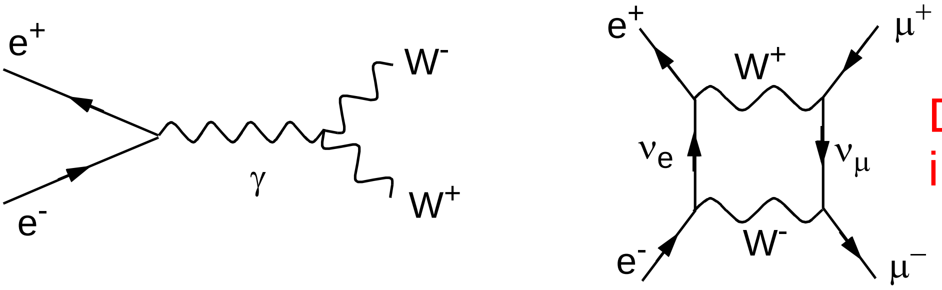
IX. Electroweak unification

The problem of divergence

- A theory of weak interactions only by means of $W^{\pm}$ bosons leads to infinities



Divergent integrals

Figure 108: Examples of divergent processes.

→ Introduction of the Z^0 boson fixes the problem because the addition of new diagrams cancel out the divergencies:

Divergent integrals

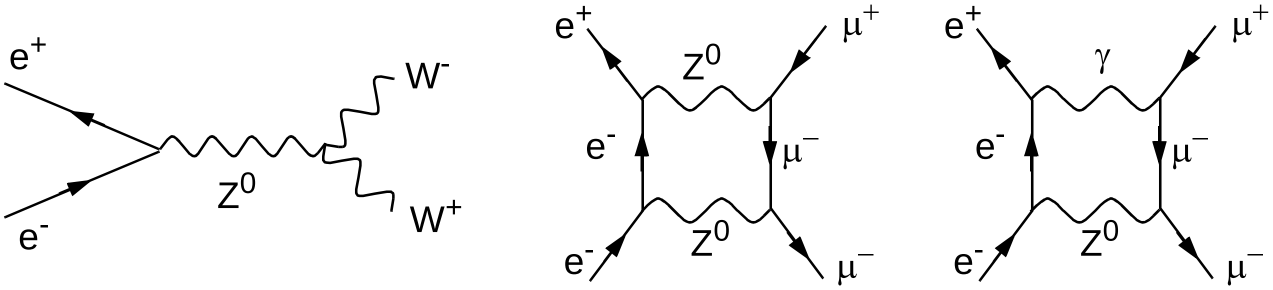


Figure 109: Additional processes which cancel the divergence.

REMINDER:

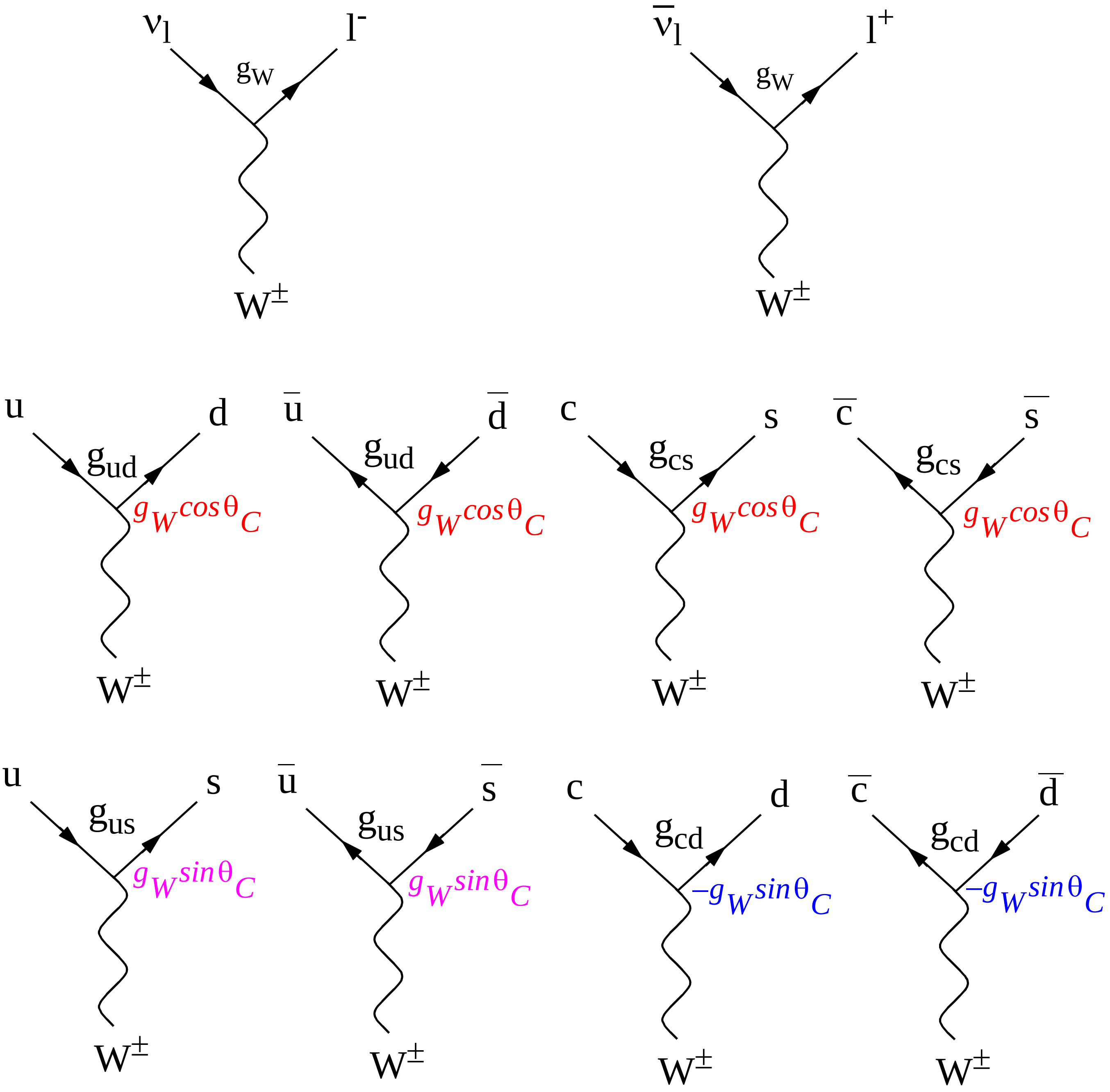


Figure 110: The basic W lepton and quark vertices (if the third generation is not taken into account).

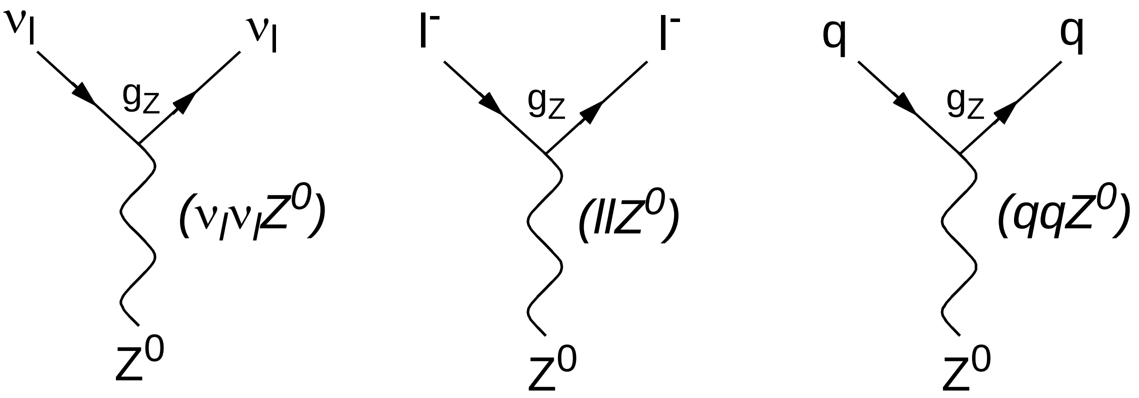
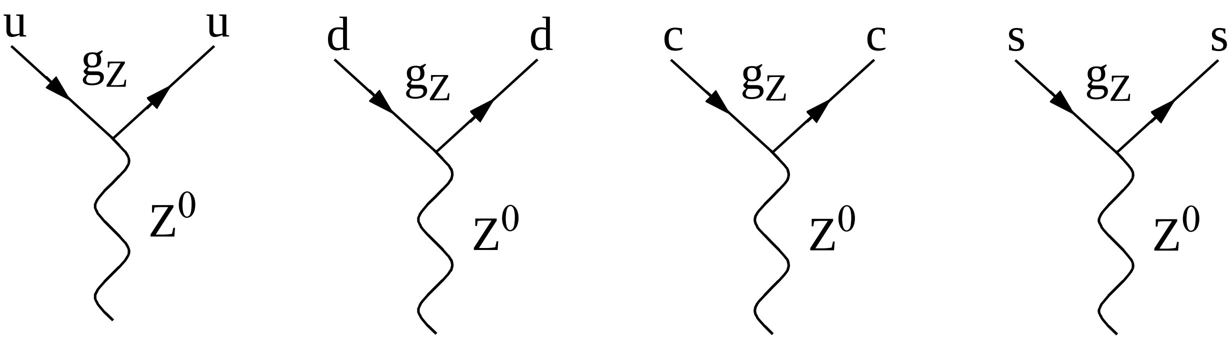


Figure 111: The basic Z^0 lepton and quark vertices.

- Basic vertices with W bosons have:
  - Conserved lepton numbers
  - Not conserved quark flavour (quark mixing)
- Basic vertices with Z^0 bosons have:
  - Conserved lepton numbers
  - Conserved flavour (no quark mixing)



Test of flavour conservation

Flavour is conserved at a Z^0 vertex (in contrast to a W vertex).This can be verified by experiments.

Consider the following two possible processes that change strangeness:

$$K^+ \to \pi^0 + \mu^+ + \nu_\mu \qquad \text{(a)}$$

and

$$K^+ \to \pi^+ + \nu_l + \bar{\nu}_l \qquad \text{(b)}$$

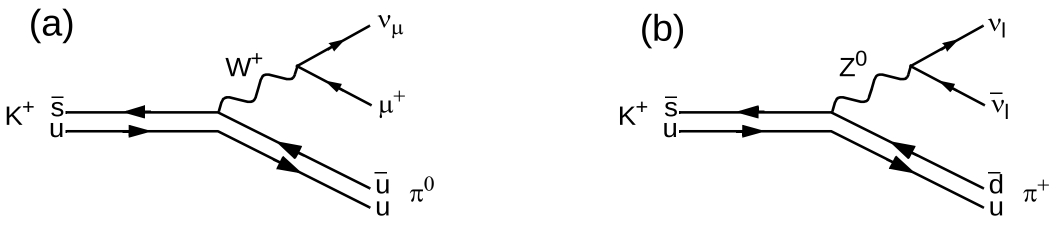
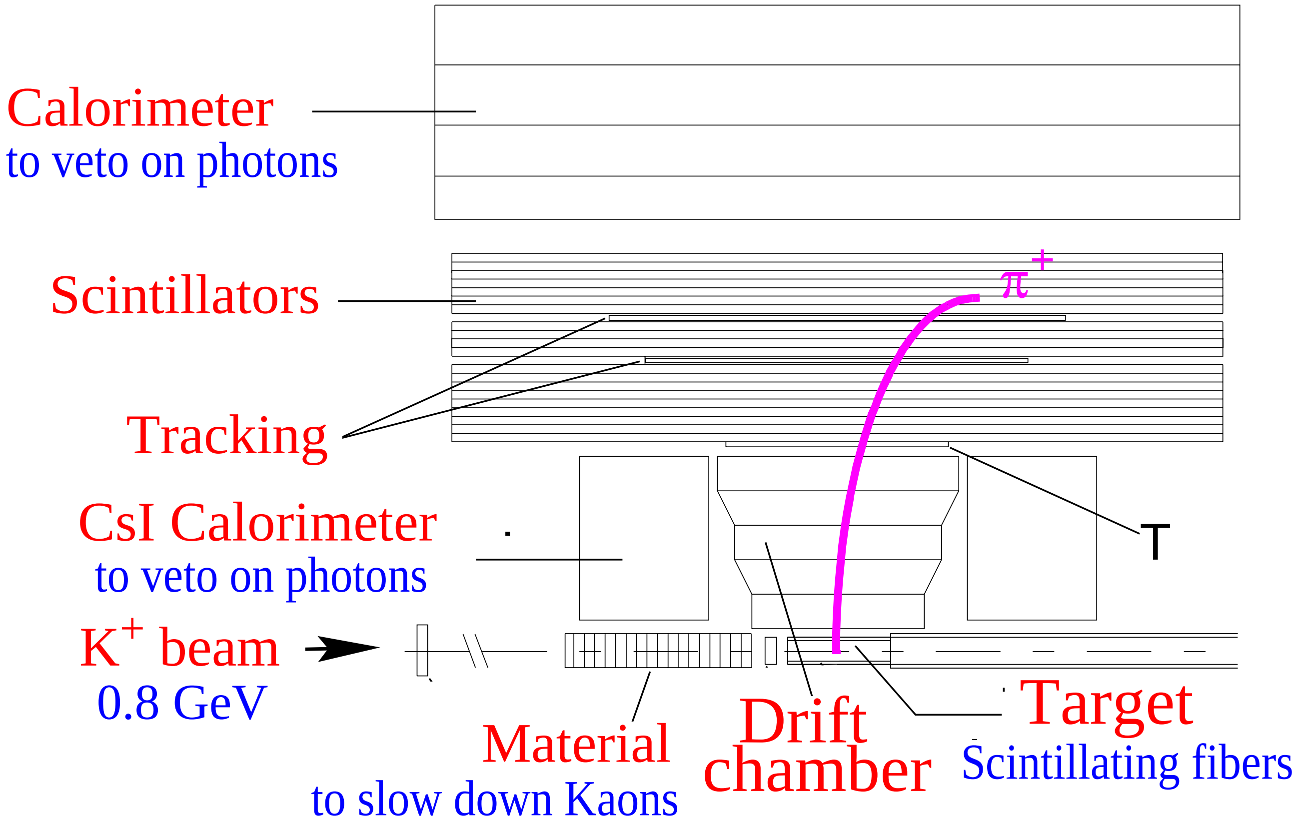


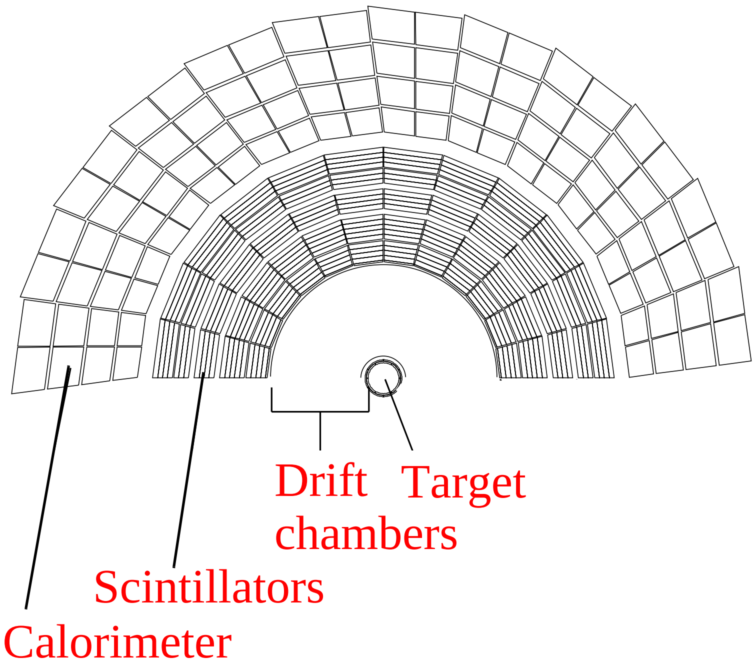
Figure 112: Decay (a) is allowed; decay (b) – forbidden

The measured upper limit on the ratio of the decay rates (b) to (a) was until recently:

$$\frac{\sum\limits_l \Gamma(K^+ \to \pi^+ + \nu_l + \bar{\nu}_l)}{\Gamma(K^+ \to \pi^0 + \mu^+ + \nu_\mu)} < 10^{-7}$$

E787 - A rare kaon decay experiment





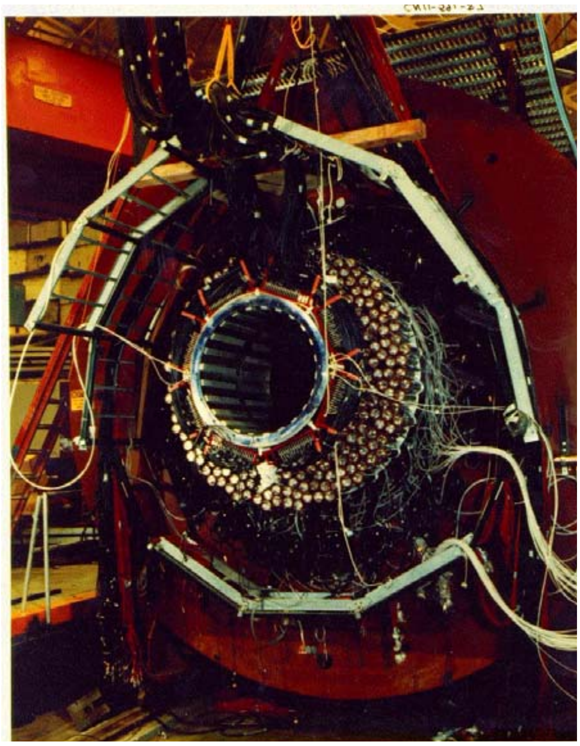


Figure 113: Side and front views of one half of the E787 experiment.

❖ The BNL experiment E787 is a fixed target experiment that uses a K^+ beam created by 24 GeV protons from the AGS accelerator.

→ The Kaons are stopped in a target made of scintillating fibers and the decay of the K^+ at rest is then studied.

→ The momentum, energy and range of the particle from the decay is measured.

→ Two candidate events for $K^+ \rightarrow \pi^+ + \nu_l + \bar{\nu}_l$ have been found after many years of running.

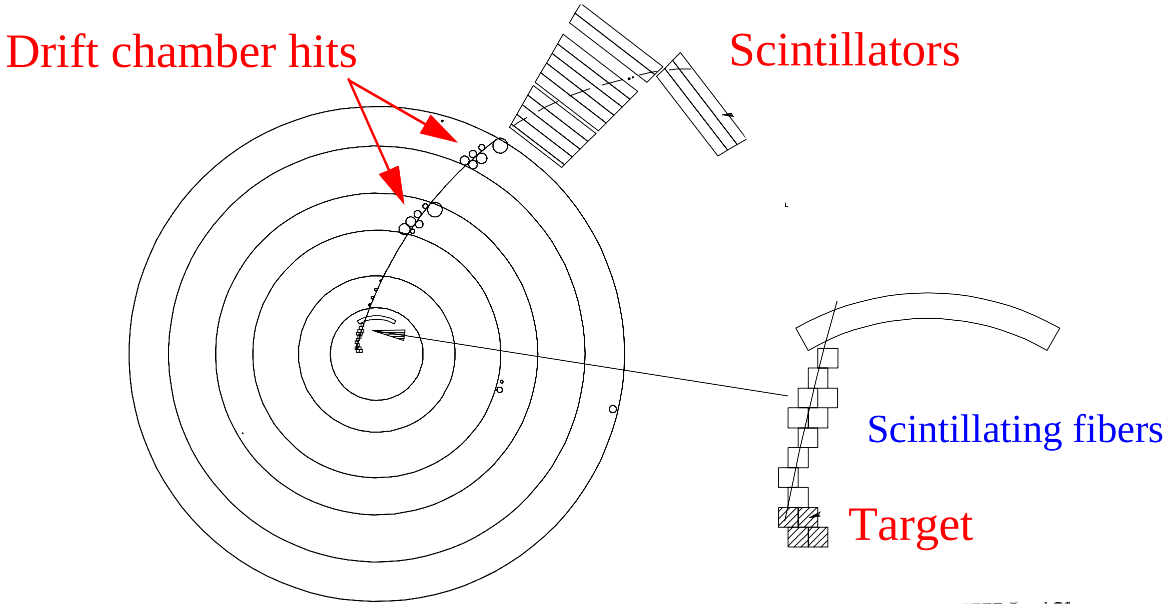


Figure 114: One of the two rare Kaon decay events found by the E787 experiment.

→ The result of the measurement was:

$$\frac{\sum_l \Gamma(K^+ \to \pi^+ + \nu_l + \bar{\nu}_l)}{\Gamma(K^+ \to \pi^0 + \mu^+ + \nu_\mu)} = \frac{1{,}6 \times 10^{-10}}{0{,}033} = 5\times10^{-9}$$

→ The two events could be explained by second-order charged current interactions:

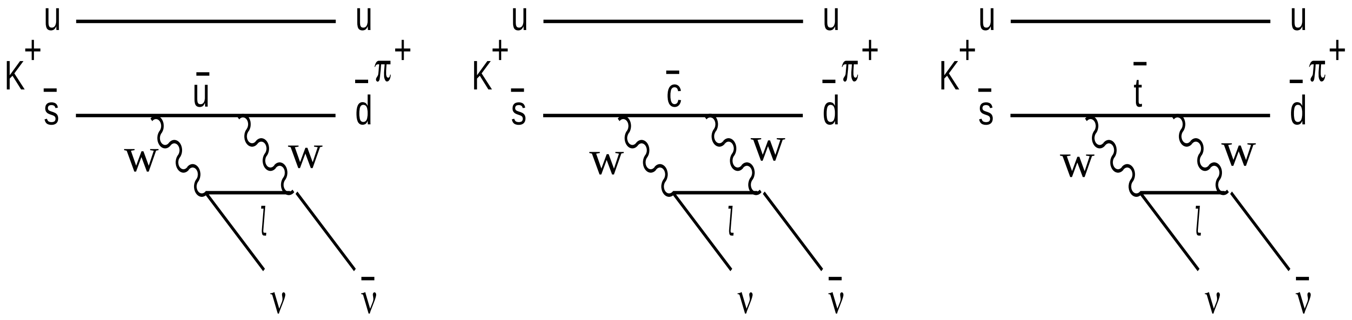


Figure 115: Feynman diagrams of higher-order charged currect interactions resulting in $K^+ \to \pi^+ + \nu_l + \bar{\nu}_l$

→ Due to the t-d vertex in the third diagram above it was also possible to set limits on the V_{td} element in the Cabibbo-Kobayashi-Maskawa matrix:

$$0{,}007 < |V_{td}| < 0{,}030$$

The unification condition and masses

The coupling constants at γ–, $W^{\pm}$– and Z^0-vertices are not independent from each other. In order for all infinities to cancel in electroweak theory, the unification relation and the anomaly condition have to be fulfilled.

→ The *unification condition* establishes a relation between the electroweak coupling constants:

$$\sqrt{\frac{\pi\cdot\alpha_{em}}{2}} = g_W \sin\theta_W = g_Z \cos\theta_W \qquad (114)$$

θ_W is the *weak mixing angle*, or *Weinberg angle*:

$$\cos\theta_W = \frac{M_W}{M_Z} \qquad (115)$$

→ The *anomaly condition* relates electric charges: $\sum_l Q_l + 3\sum_q Q_q = 0$

where the factor 3 comes from the number of colors.

Historically, the W and Z masses were predicted from low energy interactions.

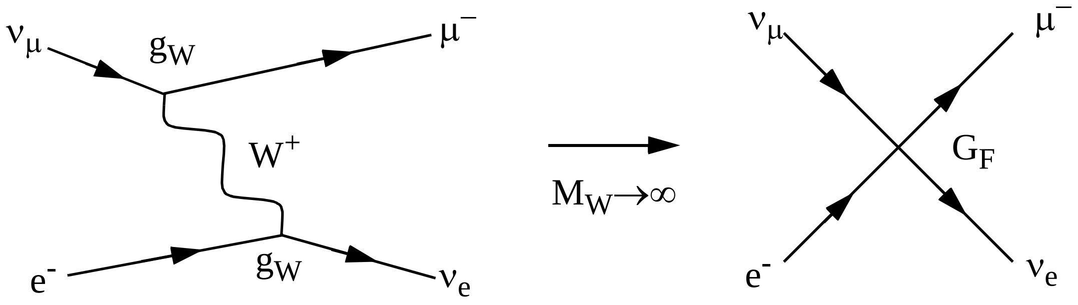


Figure 116: The low energy zero range approximation.

In the zero-range approximation i.e. in the low-energy limit, the charged current reactions are characterized by the Fermi constant (*G_F*):

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2}$$

From this expression, the unification condition and the definition of θ_W one then obtains:

$$M_W^2 = \frac{g_W^2\sqrt{2}}{G_F} = \frac{\pi\alpha_{em}}{\sqrt{2}G_F \sin^2\theta_W}$$

$$M_Z^2 = \frac{\pi\alpha_{em}}{\sqrt{2}G_F \cos^2\theta_W \sin^2\theta_W}$$

Introducing the neutral current coupling constant (G_Z) (also in the low energy zero-range approximation) one gets

$$\frac{G_Z}{\sqrt{2}} = \frac{g_Z^2}{M_Z^2}$$

and the weak mixing angle can be expressed as

$$\frac{G_Z}{G_F} = \frac{g_Z^2 M_W^2}{g_W^2 M_Z^2} = \sin^2\theta_W$$

From the measurements at low energy of rates of charged and neutral currents reactions it is therefore possible to determine that:

$$\sin^2\theta_W = 0,277 \pm 0,014$$

from this measurement at low energies (below the W and Z masses) it was possible to predict the masses of W and Z:

$$M_W = 78,3 \pm 2,4\ GeV/c^2;\ M_Z = 89,0 \pm 2,0\ GeV/c^2$$

When the W and Z boson were discovered at CERN with the masses predicted from low energy experiments it was a strong confirmation that the electroweak theory was correct.

Today the most precise estimation of the Weinberg angle using many measurements give:

$$\sin^2\theta_W = 0{,}2255 \pm 0{,}0021$$

Putting this value into the previous formulas give

$M_W = 78.5$ GeV and $M_Z = 89.3$ GeV

while the direct measurements of the masses give

$M_W = 80.4$ GeV and $M_Z = 91.2$ GeV

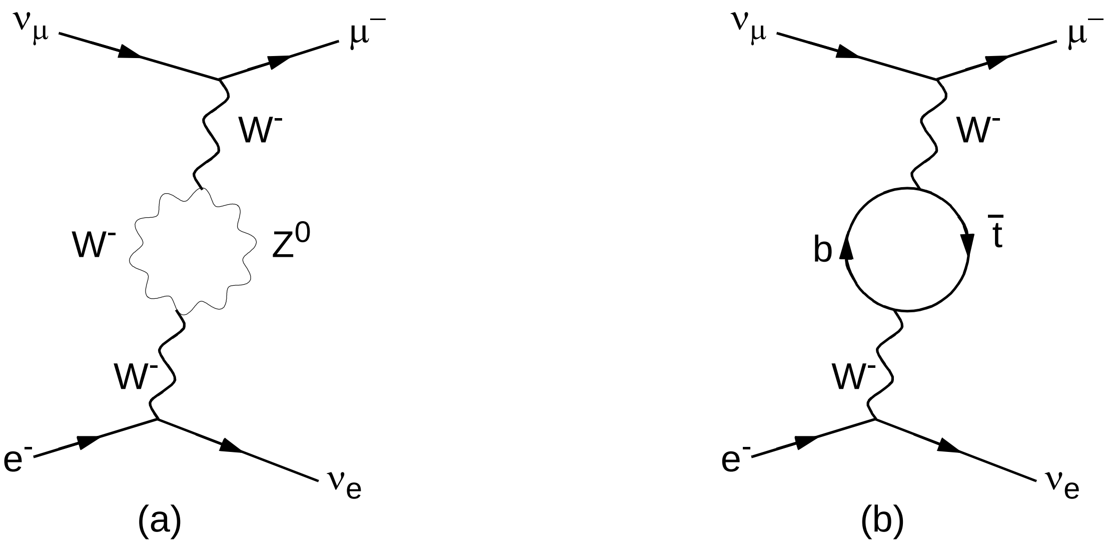


Figure 117: Examples of higher order contributions to inverse muon decay

The difference is due to higher-order diagrams which were not included in the previous low-energy formulas.

Since the top-quark is involved in higher order corrections, the measurement of electroweak processes could be used to predict the top-quark mass before it was discovered:

$$m_t = 170 \pm 30\ GeV/c^2$$

The directly measured mass of the top quark at Fermilab by CDF is today

$$m_t = 176 \pm 5\ GeV/c^2$$

in perfect agreement with the prediction !

Electroweak reactions

- In any process in which a photon can be exchanged, a Z^0 boson can be exchanged as well:

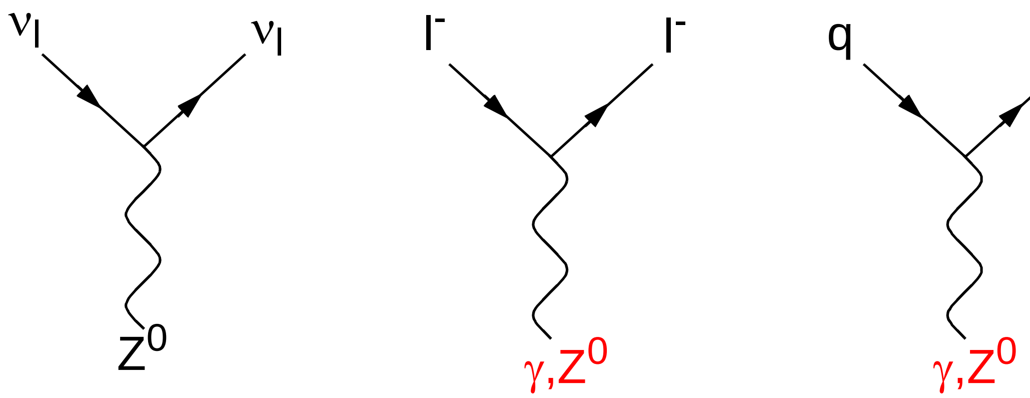


Figure 118: Z^0 and γ couplings to leptons and quarks

Example: The reaction $e^+e^- \rightarrow \mu^+\mu^-$ has two dominant contributions:

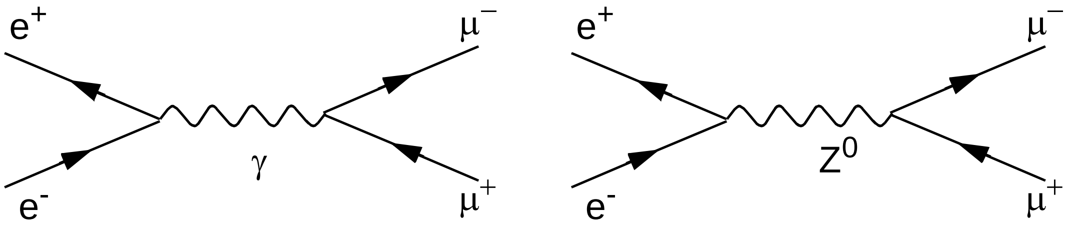


Figure 119: Dominant contributions to the e^+e^- annihilation into muons

With simple dimensional arguments one can estimate the cross section for the photon- and Z-exchange process at low energy:

$$\sigma_\gamma \approx \frac{\alpha^2}{E^2} \qquad \sigma_Z \approx G_Z^2 E^2$$

Where E is the energy of the colliding electron and positron beams.

From these expressions, the ratio of σ_Z and σ_γ is:

$$\frac{\sigma_Z}{\sigma_\gamma} \approx \frac{E^4}{M_Z^4} \qquad (116)$$

One can conclude that at low energies the photon exchange process dominates. However, at energies $E_{CM}=M_Z$, this low-energy approximation fails

The Z^0 peak is described by the Breit-Wigner formula:

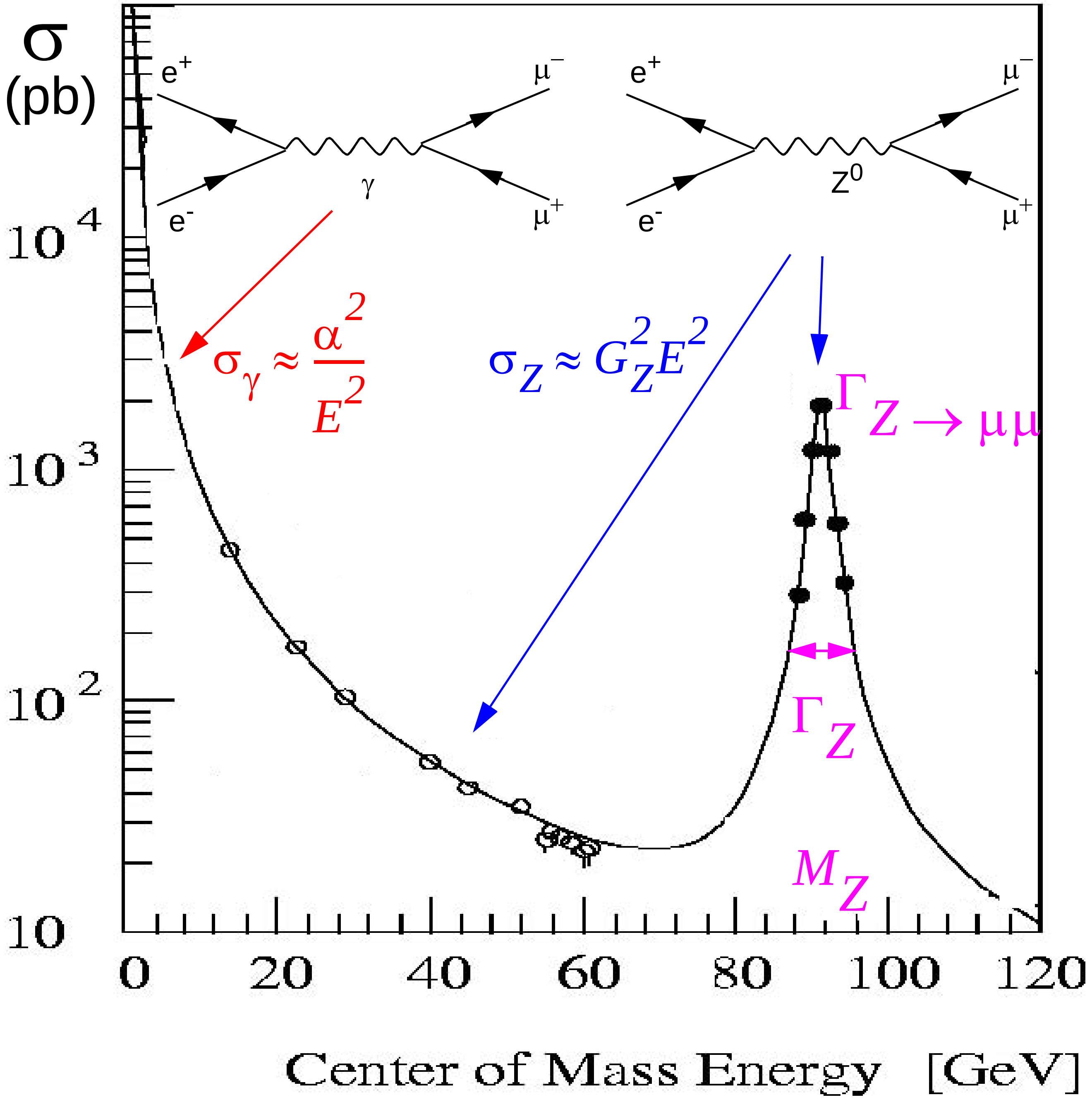
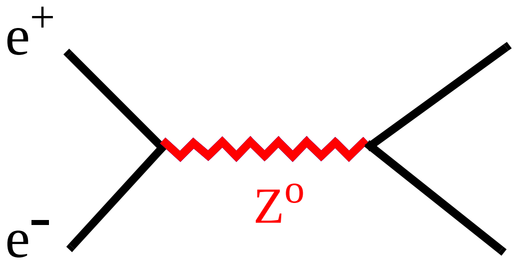


Figure 120: The cross sections of e^+e^- annihilation into $\mu\mu$

$$\sigma(ee \to \mu\mu) = \frac{12\pi M_Z^2}{E_{CM}^2}\left[\frac{\Gamma(Z^0 \to ee)\Gamma(Z^0 \to \mu\mu)}{(E_{CM}^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right]$$

The number of neutrino families

- The Z boson can decay in the following way:



	$e^-\ \mu^-\ \tau^-$ $\underbrace{e^+\ \mu^+\ \tau^+}$	ν $\bar{\nu}$	q $\bar{q}$
Branching ratio (B):	0.10	0.20	0.70
Decay width (Γ):	0.25 GeV	0.50 GeV	1.74 GeV

The lifetime (τ) , the branching ratio (B) and the partial decay width (Γ) are related to each other by

$$\tau = \frac{B}{\Gamma}$$

$$\tau = \frac{B_Z}{\Gamma_Z} = \frac{B_{had} + B_{ll} + B_{\nu\nu}}{\Gamma_{had} + \Gamma_{ll} + \Gamma_{\nu\nu}}$$

$$\tau = \frac{B_{had}}{\Gamma_{had}} = \frac{B_{ll}}{\Gamma_{ll}} = \frac{B_{\nu\nu}}{\Gamma_{\nu\nu}} = 3\times 10^{-25} s$$

Note: $1GeV^{-1} = 6, 582\times 10^{-25} s$

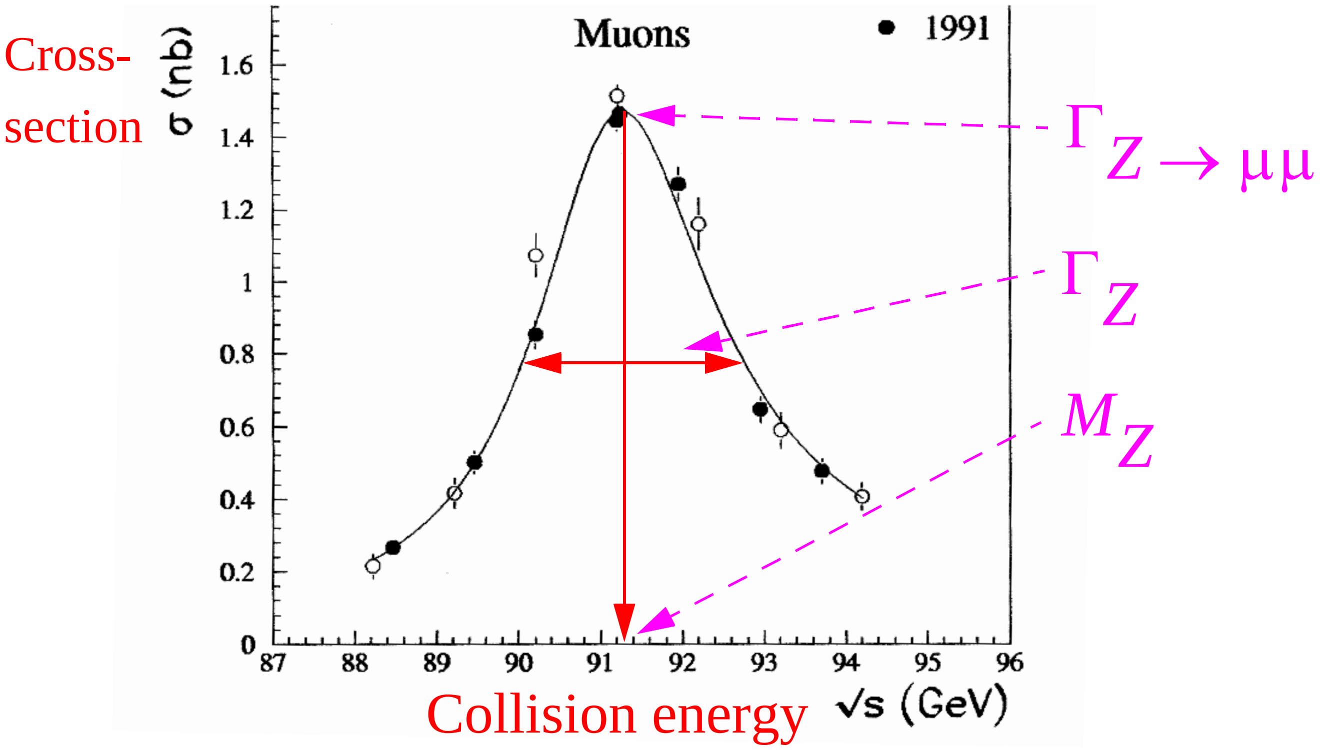


Figure 121: The leptonic decay of the Z^0 into muons.

The peak can be fitted with the Breit-Wigner formula:

$$\sigma(e^+e^- \to X) = \frac{12\pi M_Z^2}{E_{CM}^2}\left[\frac{\Gamma(Z^0 \to e^+e^-)\Gamma(Z^0 \to X)}{(E_{CM}^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right]$$

Here Γ_Z is the total Z^0 decay rate, and $\Gamma_Z(Z^0 \to X)$ is the decay rates to the final state X. The height of the peak (at $E_{CM}=M_Z$) is proportional to the product of the branching ratios:

$$B(Z^0 \to e^+e^-)B(Z^0 \to X) \equiv \frac{\Gamma(Z^0 \to e^+e^-)}{\Gamma_Z}\frac{\Gamma(Z^0 \to X)}{\Gamma_Z}$$

The fitted parameters of the Z^0 peak in the leptonic and hadronic decay modes give:

$$M_Z = 91{,}187 \pm 0{,}007\ GeV/c^2$$

$$\Gamma_Z = 2{,}490 \pm 0{,}007\ GeV$$

$$\Gamma(Z^0 \to hadrons) = 1{,}741 \pm 0{,}006\ GeV$$

$$\Gamma(Z^0 \to l^+ l^-) = 0{,}0838 \pm 0{,}0003\ GeV$$

→ The decays $Z^0 \to l^+l^-$ and $Z^0 \to hadrons$ account for only about 80% of all Z^0 decays

→ The remaining decays are those containing only neutrinos in the final state since

$$\Gamma_Z = \Gamma(Z^0 \to hadrons) + 3\Gamma(Z^0 \to l^+ l^-) + \\ + N_\nu \Gamma(Z^0 \to \nu_l \bar{\nu}_l) \qquad (117)$$

From the measurement of all other partial widths one can therefore estimate the partial decay to neutrinos which cannot be measured directly:

$$N_\nu \Gamma(Z^0 \to \nu_l \bar{\nu}_l) = 0{,}498 \pm 0{,}009\ GeV$$

The decay rate to neutrino pairs can also be calculated from the diagrams shown previously and this gives $\Gamma(Z^0 \to \nu_l\bar{\nu}_l) = 0{,}166\ GeV$ which together with $N_\nu\Gamma(Z^0 \to \nu_l\bar{\nu}_l) = 0{,}498\ GeV$ gives

$$N_\nu = 2{,}994 \pm 0{,}011$$

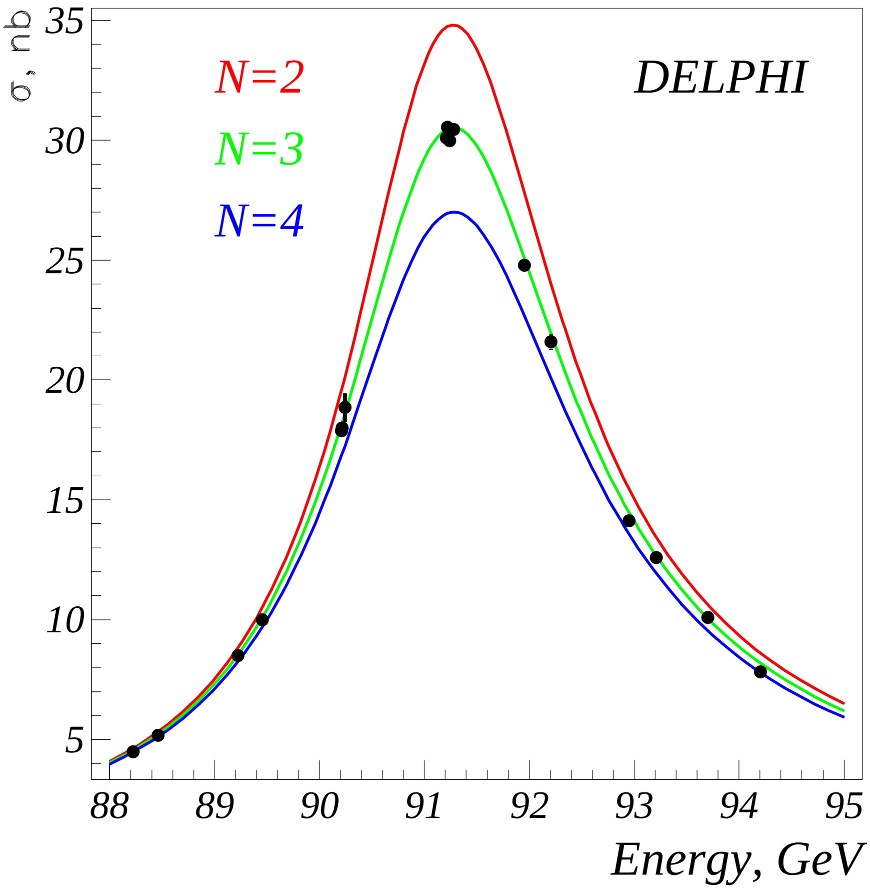


Figure 122: The decay of the Z^0 to hadrons and theoretical predictions based on different assumptions for the number of neutrino families (N)

- → There are no explicit restrictions on the number of generations in the Standard Model.

- → However, the analysis of the Z^0 line shape at LEP shows that there are 3 and only 3 kinds of <u>light</u> neutrinos.

  - ❖ If neutrinos are assumed having negligible masses as compared with the Z^0 mass, there must be only THREE generations of leptons and quarks within the Standard Model.