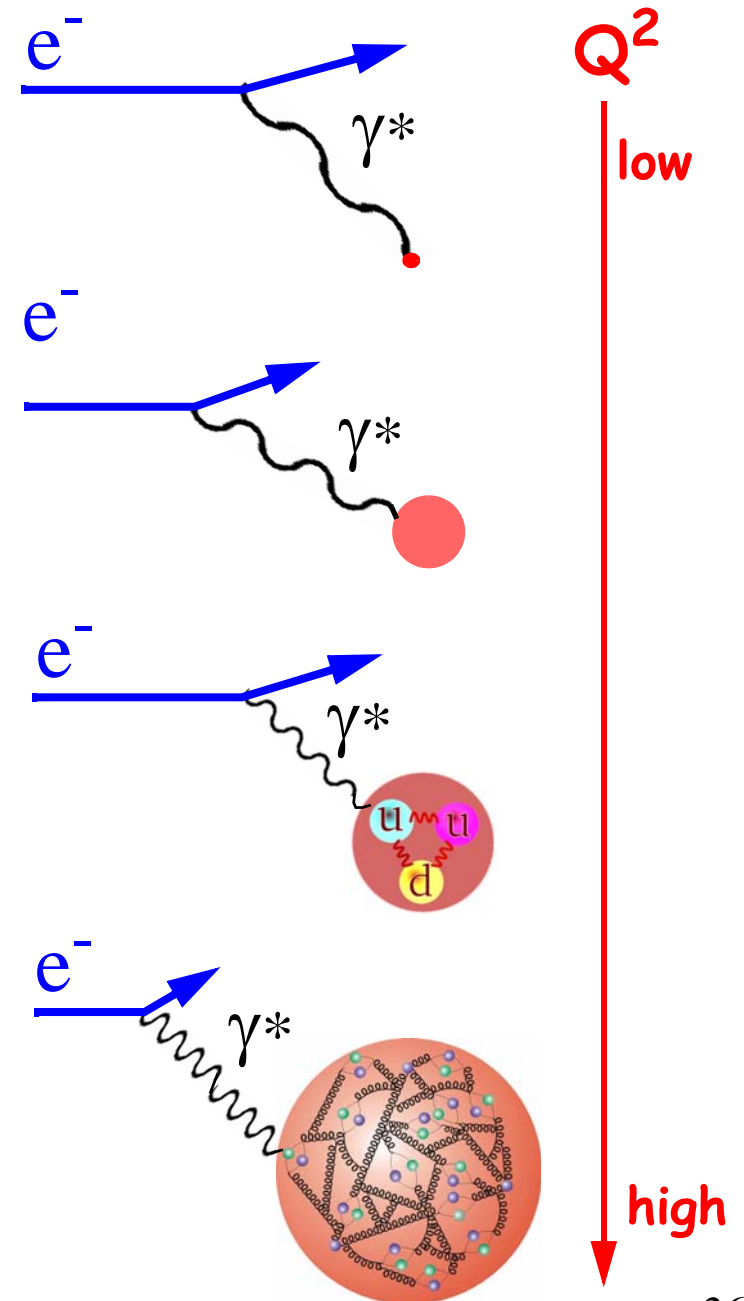
Scattering from an extended charged object.

$\lambda < r_p$ High electron energies

Interactions with the valence quarks in the proton.

$\lambda \ll r_p$ Very high electron energies

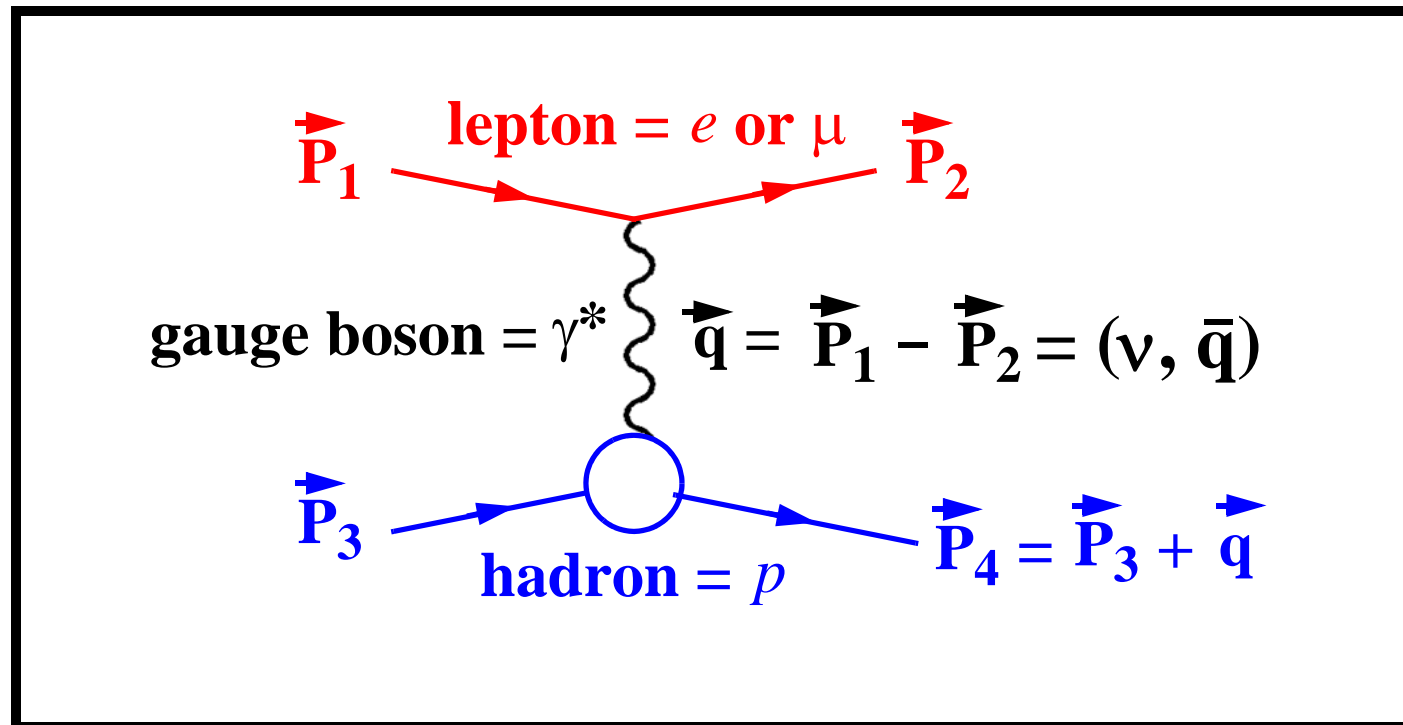
Interactions with the sea of quarks and gluons.



Electron-proton scattering

→ Elastic scattering

- **Elastic scattering:** The same type of particles before and after.



- Elastic electron-proton scattering \Rightarrow Size of the proton

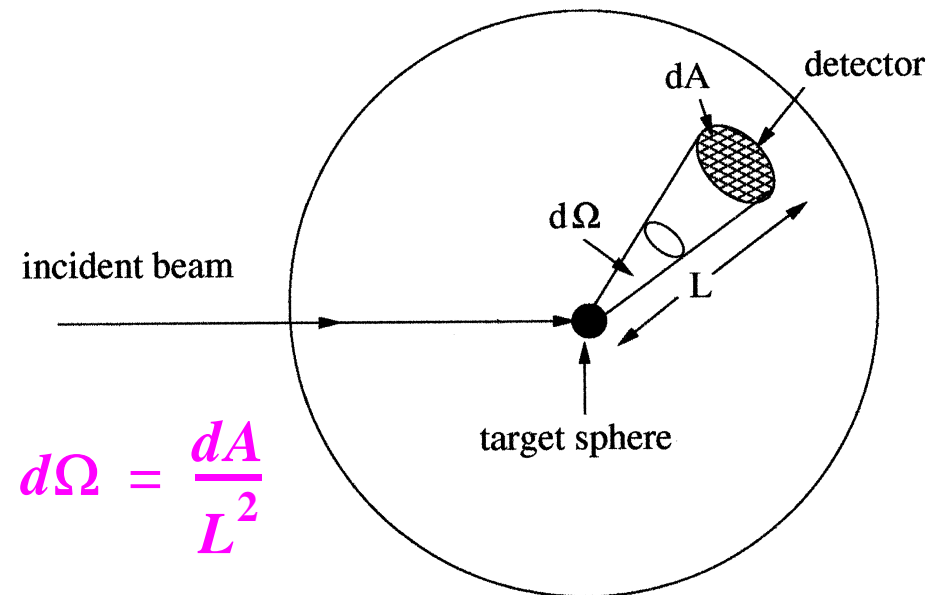
Electron-proton scattering

➔ Differential cross section

- The **differential cross section**

$$\frac{d\sigma(\theta, \varphi)}{d\Omega} \longleftarrow \sin\theta d\theta d\varphi$$

gives the angular distribution.



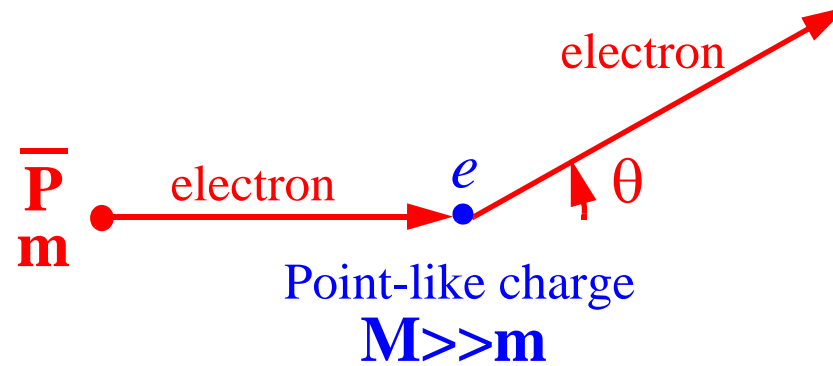
$$d\Omega = \frac{dA}{L^2}$$

- The **total cross section** by integration:

$$\sigma = \int \frac{d\sigma(\theta, \varphi)}{d\Omega} d\Omega = \int_0^\pi \int_0^{2\pi} \frac{d\sigma(\theta, \varphi)}{d\Omega} \sin\theta d\theta d\varphi$$

Electron-proton scattering

➔ Elastic scattering on a static point-like charge.



The Rutherford scattering formula

Non-relativistic electron

Point-like electric charge e .

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{m^2 \alpha^2}{4p^4 \sin^4\left(\frac{\theta}{2}\right)} \quad \alpha = \frac{e^2}{4\pi}$$

The Mott scattering formula

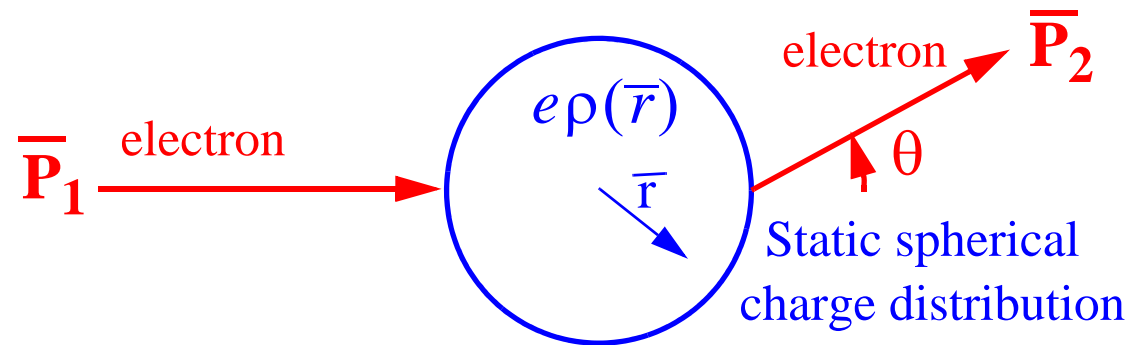
Relativistic electron

Point-like electric charge e .

$$\left(\frac{d\sigma}{d\Omega}\right)_M = \frac{\alpha^2}{4p^4 \sin^4\left(\frac{\theta}{2}\right)} \left(m^2 + p^2 \cos^2\left(\frac{\theta}{2}\right)\right)$$

Electron-proton scattering

➔ Elastic scattering on an extended charged object.



Momentum transfer

$$\bar{q} = \bar{P}_1 - \bar{P}_2$$

$$q^2 = -\bar{q} \cdot \bar{q}$$

- $\rho(r)$: a **spherically symmetric density function** with $\int \rho(r) d^3x = 1$
 $\rho(r)$: describes how the charge e is spread out.

- Not point-like interaction ➔

Rutherford formula modified by **electric form factor $G_E(q^2)$**

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_R G_E^2(q^2)$$

$$\left(\frac{d\sigma}{d\Omega} \right)_R = \frac{m^2 \alpha^2}{4p^4 \sin^4\left(\frac{\theta}{2}\right)}$$

$\bar{q} = \bar{P}_1 - \bar{P}_2$

Electron-proton scattering

- The electric form factor is the **Fourier transform** of the **charge distribution** with respect to the momentum transfer \vec{q} :

$$G_E(q^2) = \int \rho(r) e^{i\vec{q} \cdot \vec{x}} d^3\vec{x}$$

- The electric form factor has values between 0 and 1:

Low momentum transfer: $G_E(0) = 1$ for $q = 0$

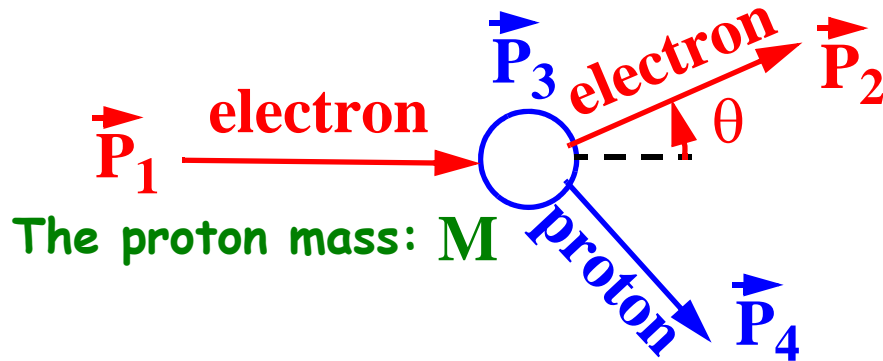
High momentum transfer: $G_E(q^2) \rightarrow 0$ for $q^2 \rightarrow \infty$

- Measurements of the cross-section \Rightarrow The form factor \Rightarrow The charge distribution.

- The mean quadratic charge radius $r_E^2 = \int r^2 \rho(r) d^3\vec{x} = -6 \left. \frac{dG_E(q^2)}{dq^2} \right|_{q^2=0}$

Electron-proton scattering

→ Elastic electron-proton scattering



4-momentum transfer

$$\vec{q} = \vec{P}_1 - \vec{P}_2$$

$$Q^2 = -\vec{q} \cdot \vec{q}$$

- Protons have charge + magnetic moment
electric formfactor (G_E) + magnetic formfactor (G_M)

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_M \times \left(G_1(Q^2) \cos^2 \frac{\theta}{2} + \frac{Q^2}{2M^2} G_2(Q^2) \sin^2 \frac{\theta}{2} \right)$$

$$G_1(Q^2) = \frac{G_E^2 + \frac{Q^2}{4M^2} G_M^2}{1 + \frac{Q^2}{4M^2}}$$

$$G_2(Q^2) = G_M^2$$

Electron-proton scattering

- Measurement of the formfactors:

i) low Q^2 ($Q \ll M$):

G_E dominates the cross section

r_E can be measured $r_E = 0.85 \pm 0.02$ fm

ii) Intermediate Q^2 ($0.02 < Q^2 < 3 \text{ GeV}^2$):

G_E and G_M give sizable contributions

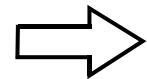
Parameterization can be used $G_E(Q^2) \approx \frac{G_M(Q^2)}{\mu_p} \approx \left(\frac{\beta^2}{\beta^2 + Q^2} \right)^2$

iii) High Q^2 ($Q^2 > 3 \text{ GeV}^2$):

G_M dominates the cross section

Electron-proton scattering

If protons are point-particles



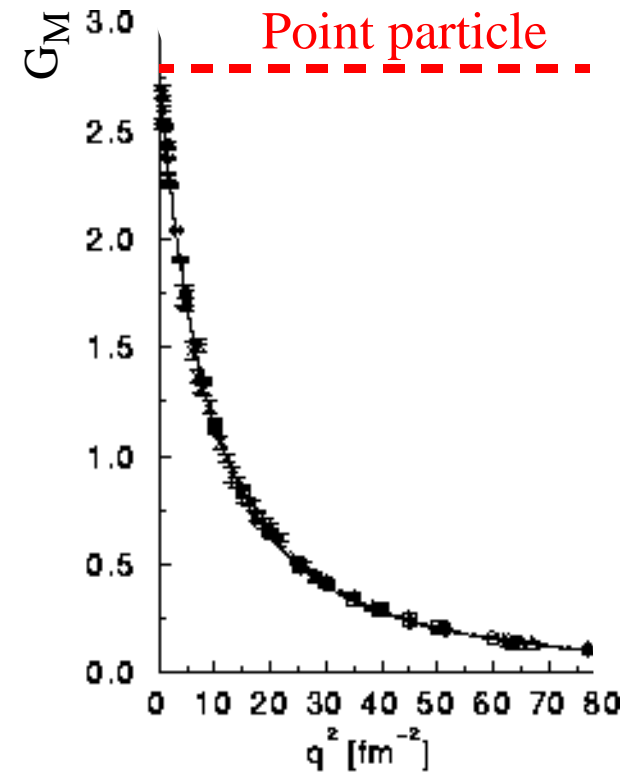
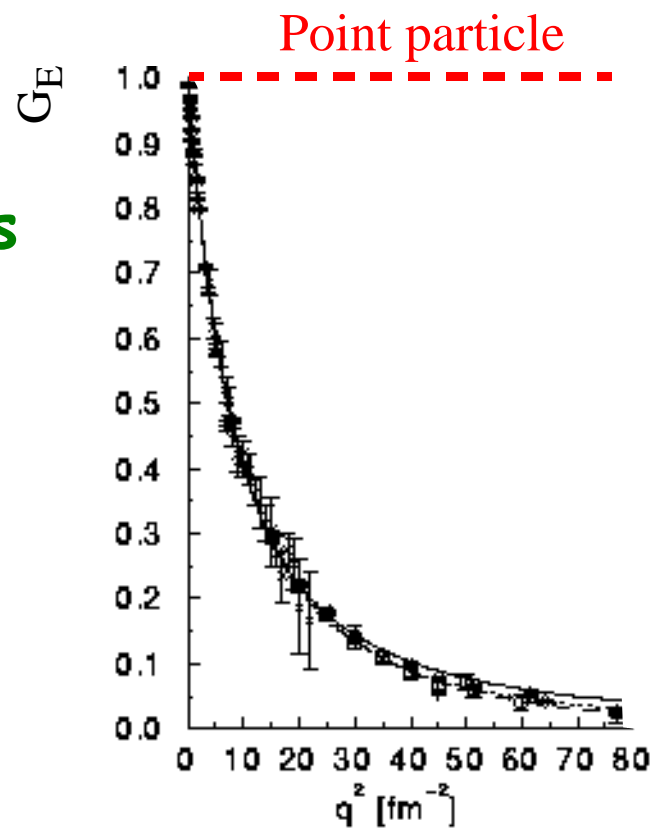
$$G_E(Q^2) = 1$$

Electric charge of a proton

$$G_M(Q^2) = 2.79$$

Magnetic moment of a proton

Measurements of G_E and G_M of the proton

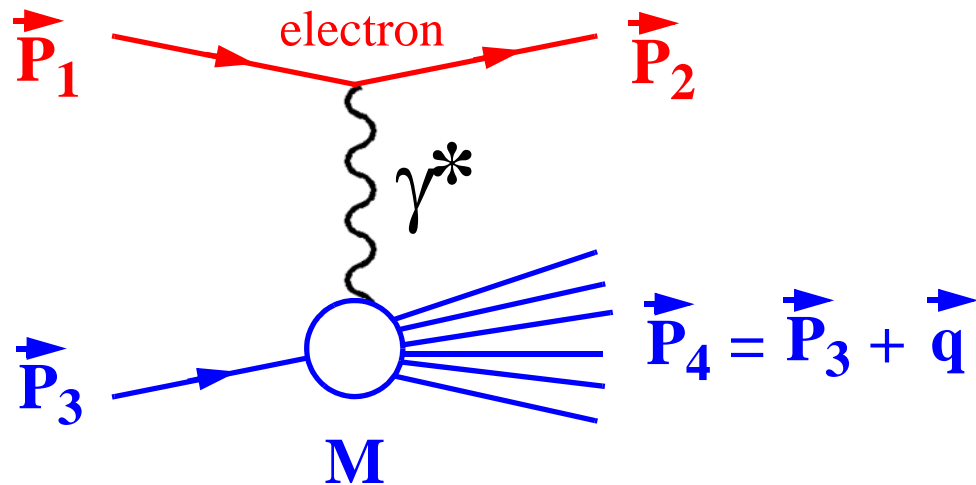


Conclusion:
The proton has an extended charge distribution !

Electron-proton scattering

➔ Inelastic electron-proton scattering

- Inelastic electron-proton scattering ➔ The proton is broken up



4-momentum transfer

$$\vec{q} = \vec{P}_1 - \vec{P}_2 = (\nu, \vec{q})$$

$$Q^2 = -\vec{q} \cdot \vec{q}$$

- Bjorken scaling variable

$$x = \frac{Q^2}{2M\nu} \quad 0 < x < 1$$

M is the mass of the proton.

Electron-proton scattering

- The differential cross section for **inelastic ep scattering**:

$$\frac{d\sigma}{dE_2 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)} \cdot \left[\frac{1}{v} F_2(x, Q^2) \cos^2\left(\frac{\theta}{2}\right) + \frac{2}{M} F_1(x, Q^2) \sin^2\left(\frac{\theta}{2}\right) \right]$$

Dimensionless **structure functions**

- $F_{1,2}(x, Q^2)$ parameterize the photon-proton interaction in the same way a $G_1(Q^2)$ and $G_2(Q^2)$ do it in elastic scattering.

- **Bjorken scaling** or scale invariance:

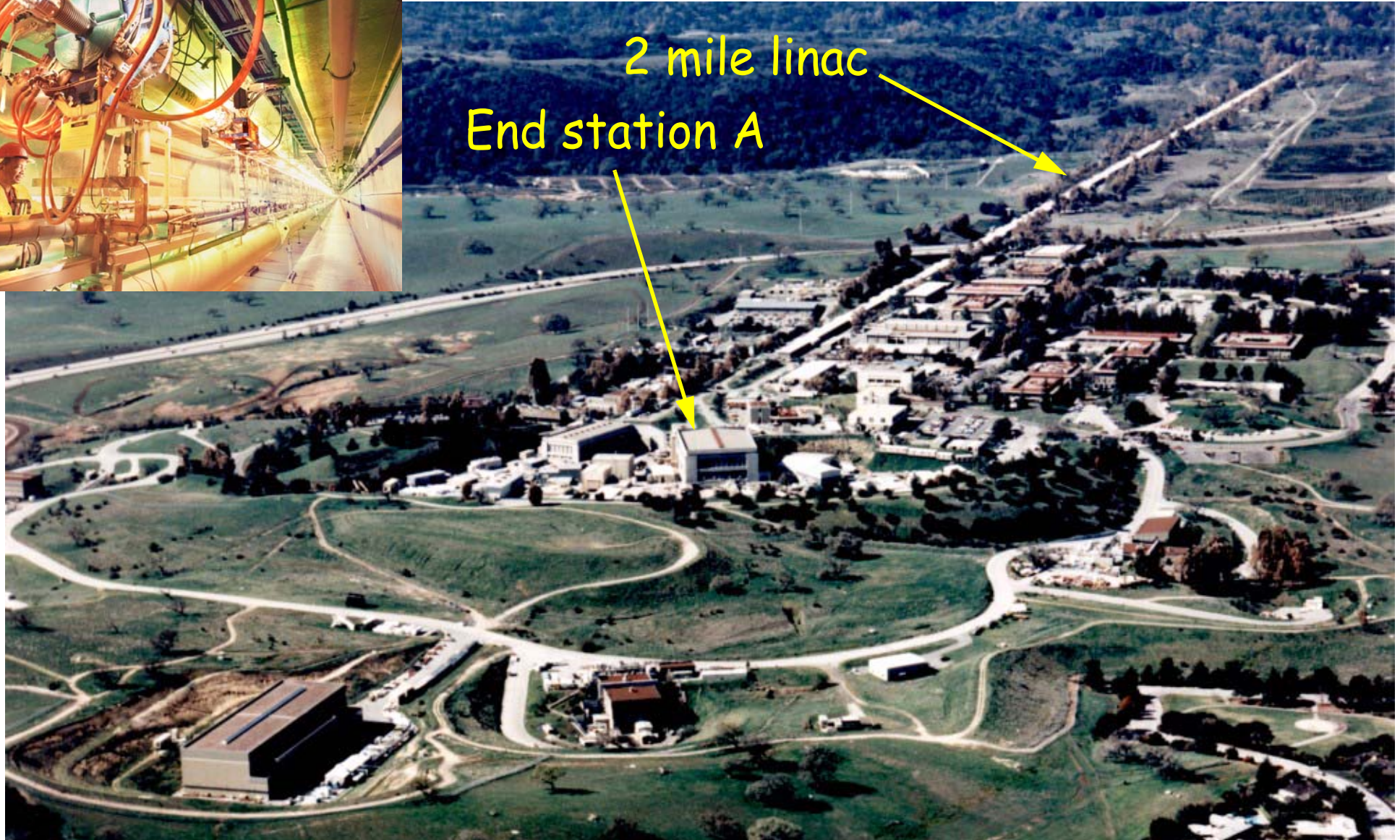
$$F_{1,2}(x, Q^2) = F_{1,2}(x) \quad \text{when } Q^2 \rightarrow \infty \text{ and } x \text{ is fixed and finite.}$$

i.e. the structure functions are **independent on Q^2** for $Q \gg M$.

- **Scaling**: $F_{1,2}(x, Q^2)$ do not change if masses, energies and momenta are multiplied by a scale factor.

Electron-proton scattering

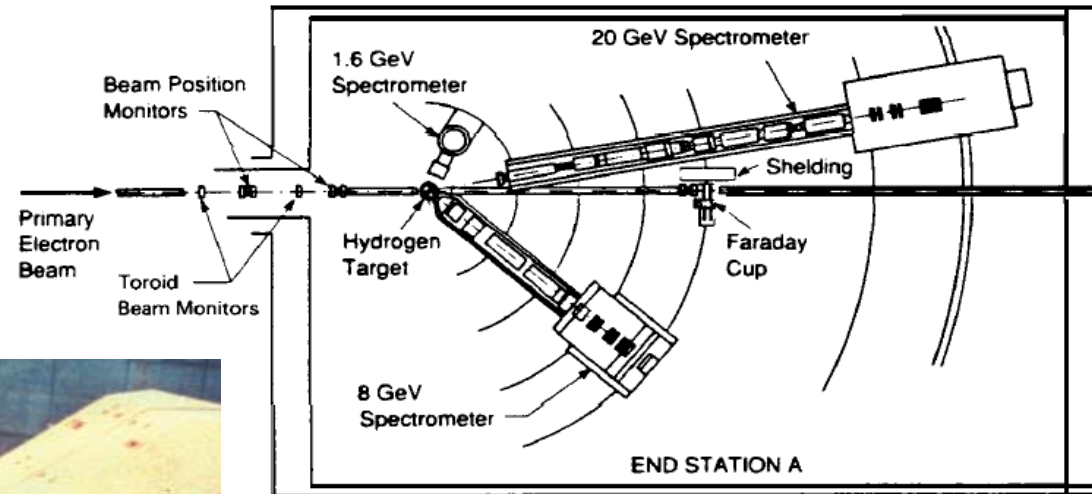
➔ The discovery of quarks at the SLAC 2 mile LINAC



Electron-proton scattering

➔ The discovery of quarks

The MIT-SLAC experiment

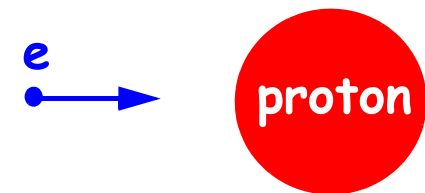


The 20 GeV spectrometer
The 8 GeV spectrometer

Magnets

Cerenkov detectors
Scintillators
Detectors for e/π separation

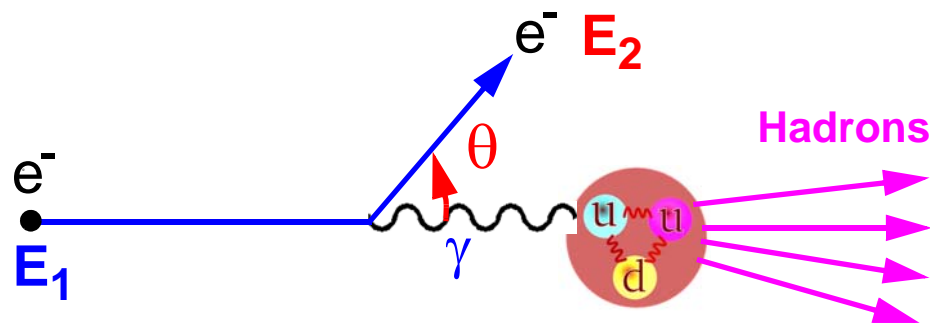
8 GeV electrons hits a hydrogen target



Electron-proton scattering

➔ The discovery of quarks

- Calculate x and Q^2 from the energy and angle of the electron.



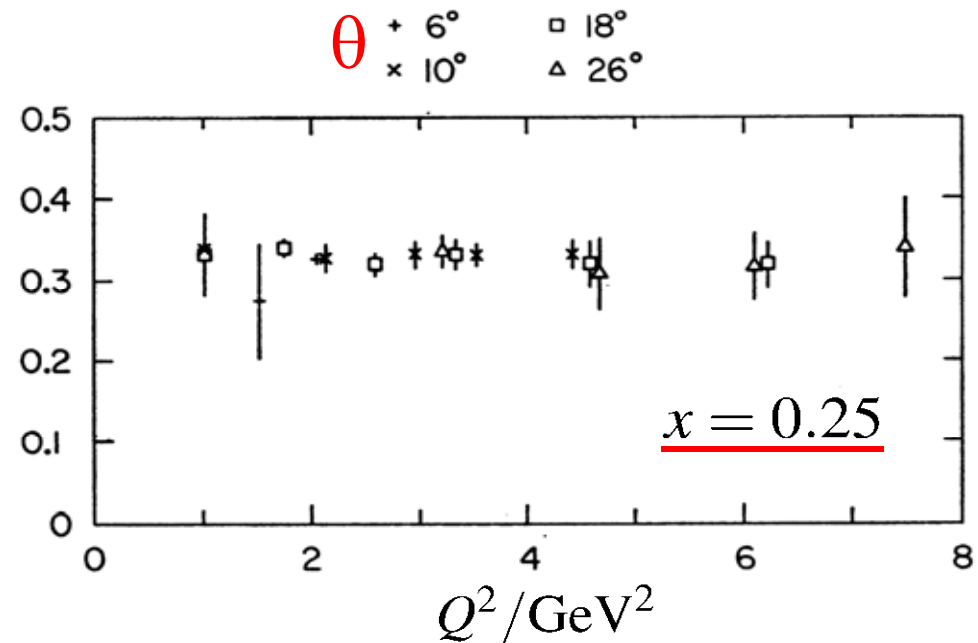
$$Q^2 = 4 \cdot E_1 \cdot E_2 \sin^2\left(\frac{\theta}{2}\right)$$

$$x = \frac{Q^2}{2 \cdot M \cdot (E_1 - E_2)}$$

- Cross section measurement ➔ F_2

- F_2 does not depend on Q^2

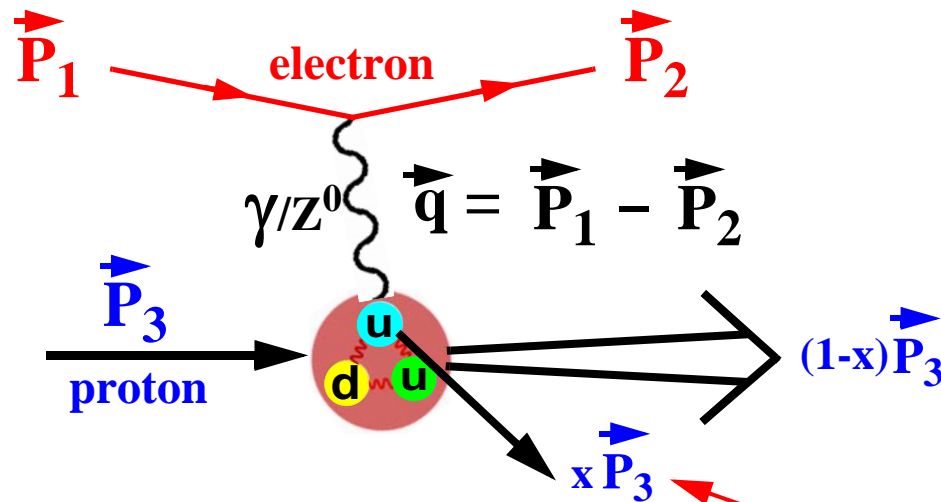
- Protons have a sub-structure (partons)



Electron-proton scattering

➔ Deep inelastic electron-proton scattering

- **The parton model:** Scale invariance \Rightarrow Scattering on point-like constituents (partons) in the proton.
- **The quark model:** Partons = Quarks



$$Q^2 = -\vec{q} \cdot \vec{q} = 4 \cdot E_1 \cdot E_2 \sin^2\left(\frac{\theta}{2}\right)$$

$$x = \frac{Q^2}{2Mv} = \frac{Q^2}{2 \cdot M \cdot (E_1 - E_2)}$$

- Parton/quark model \Rightarrow Fraction of the proton momentum carried by the struck quark is given by Bjorken x .

Electron-proton scattering

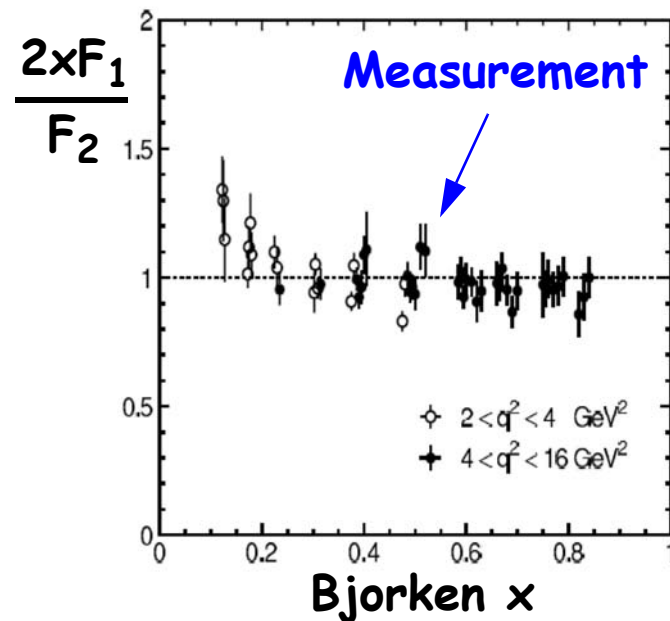
➔ Deep inelastic electron-proton scattering

- Parton model $\Rightarrow F_1$ depends on the spin of the partons (quarks)

Prediction: $F_1(x, Q^2) = 0$ quark spin = 0

The Callan-Gross relation: $2xF_1(x, Q^2) = F_2(x, Q^2)$ quark spin = 1/2

Measurement:



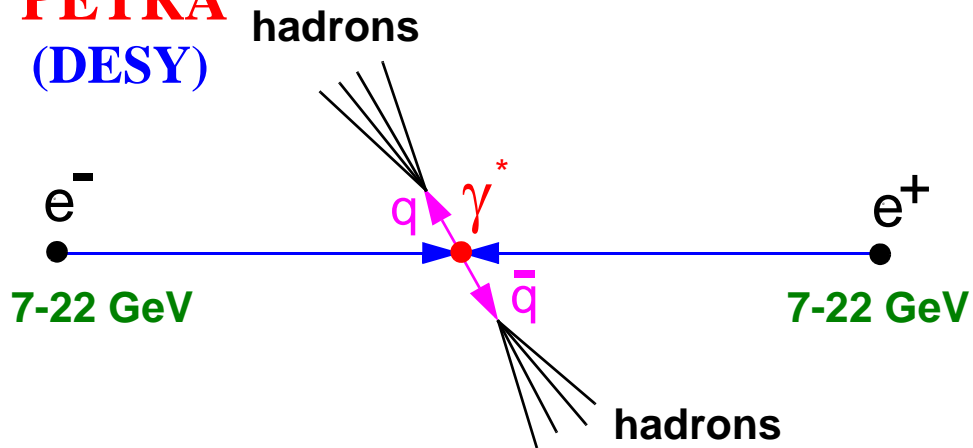
Theory prediction:

Spin = 1/2

Spin = 0

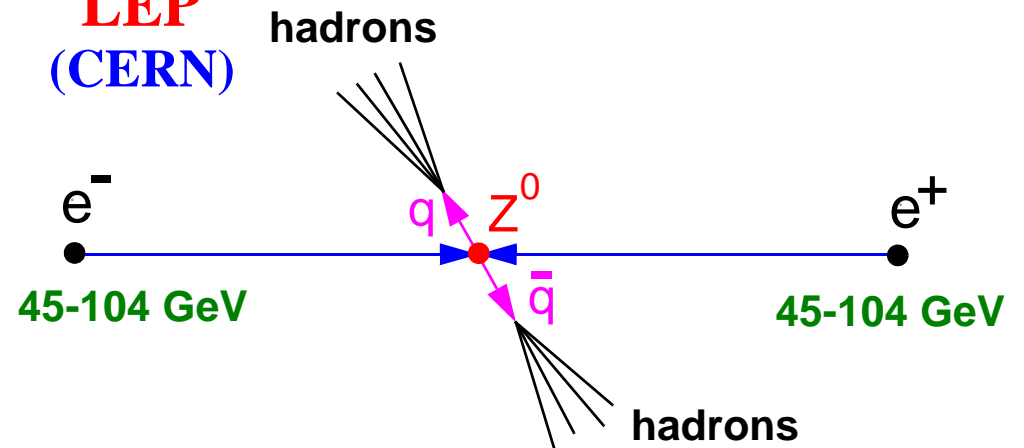
Electron-proton scattering

PETRA
(DESY)



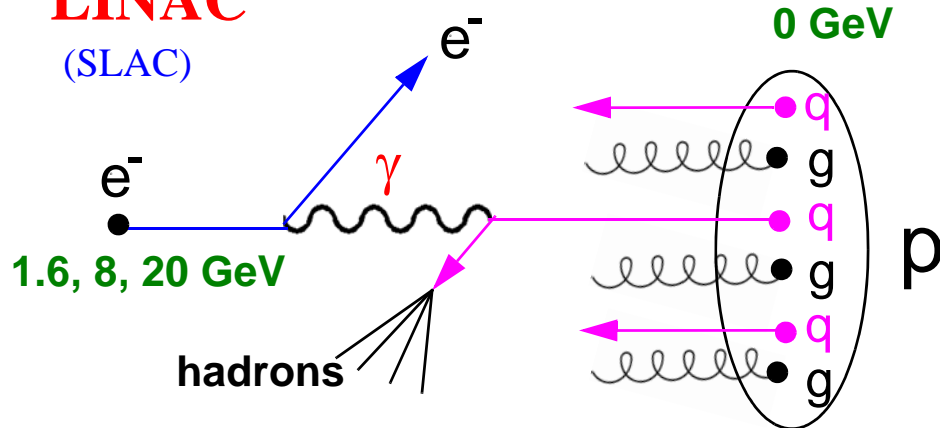
Length: 2.3 km
Experiments: Tasso, Jade, Pluto, Mark J, Cello

LEP
(CERN)



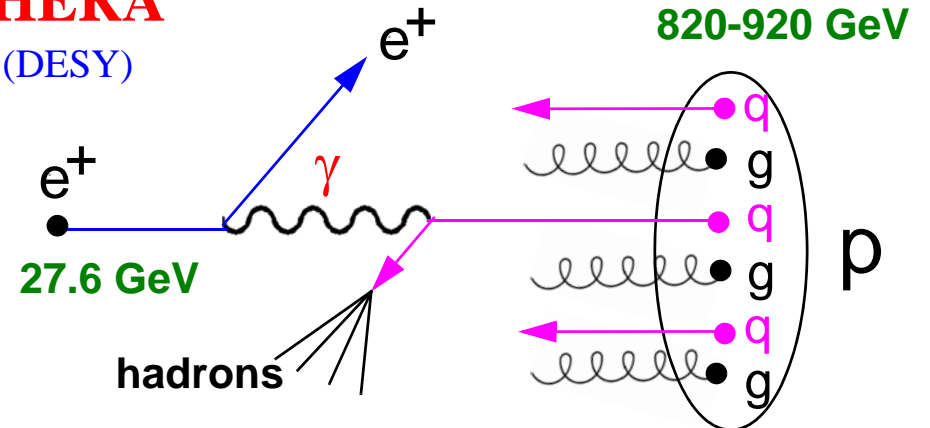
Length: 27 km (4184 magnets)
Experiments: DELPHI, OPAL, ALEPH, L3

LINAC
(SLAC)



Length: 3 km
Experiments: SLAC-MIT

HERA
(DESY)



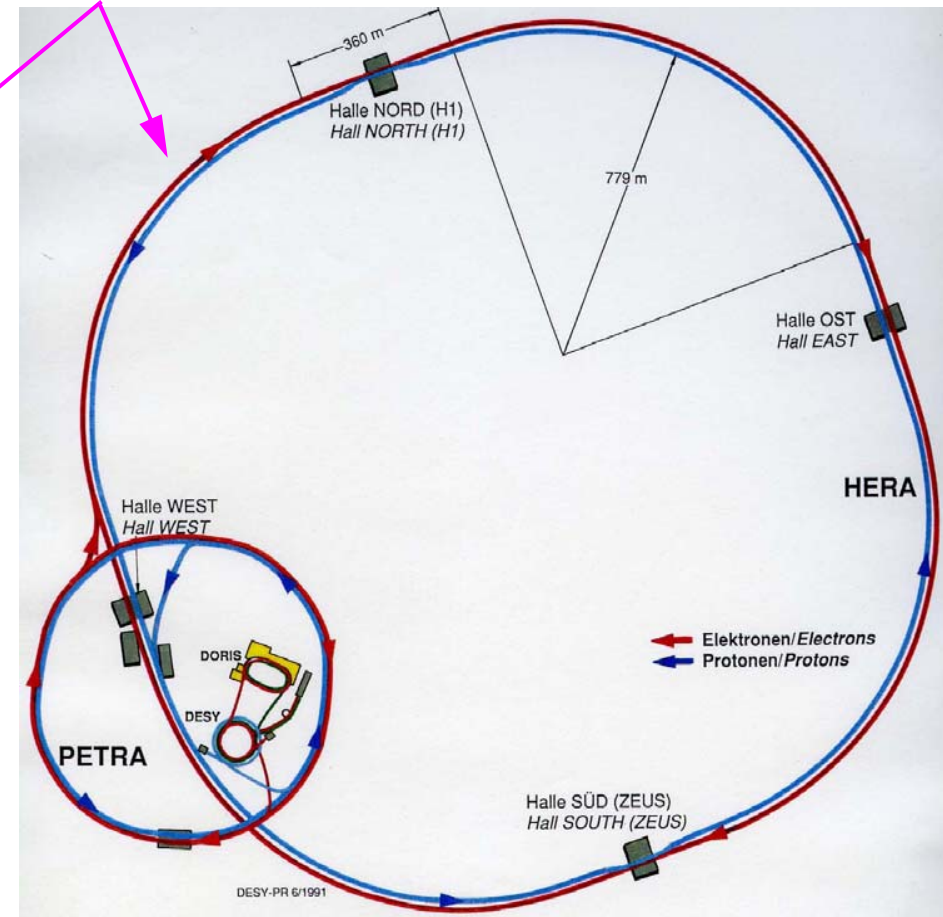
Length: 6 km (1650 magnets)
Experiments: H1, ZEUS

Electron-proton scattering

➔ The HERA accelerator at DESY

- **HERA accelerator:** only large electron-proton collider ever built.
- Petra was pre-accelerator.

6 km long



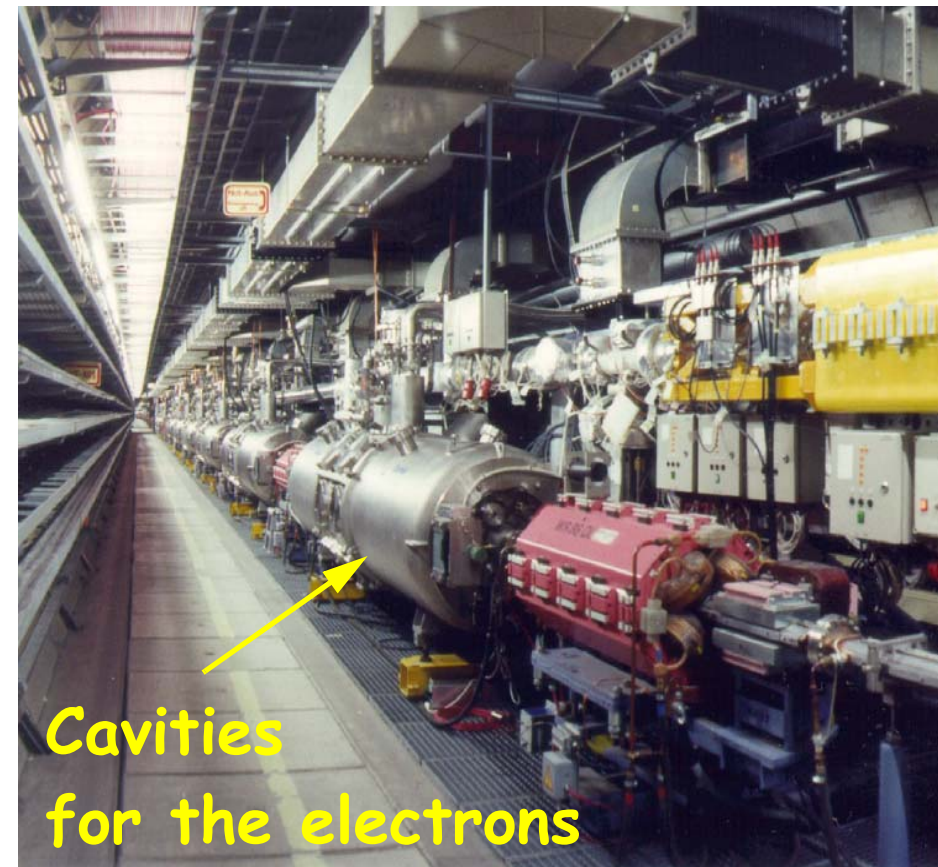
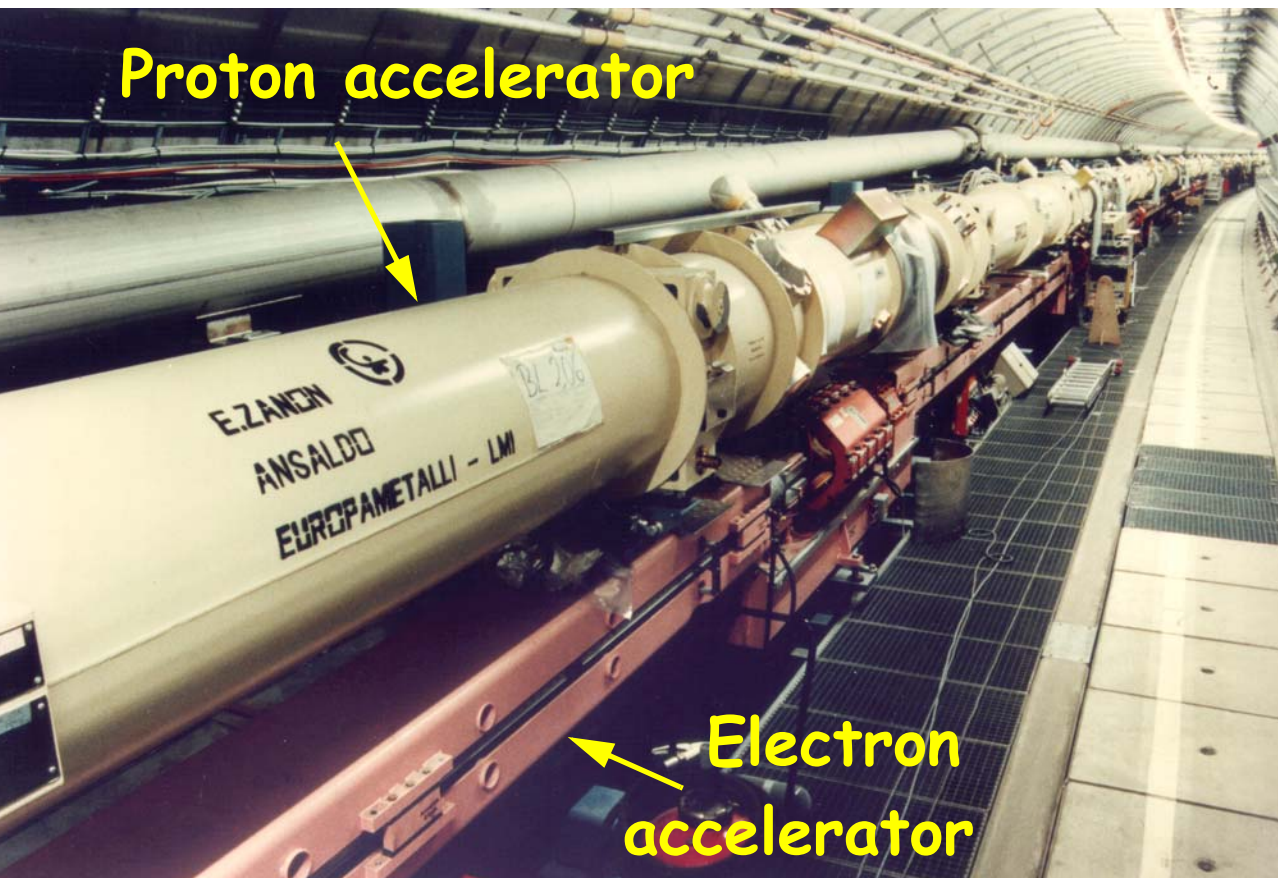
Electron-proton scattering

HERA

Proton accelerator: Super conducting magnets. Energy = 920 GeV

Electron accelerator: Normal warm magnets. Energy = 28 GeV

Collision energy = 320 GeV (54000 GeV fixed target)



Electron-proton scattering

➔ The H1 Experiment

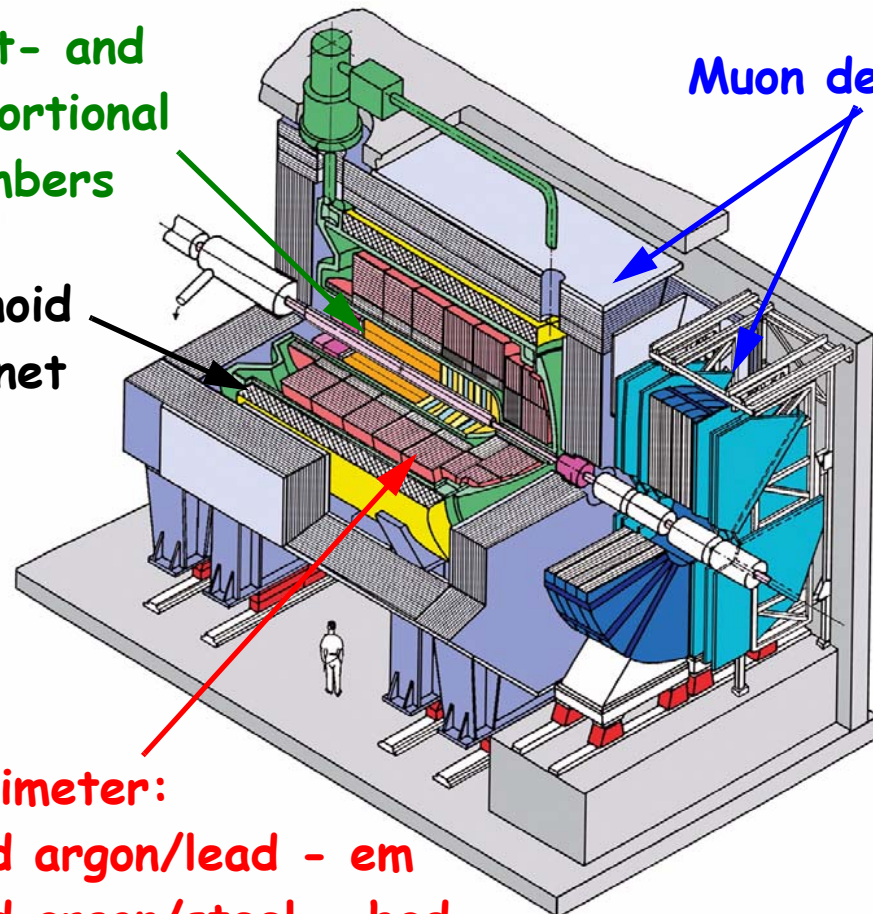
- Events at HERA were boosted in the proton direction due to the large difference in electron and proton beam energies.

Tracker:

Drift- and
Proportional
chambers

Muon detectors

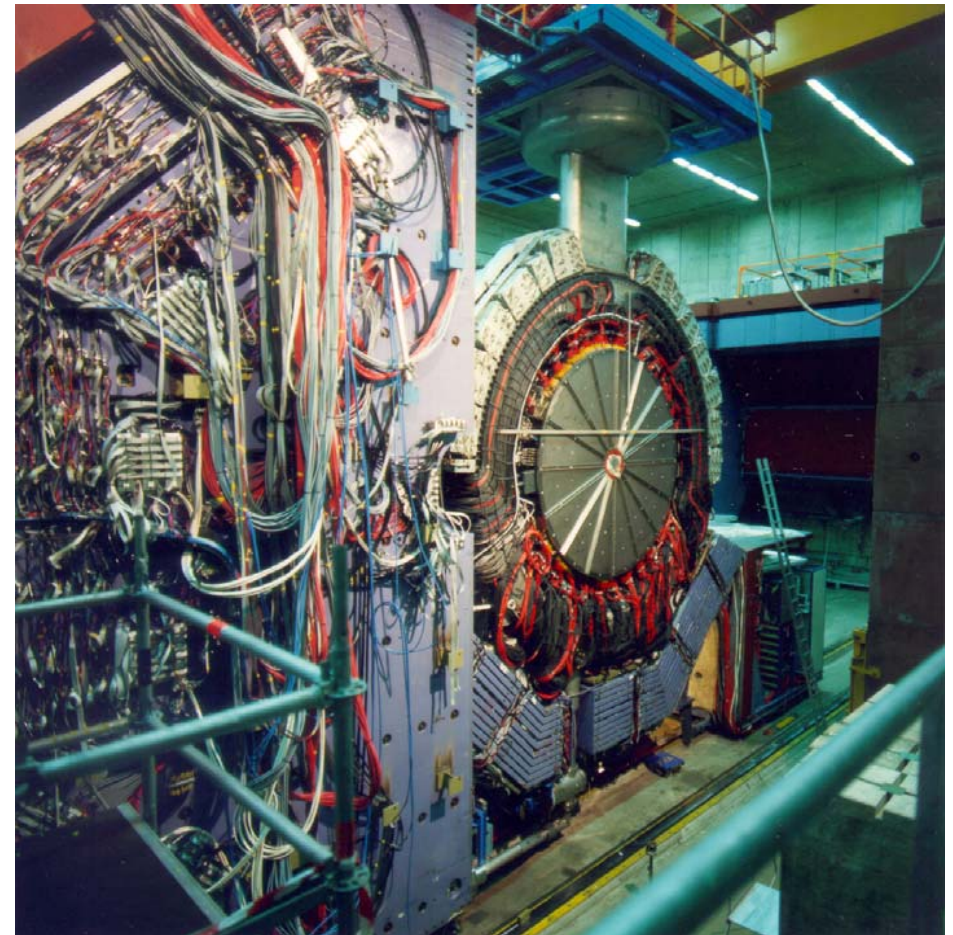
Solenoid
magnet



Calorimeter:

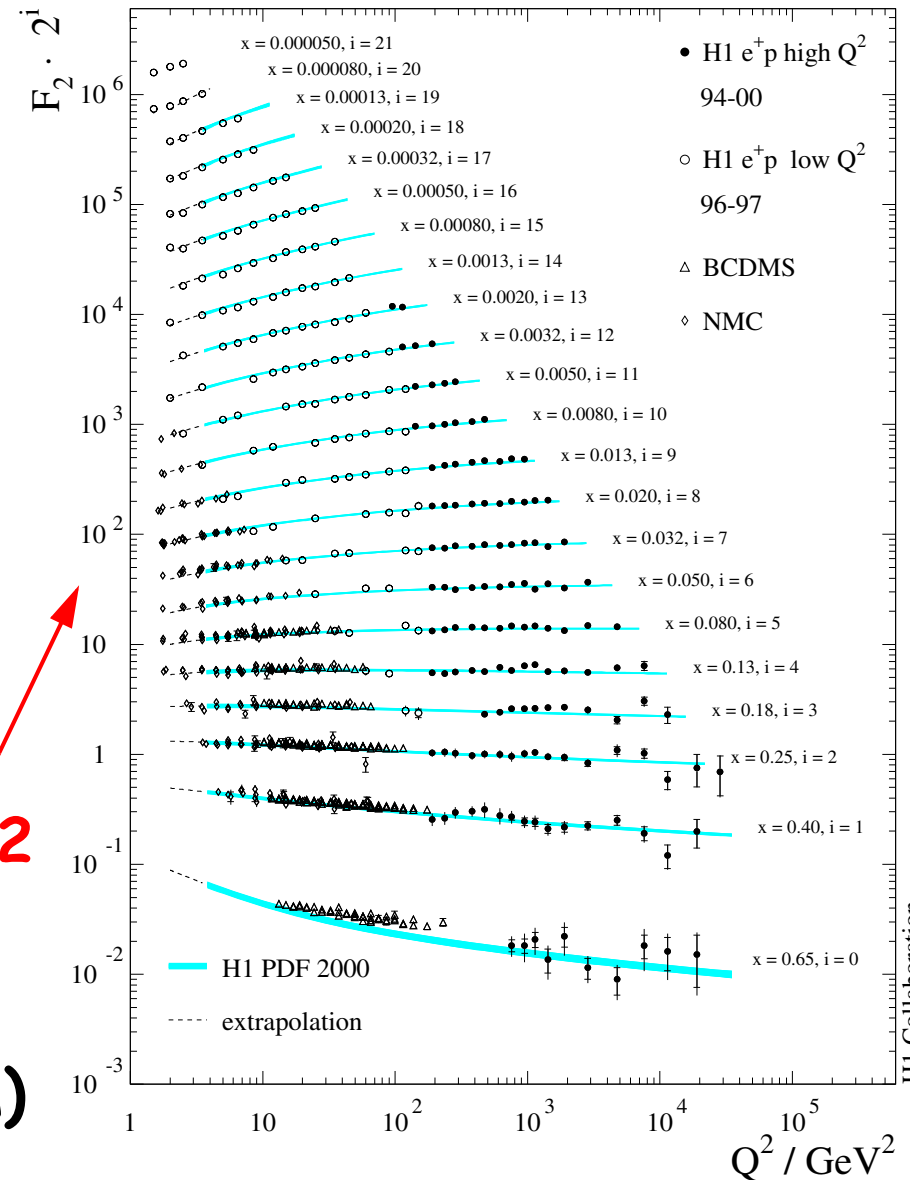
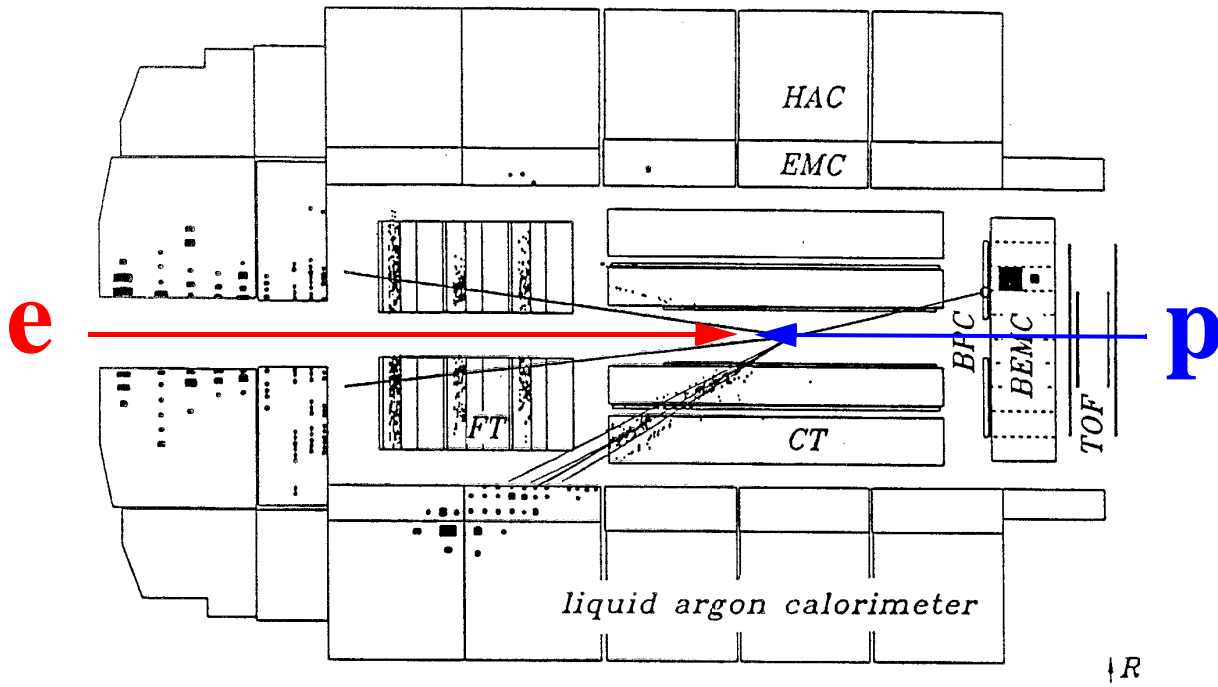
Liquid argon/lead - em

Liquid argon/steel - had



Electron-proton scattering

Measurement of structure functions

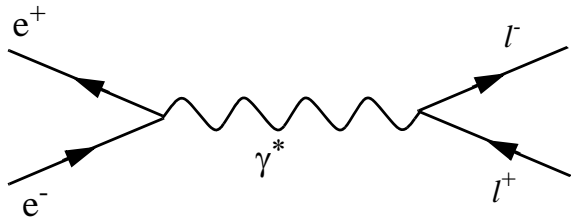


- The cross section + the energy and scattering angle of the electron $\Rightarrow F_2$
- No quark sub-structure was observed down to 10^{-18}m (1/1000th of a proton)

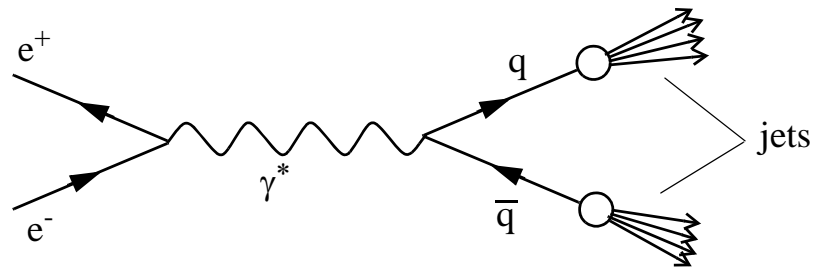
Summary of scattering formulas



SUMMARY: Electron-Positron interactions

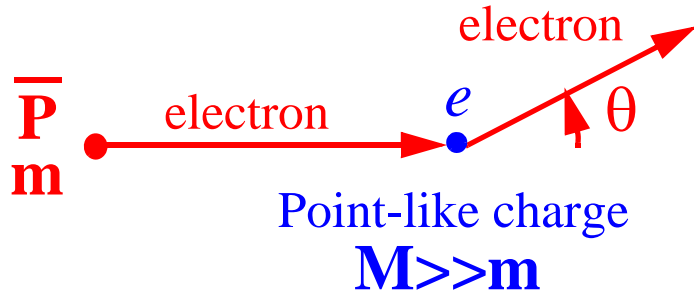


$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow l^+l^-) = \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta)$$

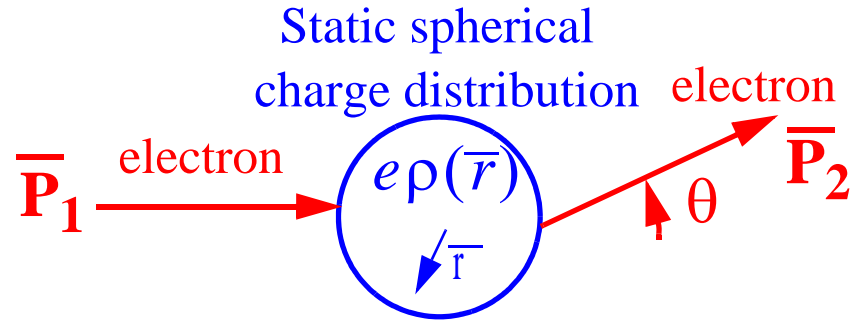


$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow q\bar{q}) = N_c e_q^2 \frac{2\pi\alpha^2}{2Q^2}(1 + \cos^2\theta)$$

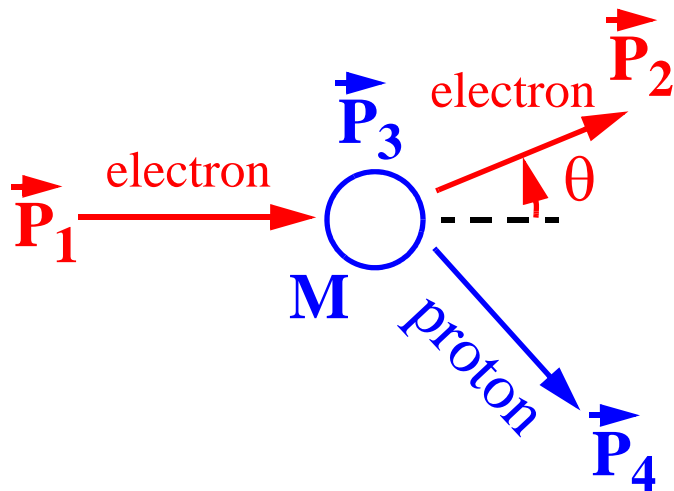
SUMMARY: Elastic electron-proton scattering



$$\left(\frac{d\sigma}{d\Omega}\right)_M = \frac{\alpha^2}{4p^4 \sin^4\left(\frac{\theta}{2}\right)} \left(m^2 + p^2 \cos^2\frac{\theta}{2}\right)$$



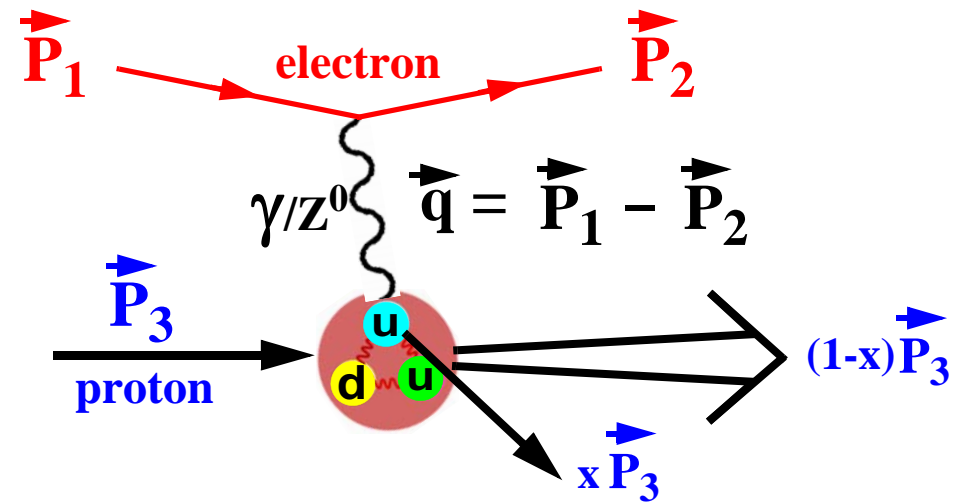
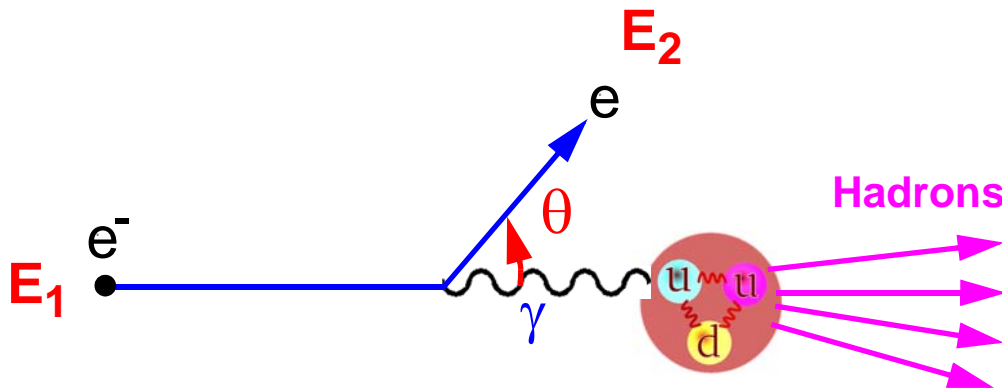
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_R G_E^2(q^2)$$



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_M \left(G_1(Q^2) \cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2} G_2(Q^2) \sin^2\frac{\theta}{2} \right)$$

$$G_1(Q^2) = \frac{G_E^2 + \frac{Q^2}{4M^2} G_M^2}{1 + \frac{Q^2}{4M^2}} \quad G_2(Q^2) = G_M^2$$

SUMMARY: Inelastic electron-proton scattering



$$Q^2 = -\vec{q} \cdot \vec{q}$$

$$x = \frac{Q^2}{2M\nu} \quad \text{Bjorken - } x$$

$$\frac{d\sigma}{dE_2 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)} \cdot \frac{1}{\nu} \cdot \left[F_2(x, Q^2) \cos^2\left(\frac{\theta}{2}\right) + \frac{Q^2}{xM^2} F_1(x, Q^2) \sin^2\left(\frac{\theta}{2}\right) \right]$$