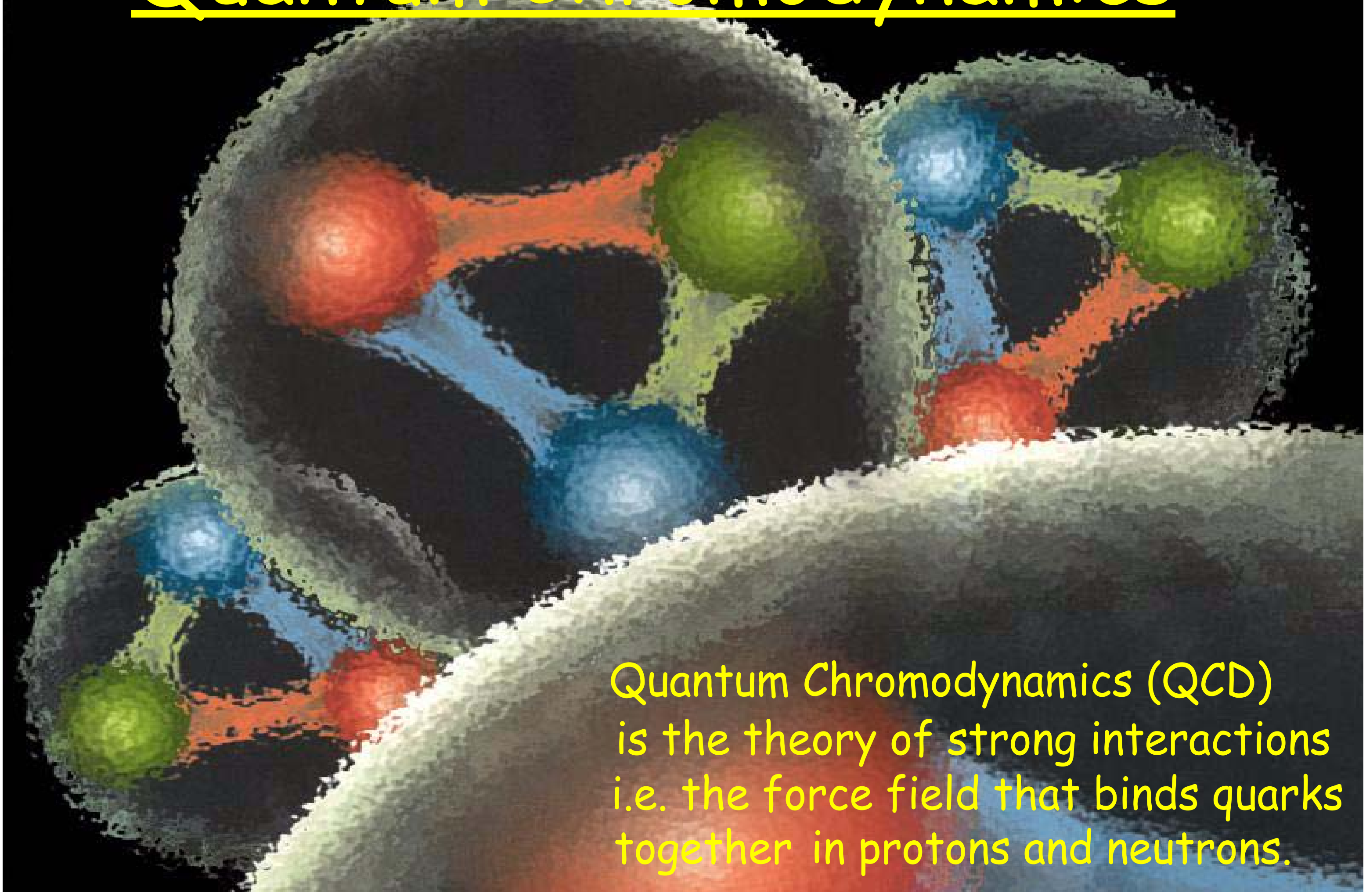
Quantum Chromodynamics



Quantum Chromodynamics

➡ Interactions are carried out by massless spin-1 particles called gauge bosons.

- In quantum electrodynamics (QED), gauge bosons are photons and in QCD they are called gluons.

- Gauge bosons couple to conserved charges:
  QED: Photons couple to electric charges (Q)
  QCD: Gluons couple to colour charges (Y^c and I_3^c).

- Y^c is called colour hypercharge.
  I_3^c is called colour isospin charge.

- The strong interaction acts the same on u,d,s,c,b and t quarks because the strong interaction is flavour-independent.

Quantum Chromodynamics

- The colour hypercharge (Y^c) and colour isospin charge (I_3^c) can be used to define three colour and three anti-colour states that the quarks can be in:

	Y^c	I_3^c		Y^c	I_3^c
r	1/3	1/2	$\bar{r}$	-1/3	-1/2
g	1/3	-1/2	$\bar{g}$	-1/3	1/2
b	-2/3	0	$\bar{b}$	2/3	0

- All observed states (all mesons and baryons) have a total colour charge that is zero. This is called colour confinement.

- Zero colour charge means that the hadrons have the following colour wave-functions:

$$q\bar{q} = \frac{1}{\sqrt{3}}(r\bar{r}+g\bar{g}+b\bar{b})$$

$$q_1q_2q_3 = \frac{1}{\sqrt{6}}(r_1g_2b_3-g_1r_2b_3+b_1r_2g_3-b_1g_2r_3+g_1b_2r_3-r_1b_2g_3)$$

Quantum Chromodynamics

- The colour hypercharge (Y^c) and colour isospin charge (I_3^c) should not be confused with the flavour hypercharge (Y) and flavour isospin (I_3) that were introduced in the quark model:

	Q	Y	I_3		Q	Y	I_3
d	-1/3	1/3	-1/2	$\bar{d}$	1/3	-1/3	1/2
u	2/3	1/3	1/2	$\bar{u}$	-2/3	-1/3	-1/2
s	-1/3	-2/3	0	$\bar{s}$	1/3	2/3	0
c	2/3	4/3	0	$\bar{c}$	-2/3	-4/3	0
b	-1/3	-2/3	0	$\bar{b}$	1/3	2/3	0
t	2/3	4/3	0	$\bar{t}$	-2/3	-4/3	0

- After introducing colour, the total wavefunction of hadrons can now be written as:

$$\Psi_{total} = \Psi_{space} \times \Psi_{spin} \times \Psi_{flavour} \times \Psi_{colour}$$

Quantum Chromodynamics

➡ Photons do not carry electric charge but gluons do carry colour charges themselves !

- The **gluons** can in fact exist in **8 different colour states** given by the following colour wave functions:

$\chi^C_{g1} = r\,\bar{g}$	$I^C_3 = 1$	$Y^C = 0$
$\chi^C_{g2} = \bar{r}\,g$	$I^C_3 = -1$	$Y^C = 0$
$\chi^C_{g3} = r\,\bar{b}$	$I^C_3 = 1/2$	$Y^C = 1$
$\chi^C_{g4} = \bar{r}\,b$	$I^C_3 = -1/2$	$Y^C = -1$
$\chi^C_{g5} = g\,\bar{b}$	$I^C_3 = -1/2$	$Y^C = 1$
$\chi^C_{g6} = \bar{g}\,b$	$I^C_3 = 1/2$	$Y^C = -1$
$\chi^C_{g7} = 1/\sqrt{2}\,(g\bar{g} - \bar{r}r)$	$I^C_3 = 0$	$Y^C = 0$
$\chi^C_{g8} = 1/\sqrt{6}\,(g\bar{g} - r\bar{r} - 2\,b\bar{b})$	$I^C_3 = 0$	$Y^C = 0$

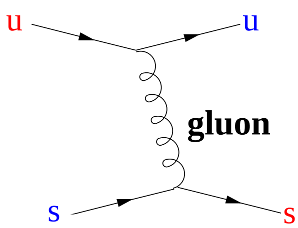
- Gluons do **not** exist a **free particles** since they have colour charge.

Quantum Chromodynamics

- The colour hypercharge and colour isospin charge are **additive quantum numbers** like the electric charge. The gluon colour charge in the following process can therefore be easily calculated:

Example:

$I^C_3 = 1/2 \quad Y^C = 1/3$ (u in) $\qquad I^C_3 = 0 \quad Y^C = -2/3$ (u out)



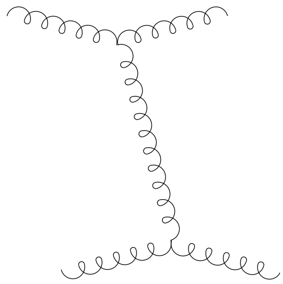
$I^C_3 = 0 \quad Y^C = -2/3$ (s in) $\qquad I^C_3 = 1/2 \quad Y^C = 1/3$ (s out)

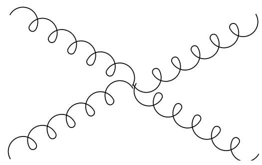
Gluon:
$$I^C_3 = I^C_3(r) - I^C_3(b) = \frac{1}{2}$$
$$Y^C = Y^C(r) - Y^C(b) = 1$$

$$\chi^C_{g3} = r\,\bar{b}$$

Quantum Chromodynamics

➡ Gluons can couple to other gluons since they carry colour charge.



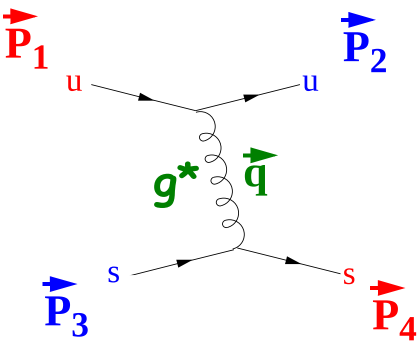


- This means that gluons can in principle bind together to form colourless states.
- These gluon states are called glueballs.

Quantum Chromodynamics

➡ The strong coupling constant

- The strong couplings constant α_s is the analogue in QCD of α_{em} in QED and it is a measure of the strength of the interaction.
- It is not a true constant but a "running constant" since it decreases with increasing Q^2.
- What is Q^2 ?



Assume that the 4-vectors of the interacting quarks are given by $\vec{P} = (E,\bar{p}) = (E,p_x,p_y,p_z)$

The 4-vector energy-momentum tranfer is then

$$Q^2 = -\vec{q} \bullet \vec{q}$$ (i.e. Q is the "mass" of the gluon)

which can be calculated from the 4-vectors of the quarks

$$\vec{q} = (E_q,\bar{q}) = \vec{P}_1 - \vec{P}_2 = (E_1 - E_2, \overline{P_1} - \overline{P_2})$$

Quantum Chromodynamics

➡ The strong coupling constant

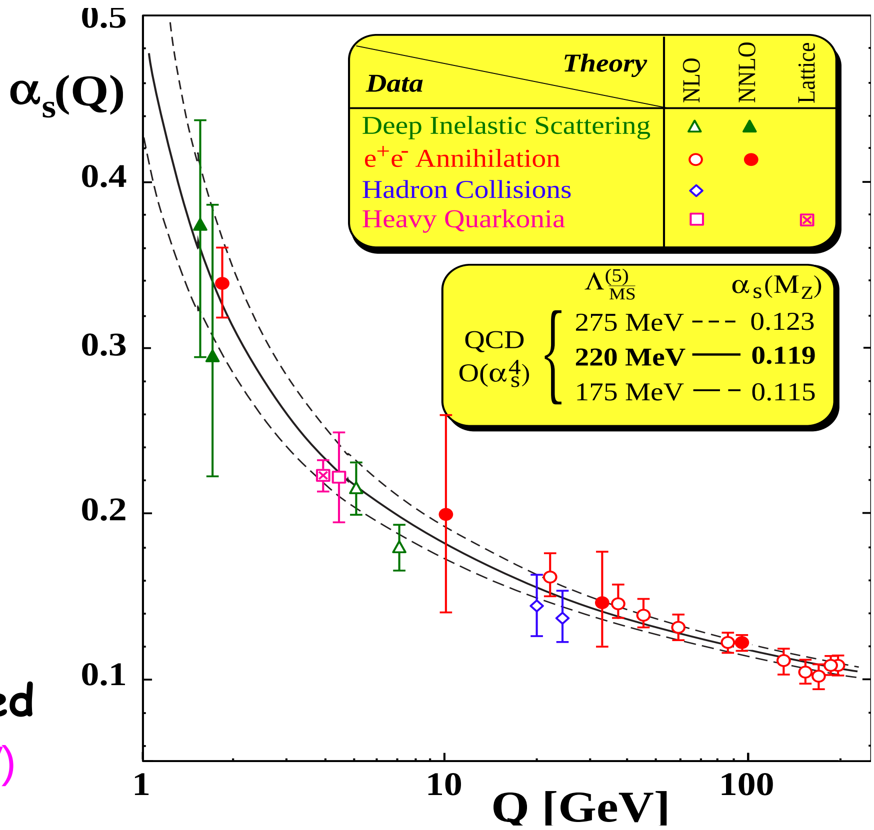
In leading order of QCD, α_s is given by:

$$\alpha_s = \frac{12\pi}{(33 - 2N_f)\ln(Q^2/\Lambda^2)}$$

where

N_f: Number of allowed quark flavours

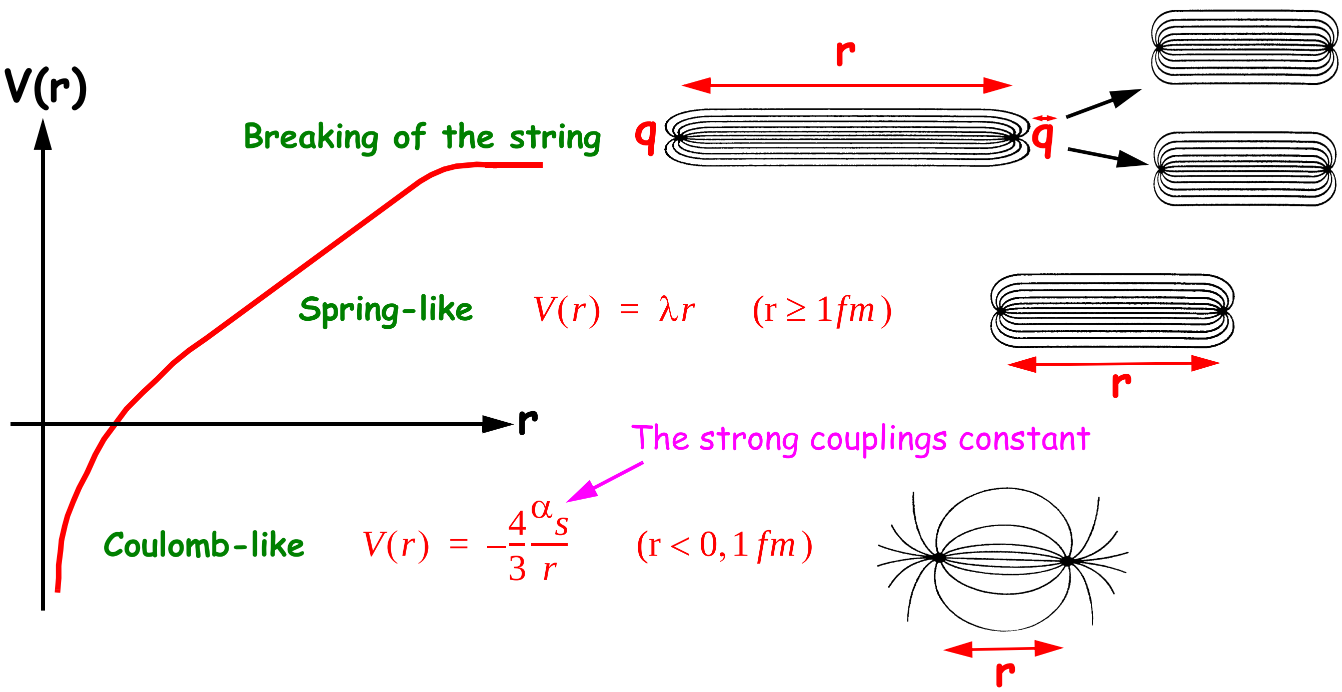
Λ: QCD scale parameter that has to be determined experimentally ($\Lambda \approx 0.2$ GeV)



Quantum Chromodynamics

➡ The quark-antiquark potential (mesons)

- The **quark-antiquark potential** can be described in the following simplified way:

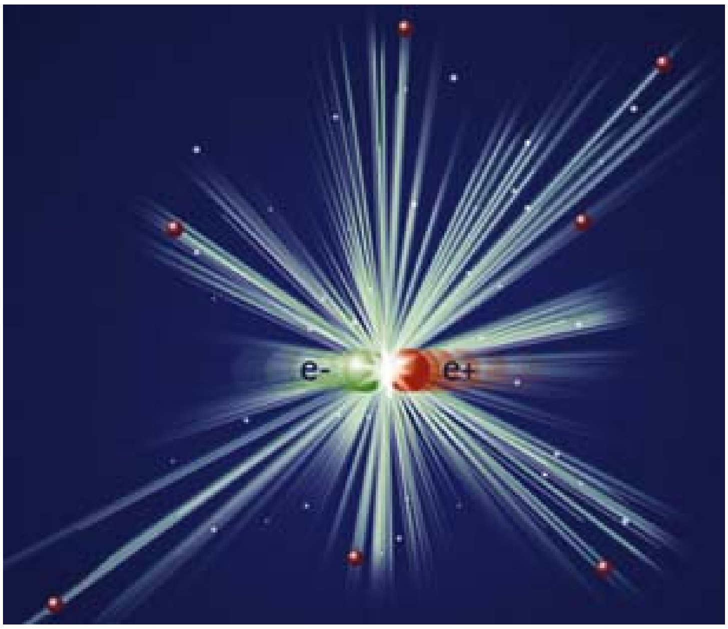


Quantum Chromodynamics

➡ The principle of asymptotic freedom.

- At short distances the strong interaction is weaker and at large distance the interaction gets stronger.
- The combination of a Coulomb-like potential at small distances and a small α_s at large Q^2 (i.e. small distances) means that quarks and gluons act as essentially free particles and interactions can be described by the lowest order diagrams.
- At large distances the strong interaction can, however, only be described by higher order diagrams.
- Due to the complexity of the higher-order diagrams, the very process of confinement cannot be calculated analytically. Only numerical models can be used !

Electron-positron collisions



Electron-positron annihilation

➡ The R-value

- At e+e- colliders one has traditionally studied the ratio of the number of events with hadrons to those with muons:

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- The cross section for hadron and muon production would be almost the same if it was not for quark flavours and colours i.e.

$$R = N_c \sum e_q^2$$

where N_c is the number of colours (=3) and e_q the charge of the quarks.

Electron-positron annihilation

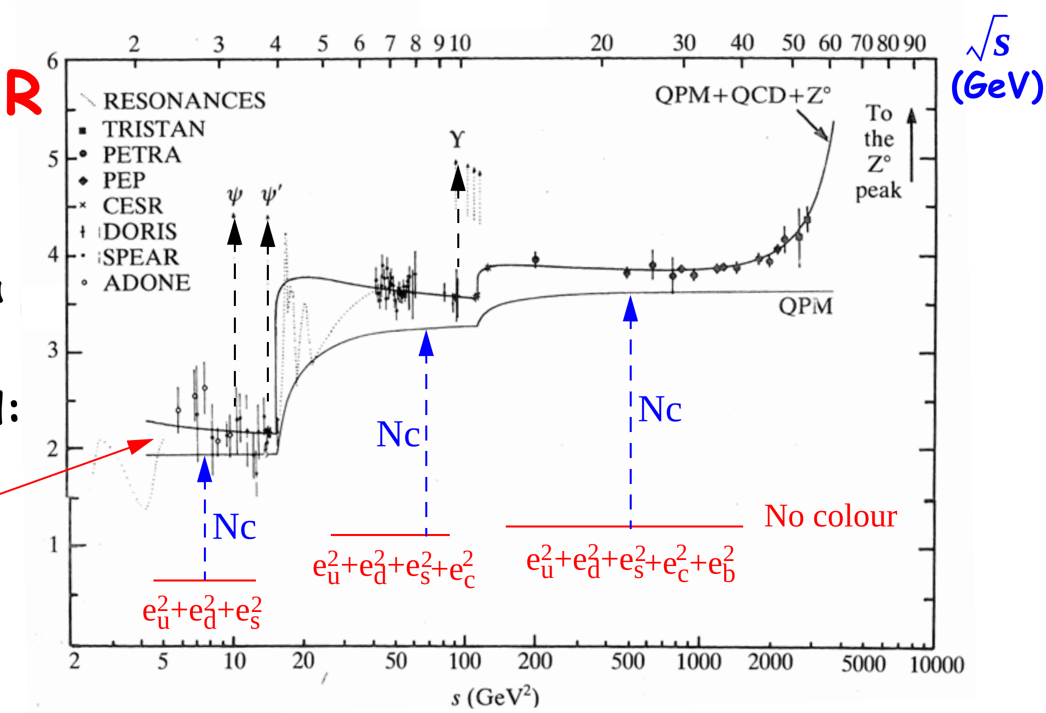
$$R = N_c(e_u^2+e_d^2+e_s^2) = 3\,((-1/3)^2+(-1/3)^2+(2/3)^2) = 2 \quad \text{if } \sqrt{s} < m_\psi$$

$$R = N_c(e_u^2+e_d^2+e_s^2+e_c^2) = 10/3 \quad \text{if } \sqrt{s} < m_\Upsilon$$

$$R = N_c(e_u^2+e_d^2+e_s^2+e_c^2+e_b^2) = 11/3 \quad \text{if } \sqrt{s} > m_\Upsilon$$

If the radiation of hard gluons is taken into account, an extra factor proportional to α_s has to be added:

$$R = 3\sum_q e_q^2\left(1 + \frac{\alpha_s(Q^2)}{\pi}\right)$$



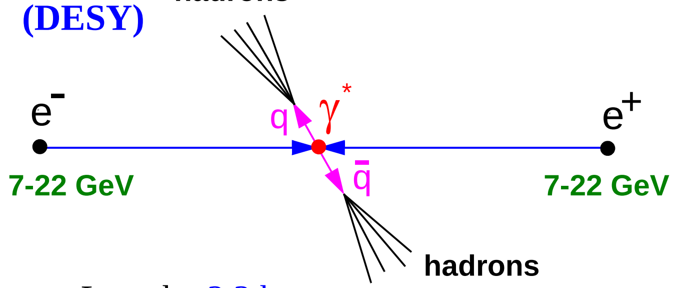
Electron-positron annihilation

➡ Jets of particles

- **In the lowest order e^+e^- annihilation process, a photon or a Z^0 is produced which then converts into a quark-antiquark pair.**

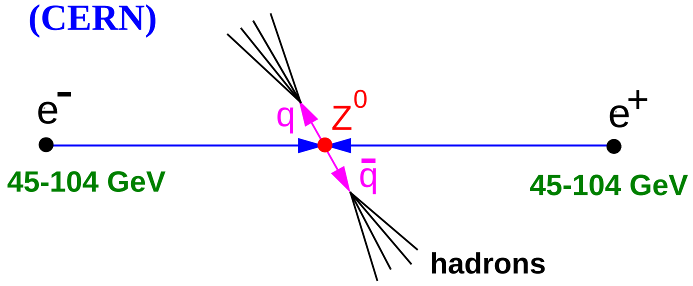
- **The quark and the antiquark fragment into observable hadrons.**

PETRA (DESY)



Length: 2.3 km
Experiments: Tasso, Jade, Pluto, Mark J, Cello

LEP (CERN)



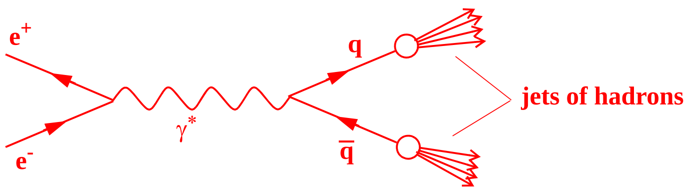
Length: 27 km (4184 magnets)
Experiments: DELPHI, OPAL, ALEPH, L3

Electron-positron annihilation

➡ Jets of particles

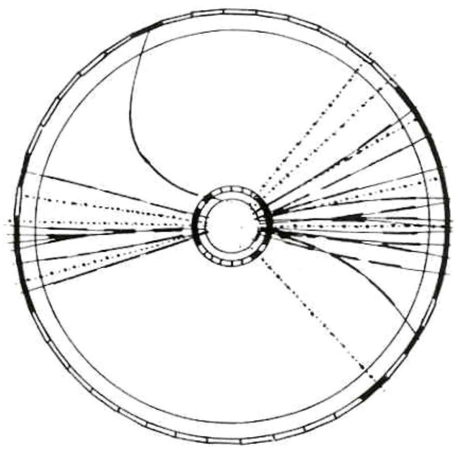
- **Since the quark and antiquark momenta are equal and counter-paralell, the hadrons are produced in two jets of equal energy going in the opposite direction.**

- **The direction of the jet reflects the direction of the corresponding quark.**



$$e^+ + e^- \to \gamma^* \to \text{hadrons}$$

Diagram for two-jet events.



Two-jet event recorded by the Jade experiment at PETRA.

Electron-positron annihilation

➡ A study of the angular distribution of jets give information about the spin of the quarks.

- The angular distribution of $e^+ + e^- \to \gamma^* \to \mu^+ + \mu^-$ is

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to \mu^+\mu^-) = \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta)$$

where θ is the production angle with respect to the direction of the colliding electrons.

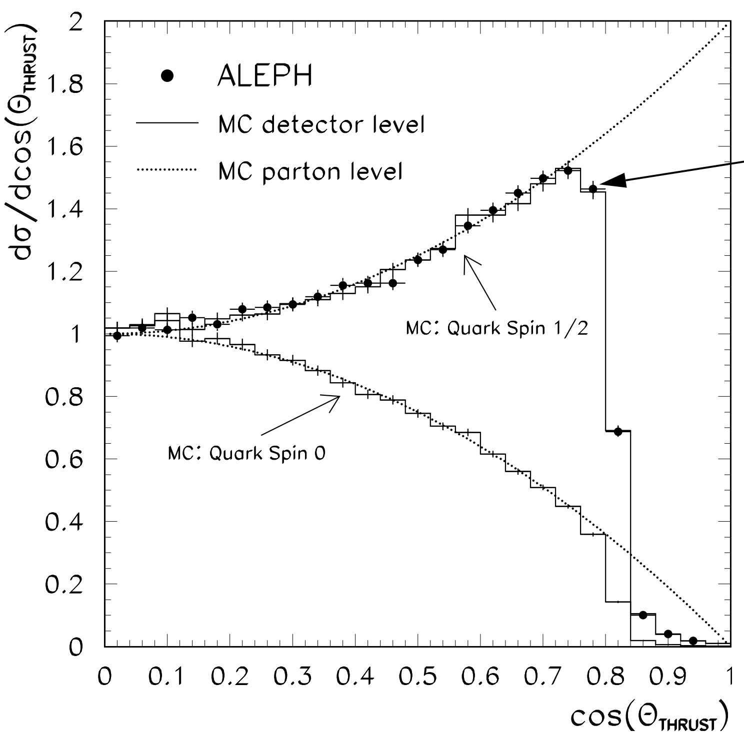
- The angular distribution of $e^+ + e^- \to \gamma^* \to q + \bar{q}$ is

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to q\bar{q}) = N_c e_q^2 \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta)$$ if the quark spin = 1/2

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to q\bar{q}) = N_c e_q^2 \frac{\pi\alpha^2}{2Q^2}(1 - \cos^2\theta)$$ if the quark spin = 0

where e_q is the fractional quark charge and N_c is the number of colours (=3).

Electron-positron annihilation



The experimentally measured angular distribution of jets is clearly following $(1+\cos^2\theta)$.

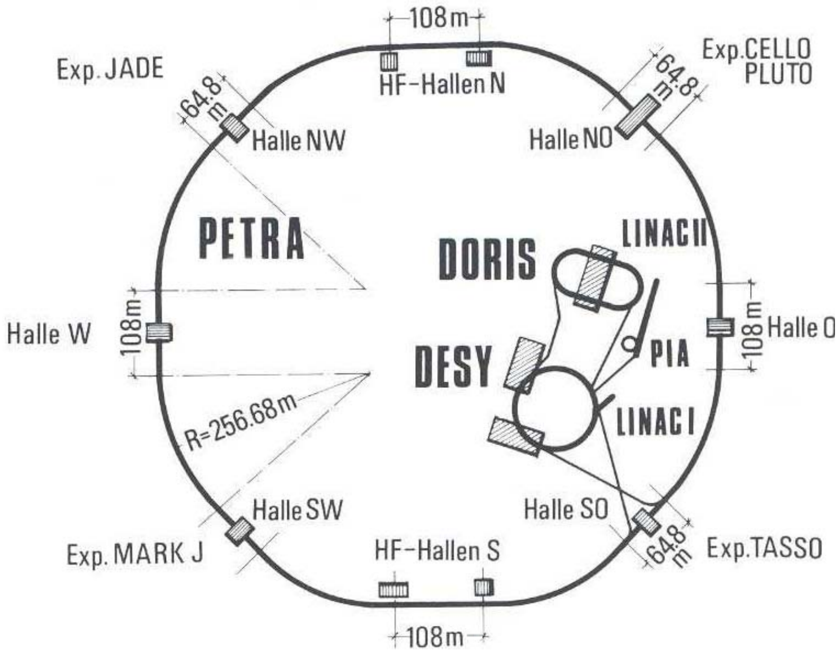
The jets are therefore associated with spin 1/2 particles.

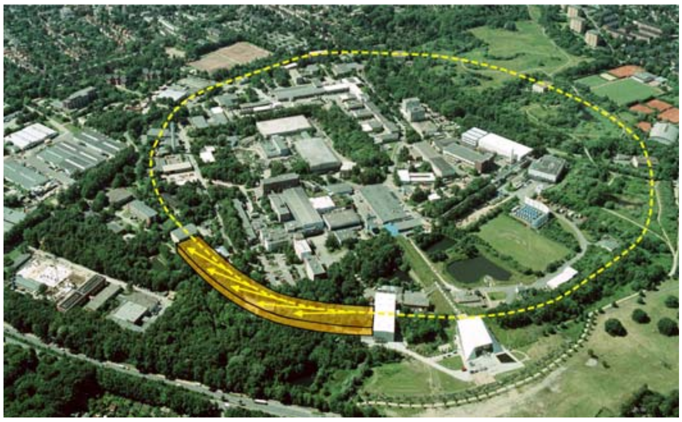
Quarks have spin = 1/2 !

The angular distribution of the quark jets in e^+e^- annihilations, compared with models with spin=0 and 1/2.

The discovery of the gluon

➡ The accelerator: PETRA at the German laboratory DESY.

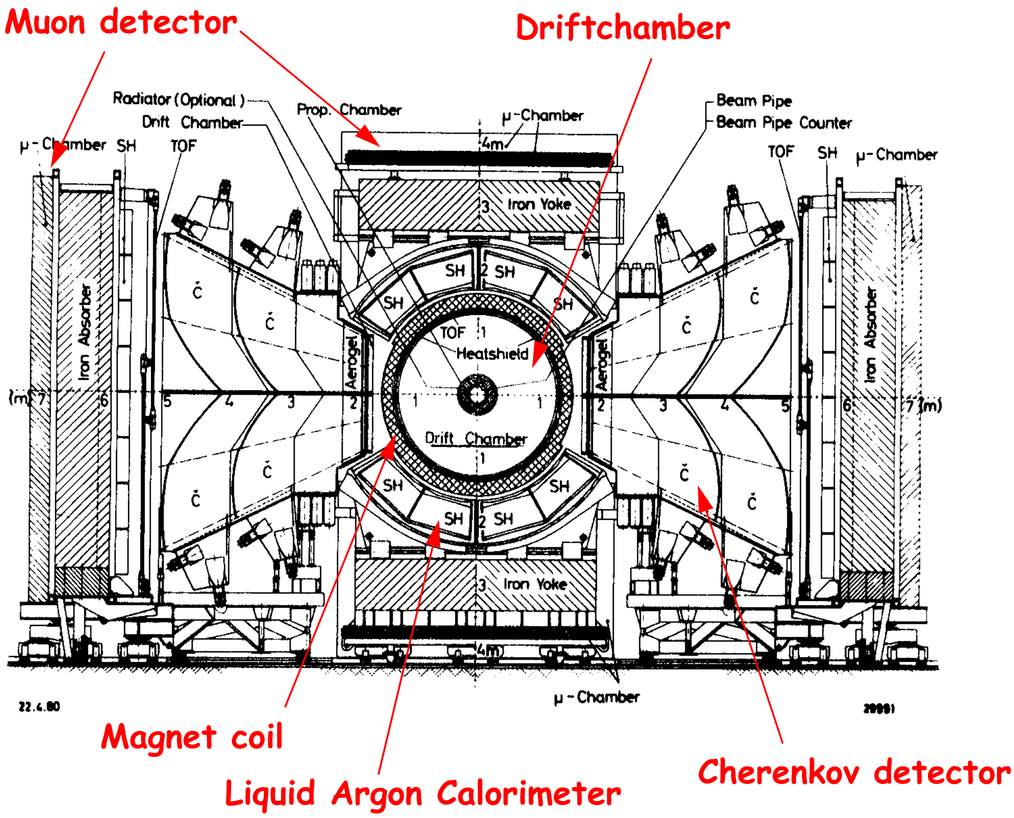




The discovery of the gluon

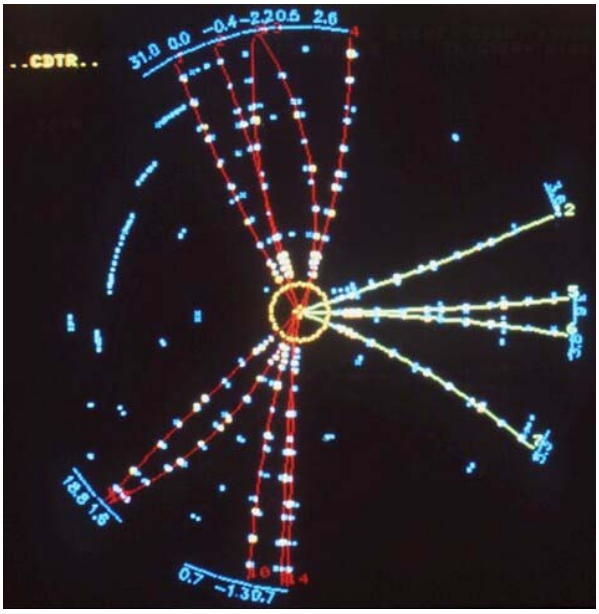
➡ The experiment: TASSO

The central part of the TASSO experiment.

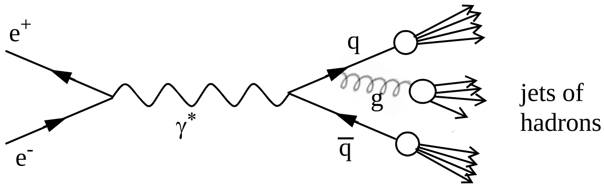


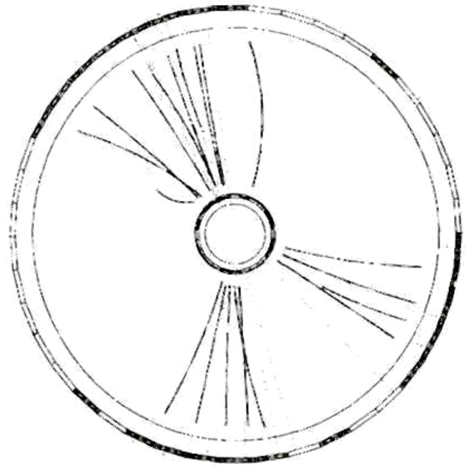
The discovery of the gluon

When the PETRA accelerator started up, one began to see **three-jet events** in the experiments. The interpretation was that the quark or antiquark emitted a high-momentum **gluon** that fragmented to a jet.



A Tasso 3-jet event.





A Jade 3-jet event.

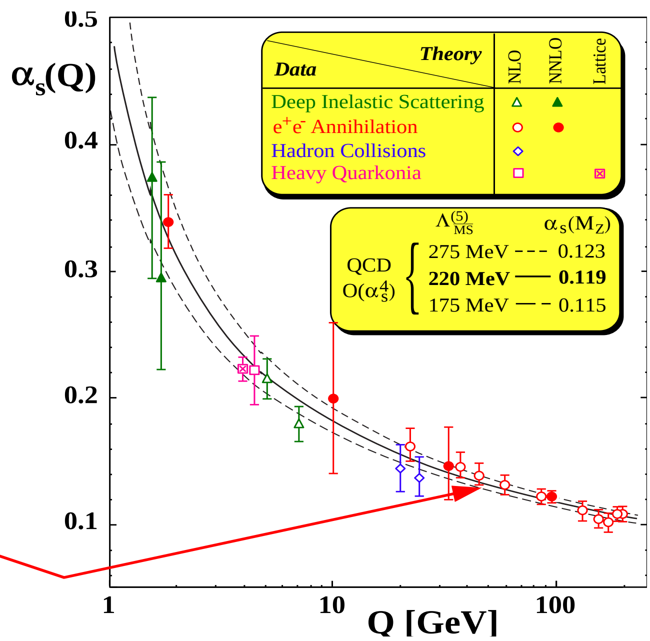
The discovery of the gluon

- The **probability** for a quark to emit a **gluon** is proportional to α_s and by comparing the rate of two-jet with three-jet events one can determine α_s.

$$\alpha_s = \frac{\text{Number of three-jet events}}{\text{Number of two-jet events}}$$

- At PETRA one measured:

$\alpha_s = 0.15 \pm 0.03$ for $\sqrt{s}$ = 30-40 GeV



Electron-positron annihilation

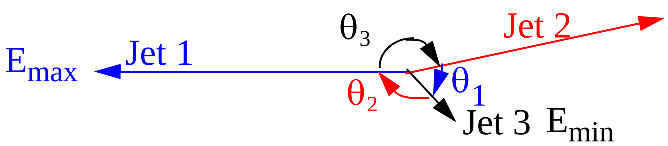
➡ The spin of the gluon.

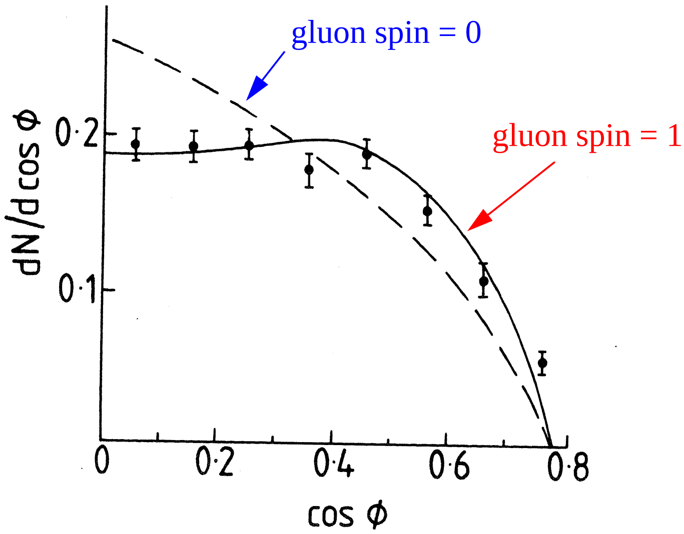
- It is possible to determine the **spin** of the **gluon** by measuring the angular distribution of jets in three-jet events.

- This is done by measuring:

$$\cos\phi = \frac{\sin\theta_2 - \sin\theta_3}{\sin\theta_1}$$

where the angles are defined in the following way.

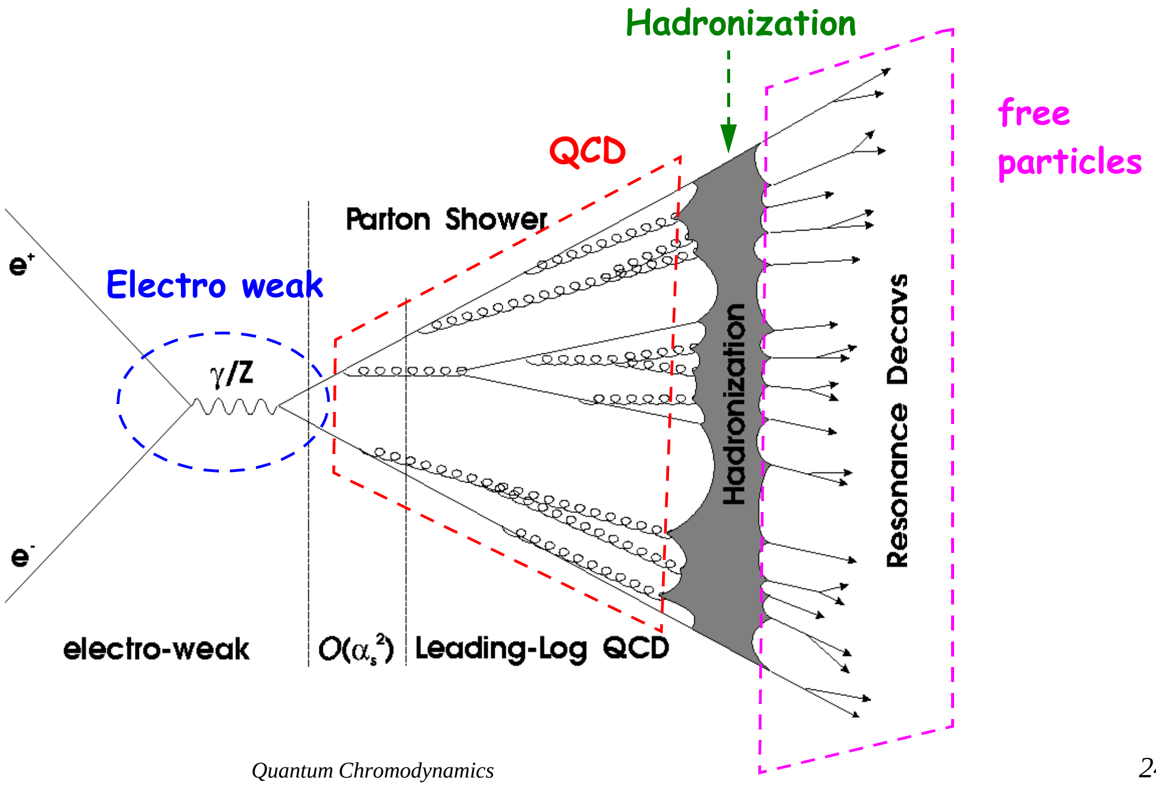




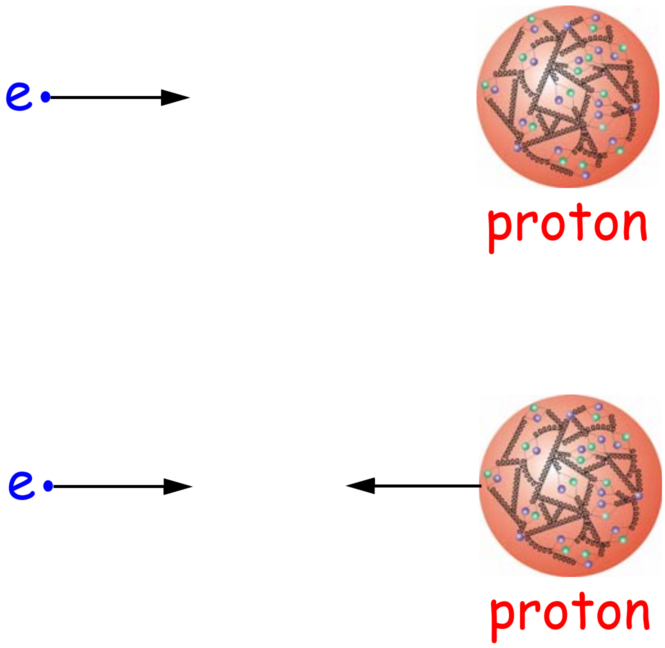
Conclusion: Gluons have spin = 1 !

Electron-positron annihilation

- The process of turning quarks and gluons into hadrons is called **hadronization**



Electron-proton collisions



Electron-proton scattering

Electrons are good tools for investigating the properties of hadrons since electrons do not have a substructure. The wavelength of the exchanged photon determines how the proton is being probed.

$\lambda >> r_p$ Very low electron energies

The scattering is equivalent to that from a "point-like" spin-less object.

$\lambda = r_p$ Low electron energies

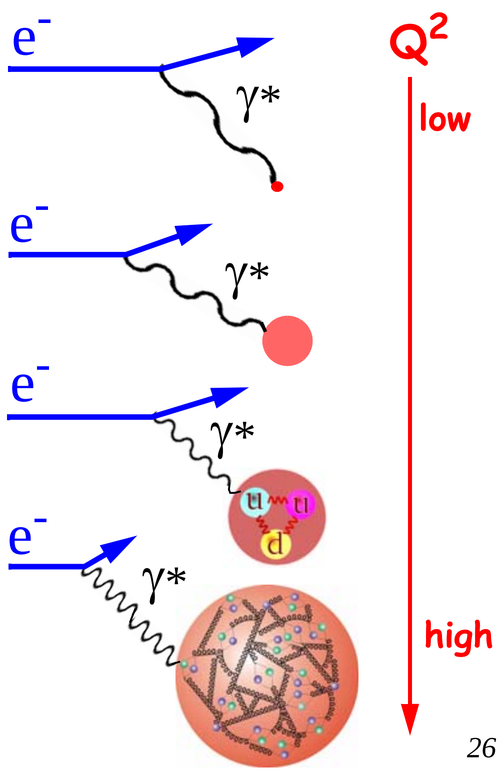
The scattering is equivalent to that from an extended charged object.

$\lambda < r_p$ High electron energies

The wavelength is short enough to make it possible to interact with the valence quarks in the proton.

$\lambda << r_p$ Very high electron energies

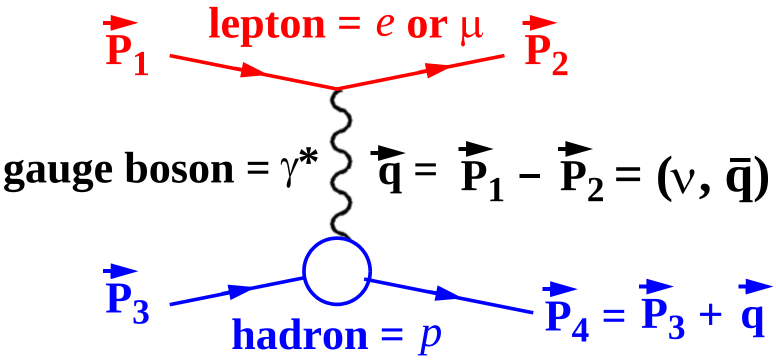
The electron can at these short wavelengths interact with the sea of quarks and gluons.



Electron-proton scattering

➡ Elastic scattering

- **Elastic scattering** means that the same type of particles goes into and comes out of the collision.



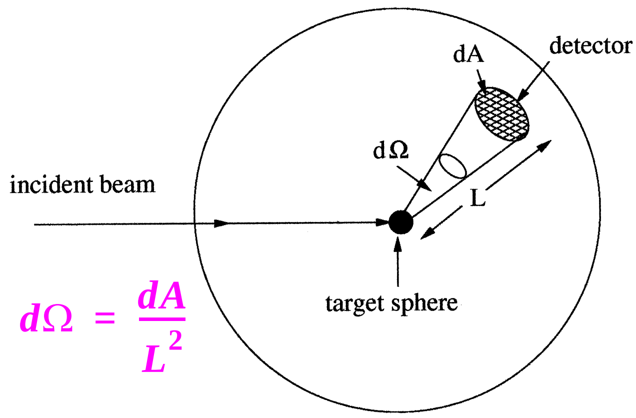
- Elastic electron-proton scattering can be used to measure the **size of the proton.**

Electron-proton scattering

➡ Differential cross section

- The **angular distribution** of the particles emerging from a scattering reaction is given by the differential cross section:

$$\frac{d\sigma(\theta, \varphi)}{d\Omega} \quad \text{where } d\Omega = \sin\theta\, d\theta\, d\varphi$$



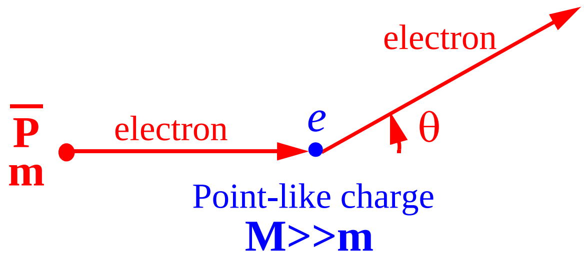
$$d\Omega = \frac{dA}{L^2}$$

- The **total cross section** of the reaction is obtained by integrating the differential cross section:

$$\sigma = \int \frac{d\sigma(\theta, \varphi)}{d\Omega}\, d\Omega = \int_0^{\pi}\int_0^{2\pi} \frac{d\sigma(\theta, \varphi)}{d\Omega} \sin\theta\, d\theta\, d\varphi$$

Electron-proton scattering

➡ Elastic scattering on a static point-like charge.



The Mott scattering formula describes the angular distribution of a **relativistic electron** of momentum p which is scattered by a point-like electric charge e.

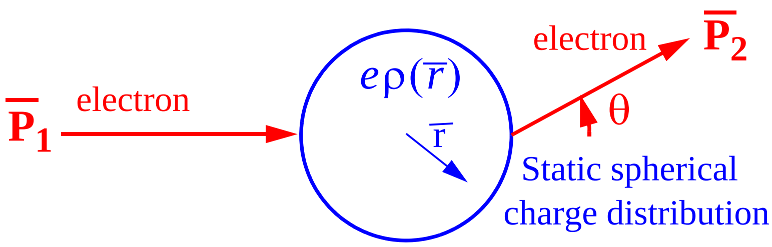
$$\left(\frac{d\sigma}{d\Omega}\right)_M = \frac{\alpha^2}{4p^4\sin^4(\frac{\theta}{2})}\left(m^2 + p^2\cos^2\frac{\theta}{2}\right)$$

The Rutherford scattering formula describes the same for a **non-relativistic electron** with a momentum p<<m , i.e., it is obtained from the Mott formula by assuming p=0.

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{m^2\alpha^2}{4p^4\sin^4\left(\frac{\theta}{2}\right)} \quad \text{where} \quad \alpha = \frac{e^2}{4\pi}$$

Electron-proton scattering

➡ Elastic scattering on an extended charged object.



Momentum transfer

$\overline{q} = \overline{P}_1 - \overline{P}_2$

$q^2 = -\overline{q}\cdot\overline{q}$

- If the electric **charge is not point-like**, but spread out with a spherically symmetric density function ($e \rightarrow e\rho(r)$) that is normalized to one ($\int\rho(r)d^3\bar{x} = 1$) then the Rutherford scattering formula has to be modified by an **electric form factor** $G_E^2(q^2)$:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_R G_E^2(q^2) \qquad \text{where} \qquad \left(\frac{d\sigma}{d\Omega}\right)_R = \frac{m^2\alpha^2}{4p^4\sin^4\left(\frac{\theta}{2}\right)}$$

Electron-proton scattering

- The electric form factor is the **Fourier transform** of the **charge distribution** with respect to the momentum transfer $\underline{q}$:

$$G_E(q^2) = \int \rho(r) e^{i\bar{q}\cdot\bar{x}} d^3\bar{x}$$

- The electric form factor has values between 0 and 1:

Low momentum transfer: $G_E(0) = 1 \quad \text{for} \quad q = 0$

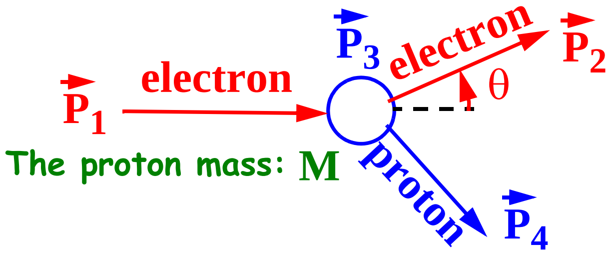
High momentum transfer: $G_E(q^2) \to 0 \quad \text{for} \quad q^2 \to \infty$

- Measurements of the cross-section can be used to determine the form-factor and hence the charge distribution. The mean **quadratic charge radius** is for example given by:

$$r_E^2 = \int r^2 \rho(r) d^3\bar{x} = -6 \left.\frac{dG_E(q^2)}{dq^2}\right|_{q^2=0}$$

Electron-proton scattering

➡ Elastic electron-proton scattering



4-momentum transfer

$$\vec{q} = \vec{P}_1 - \vec{P}_2$$

$$Q^2 = -\vec{q} \cdot \vec{q}$$

- Scattering of electrons on protons depends not only on the **electric formfactor (G_E)** but also on a **magnetic formfactor (G_M)** which is associated with the magnetic moment distribution:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_M \times \left(G_1(Q^2)\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2} G_2(Q^2)\sin^2\frac{\theta}{2}\right)$$

$$G_1(Q^2) = \frac{G_E^2 + \frac{Q^2}{4M^2} G_M^2}{1 + \frac{Q^2}{4M^2}} \qquad G_2(Q^2) = G_M^2$$

Electron-proton scattering

- Measurement of the formfactors are conveniently divided up into three regions of Q^2:

i) low Q^2 ($Q<<M$):

G_E dominates the cross section and r_E can be precisely measured: $r_E = 0.85 \pm 0.02$ fm

ii) Intermediate Q^2 ($0.02 < Q^2 < 3$ GeV^2):

Both G_E and G_M give sizable contributions and the form-factors can be described by the parameterization:

$$G_E(Q^2) \approx \frac{G_M(Q^2)}{\mu_p} \approx \left(\frac{\beta^2}{\beta^2 + Q^2}\right)^2$$

iii) High Q^2 ($Q^2 > 3$ GeV^2):

G_M dominates the cross section.

Electron-proton scattering

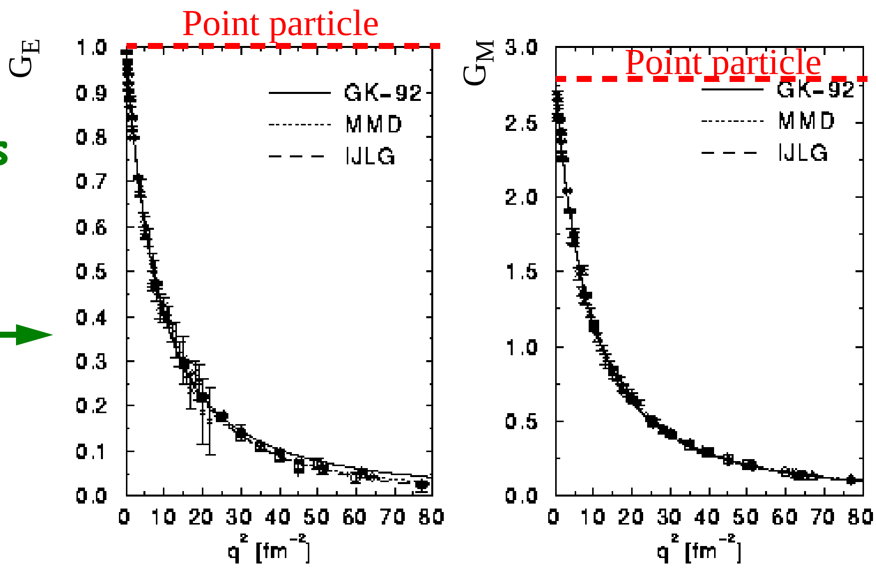
- The form factors are normalized so that

Protons: $G_E(0)$ = **total charge** = 1 $G_M(0)$ = **magnetic moment** = μ_p = +2.79

Neutrons: $G_E(0)$ = **total charge** = 0 $G_M(0)$ = **magnetic moment** = μ_n = -1.91

- If the proton is a point particle then G_E and G_M do not depend on Q^2 and they should be constants with $G_E=1$ and $G_M=2.79$.

Measurements of G_E and G_M of the proton gives: →

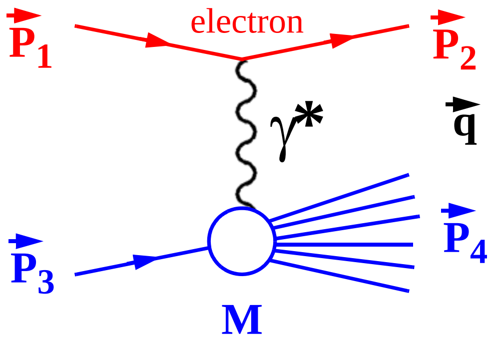


Conclusion: The proton has an extended charge distribution !

Electron-proton scattering

➡ Inelastic electron-proton scattering

- In inelastic electron-proton scattering, the proton is broken up into new hadrons:



$$\vec{q} = \vec{P}_1 - \vec{P}_2 = (\nu, \bar{q}) \qquad Q^2 = -\vec{q} \cdot \vec{q}$$

$$\vec{P}_4 = \vec{P}_3 + \vec{q}$$

- A new dimensionless variable called the Bjorken scaling variable (x) is introduced which can take values between 0 and 1:

$$x = \frac{Q^2}{2M\nu}$$ where M is the mass of the proton.

Electron-proton scattering

- The differential cross section for inelastic electron-proton scattering can be written as:

$$\frac{d\sigma}{dE_2 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)} \bullet \left[\frac{1}{\nu} F_2(x, Q^2) \cos^2\left(\frac{\theta}{2}\right) + \frac{2}{M} F_1(x, Q^2) \sin^2\left(\frac{\theta}{2}\right)\right]$$

  where two dimensionless structure functions $F_1(x,Q^2)$ and $F_2(x,Q^2)$ parameterize the photon-proton interaction in the same way a $G_1(Q^2)$ and $G_2(Q^2)$ do it in elastic scattering.

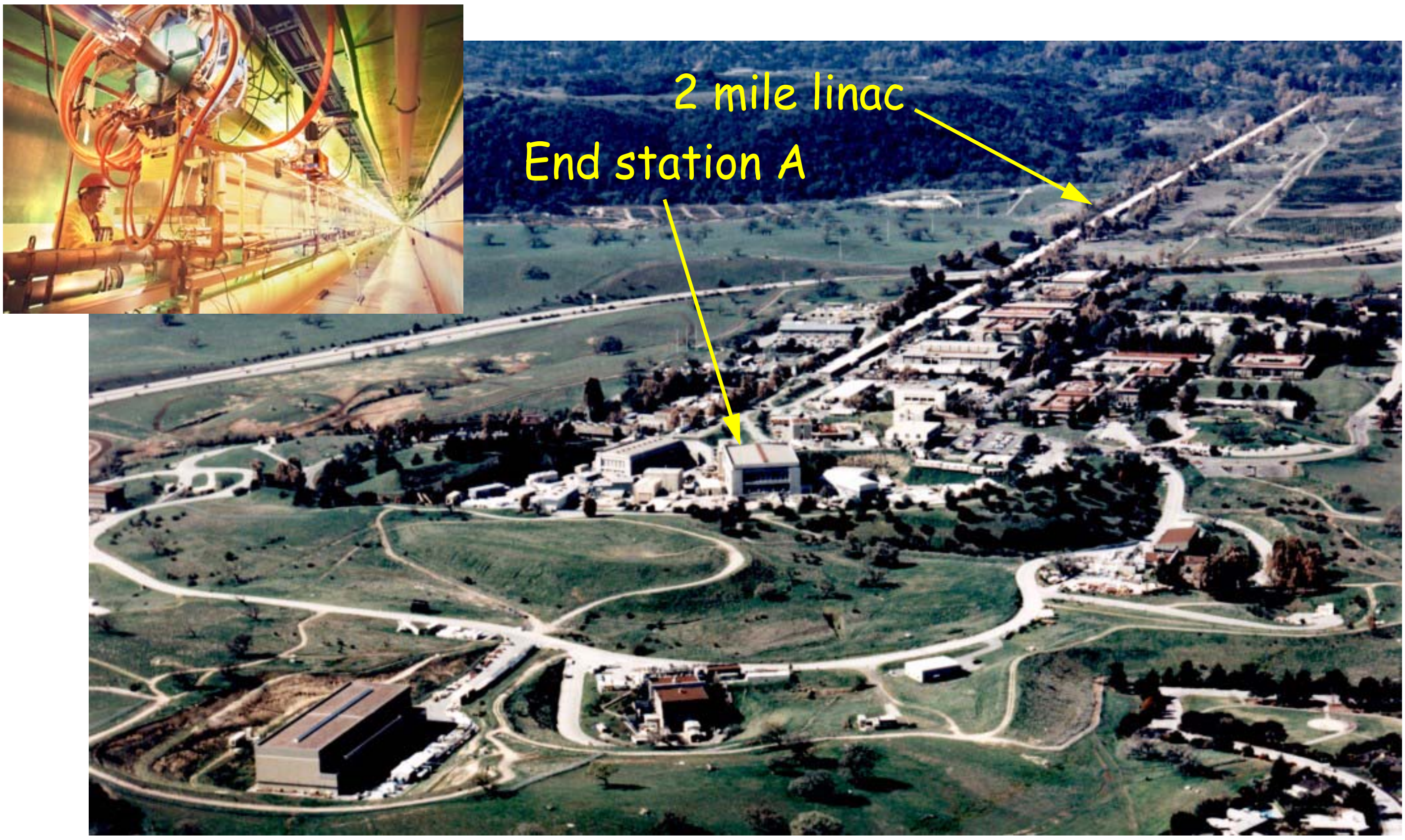
- An important concept is that of Bjorken scaling or scale invariance:

  $F_{1,2}(x, Q^2) = F_{1,2}(x)$ when $Q^2 \to \infty$ and x is fixed and finite.

  i.e. the structure functions are almost independent on Q^2 when Q>>M. It is called scaling because structure functions at a given x remain unchanged if all particle masses, energies and momenta are multiplied by a scale factor.

Electron-proton scattering

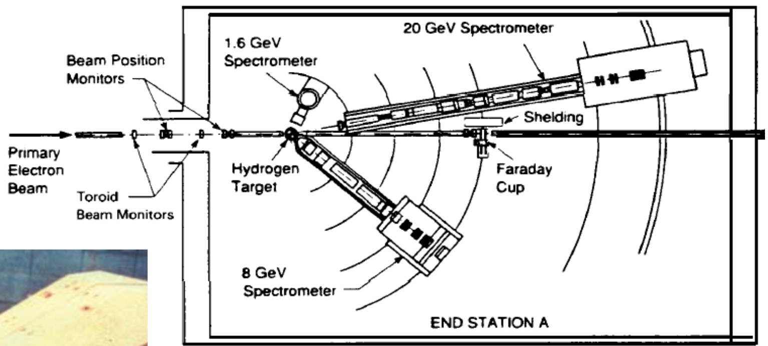
➡ The discovery of quarks at the SLAC 2 mile LINAC

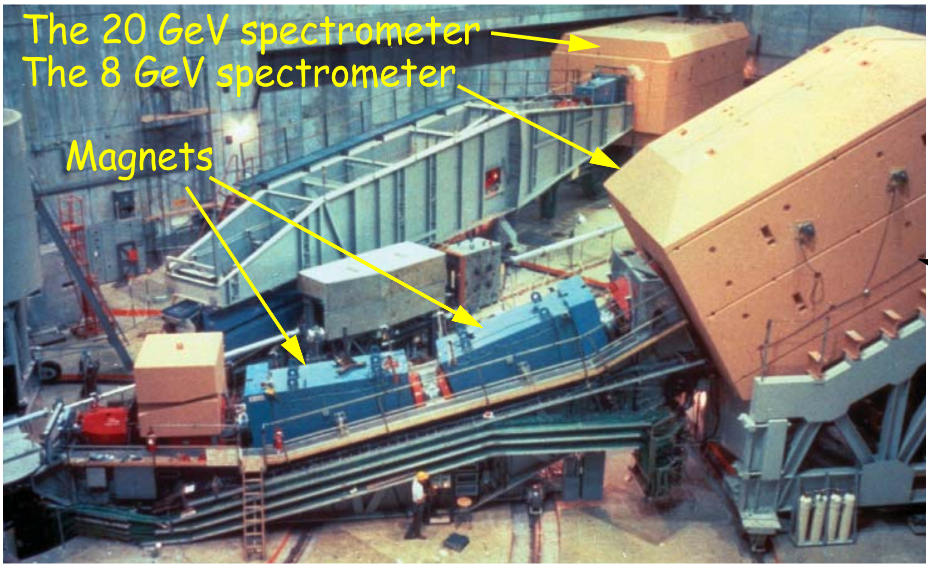


Electron-proton scattering

➡ The discovery of quarks

The MIT-SLAC experiment





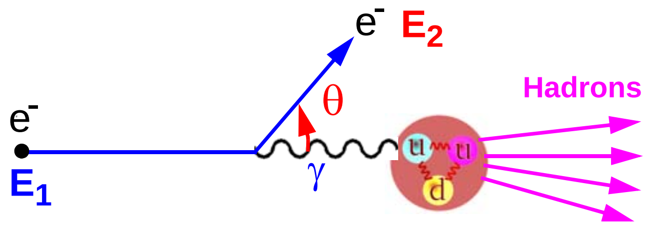
Inside the shielding here were Cerenkov detectors, scintillators and detectors for e/π separation.

8 GeV electrons were hitting a hydrogen target. The scattered electrons were selected by magnets at different angles and identified by detectors inside the brown shielding.

Electron-proton scattering

The discovery of quarks: The measurements

- It is possible to calculate x and Q^2 from the energies and scattering angle of the electron:

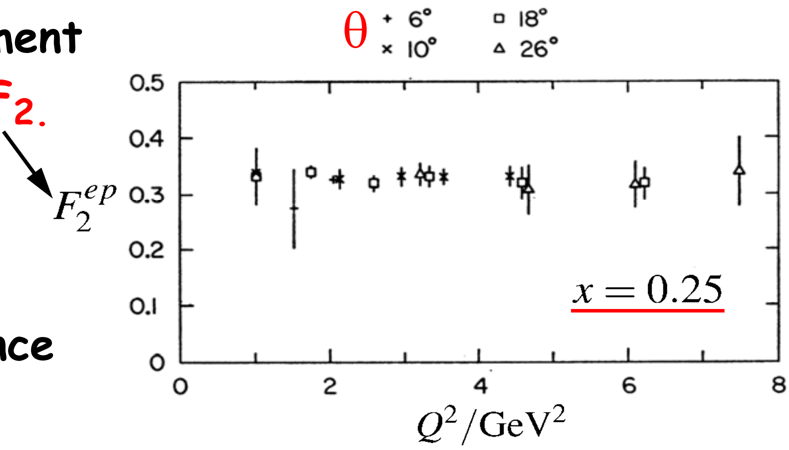


$$Q^2 = 4 \cdot E_1 \cdot E_2 \sin^2\left(\frac{\theta}{2}\right)$$

$$x = \frac{Q^2}{2 \cdot M \cdot (E_1 - E_2)}$$

- From a cross section measurement it is then possible to extract F_2.

- The result that F_2 does not depend on Q^2 was later interpreted as the first evidence for the existance of quarks.

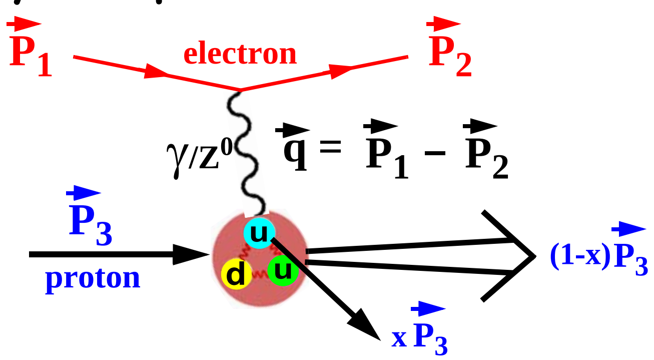


Electron-proton scattering

Deep inelastic electron-proton scattering

- The scale invariance is explained in the parton model by the scattering on point-like constituents (partons) in the proton.

- These partons are identical to the quarks that were postulated by the quark model.



$$Q^2 = -\vec{q} \bullet \vec{q} = 4 \cdot E_1 \cdot E_2 \sin^2\left(\frac{\theta}{2}\right)$$

$$x = \frac{Q^2}{2M\nu} = \frac{Q^2}{2 \cdot M \cdot (E_1 - E_2)}$$

- The parton model is valid if the proton momentum is sufficiently large so that the fraction of the proton momentum carried by the struck quark is given by the Bjorken x.

Electron-proton scattering

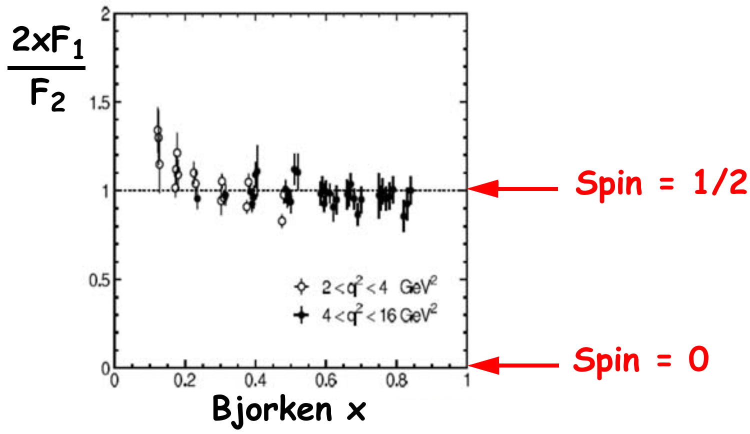
➡ Deep inelastic electron-proton scattering

- The structure function F_1 depends on the spin of the partons (quarks) in the parton model:

$$F_1(x, Q^2) = 0 \qquad \text{(spin-0)}$$

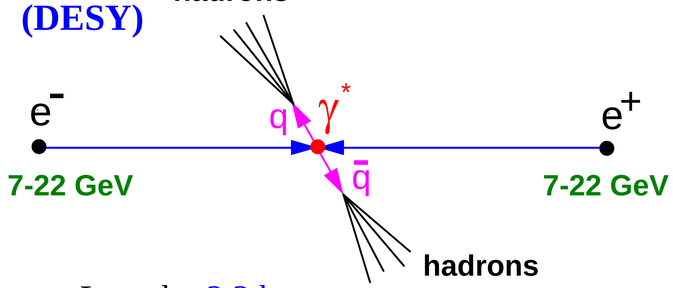
The Callan-Gross relation: $$2xF_1(x, Q^2) = F_2(x, Q^2) \qquad \text{(spin-1/2)}$$

- Measurements shows that the partons have spin 1/2:



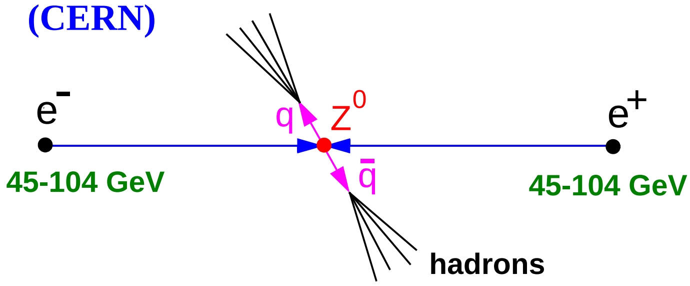
Electron-proton scattering

PETRA (DESY)



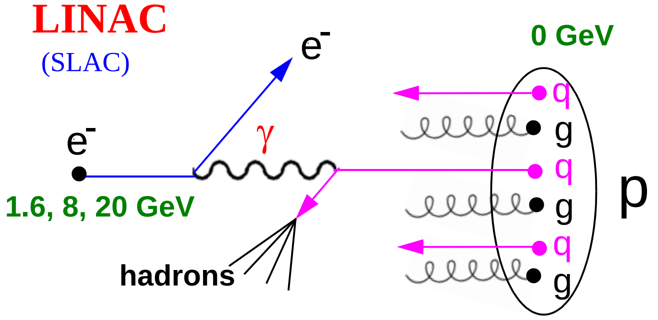
Length: 2.3 km
Experiments: Tasso, Jade, Pluto, Mark J, Cello

LEP (CERN)



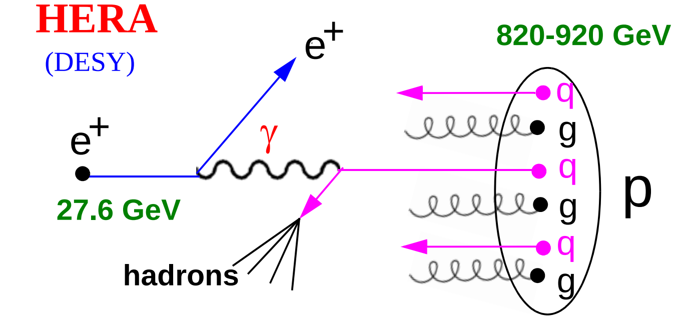
Length: 27 km (4184 magnets)
Experiments: DELPHI, OPAL, ALEPH, L3

LINAC (SLAC)



Length: 3 km
Experiments: SLAC-MIT

HERA (DESY)

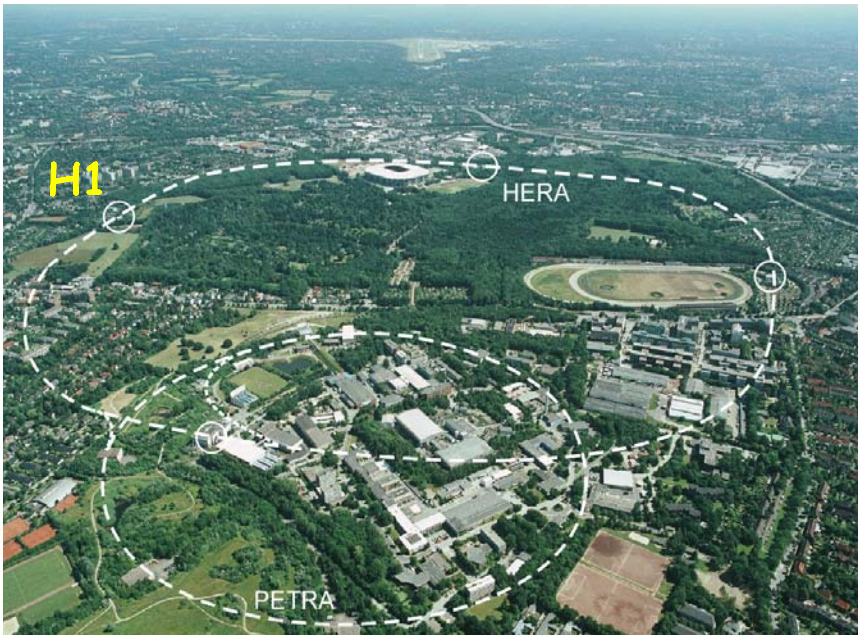


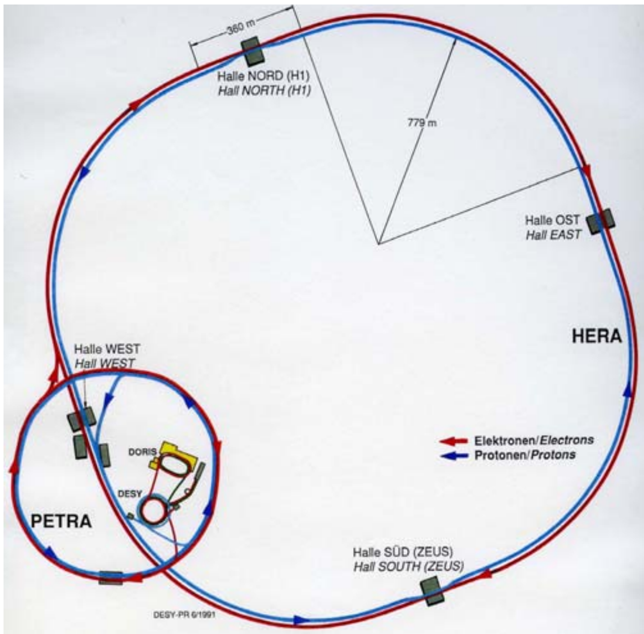
Length: 6 km (1650 magnets)
Experiments: H1, ZEUS

Electron-proton scattering

➡ The HERA accelerator

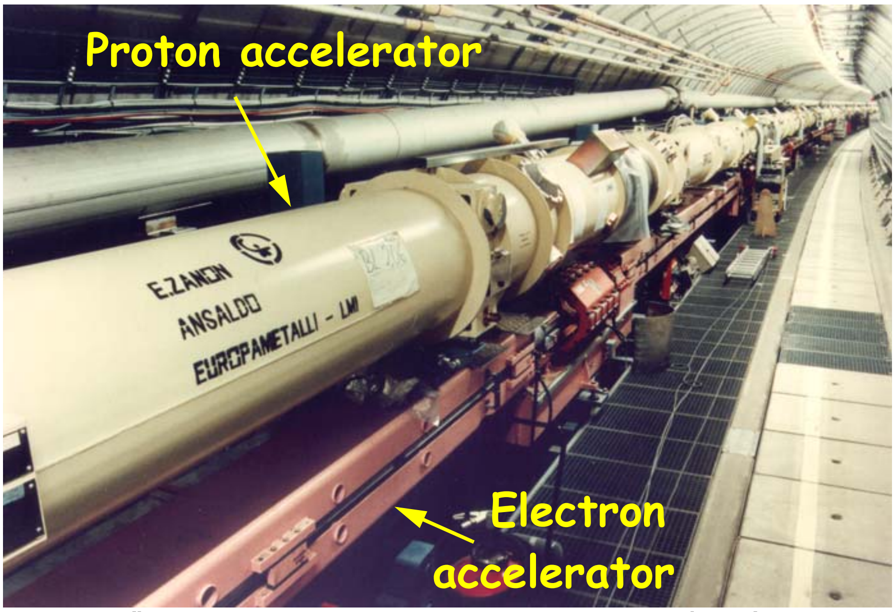
- The HERA accelerator at the German DESY laboratory is the only large electron-proton collider ever built. It used PETRA as a pre-accelerator.

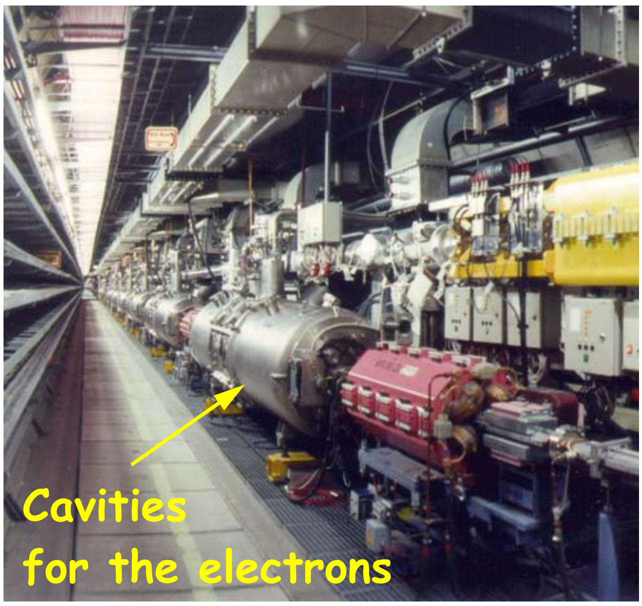




Electron-proton scattering

- HERA, which was 6 km long, had a ring of superconducting magnets for the protons and a ring of warm magnets for the electrons. The center-of-mass energy of the collision of 28 GeV electrons on 920 GeV protons was 320 GeV. This is equivalent to a fix target accelerator with a 54 TeV electron beam.

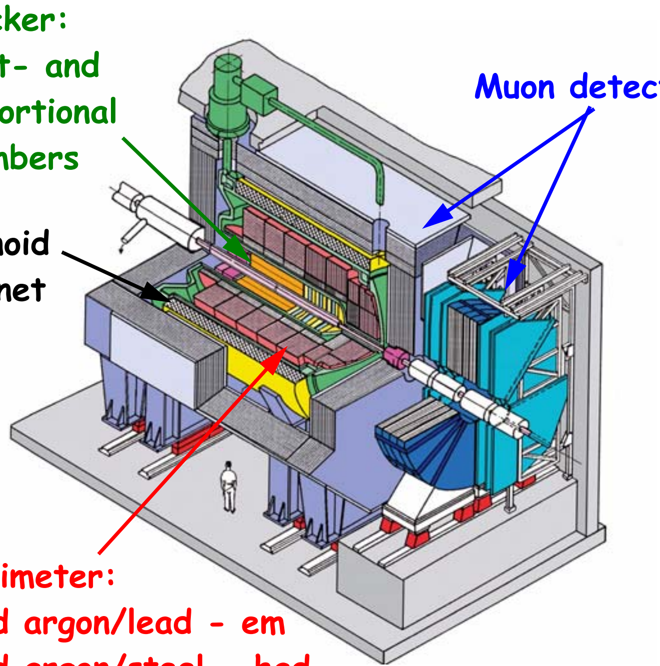


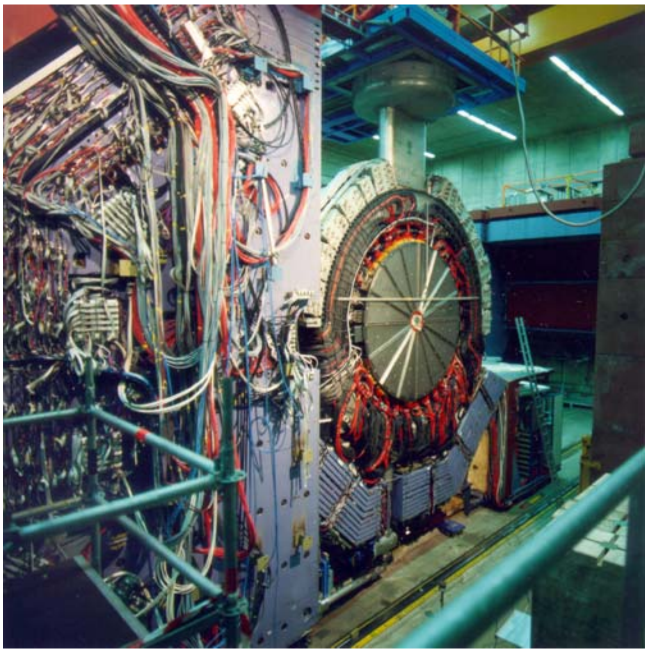


Electron-proton scattering

➡ The H1 Experiment

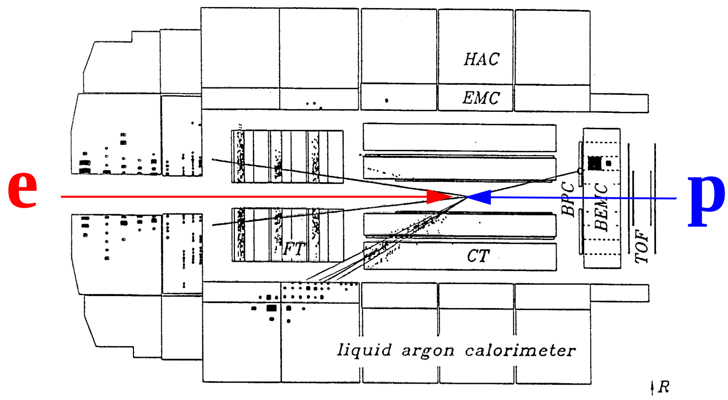
- The events at HERA were boosted in the proton direction due to the large difference in electron and proton beam energies.



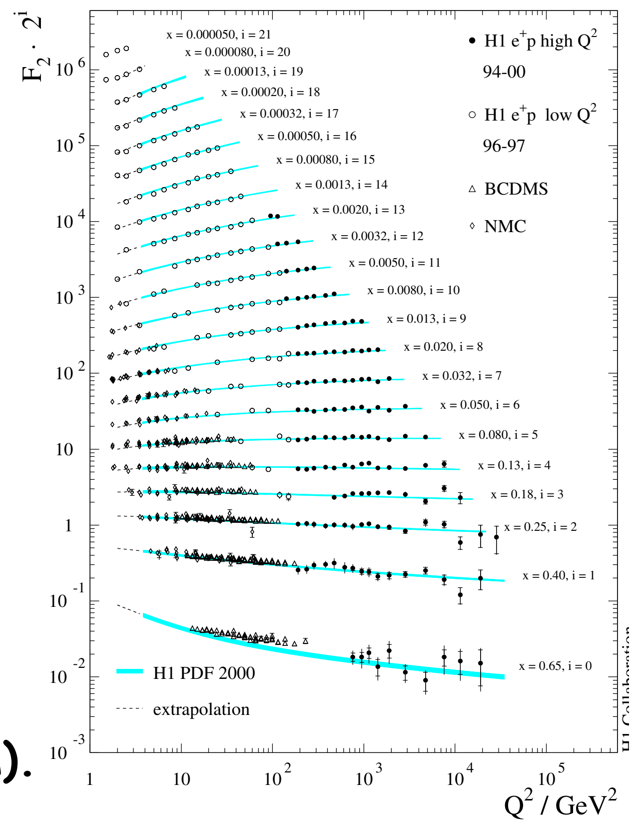


Electron-proton scattering

➡ Measurement of structure functions

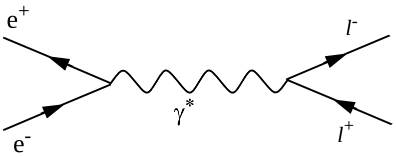


- A measurement of the cross section + the energy and scattering angle of the electron made it possible to measure F_2.
- No quark sub-structure was observed down to 10^{-18}m (1/1000th of a proton).

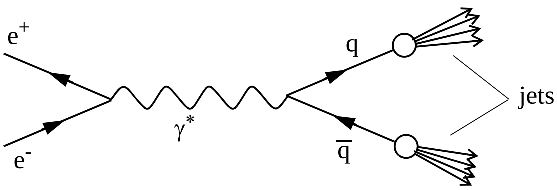


Summary of scattering formulas

SUMMARY: Electron-Positron interactions

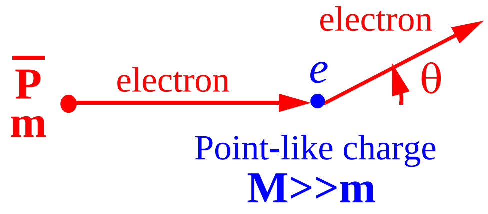


$$\frac{d\sigma}{d\cos\theta}\left(e^+e^- \rightarrow l^+l^-\right) = \frac{\pi\alpha^2}{2Q^2}(1+\cos^2\theta)$$

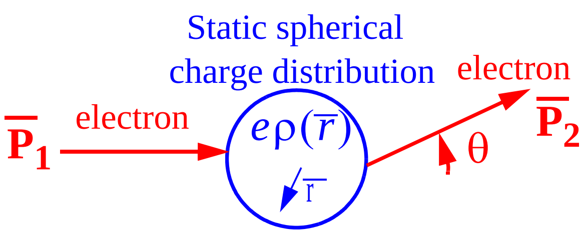


$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow q\bar{q}) = N_c e_q^2 \frac{\pi\alpha^2}{2Q^2}(1+\cos^2\theta)$$

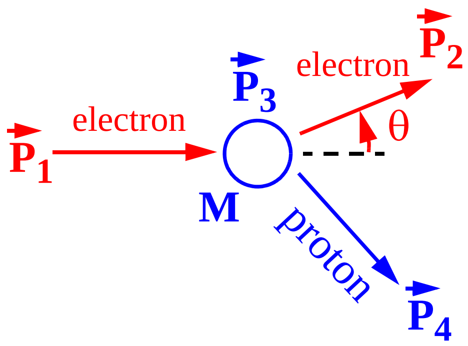
SUMMARY: Elastic electron-proton scattering



$$\left(\frac{d\sigma}{d\Omega}\right)_M = \frac{\alpha^2}{4p^4\sin^4(\frac{\theta}{2})}\left(m^2 + p^2\cos^2\frac{\theta}{2}\right)$$



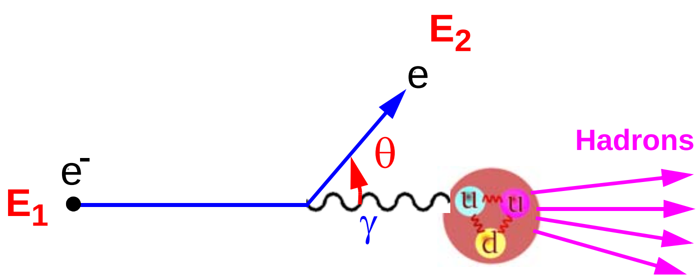
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_R G_E^2(q^2)$$

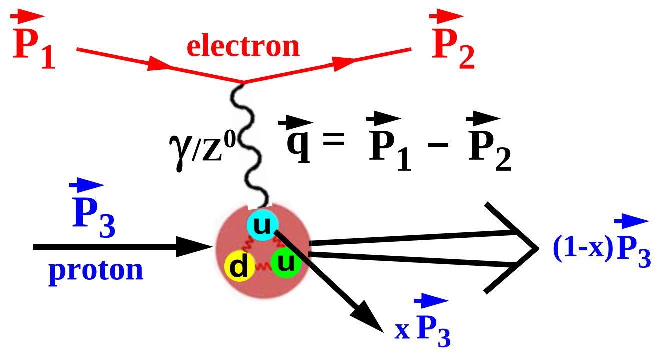


$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_M \times \left(G_1(Q^2)\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}G_2(Q^2)\sin^2\frac{\theta}{2}\right)$$

$$G_1(Q^2) = \frac{G_E^2 + \frac{Q^2}{4M^2}G_M^2}{1 + \frac{Q^2}{4M^2}} \qquad G_2(Q^2) = G_M^2$$

SUMMARY: Inelastic electron-proton scattering





$$Q^2 = -\vec{q}\cdot\vec{q}$$

$$x = \frac{Q^2}{2M\nu} \qquad \textbf{Bjorken - x}$$

$$\frac{d\sigma}{dE_2 d\Omega} = \frac{\alpha^2}{4E_1^2\sin^4\left(\frac{\theta}{2}\right)} \bullet \frac{1}{\nu} \bullet \left[F_2(x, Q^2)\cos^2\left(\frac{\theta}{2}\right) + \frac{Q^2}{xM^2}F_1(x, Q^2)\sin^2\left(\frac{\theta}{2}\right)\right]$$