

Weak interaction of hadrons.

❖ Weak interactions of hadrons: constituent quarks emit or absorb W bosons

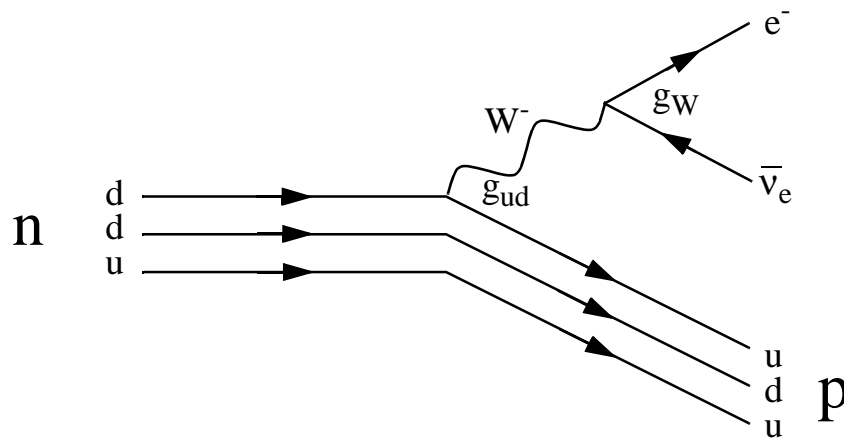


Figure 99: Neutron β -decay.

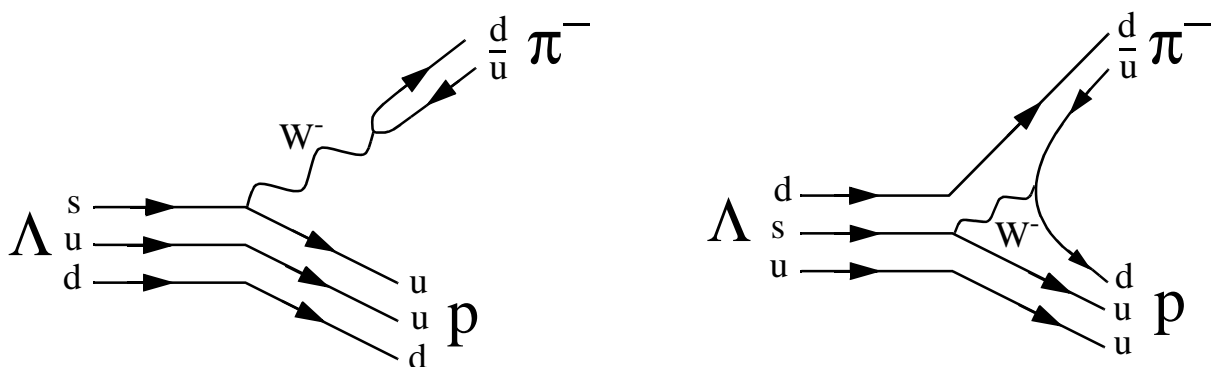


Figure 100: The dominant quark diagrams for Λ decay.

❖ **Lepton-quark symmetry:** assumption that corresponding generations of quarks and leptons have identical weak interactions:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \leftrightarrow \begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \leftrightarrow \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \leftrightarrow \begin{pmatrix} t \\ b \end{pmatrix}$$

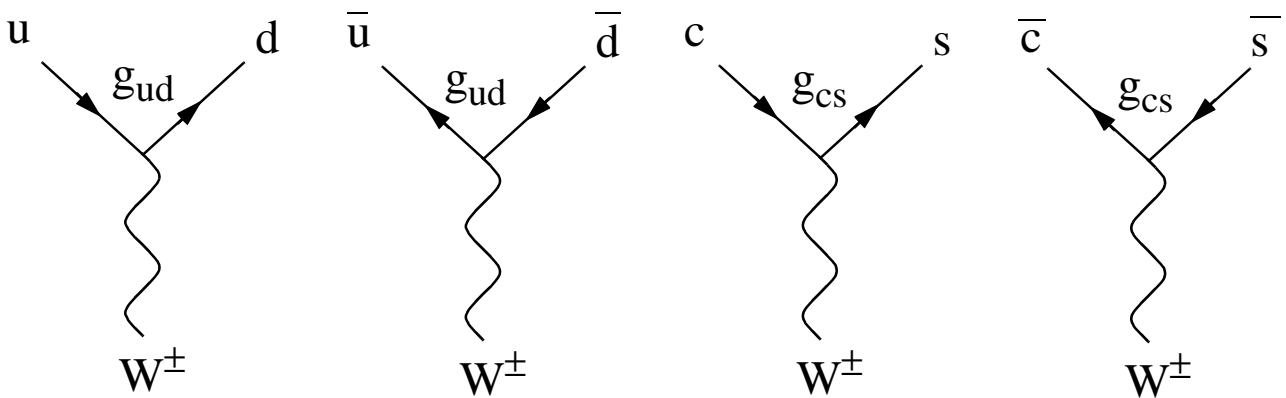


Figure 101: The W-quark vertices obtained from lepton-quark symmetry (if quark mixing and the third family is ignored).

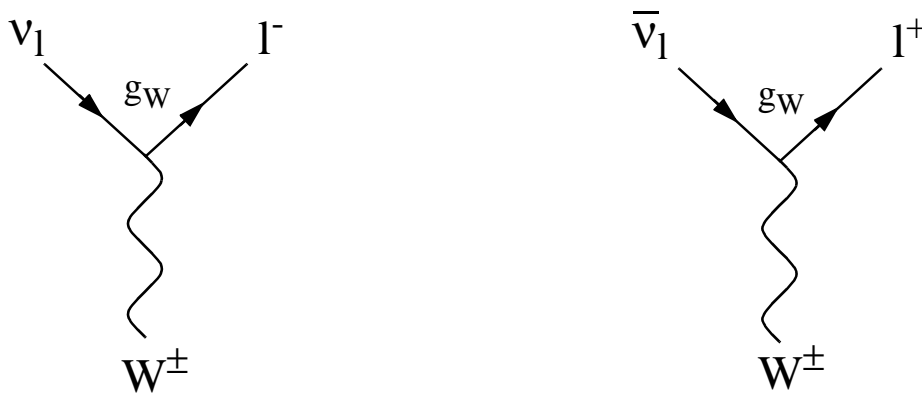


Figure 102: REMINDER: The two basic vertices for W^\pm -lepton interactions.

→ Examples of reactions that are allowed and not allowed according to the lepton-quark symmetry scheme are pion and kaon decay:

$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ $d\bar{u} \rightarrow \mu^- + \bar{\nu}_\mu$ allowed !

$K^- \rightarrow \mu^- + \bar{\nu}_\mu$ $s\bar{u} \rightarrow \mu^- + \bar{\nu}_\mu$ not allowed !

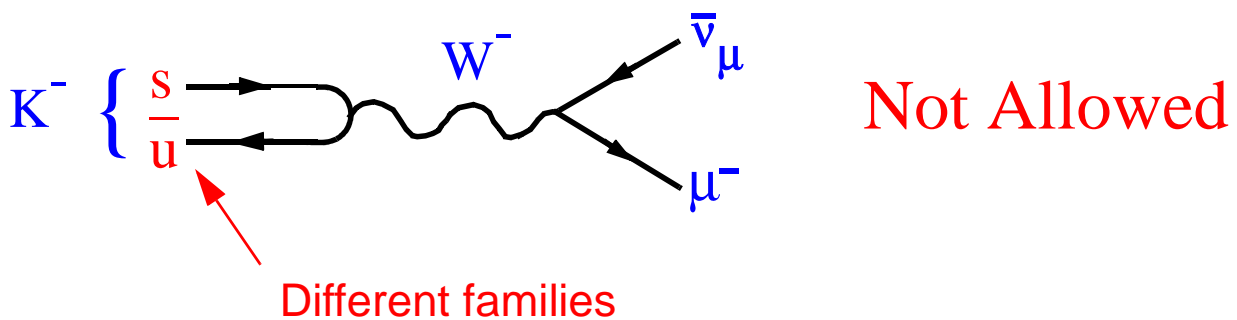
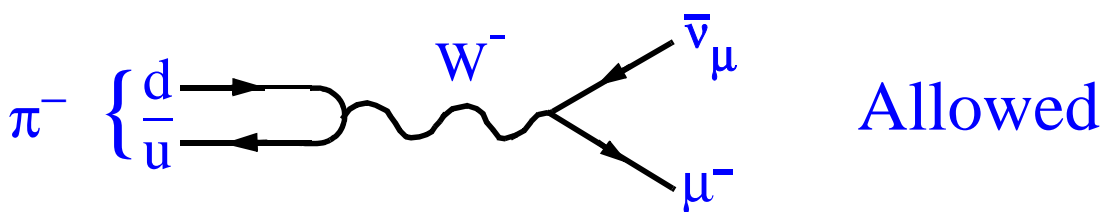


Figure 103: Decays that are allowed and not allowed if there is a lepton-quark symmetry in nature.

→ The kaon decay can be made allowed by introducing **quark mixing** (originally proposed by Cabibbo).

→ According to the quark mixing scheme, d- and s-quarks participate in the weak interactions via the linear combinations:

$$d' = d \cos \theta_C + s \sin \theta_C$$

$$s' = -d \sin \theta_C + s \cos \theta_C$$

where the parameter θ_C is called the **Cabibbo angle**.

→ With quark mixing the quark-lepton symmetry

applies to doublets like $\begin{pmatrix} u \\ d' \end{pmatrix}$ and $\begin{pmatrix} c \\ s' \end{pmatrix}$

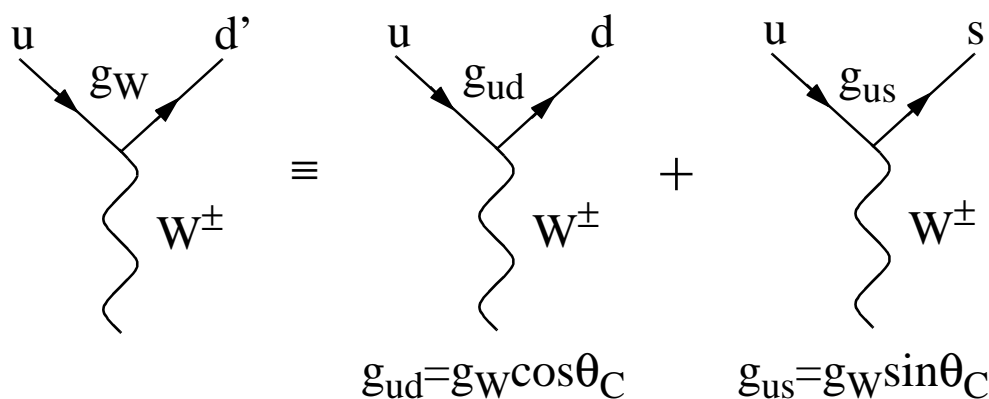


Figure 104: The $ud'W$ vertex can be interpreted as a sum of the udW and usW vertices.

→ With the quark mixing hypothesis some more W-quark vertices are allowed:

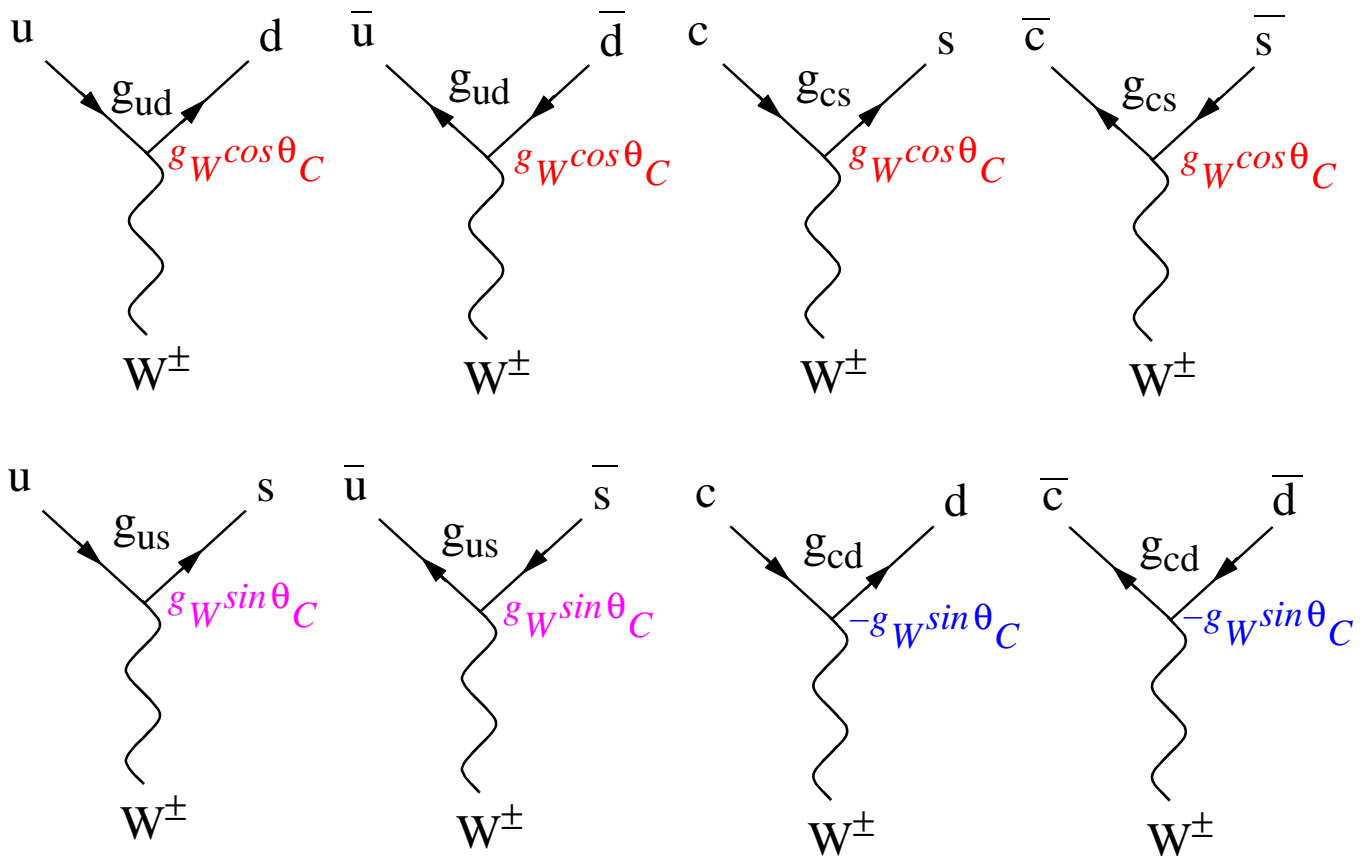


Figure 105: The basic W-quark vertices if quark mixing is taken into account.

The top row of diagrams have couplings given by

$$g_{ud} = g_{cs} = g_W \cos \theta_C \tag{97}$$

while the bottom row diagrams have the couplings

$$g_{us} = -g_{cd} = g_W \sin \theta_C \tag{98}$$



The Cabibbo angle is measured experimentally by comparing decay rates like:

$$\frac{\Gamma(K^- \rightarrow \mu^- \bar{\nu}_\mu)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} \propto \frac{g_{us}^2}{g_{ud}^2} = \tan^2 \theta_C$$

which give the result

$$\theta_C = 12,7^\circ \pm 0,1^\circ \quad (99)$$

$$g_W \cos \theta_C = 0,98 g_W$$

$$g_W \sin \theta_C = 0,22 g_W$$

→ The charmed quark couplings g_{cd} and g_{cs} are measured in **neutrino scattering experiments** and this give the same result:

$$\theta_C = 12^\circ \pm 1^\circ$$

❖ Experimentally it has been observed that **charmed hadrons** almost always **decays** into **strange hadrons**.

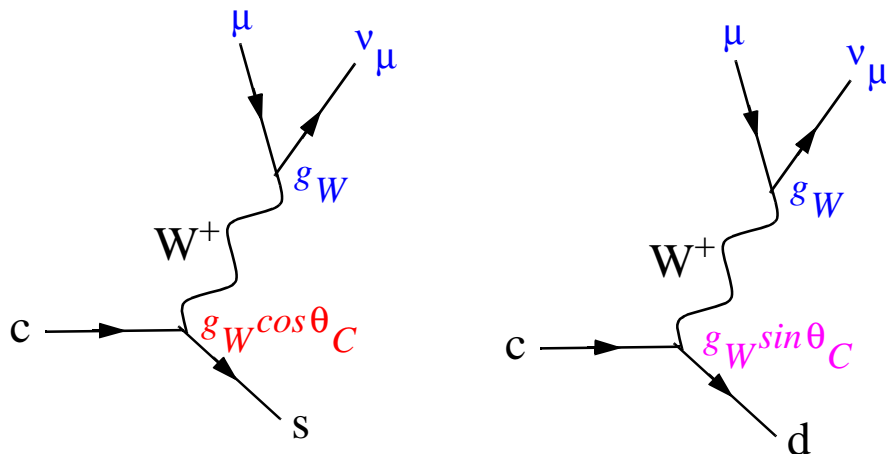


Figure 106: Cabibbo-allowed and Cabibbo-suppressed semi-leptonic decay of a charmed quark.

→ The ratio of semi-leptonic decays with Cabibbo-allowed and Cabibbo suppressed couplings are given by

$$\frac{g_{cd}^2}{g_{cs}^2} = \tan^2 \theta_C = \frac{1}{20}$$

The third generation

- The existence of the **c-quark** was first **predicted** from the **lepton-quark symmetry**.
- After the discovery of τ , ν_τ , and b , the sixth quark was predicted to complete the symmetry and the **top-quark** was discovered in 1994 with a **mass** of about **180 GeV/c²**.
- ❖ The **two generation** quark mixing is conveniently written in matrix form as:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \quad (100)$$

- A **third generation**: gives rise to the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V_{\alpha\beta}$:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (101)$$

→ The **coupling constants** are then:

$$g_{\alpha\beta} = g_W V_{\alpha\beta} \quad (\alpha = u, c, t; \beta = d, s, b) \quad (102)$$

→ If the mixing between the b quark and (d,s) quarks can be neglected, the CKM matrix is reduced to

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0 \\ -\sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (103)$$

and hence $b'=b$.

→ Since the two-generation mixing model agree well with data V_{ub} , V_{cb} , V_{td} and V_{ts} must be **small**.

→ The **b-quark** is heavy (mass=4.5 GeV) and it can **decay** to the lighter **u and c** quarks as in the following decay:

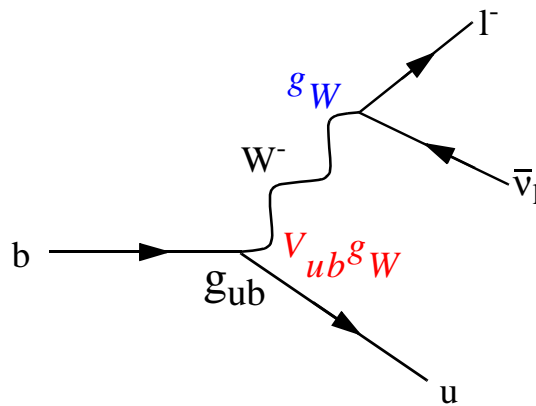


Figure 107: The semi-leptonic decay of the b-quark to a lighter quark.

→ This decay modes has a **rate** proportional to the squared couplings:

$$|g_{ub}|^2 = |V_{ub}|^2 g_W^2 \quad (104)$$

→ If V_{ub} and V_{cb} are 0, **b-quark** must be **stable**. Experimentally, it is clear that the b lifetime is long but it is **NOT stable**.

❖ The weak b-quark decay can be compared to that of the τ -lepton which have a decay rate proportional to g_W^2 .

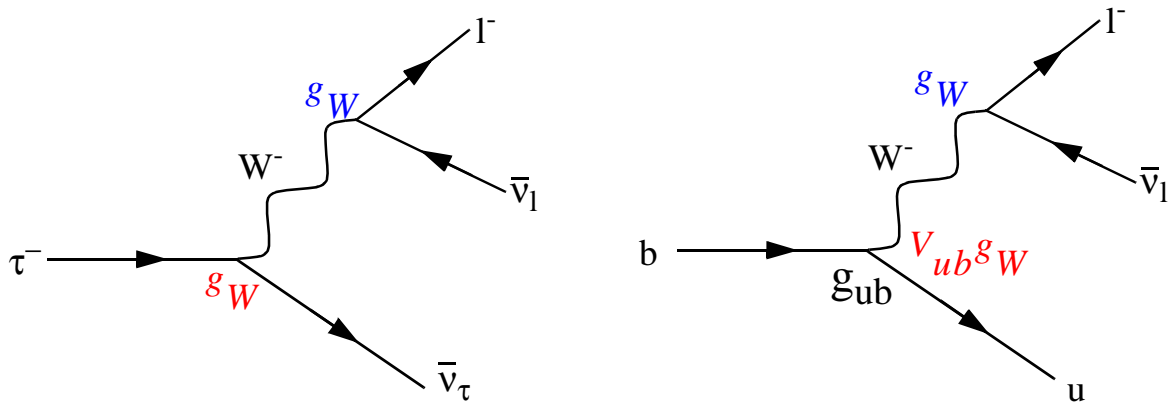


Figure 108: The decay of the τ -lepton can be compared to that of the b-quark.

If one assumes that $|V_{ub}| = 1$ then one can predict what lifetime this would result in for the b-meson:

$$\tau_b \approx \frac{1}{N} \left(\frac{m_\tau}{m_b} \right)^5 \cdot \tau_\tau \approx 10^{-15} s$$

where $\tau_\tau \approx 3 \times 10^{-13}$ s is the lifetime of the τ -lepton, N is the number of possible b-quark decays per analogous τ -decays (3 or 4) and m_τ and m_b are the masses of the τ and the b.

$$\begin{array}{l}
 \text{Conclusion:} \\
 \text{Experimentally:}
 \end{array}
 \begin{array}{l}
 |V_{ub}| = |V_{cb}| = 0 \\
 |V_{ub}| = 1 \\
 \tau_b \approx \infty \\
 \tau_b \approx 10^{-15} \text{ s} \\
 \tau_b \approx 10^{-12} \text{ s}
 \end{array}$$

The most precise measurements at present yield

$$|V_{ub}| \approx 0,004 \quad \text{and} \quad |V_{cb}| \approx 0,04 \quad (105)$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0,004 \\ -\sin\theta_C & \cos\theta_C & 0,04 \\ \sim 0 & \sim 0 & \sim 1 \end{pmatrix} \\
 \approx \begin{pmatrix} 0,98 & 0,22 & 0,004 \\ -0,22 & 0,98 & 0,04 \\ \sim 0 & \sim 0 & \sim 1 \end{pmatrix}$$

The top-quark

❖ The top-quark is much heavier than even W bosons and can decay by

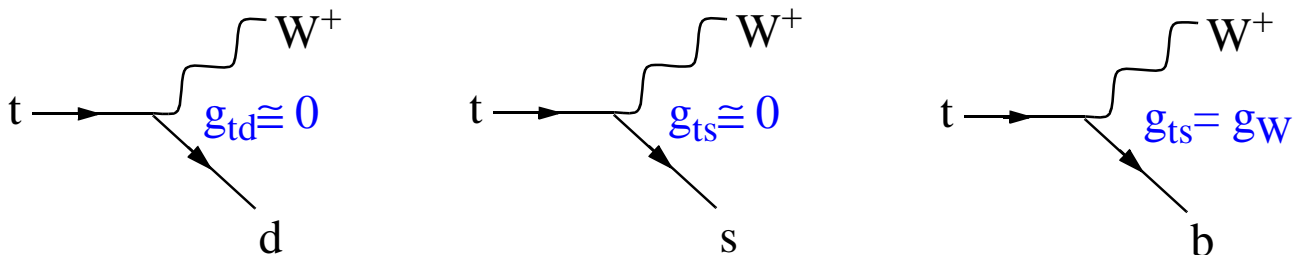


Figure 109: Weak interactions involving the top-quark.

❖ Since $g_{td} \approx 0$ and $g_{ts} \approx 0$ the only significant decay mode of the t-quark is

$$t \rightarrow W^+ + b \quad (106)$$

with a **rate proportional to**

$$\alpha_W = g_W^2 / 4\pi \approx 4,2 \times 10^{-3}$$

Estimation of the decay width: $\Gamma \sim \alpha_W m_t \sim 1 \text{ GeV}$
suggests a very short lifetime for the top-quark:

$$\tau_t \approx 4 \times 10^{-25} \text{ s} \quad (107)$$

Discovery of the top quark.

❖ The **top-quark** was discovered in $p\bar{p}$ -collisions in the Collider Detector at Fermilab (**CDF**) in 1994. The **Tevatron** accelerator with a collision energy = 1.8 TeV was used.

➔ **PRODUCTION:** In proton colliders, pairs of top quarks are produced by the quark-antiquark annihilation process:

$$q + \bar{q} \rightarrow g \rightarrow t + \bar{t} \quad (108)$$

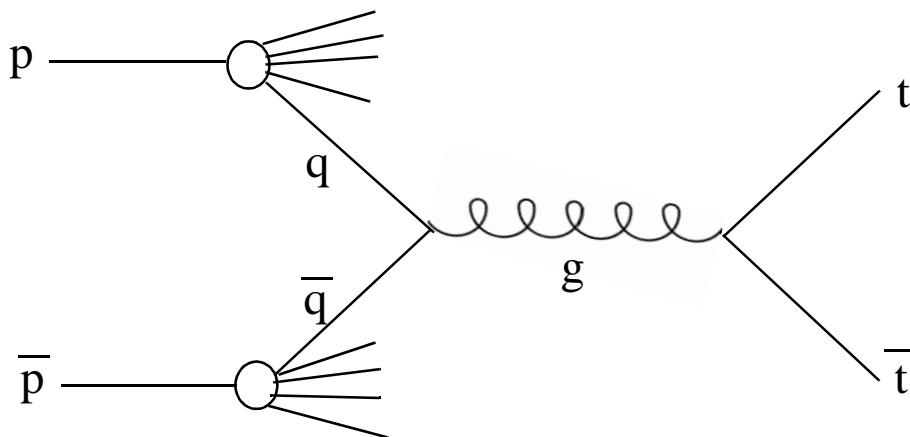


Figure 110: The production of top quarks in $p\bar{p}$ annihilation.

→ **DECAY:** The **top quark** decay in most cases to a b-quark and to a W which in turn decays to leptons or quarks:

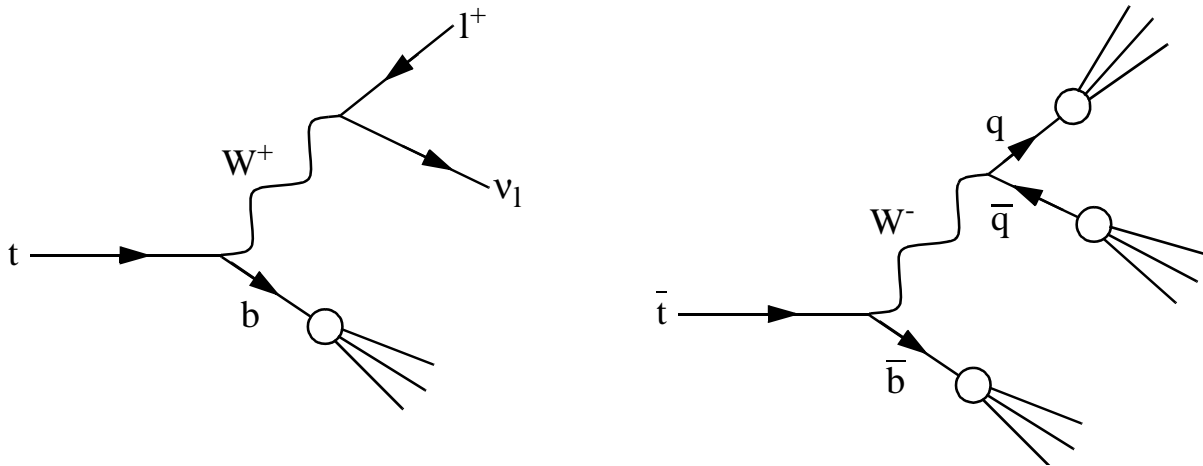


Figure 111: The decay of top quarks.

The final state is a complex mix of jets and leptons:

$$\begin{array}{c}
 p + \bar{p} \longrightarrow t + \bar{t} + X \\
 \begin{array}{l}
 \left. \begin{array}{l} \longrightarrow \\ \longrightarrow \end{array} \right\} \begin{array}{l} l^- + \bar{\nu}_1 \\ q + \bar{q} \end{array} \\
 \longrightarrow W^- + \bar{b} \\
 \left. \begin{array}{l} \longrightarrow \\ \longrightarrow \end{array} \right\} \begin{array}{l} l^+ + \nu_1 \\ q + \bar{q} \end{array} \\
 \longrightarrow W^+ + b
 \end{array}
 \end{array}$$

Figure 112: The production of $t\bar{t}$ -pairs in $p\bar{p}$ -collisions.

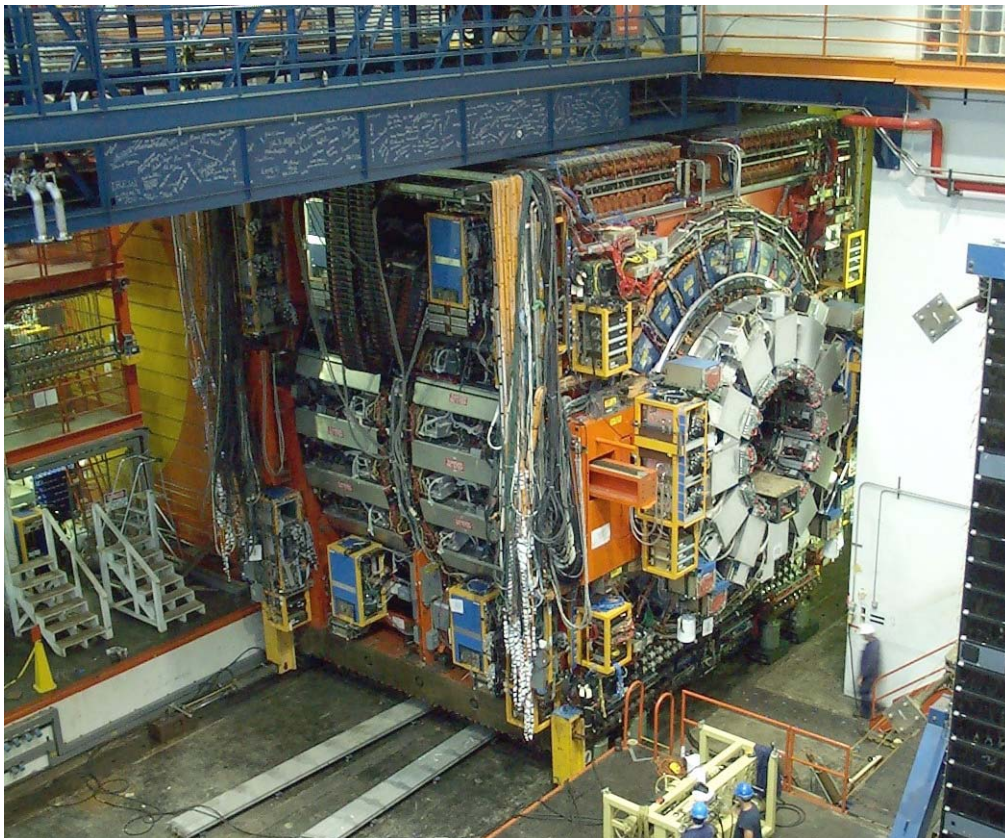
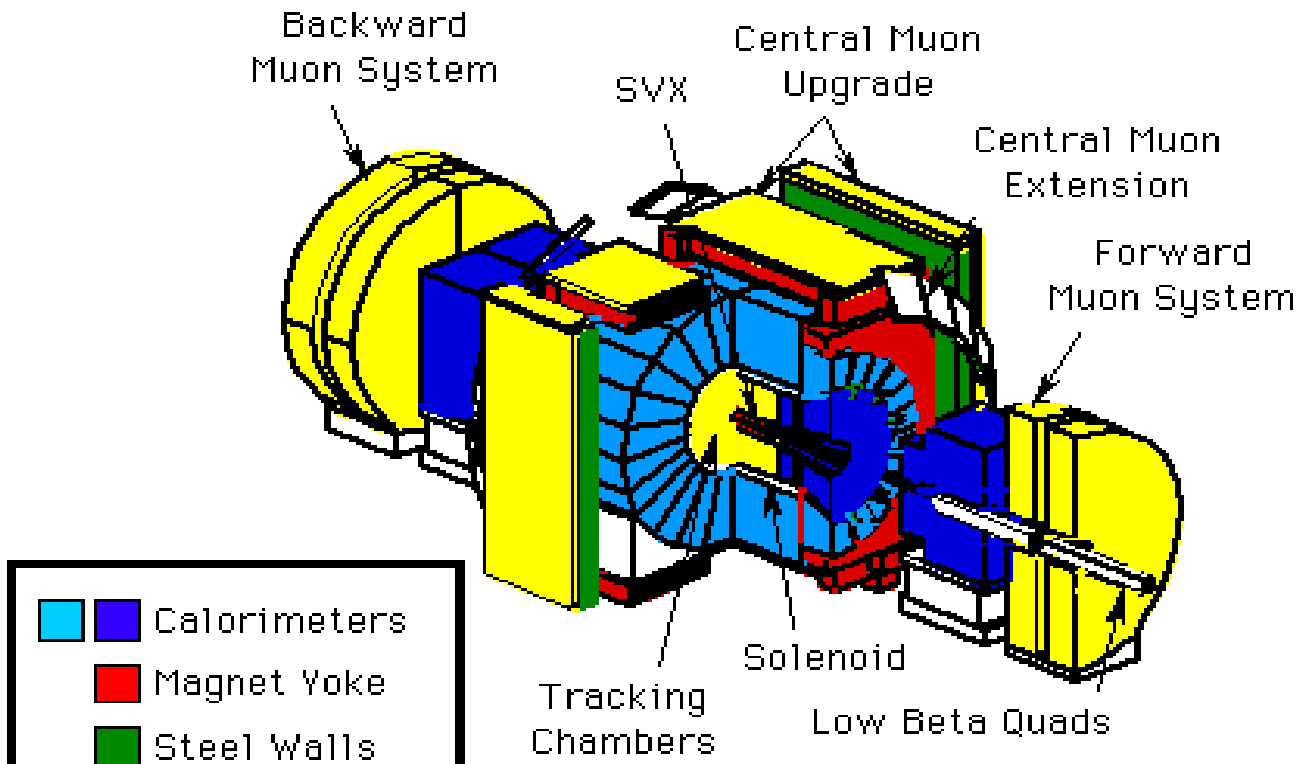


Figure 113: The CDF experiment which discovered the top quark.

→ The events that the CDF collaboration searched for were:

$$t + \bar{t} \rightarrow b + l + \nu + \bar{b} + q + \bar{q}$$

where the lepton l was either an electron or a muon.

→ SIGNATURE:

Two b-jets

Two light quark jets

Missing energy from the neutrino

Lepton with large transverse energy

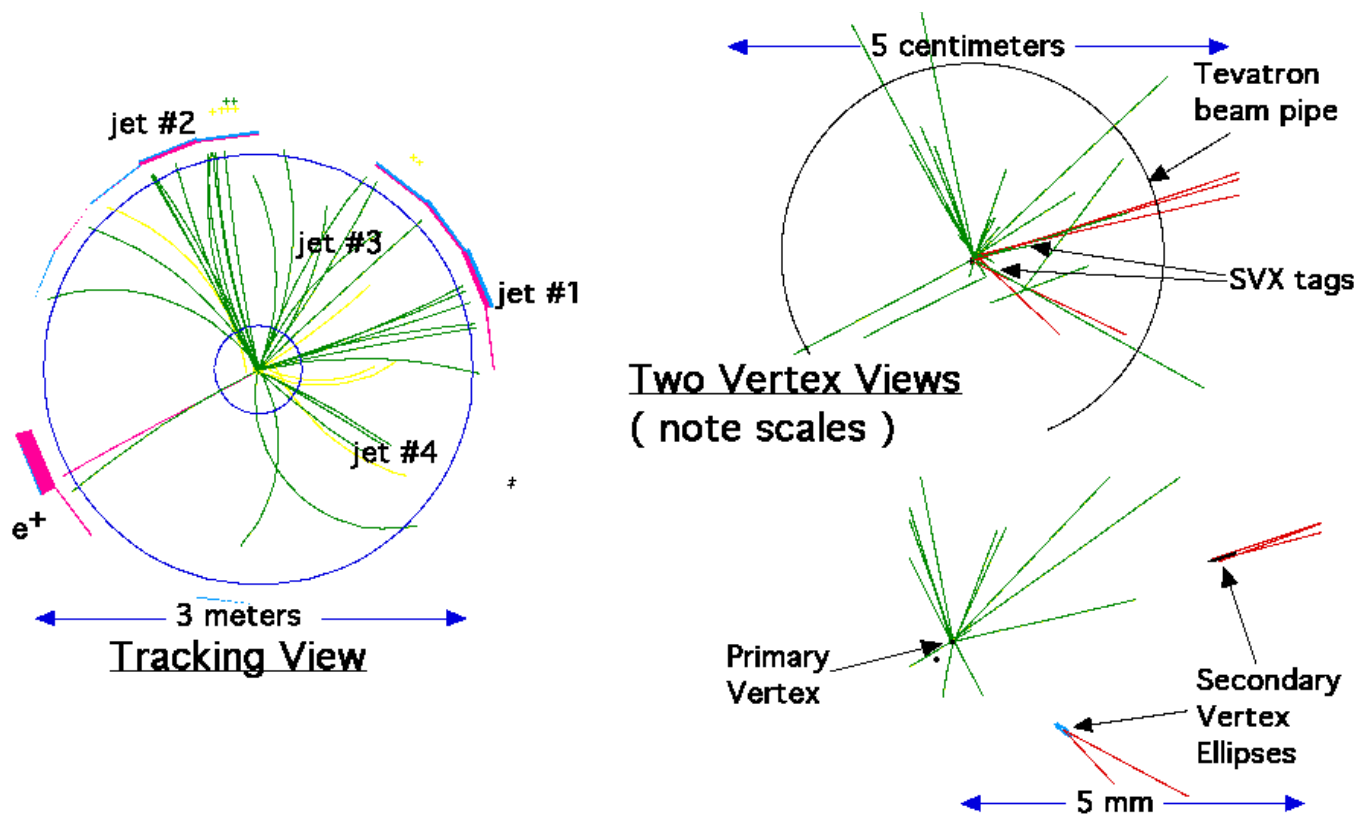


Figure 114: A top event in the CDF experiment.

→ The measured **mass distribution** (by the CDF experiment) of the events with the required signature was:

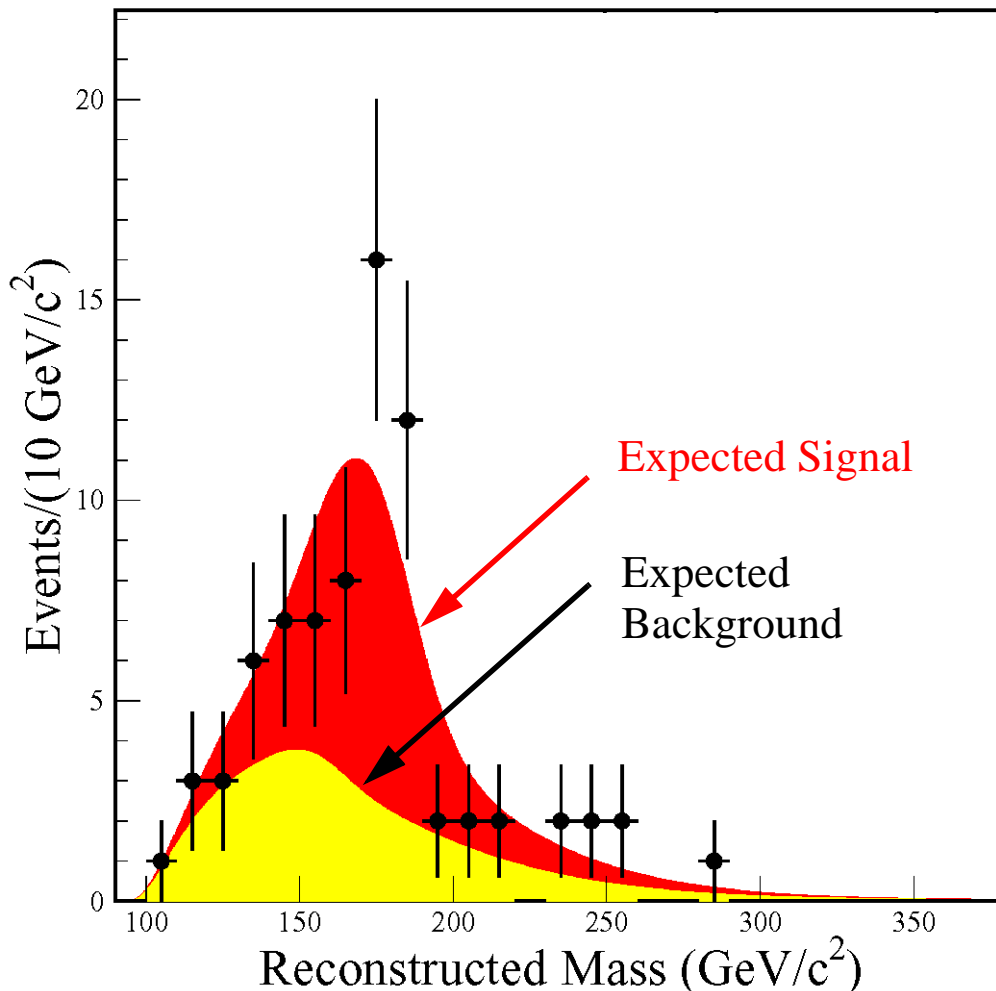


Figure 115: Mass distribution of top events.

→ The top-quark's mass was measured to be

$$M_t = 176 \pm 5 \text{ GeV}$$

Summary

• **W and Z bosons.**

- a) W and Z are the force carriers in weak interaction.
- b) They are very massive spin-1 bosons.
- c) Processes with W exchange are called charged current reactions and processes with Z exchange neutral current reactions.

• **Discovery of the W and Z bosons.**

- d) The W and Z were discovered in $p\bar{p}$ -collisions by the UA1 and UA2 experiments at CERN.
- e) The W and Z decays to pairs of leptons or quarks.
- f) One has measured the W and Z mass with a large accuracy at the LEP accelerator:
 $M_W=80\text{GeV}$ and $M_Z=91\text{ GeV}$.

- **Charged current reactions.**

- g) Leptonic weak interactions can be built from a set of basic vertices.
- h) The weak strength parameter is of the same size as that of the electromagnetic interaction.

- **Weak interactions of hadrons.**

- i) There is a lepton-quark symmetry between particles in the same generation.
- j) The quark mixing scheme describe how quarks with different flavours interact with each other.

- **The third generation.**

- k) Quark mixing is described by the CKM-matrix.
- l) The mixing is much smaller for the heavy quarks than the light quarks.

- **The top quark.**

- m) The top decay in most cases to a W and a b .
- n) It was discovered in the CDF experiment.