

IX. Electroweak unification

The problem of divergence

❖ A theory of weak interactions only by means of W^\pm bosons leads to **infinities**

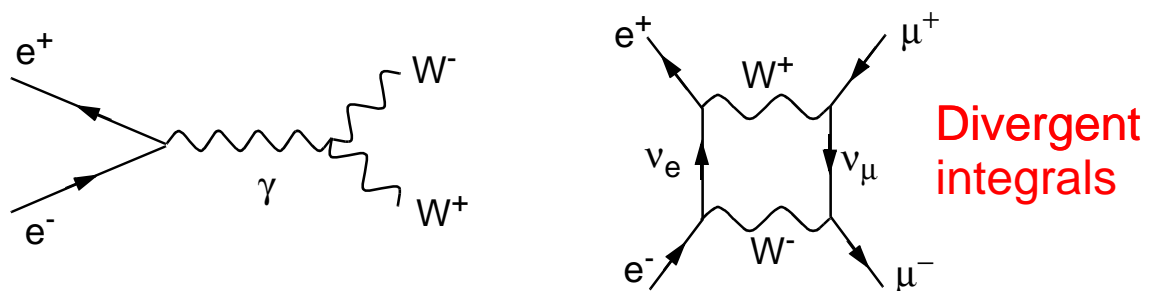


Figure 108: Examples of divergent processes.

➔ Introduction of the Z^0 boson fixes the problem because the addition of new diagrams **cancel** out the **divergencies**:

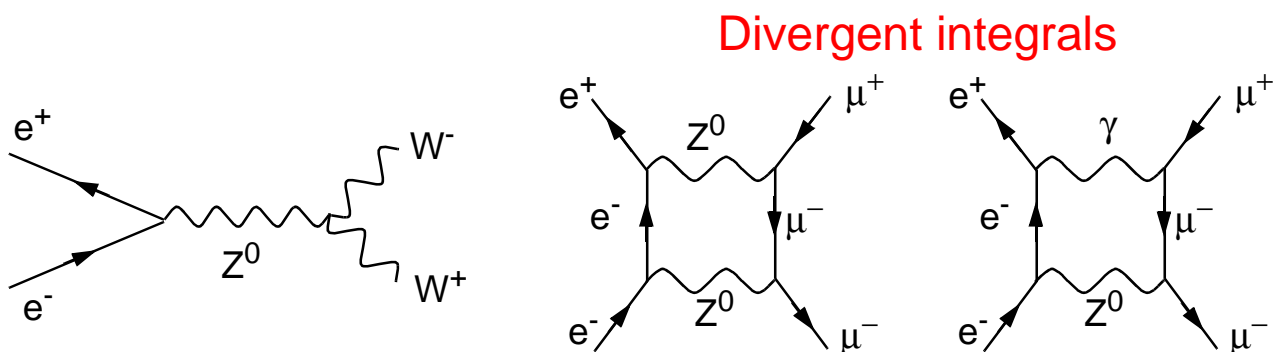


Figure 109: Additional processes which cancel the divergence.

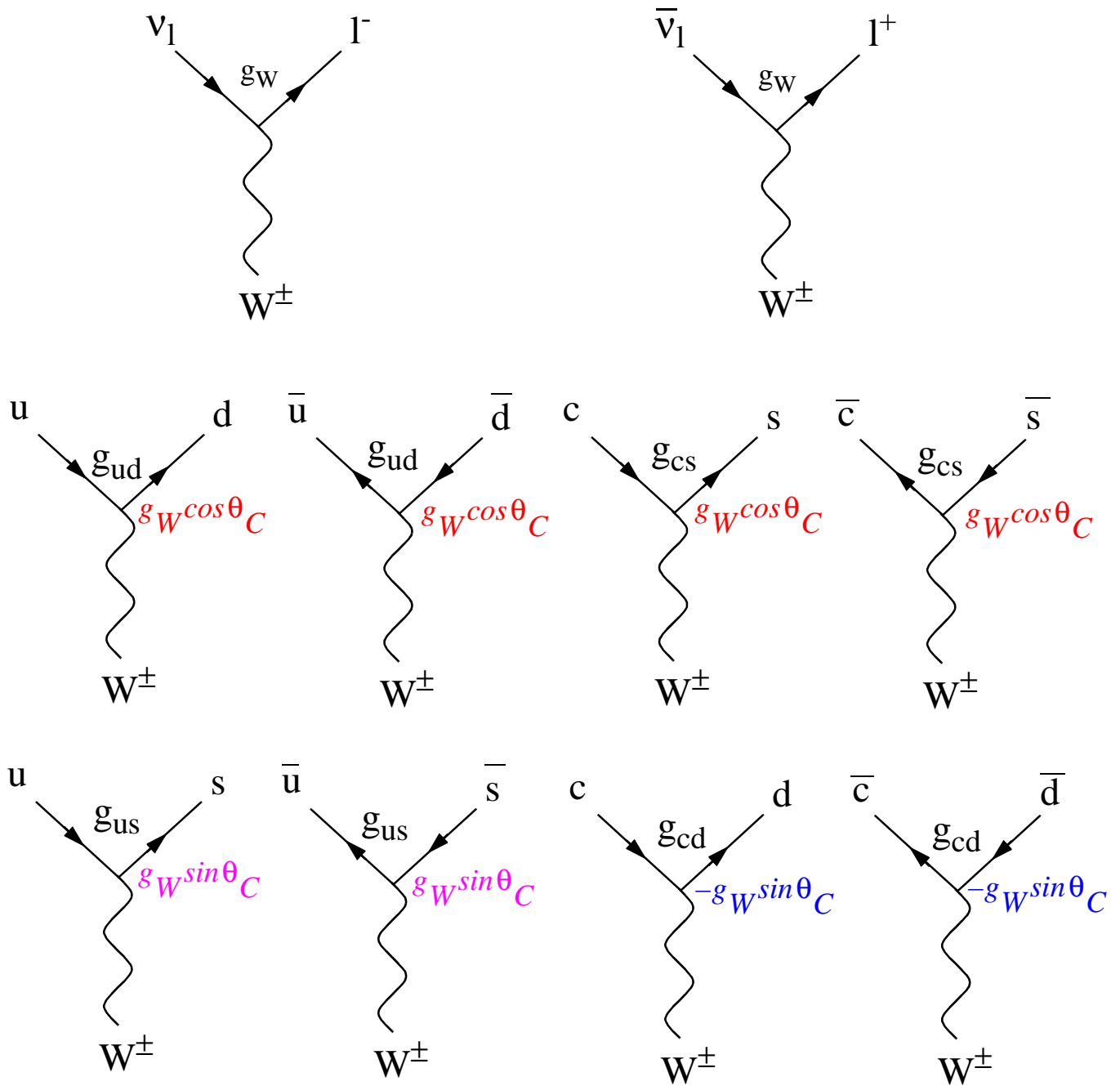


Figure 110: REMINDER: The basic W lepton and quark vertices (if the third generation is not taken into account).

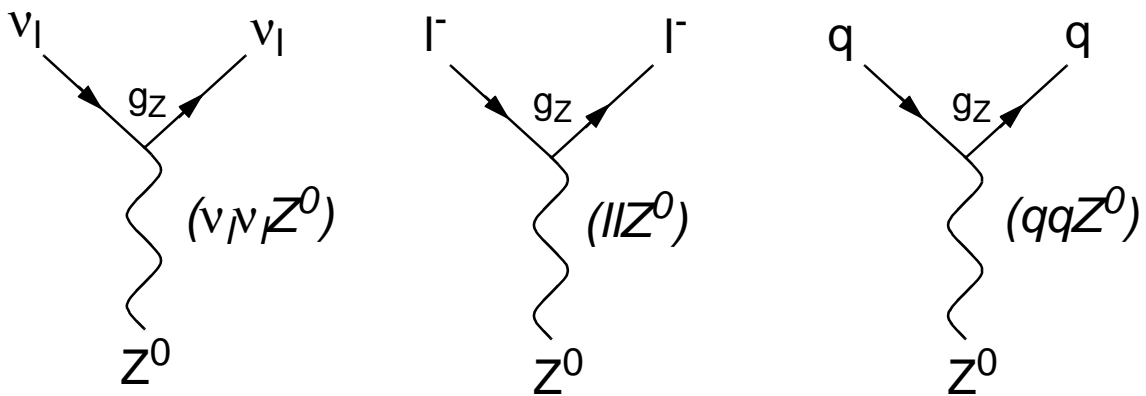


Figure 111: The basic Z^0 lepton and quark vertices.



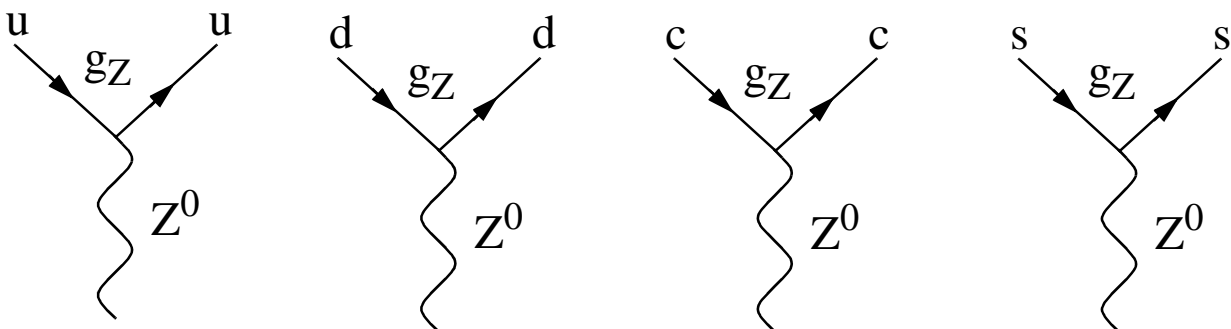
Basic vertices with W bosons have:

- Conserved lepton numbers
- Not conserved quark flavour (quark mixing)



Basic vertices with Z^0 bosons have:

- Conserved lepton numbers
- Conserved flavour (no quark mixing)



Test of flavour conservation

Flavour is **conserved** at a Z^0 vertex (in contrast to a W vertex). This can be verified by experiments.

Consider the following two possible processes that change strangeness:

$$K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu \quad (\text{a})$$

and

$$K^+ \rightarrow \pi^+ + \nu_l + \bar{\nu}_l \quad (\text{b})$$

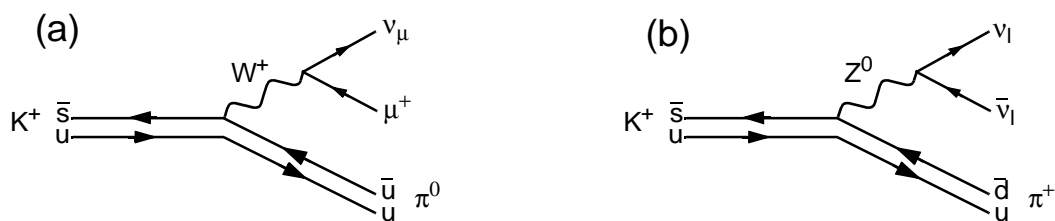


Figure 112: Decay (a) is allowed; decay (b) – forbidden

The measured upper limit on the ratio of the decay rates (b) to (a) is:

$$\frac{\sum_l \Gamma(K^+ \rightarrow \pi^+ + \nu_l + \bar{\nu}_l)}{\Gamma(K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu)} < 10^{-7}$$

The unification condition and masses

The coupling constants at γ -, W^\pm - and Z^0 -vertices are not independent from each other. In order for all **infinities to cancel** in electroweak theory, the **unification relation** and the **anomaly condition** have to be fulfilled.

→ The *unification condition* establishes a relation between the coupling constants ($\alpha_{em}=e^2/4\pi\epsilon_0$):

$$\sqrt{\frac{\pi \cdot \alpha}{2}} = g_W \sin \theta_W = g_Z \cos \theta_W \quad (114)$$

θ_W is the *weak mixing angle*, or *Weinberg angle*:

$$\cos \theta_W = \frac{M_W}{M_Z} \quad (115)$$

→ The *anomaly condition* relates electric

charges:
$$\sum_l Q_l + 3 \sum_q Q_q = 0$$

where the factor 3 comes from the number of colors.

Historically, the **W and Z masses** were predicted from **low energy interactions**.

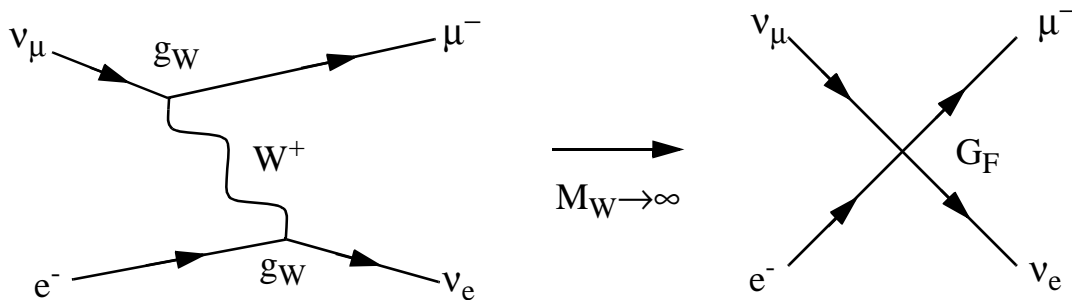


Figure 113: The low energy zero range approximation.

In the zero-range approximation i.e. in the low-energy limit, the charged current reactions are characterized by the **Fermi constant** (G_F):

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2}$$

From this expression, the unification condition and the definition of θ_W one then obtains:

$$M_W^2 = \frac{g_W^2 \sqrt{2}}{G_F} = \frac{\pi \alpha}{\sqrt{2} G_F \sin^2 \theta_W}$$

$$M_Z^2 = \frac{\pi \alpha}{\sqrt{2} G_F \cos^2 \theta_W \sin^2 \theta_W}$$

Introducing the **neutral current coupling** constant (G_Z) (also in the low energy zero-range approximation) one gets

$$\frac{G_Z}{\sqrt{2}} = \frac{g_Z^2}{M_Z^2}$$

and the weak mixing angle can be expressed as

$$\frac{G_Z}{G_F} = \frac{g_Z^2 M_W^2}{g_W^2 M_Z^2} = \sin^2 \theta_W$$

From the **measurements at low energy** of rates of charged and neutral currents reactions it is therefore possible to determine that:

$$\sin^2 \theta_W = 0,277 \pm 0,014$$

from this measurement at low energies (below the W and Z masses) it was possible to **predict the masses** of W and Z:

$$M_W = 78,3 \pm 2,4 \text{ GeV}/c^2; M_Z = 89,0 \pm 2,0 \text{ GeV}/c^2$$

When the W and Z boson were discovered at CERN with the masses predicted from low energy experiments it was a strong **confirmation** that the **electroweak theory** was correct.

Today the most precise estimation of the Weinberg angle using many measurements give:

$$\sin^2 \theta_W = 0,2255 \pm 0,0021$$

Putting this value into the previous formulas give

$$M_W = 78.5 \text{ GeV} \text{ and } M_Z = 89.3 \text{ GeV}$$

while the direct measurements of the masses give

$$M_W = 80.4 \text{ GeV} \text{ and } M_Z = 91.2 \text{ GeV}$$

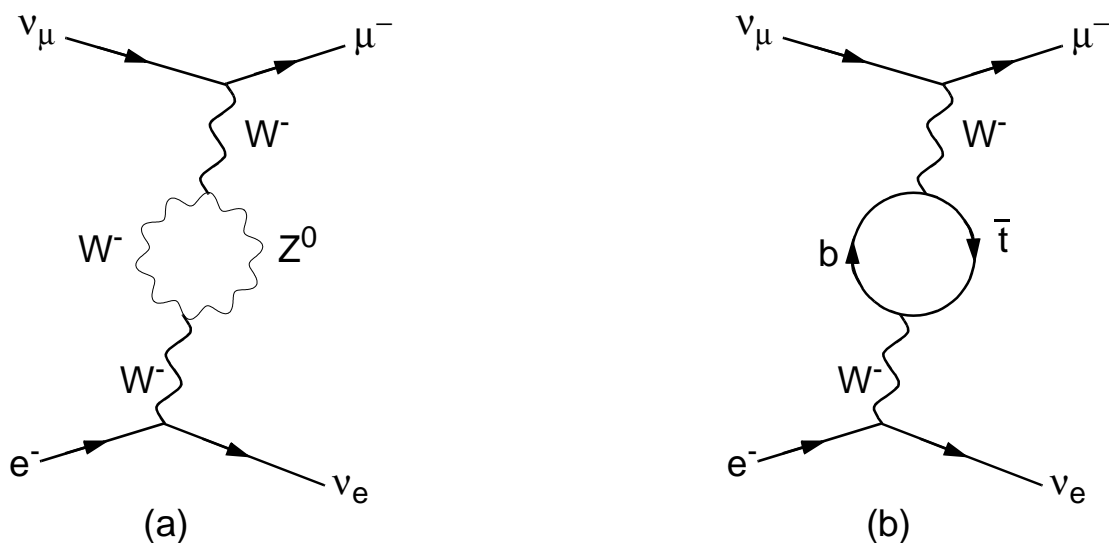


Figure 114: Examples of higher order contributions to inverse muon decay

The difference is due to **higher-order diagrams** which were not included in the previous low-energy formulas.

Since the top-quark is involved in higher order corrections, the measurement of electroweak processes could be used to **predict the top-quark mass** before it was discovered:

$$m_t = 170 \pm 30 \text{ GeV}/c^2$$

The directly measured mass of the top quark at Fermilab by CDF is today

$$m_t = 176 \pm 5 \text{ GeV}/c^2$$

in perfect agreement with the prediction !

Electroweak reactions

❖ In any process in which a **photon** can be exchanged, a **Z^0** boson can be **exchanged** as well:

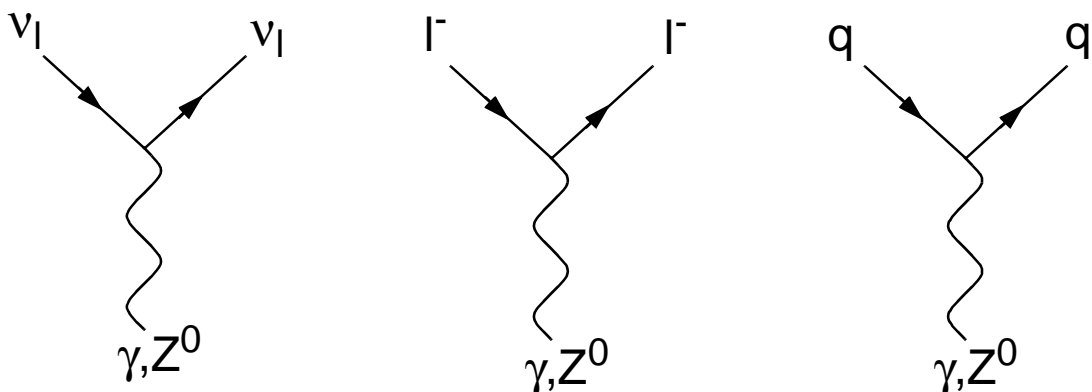


Figure 115: Z^0 and γ couplings to leptons and quarks

Example: The reaction $e^+e^- \rightarrow \mu^+\mu^-$ has two dominant contributions:

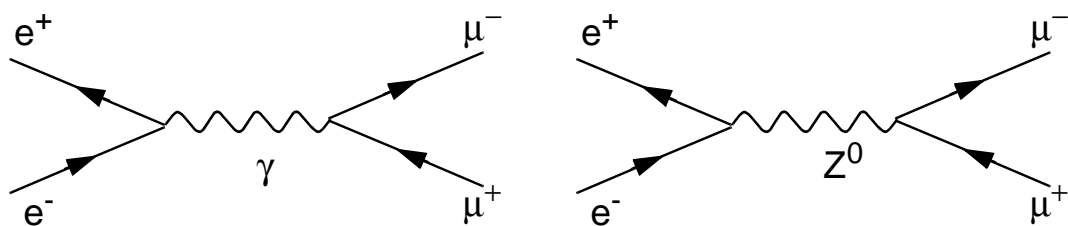


Figure 116: Dominant contributions to the e^+e^- annihilation into muons

With simple dimensional arguments one can estimate the **cross section** for the photon- and Z-exchange process at low energy:

$$\sigma_{\gamma} \approx \frac{\alpha^2}{E^2} \quad \sigma_Z \approx G_Z^2 E^2$$

Where E is the energy of the colliding electron and positron beams.

From these expressions, the ratio of σ_Z and σ_{γ} is:

$$\frac{\sigma_Z}{\sigma_{\gamma}} \approx \frac{E^4}{M_Z^4} \quad (116)$$

One can conclude that at low energies the **photon exchange** process **dominates**. However, at energies $E_{CM}=M_Z$, this low-energy approximation fails

The Z^0 peak is described by the **Breit-Wigner** formula:

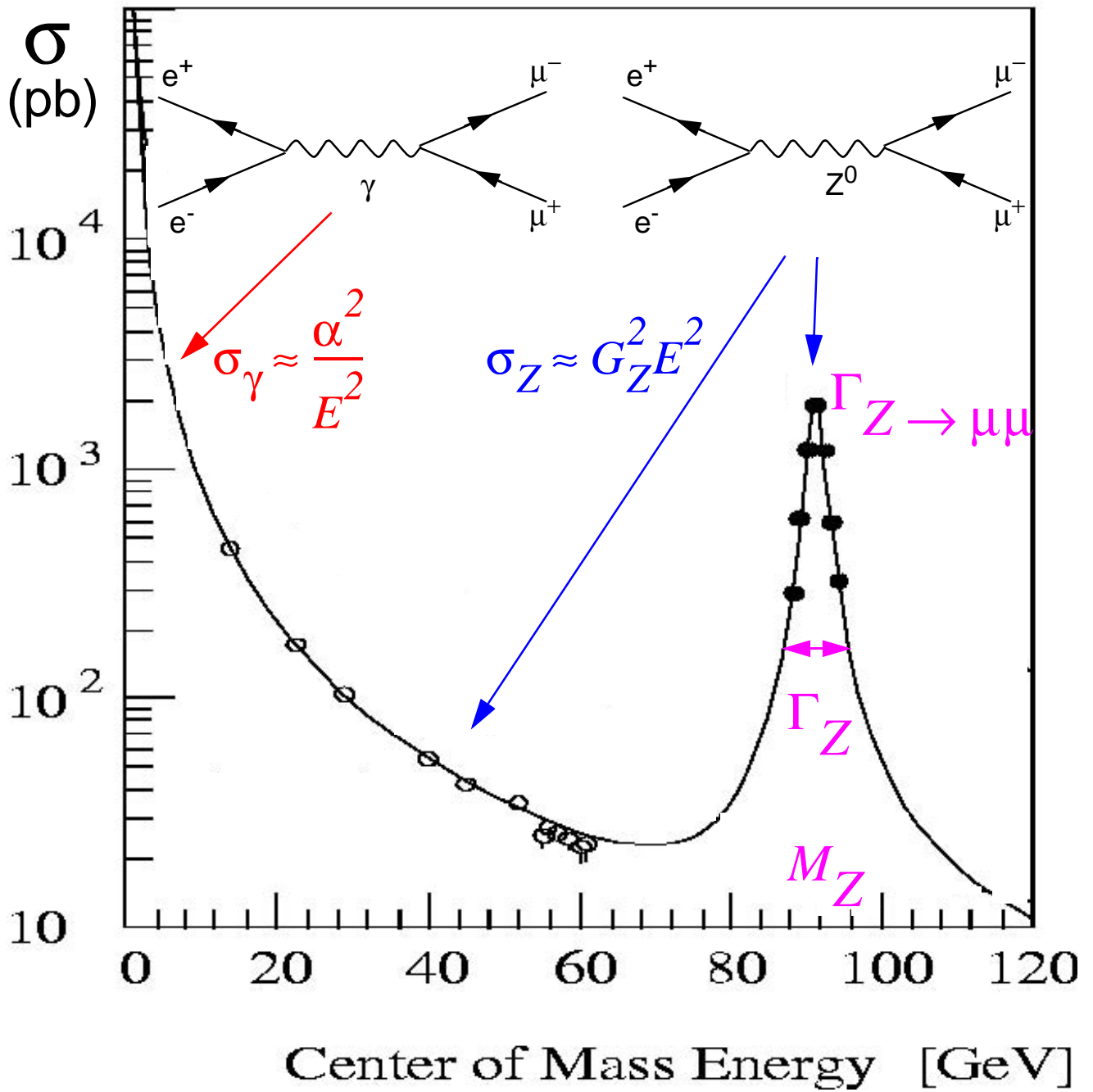
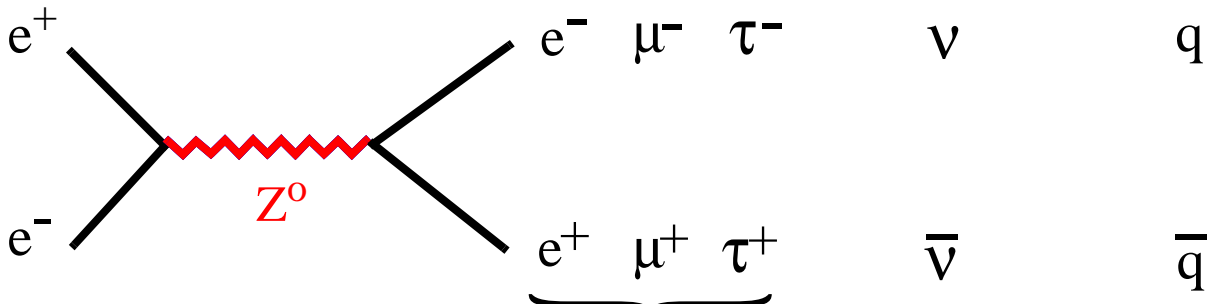


Figure 117: The cross sections of e^+e^- annihilation into $\mu\mu$

$$\sigma(ee \rightarrow \mu\mu) = \frac{12\pi M_Z^2}{E_{CM}^2} \left[\frac{\Gamma(Z^0 \rightarrow ee)\Gamma(Z^0 \rightarrow \mu\mu)}{(E_{CM}^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \right]$$

The number of neutrino families

❖ The Z boson can decay in the following way:



Branching ratio (B):	0.10	0.20	0.70
Decay width (Γ):	0.25 GeV	0.50 GeV	1.74 GeV

The lifetime (τ), the branching ratio (B) and the partial decay width (Γ) are related to each other by

$$\tau = \frac{B}{\Gamma}$$

$$\tau = \frac{B_Z}{\Gamma_Z} = \frac{B_{had} + B_{ll} + B_{\nu\nu}}{\Gamma_{had} + \Gamma_{ll} + \Gamma_{\nu\nu}}$$

$$\tau = \frac{B_{had}}{\Gamma_{had}} = \frac{B_{ll}}{\Gamma_{ll}} = \frac{B_{\nu\nu}}{\Gamma_{\nu\nu}} = 3 \times 10^{-25} s$$

Note: $1 GeV^{-1} = 6,582 \times 10^{-25} s$

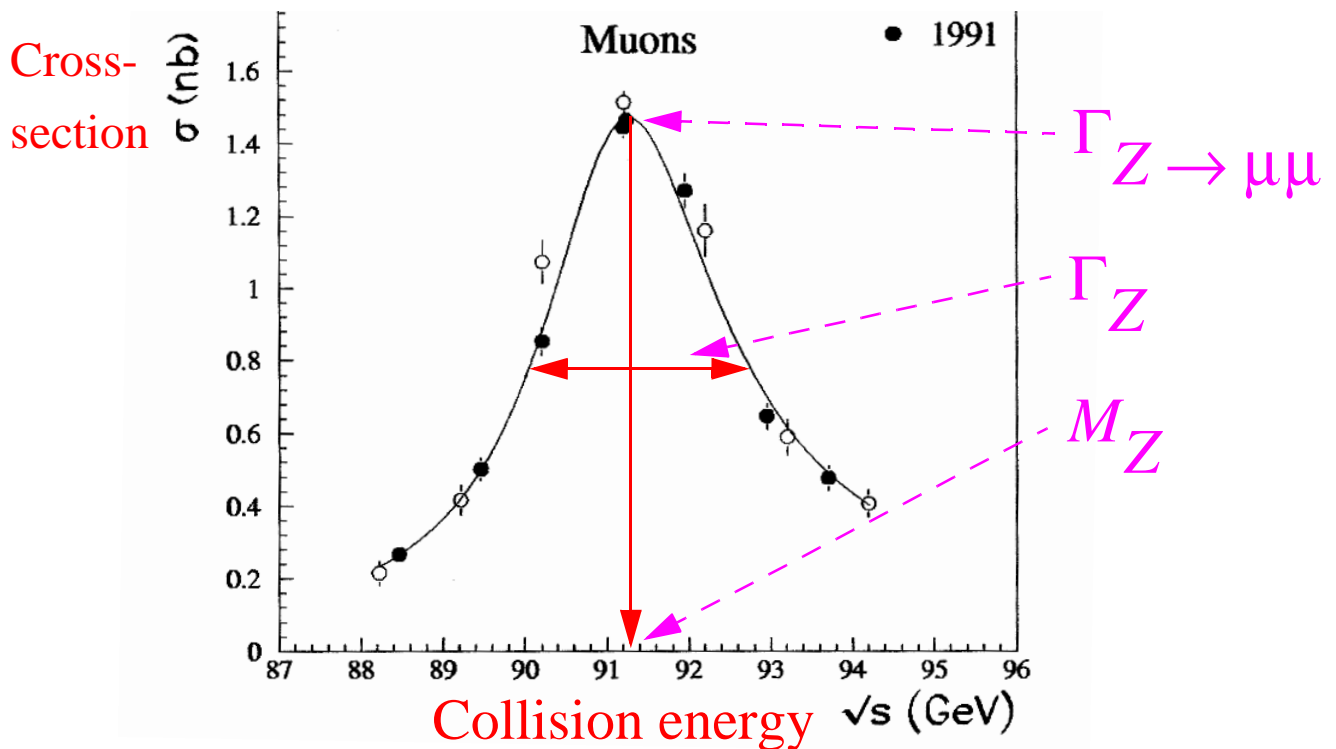


Figure 118: The leptonic decay of the Z^0 into muons.

The peak can be fitted with the Breit-Wigner formula:

$$\sigma(e^+e^- \rightarrow X) = \frac{12\pi M_Z^2}{E_{CM}^2} \left[\frac{\Gamma(Z^0 \rightarrow e^+e^-)\Gamma(Z^0 \rightarrow X)}{(E_{CM}^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2} \right]$$

Here Γ_Z is the total Z^0 decay rate, and $\Gamma_Z(Z^0 \rightarrow X)$ is the decay rates to the final state X . The height of the peak (at $E_{CM}=M_Z$) is proportional to the product of the branching ratios:

$$B(Z^0 \rightarrow e^+e^-)B(Z^0 \rightarrow X) \equiv \frac{\Gamma(Z^0 \rightarrow e^+e^-)\Gamma(Z^0 \rightarrow X)}{\Gamma_Z^2}$$

The fitted parameters of the Z^0 peak in the leptonic and hadronic decay modes give:

$$M_Z = 91,187 \pm 0,007 \text{ GeV}/c^2$$

$$\Gamma_Z = 2,490 \pm 0,007 \text{ GeV}$$

$$\Gamma(Z^0 \rightarrow \text{hadrons}) = 1,741 \pm 0,006 \text{ GeV}$$

$$\Gamma(Z^0 \rightarrow l^+ l^-) = 0,0838 \pm 0,0003 \text{ GeV}$$

→ The decays $Z^0 \rightarrow l^+ l^-$ and $Z^0 \rightarrow \text{hadrons}$ account for only about **80%** of all Z^0 decays

→ The remaining decays are those containing only **neutrinos** in the final state since

$$\Gamma_Z = \Gamma(Z^0 \rightarrow \text{hadrons}) + 3\Gamma(Z^0 \rightarrow l^+ l^-) + N_\nu \Gamma(Z^0 \rightarrow \nu_l \bar{\nu}_l) \quad (117)$$

From the measurement of all other partial widths one can therefore estimate the **partial decay to neutrinos** which cannot be measured directly:

$$N_\nu \Gamma(Z^0 \rightarrow \nu_l \bar{\nu}_l) = 0,498 \pm 0,009 \text{ GeV}$$

The decay rate to neutrino pairs can also be **calculated** from the diagrams shown previously and this gives $\Gamma(Z^0 \rightarrow \nu_l \bar{\nu}_l) = 0,166 \text{ GeV}$ which together with $N_\nu \Gamma(Z^0 \rightarrow \nu_l \bar{\nu}_l) = 0,498 \text{ GeV}$ gives

$$N_\nu = 2,994 \pm 0,011$$

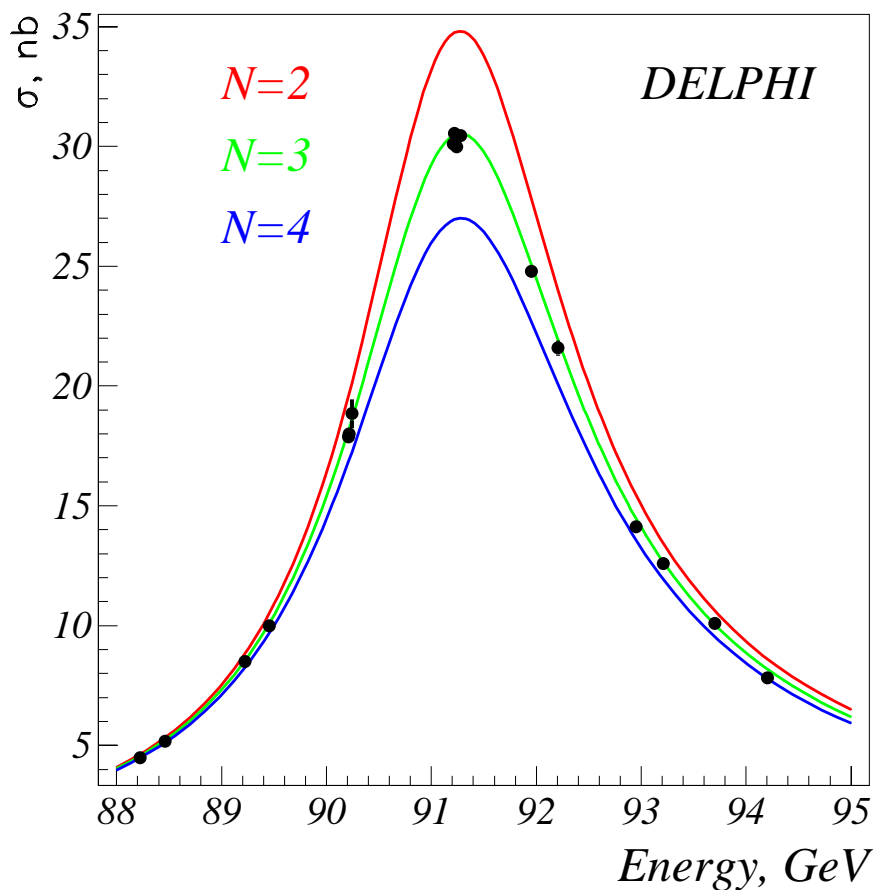


Figure 119: The decay of the Z^0 to hadrons and theoretical predictions based on different assumptions for the number of neutrino families (N)

- There are **no explicit restrictions** on the number of generations in the Standard Model.
- However, the analysis of the Z^0 line shape at LEP shows that there are 3 and only 3 kinds of light neutrinos.
- ❖ If neutrinos are assumed having negligible masses as compared with the Z^0 mass, there must be only **THREE generations** of leptons and quarks within the Standard Model.