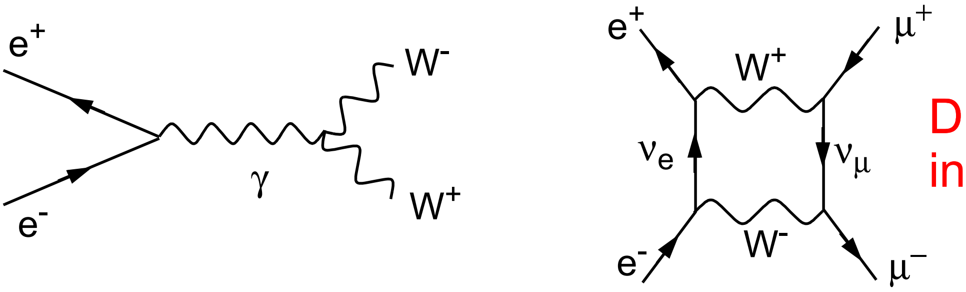
IX. Electroweak unification

The problem of divergence

❖ A theory of weak interactions only by means of $W^{\pm}$ bosons leads to infinities



Divergent integrals

Figure 108: Examples of divergent processes

→ Introduction of the Z^0 boson fixes the problem because the addition of new diagrams cancel out the divergencies:

Divergent integrals

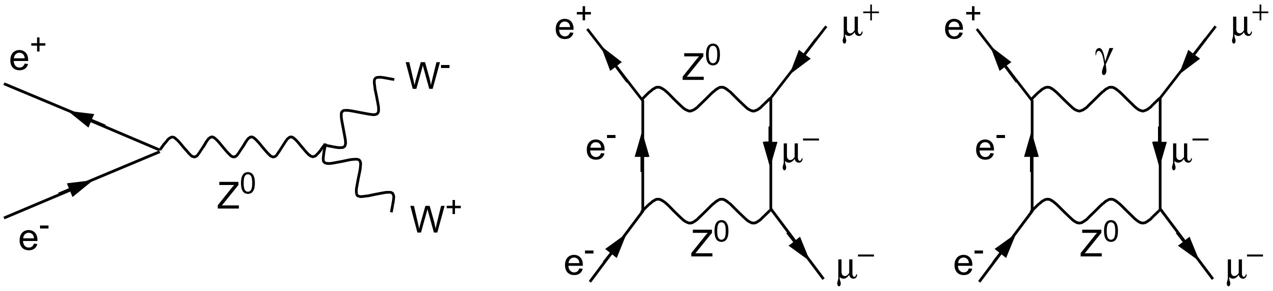


Figure 109: Additional processes which cancel the divergence

Basic vertices with Z^0 bosons have:

– Conserved lepton numbers

– Conserved flavour

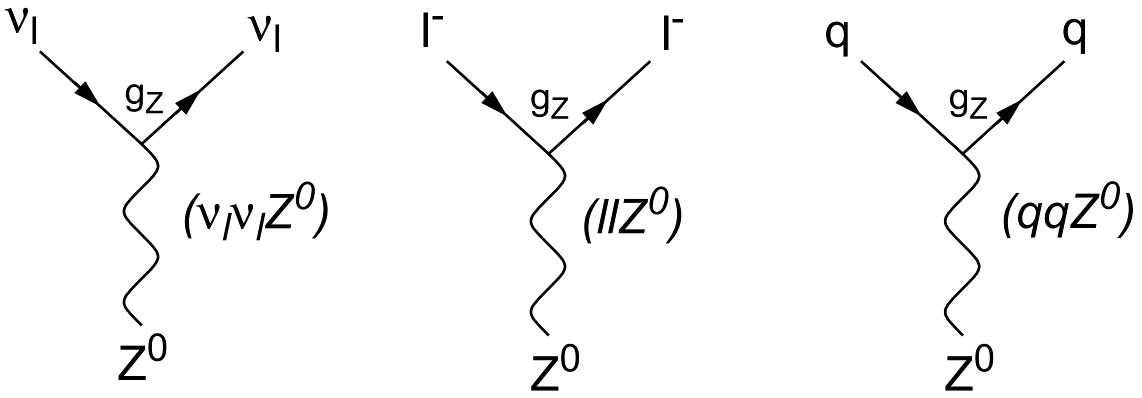


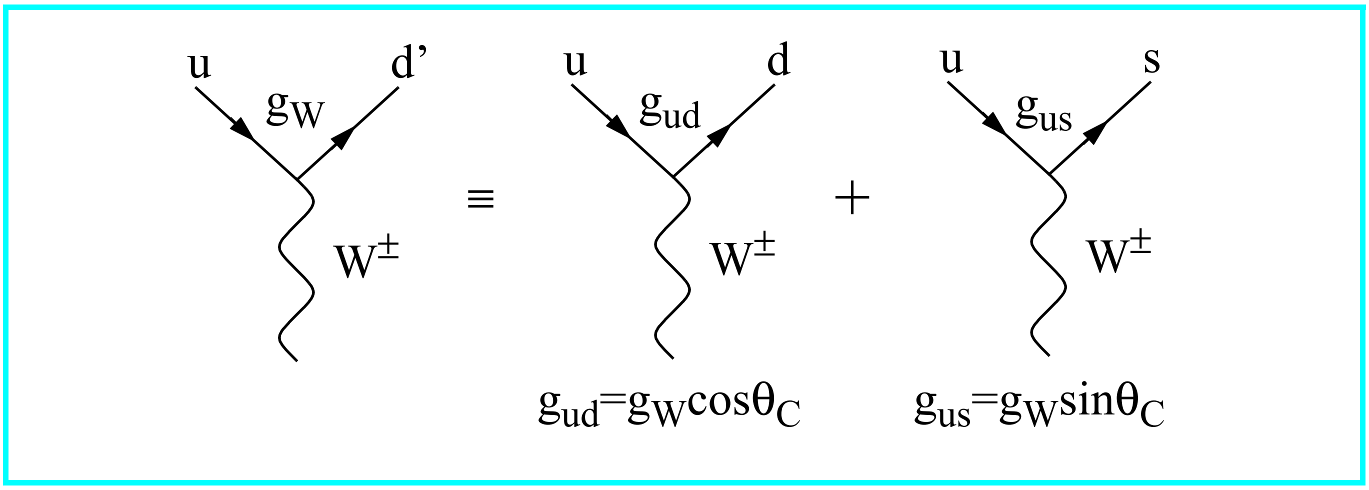
Figure 110: Z^0-lepton and Z^0-quark basic vertices

Reminder of quark mixing in W vertices:

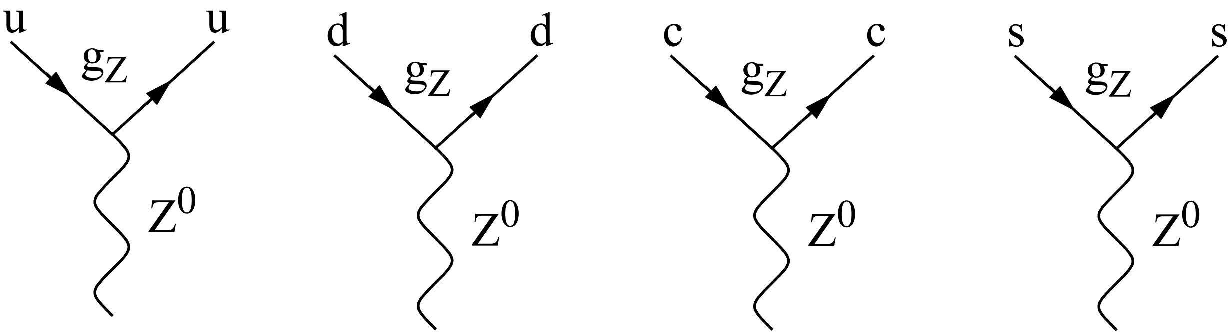
$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix}$$

$$\begin{aligned} d' &= d\cos\theta_C + s\sin\theta_C \\ s' &= -d\sin\theta_C + s\cos\theta_C \end{aligned}$$

where u , c , d’ and s’ are wave functions



At Z^0 vertices it is not necessary to introduce quark mixing:



Note that the flavour is conserved in neutral current interactions but not charge current interactions.

Test of flavour conservation

Flavour is conserved at a Z^0 vertex (in contrast to a W vertex).This can be verified by experiments.

Consider the following two possible processes that change strangeness:

$$K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu \qquad \text{(a)}$$

and

$$K^+ \rightarrow \pi^+ + \nu_l + \bar{\nu}_l \qquad \text{(b)}$$

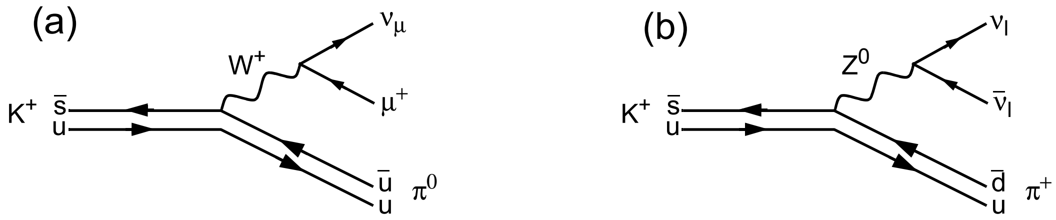


Figure 111: Decay (a) is allowed; decay (b) – forbidden

The measured upper limit on the ratio of the decay rates (b) to (a) is:

$$\frac{\sum_l \Gamma(K^+ \rightarrow \pi^+ + \nu_l + \nu_l)}{\Gamma(K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu)} < 10^{-7}$$

The unification condition and masses

The coupling constants at γ–, $W^{\pm}$– and Z^0-vertices are not independent from each other. In order for all infinities to cancel in electroweak theory, the unification relation and the anomaly condition have to be fulfilled.

→ The *unification condition* establishes a relation between the coupling constants ($\alpha_{em}=e^2/4\pi\varepsilon_0$):

$$\frac{e}{2\sqrt{2\varepsilon_0}} = g_W \sin\theta_W = g_Z \cos\theta_W \tag{114}$$

θ_W is the *weak mixing angle*, or *Weinberg angle*:

$$\cos\theta_W = \frac{M_W}{M_Z} \tag{115}$$

→ The *anomaly condition* relates electric charges: $\sum_l Q_l + 3\sum_q Q_q = 0$

Historically, the W and Z masses were predicted from low energy interactions.

In the zero-range approximation i.e. in the low-energy limit, the charged current reactions are characterized by the Fermi constant (G_F):

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2} \Rightarrow M_W^2 = \frac{g_W^2\sqrt{2}}{G_F} = \frac{\pi\alpha}{\sqrt{2}G_F \sin^2\theta_W}$$

Introducing the neutral current coupling constant (G_Z) (also in the low energy zero-range approximation)

$$\frac{G_Z}{\sqrt{2}} = \frac{g_Z^2}{M_Z^2} \Rightarrow M_Z^2 = \frac{\pi\alpha}{\sqrt{2}G_F \cos^2\theta_W \sin^2\theta_W}$$

and the weak mixing angle can be expressed as

$$\frac{G_Z}{G_F} = \frac{g_Z^2 M_W^2}{g_W^2 M_Z^2} = \sin^2\theta_W$$

From the measurements at low energy of rates of charged and neutral currents reactions it is therefore possible to determine that:

$$\sin^2\theta_W = 0,277 \pm 0,014$$

from this measurement at low energies (below the W and Z masses) it was possible to predict the masses of W and Z:

$$M_W = 78,3 \pm 2,4\ GeV/c^2;\ M_Z = 89,0 \pm 2,0\ GeV/c^2$$

When the W and Z boson were discovered at CERN with the masses predicted from low energy experiments it was a strong confirmation that the electroweak theory was correct.

Today the most precise estimation of the Weinberg angle using many measurements give:

$$\sin^2\theta_W = 0{,}2255 \pm 0{,}0021$$

Putting this value into the previous formulas give

M_W = 78.5 GeV and M_Z = 89.3 GeV

while the direct measurements of the masses give

M_W = 80.4 GeV and M_Z = 91.2 GeV

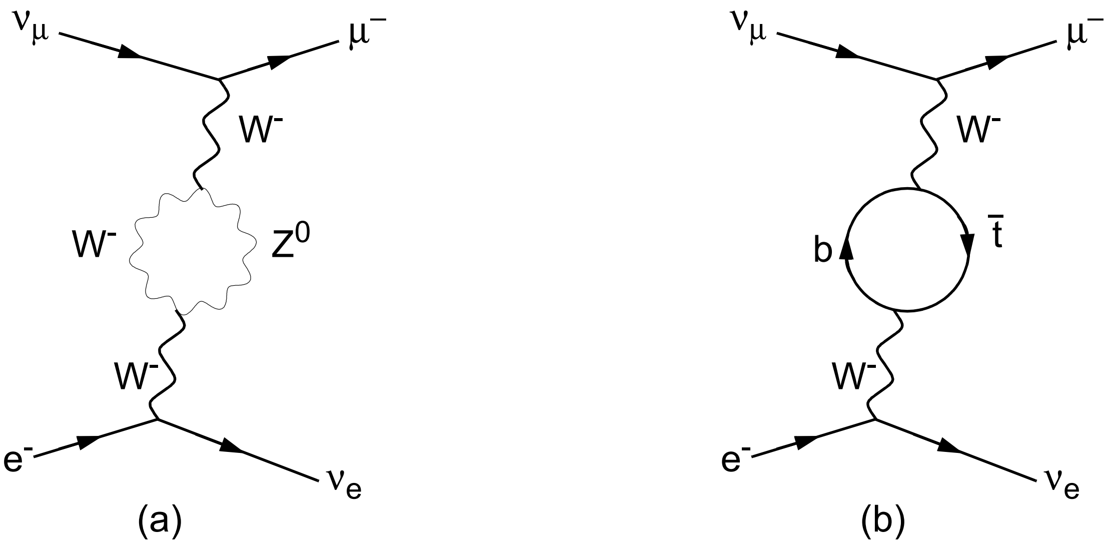


Figure 112: Examples of higher order contributions to inverse muon decay

The difference is due to higher-order diagrams which were not included in the previous low-energy formulas.

Since the top-quark is involved in higher order corrections, the measurement of electroweak processes could be used to predict the top-quark mass before it was discovered:

$$m_t = 170 \pm 30\ GeV/c^2$$

The directly measured mass of the top quark at Fermilab by CDF is today

$$m_t = 176 \pm 5\ GeV/c^2$$

in perfect agreement with the prediction !

Electroweak reactions

In any process in which a photon can be exchanged, a Z^0 boson can be exchanged as well

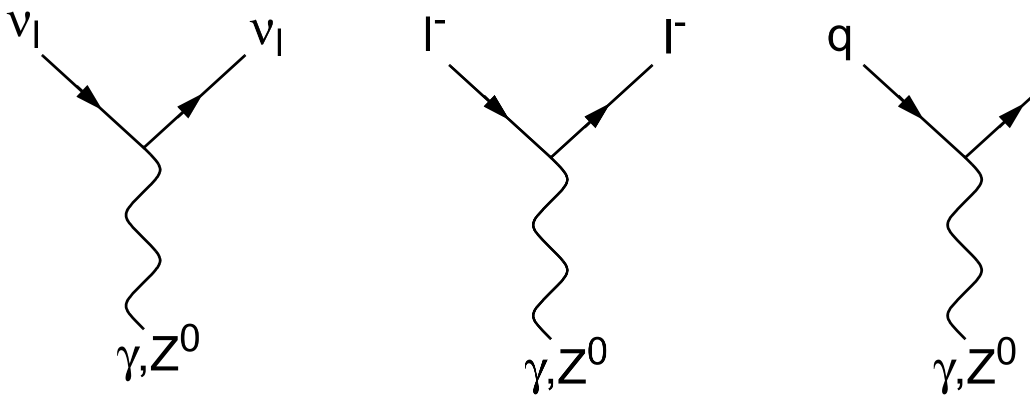


Figure 113: Z^0 and γ couplings to leptons and quarks

Example: The reaction $e^+e^- \rightarrow \mu^+\mu^-$ has two dominant contributions:

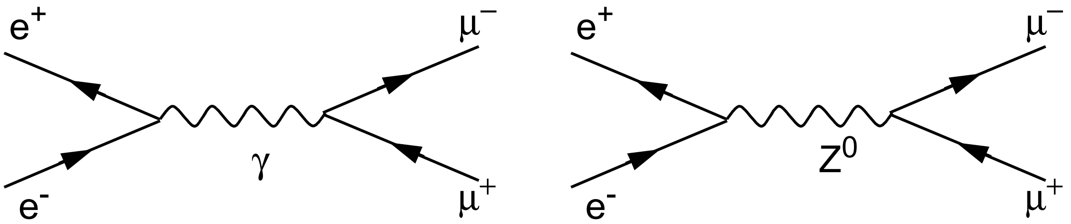


Figure 114: Dominant contributions to the e^+e^- annihilation into muons

With simple dimensional arguments one can estimate the cross section for the photon- and Z-exchange process at low energy:

$$\sigma_\gamma \approx \frac{\alpha^2}{E^2} \qquad \sigma_Z \approx G_Z^2 E^2$$

Where E is the energy of the colliding electron and positron beams.

From these expressions, the ratio of σ_Z and σ_γ is:

$$\frac{\sigma_Z}{\sigma_\gamma} \approx \frac{E^4}{M_Z^4} \qquad (116)$$

One can conclude that at low energies the photon exchange process dominates. However, at energies $E_{CM}=M_Z$, this low-energy approximation fails

The Z^0 peak is described by the Breit-Wigner formula:

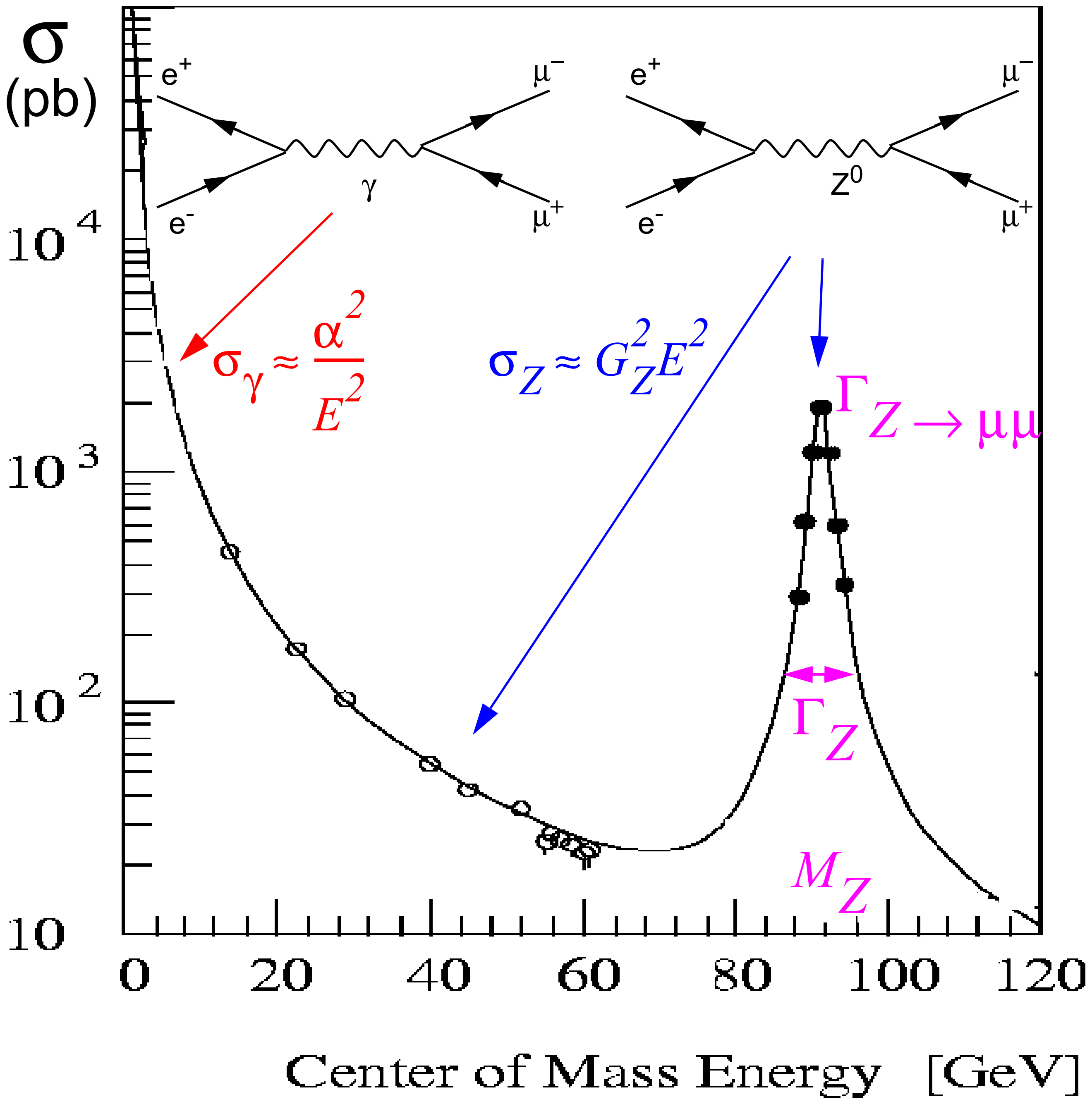


Figure 115: The cross sections of e^+e^- annihilation into μμ

$$\sigma(ee \to \mu\mu) = \frac{12\pi M_Z^2}{E_{CM}^2}\left[\frac{\Gamma(Z^0 \to ee)\Gamma(Z^0 \to \mu\mu)}{(E_{CM}^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right]$$

The number of neutrino families

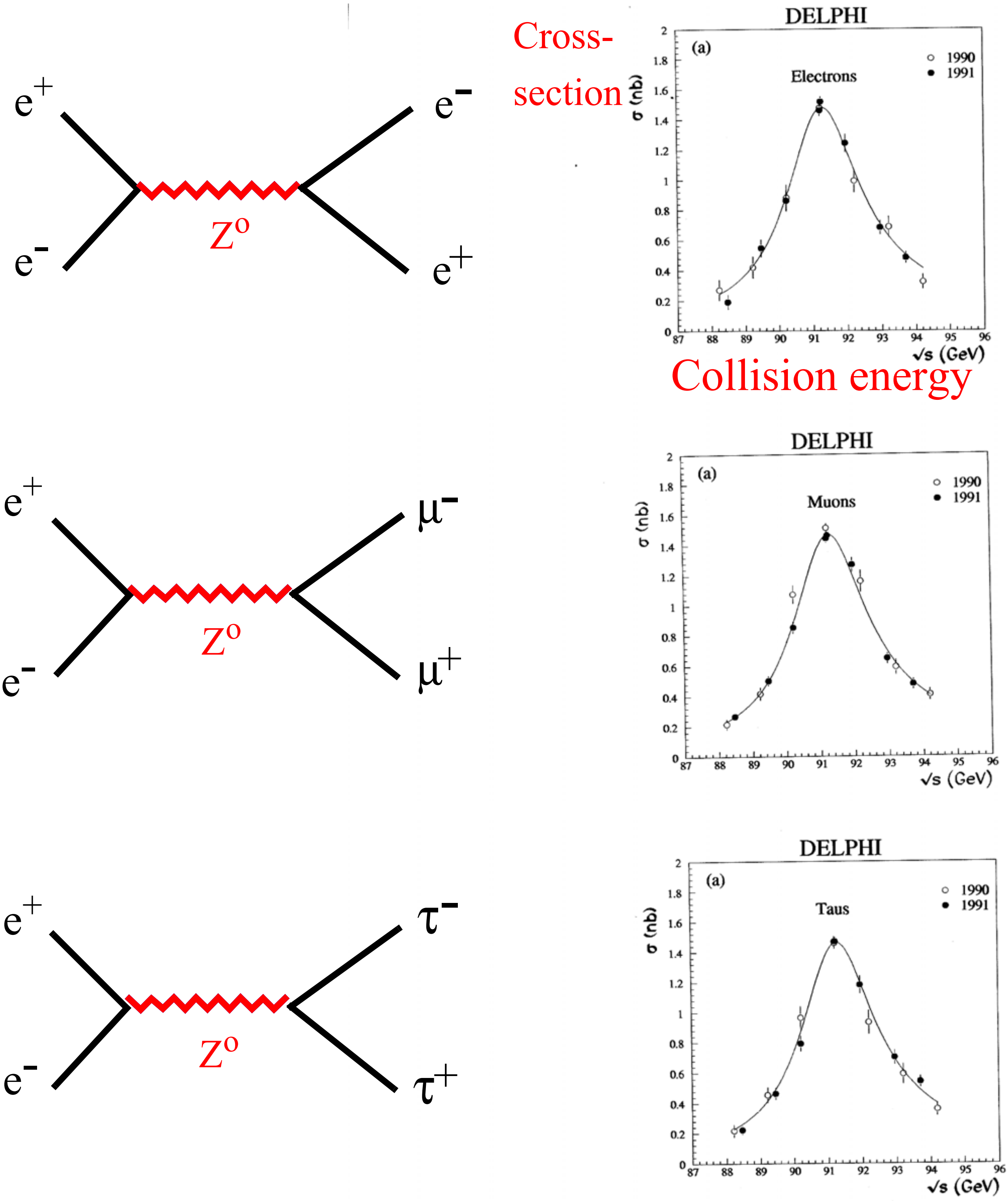


Figure 116: The leptonic decay of the Z^0.

- All these peaks can be fitted with the Breit-Wigner formula:

$$\sigma(e^+e^- \to X) = \frac{12\pi M_Z^2}{E_{CM}^2}\left[\frac{\Gamma(Z^0 \to e^+e^-)\Gamma(Z^0 \to X)}{(E_{CM}^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right]$$

Here Γ_Z is the total Z^0 decay rate, and $\Gamma_Z(Z^0 \to X)$ is the decay rates to the final state X.

The height of the peak (at $E_{CM}=M_Z$) is proportional to the product of the branching ratios:

$$B(Z^0 \to e^+e^-)B(Z^0 \to X) \equiv \frac{\Gamma(Z^0 \to e^+e^-)}{\Gamma_Z}\frac{\Gamma(Z^0 \to X)}{\Gamma_Z}$$

Fitted parameters of the Z^0 peak:

$$\begin{aligned} M_Z &= 91{,}187 \pm 0{,}007\ GeV/c^2 \\ \Gamma_Z &= 2{,}490 \pm 0{,}007\ GeV \end{aligned} \qquad (117)$$

❖ Fitting the peak give not only M_Z and Γ_Z but also partial decay rates:

$$\Gamma(Z^0 \to hadrons) = 1{,}741 \pm 0{,}006\ GeV \tag{118}$$

$$\Gamma(Z^0 \to l^+ l^-) = 0{,}0838 \pm 0{,}0003\ GeV \tag{119}$$

→ The decays $Z^0 \to l^+ l^-$ and $Z^0 \to hadrons$ account for only about 80% of all Z^0 decays

→ The remaining decays are those containing only neutrinos in the final state

$$\begin{gathered} \Gamma_Z = \Gamma(Z^0 \to hadrons) + 3\Gamma(Z^0 \to l^+ l^-) + \\ + N_\nu \Gamma(Z^0 \to \nu_l \bar{\nu}_l) \end{gathered} \tag{120}$$

From the measurement of all other partial widths one can estimate the partial decay to neutrinos which cannot be measured directly.:

$$N_\nu \Gamma(Z^0 \to \nu_l \bar{\nu}_l) = 0{,}498 \pm 0{,}009\ GeV$$

The decay rate to neutrino pairs can also be calculated from the diagrams shown previously:

$$\Gamma(Z^0 \to \nu_l \bar{\nu}_l) = 0{,}166\ GeV \qquad (121)$$

which means that $N_\nu \approx 3$. More precisely,

$$N_\nu = 2{,}994 \pm 0{,}011 \qquad (122)$$

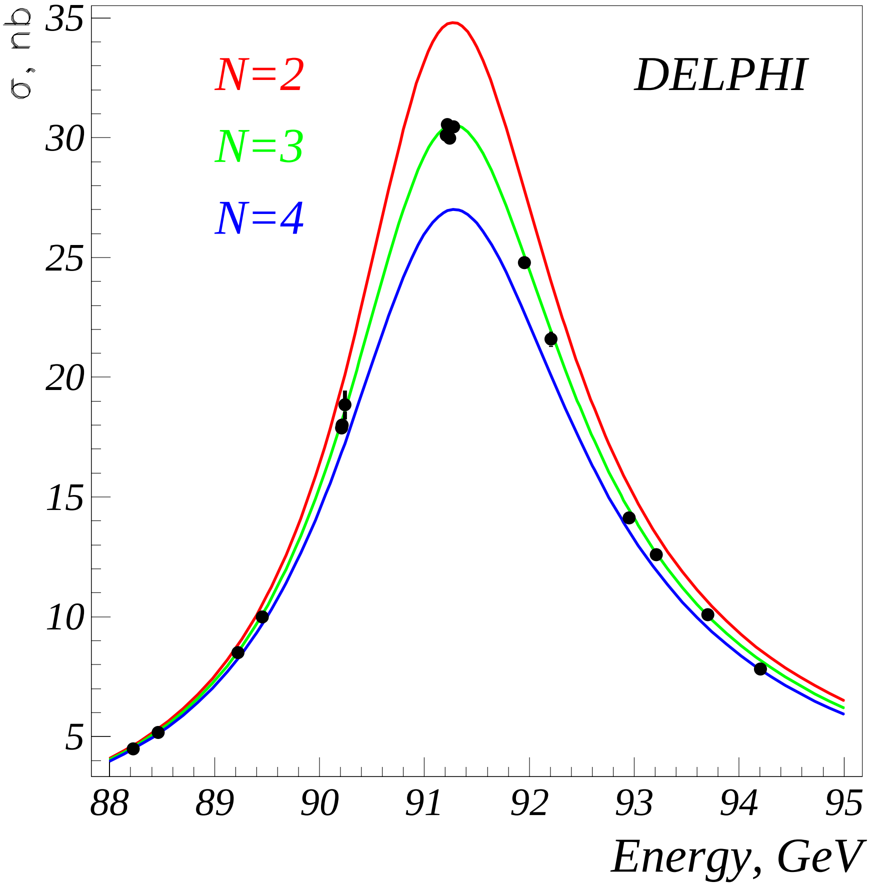


Figure 117: The decay of the Z^0 to hadrons and theoretical predictions based on different assumptions for the number of neutrino families (N)

- → There are no explicit restrictions on the number of generations in the Standard Model

- → However, the analysis of the Z^0 line shape shows that there are 3 and only 3 kinds of light neutrinos.

  - ❖ If neutrinos are assumed having negligible masses as compared with the Z^0 mass, there must be only THREE generations of leptons and quarks within the Standard Model.

Reminder:

The lifetime (τ) , the branching ratio (B) and the partial decay width (Γ) are related to each other by

$$\tau = \frac{B}{\Gamma}$$

$$\tau = \frac{B_Z}{\Gamma_Z} = \frac{B_{had} + B_{ll} + B_{\nu\nu}}{\Gamma_{had} + \Gamma_{ll} + \Gamma_{\nu\nu}}$$

$$\tau = \frac{B_{had}}{\Gamma_{had}} = \frac{B_{ll}}{\Gamma_{ll}} = \frac{B_{\nu\nu}}{\Gamma_{\nu\nu}} = 3 \times 10^{-25} s$$

since

$$B_{had} = 0,70 \qquad \Gamma_{had} = 1,74 GeV$$

$$B_{ll} = 0,10 \qquad \Gamma_{ll} = 0,25 GeV$$

$$B_{\nu\nu} = 0,20 \qquad \Gamma_{\nu\nu} = 0,50 GeV$$

and

$$1 GeV^{-1} = 6,582 \times 10^{-25} s$$