

Elastic electron-proton scattering

- ❖ Beams of **leptons** are good **tools** for investigating the properties of hadrons since leptons have **no substructure**.
- **Elastic lepton-hadron scattering** can be used to measure the **size** of the **hadron**.
- **Elastic scattering** means that the same type of particles goes into and comes out of the scattering process.

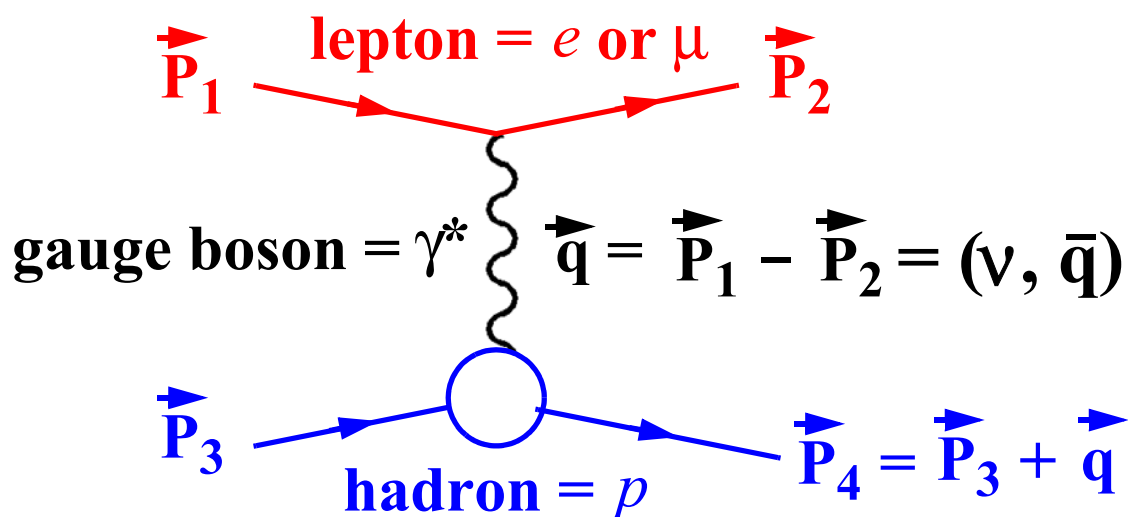
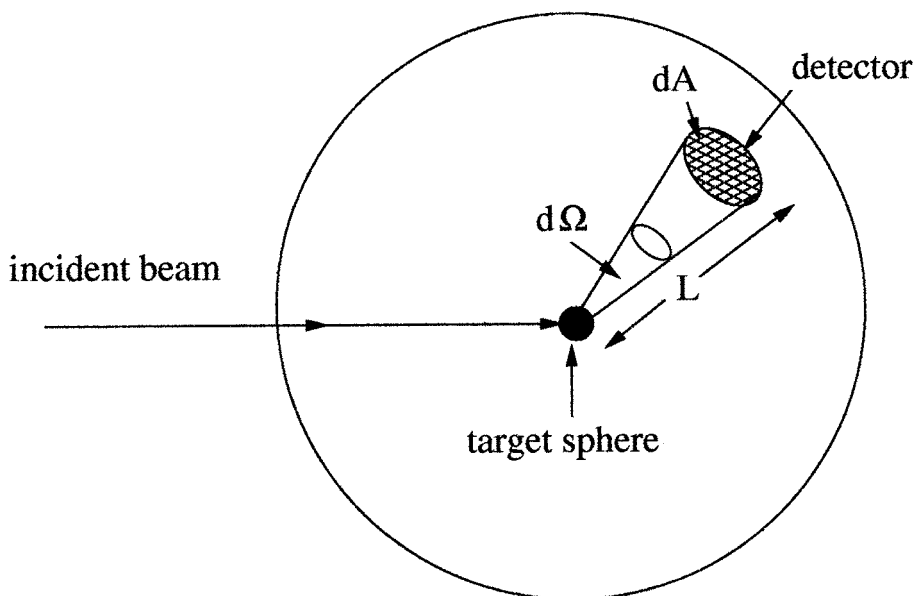


Figure 68: The dominant one-photon exchange mechanism in elastic lepton-proton scattering.

❖ The angular distribution of the particles emerging from a scattering reaction is given by the **differential cross-section**

$$\frac{d\sigma(\theta, \varphi)}{d\Omega} \quad \text{where} \quad d\Omega = \sin\theta d\theta d\varphi$$



$$d\Omega = \frac{dA}{L^2}$$

Figure 69: The definition of the solid angle $d\Omega$ in scattering experiments.

➔ The **total cross section** of the reaction is obtained by integrating the differential cross section:

$$\sigma = \int \frac{d\sigma(\theta, \varphi)}{d\Omega} d\Omega = \int_0^\pi \int_0^{2\pi} \frac{d\sigma(\theta, \varphi)}{d\Omega} \sin\theta d\theta d\varphi$$

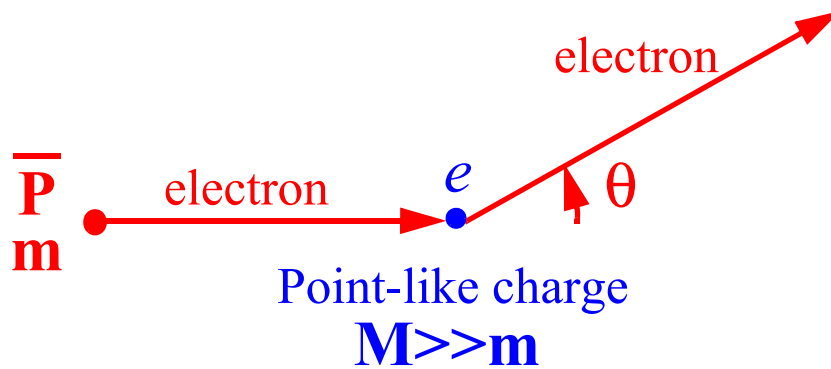


Figure 70: The scattering of an electron on a static point-like electrical charge.

→ The angular distribution of a relativistic electron of momentum p which is scattered by a point-like static electric charge e is described by the **Mott scattering formula**:

$$\left(\frac{d\sigma}{d\Omega}\right)_M = \frac{\alpha^2}{4p^4 \sin^4\left(\frac{\theta}{2}\right)} \left(m^2 + p^2 \cos^2\frac{\theta}{2}\right)$$

In the low energy limit $p \ll m$, the Mott scattering formula is reduced to the non-relativistic **Rutherford scattering formula**:

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{m^2 \alpha^2}{4p^4 \sin^4\left(\frac{\theta}{2}\right)} \quad \text{where } \alpha = \frac{e^2}{4\pi}$$

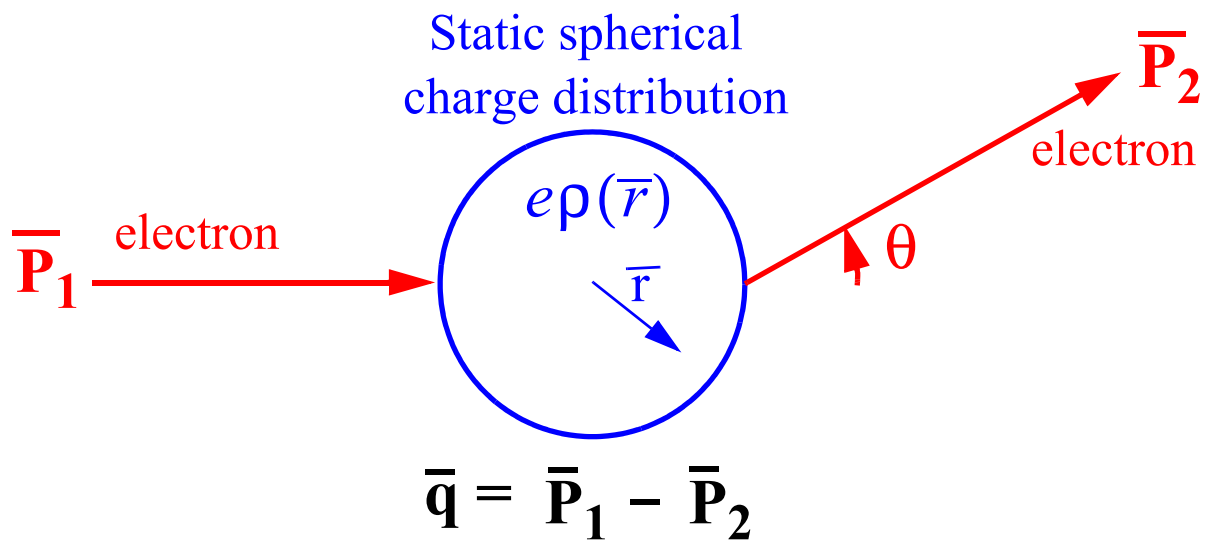


Figure 71: The scattering of an electron on a static spherical charge distribution.

If the electric charge is not point-like, but it is spread out with a **spherically symmetric density distribution**, i.e., $e \rightarrow e\rho(r)$, where $\rho(r)$ is normalized:

$$\int \rho(r) d^3\vec{x} = 1$$

then the Rutherford scattering formula has to be modified by an **electric form factor** $G_E^2(q^2)$:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_R G_E^2(q^2) \quad (89)$$

→ The **electric form factor** is the **Fourier transform of the charge distribution** with respect to the momentum transfer \vec{q} :

$$G_E(q^2) = \int \rho(r) e^{i\vec{q} \cdot \vec{x}} d^3\vec{x} \quad (90)$$

– For $q = 0$, $G_E(0) = 1$ (low momentum transfer)

– For $q^2 \rightarrow \infty$, $G_E(q^2) \rightarrow 0$ (large momentum transfer)

→ Measurements of the **cross-section** can be used to determine the **form-factor** and hence the charge distribution.

The **mean quadratic charge radius** is for example given by

$$r_E^2 = \int r^2 \rho(r) d^3\vec{x} = -6 \left. \frac{dG_E(q^2)}{dq^2} \right|_{q^2=0} \quad (91)$$

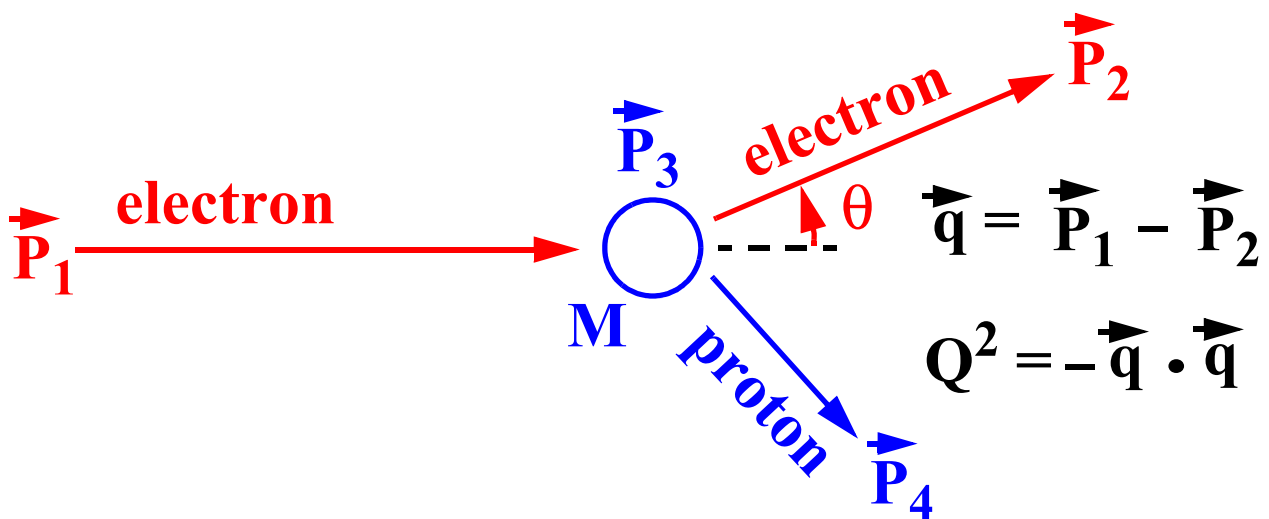


Figure 72: Elastic electron-proton scattering when the recoil energy of the proton is taken into account.

→ Scattering of electrons on protons depend not only on the **electric form factor G_E** but also on a **magnetic form factor G_M** which is associated with the magnetic moment distribution.

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_M \times \left(G_1(Q^2) \cos^2 \frac{\theta}{2} + \frac{Q^2}{2M^2} G_2(Q^2) \sin^2 \frac{\theta}{2} \right)$$

$$G_1(Q^2) = \frac{G_E^2 + \frac{Q^2}{4M^2} G_M^2}{1 + \frac{Q^2}{4M^2}} \quad G_2(Q^2) = G_M^2$$

→ Measurement of the form factors are conveniently divided into three Q^2 regions:

1) **low** $Q^2 \Rightarrow Q \ll M \Rightarrow G_E$ dominates the cross-section and r_E can be precisely measured:

$$r_E = 0,85 \pm 0,02 \text{ fm} \quad (92)$$

2) An **intermediate** range: $0.02 \leq Q^2 \leq 3 \text{ GeV}^2 \Rightarrow$ both G_E and G_M give sizeable contribution \Rightarrow the result can be given by the parameterization:

$$G_E(Q^2) \approx \frac{G_M(Q^2)}{\mu_p} \approx \left(\frac{\beta^2}{\beta^2 + Q^2} \right)^2 \quad (93)$$

with $\beta^2 = 0.84 \text{ GeV}^2$

3) **high** $Q^2 > 3 \text{ GeV}^2 \Rightarrow G_M$ dominates the cross section:

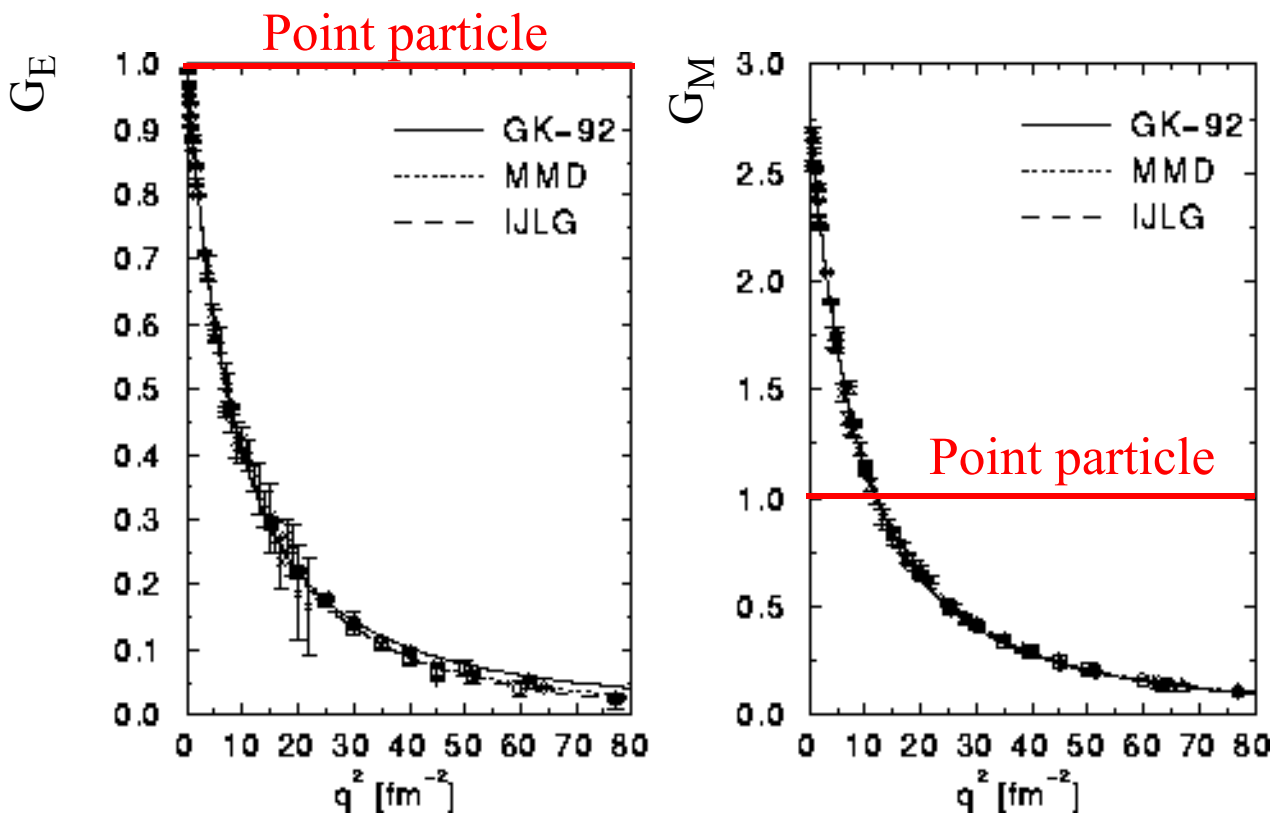


Figure 73: Electric and magnetic proton form-factors, compared with different parameterizations.

The form factors are normalized so that

$$\begin{aligned}
 G_E(0) &= \text{total charge} && = 1 \text{ (p)} \\
 &&& = 0 \text{ (n)} \\
 G_M(0) &= \text{magnetic moment} = \mu_p = +2.79 \text{ (p)} \\
 &&& = \mu_n = -1.91 \text{ (n)}
 \end{aligned}$$

→ If the proton is a point particle then

$$G_E(Q^2) = 1 \text{ and } G_M(Q^2) = 1$$

Inelastic lepton-proton scattering

❖ Inelastic electron-proton scattering can be used to **probe the photon structure** and gave the first evidence for the existence of quarks.

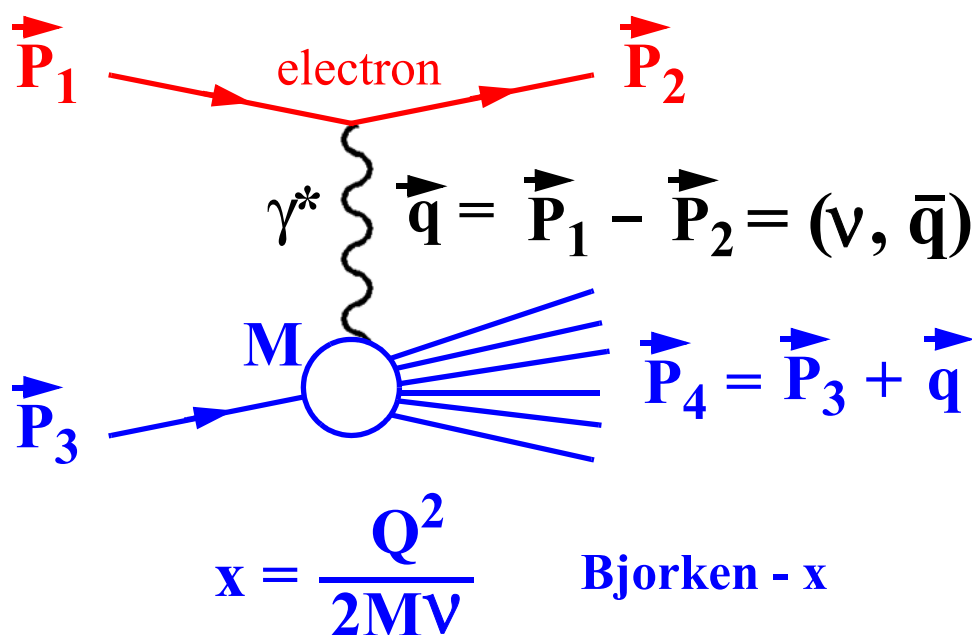


Figure 74: One-photon exchange in inelastic lepton-proton scattering.

➔ In inelastic lepton-proton scattering a new dimensionless variable called the **Bjorken scaling variable x** is introduced where $0 < x < 1$.

→ The differential cross section for inelastic electron-proton scattering can be written as:

$$\frac{d\sigma}{dE_2 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)} \cdot \frac{1}{v} \cdot \left[F_2(x, Q^2) \cos^2\left(\frac{\theta}{2}\right) + \frac{Q^2}{xM^2} F_1(x, Q^2) \sin^2\left(\frac{\theta}{2}\right) \right]$$

→ The two dimensionless **structure functions** $F_1(x, Q^2)$ and $F_2(x, Q^2)$ parameterize the photon-proton interaction in the same way as $G_1(Q^2)$ and $G_2(Q^2)$ in elastic scattering.

→ **Bjorken scaling or scale invariance:**

$$F_{1,2}(x, Q^2) \approx F_{1,2}(x)$$

i.e. for $Q \gg M$, **structure functions** are almost **independent of Q^2** . If all particle masses, energies and momenta are multiplied by a scale factor, structure functions at any given x remain unchanged.

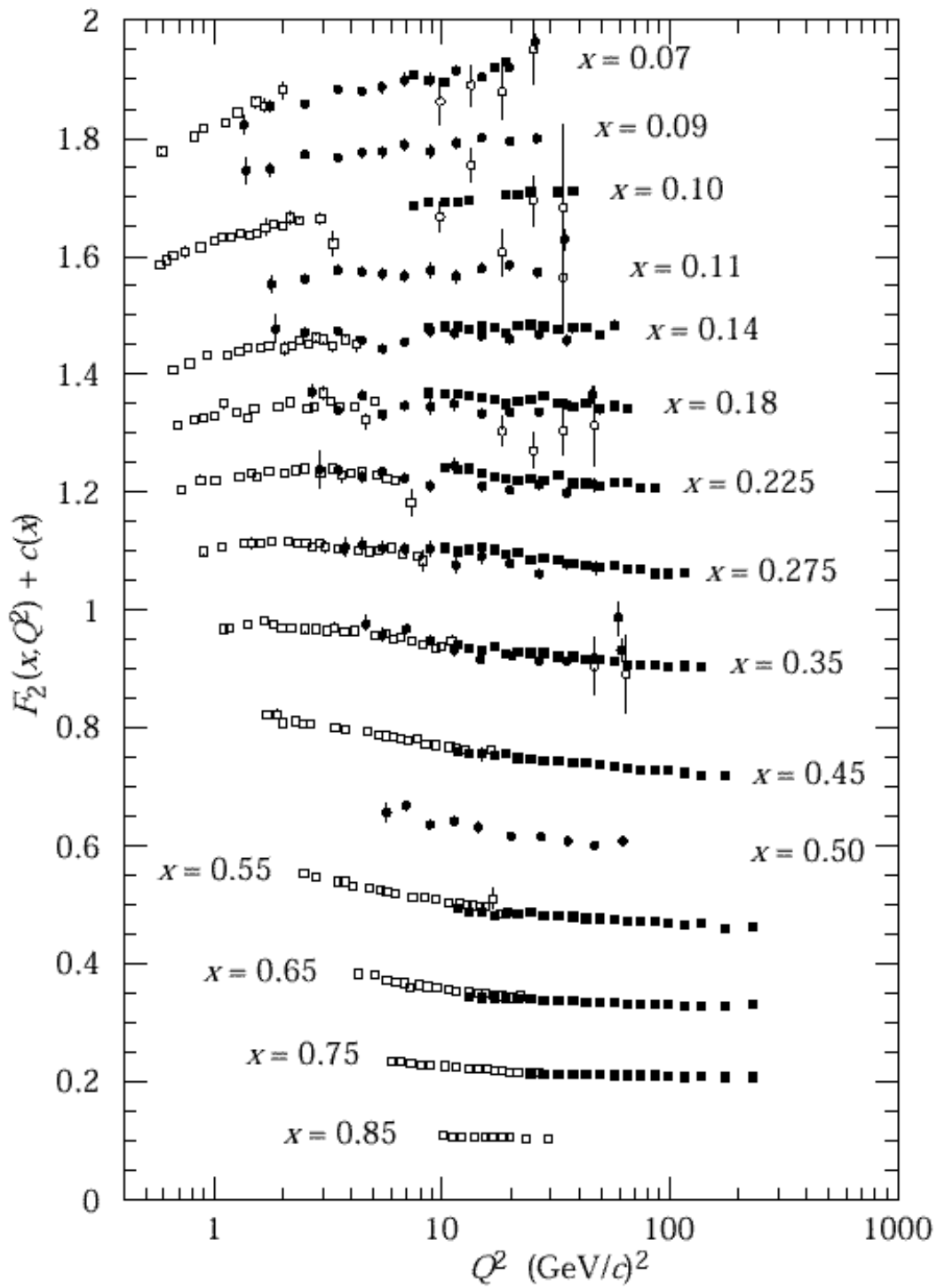


Figure 75: The measured structure function F_2 (compilation of data from different experiments).

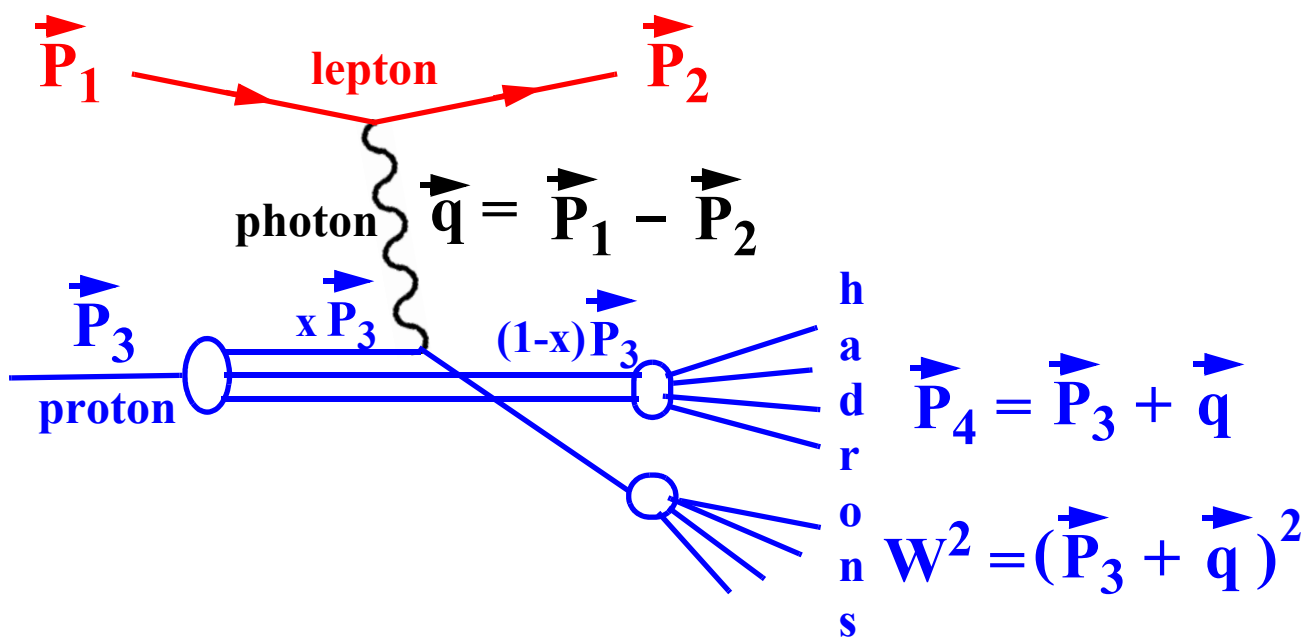
→ The **first observation** of scale invariance in inelastic scattering was observed at **SLAC** in 1969 and was later interpreted as the first evidence for quarks.



Figure 76: Two spectrometers in SLAC's End Station A that were used to discover quarks in the late 1960s.

Deep inelastic electron-proton scattering.

❖ In the **parton model** the scale invariance is explained by scattering on point-like constituents (partons) in the proton.



→ The parton model is valid if the target proton has a sufficiently large momentum, so that the fraction of the proton **momentum carried by the struck quark** is given by Bjorken x .

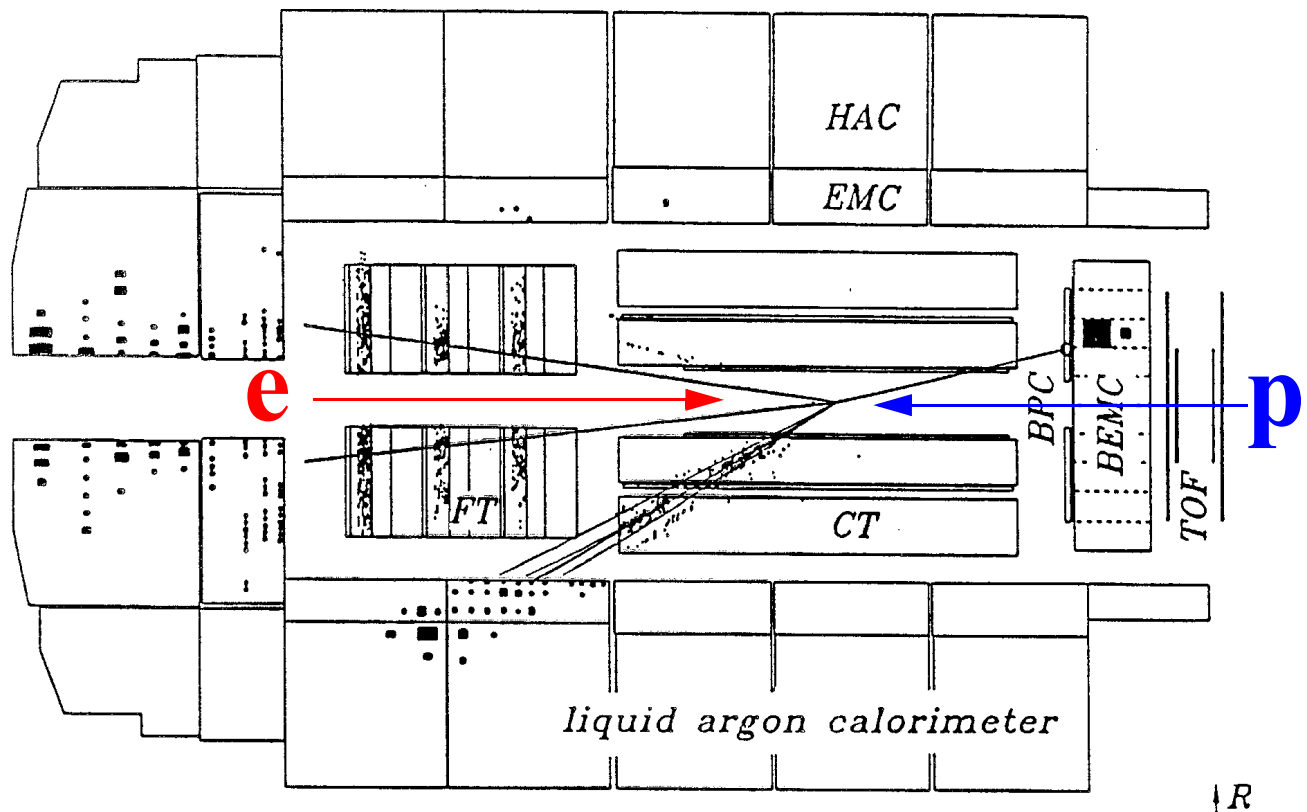


Figure 77: A computer reconstruction of a deep inelastic electron-proton scattering event recorded by the H1 experiment at DESY.

→ In the parton model, the structure function F_1 depends on the spin of the partons:

$$F_1(x, Q^2) = 0 \quad (\text{spin-0})$$

$$2xF_1(x, Q^2) = F_2(x, Q^2) \quad (\text{spin-1/2})$$

The data favours the second relation (called the Callan-Gross relation) i.e. **quarks** have **spin 1/2**.

Summary

• Quantum Chromodynamics

- a) The gauge bosons in QCD are called gluons and are spin 1 particles.
- b) The charge in QCD is called colour and gluons carry colour charge but not electric charge.
- c) The strong interaction is flavour independent.
- d) Colour confinement means that a particle with a colour charge (such as a gluon or a quark) cannot exist as a free particle.

• The strong coupling constant.

- e) The strong coupling constant α_s gives the strength of the strong interaction.
- f) α_s is not a true constant since it depends on Q^2 .

- **Electron-positron interactions.**

- g) Quarks are seen as jets of hadrons in electron-positron interactions.
- h) The measured cross section ratio R can only be explained if there are 3 colours.
- i) A measurement of the angular distribution of jets in two-jet events show that the quark is a spin $1/2$ particle.
- j) Three-jet events can be used to measure α_s and to show that the gluon is a spin 1 particle.

- **Elastic electron-proton scattering.**

- k) Elastic electron-proton scattering can be used to measure the size of the proton.
- l) Scattering of electrons on protons depends on an electric and a magnetic form factor.
- m) The measurement of these form factors show that the proton is not a point particle.

- **Inelastic lepton-proton scattering.**

- n) Inelastic scattering of electrons on protons depends on two structure functions F_1 and F_2 .
- o) Scale invariance means that these structure functions are almost independent on Q^2 . The scale invariance of F_2 is evidence for the existence of quarks in the proton.

- **Deep inelastic electron-proton scattering.**

- p) The measurement of F_1 show that the quarks have to be spin 1/2 particles.