

VII. QCD, jets and gluons

Quantum Chromodynamics (QCD)

❖ *Quantum Chromodynamics (QCD)* is the theory of strong interactions.

- Interactions are carried out by a massless spin-1 particle – *gauge boson*
- In quantum electrodynamics (**QED**) gauge bosons are *photons*, in **QCD** – *gluons*
- Gauge bosons couple to conserved charges: photons in **QED** – to *electric charges*, and gluons in **QCD** – to *colour charges*
- The strong interaction acts the same on u,d,s,c,b and t quarks because the strong interaction is **flavour-independent**.



Gluons carry colour charges themselves !

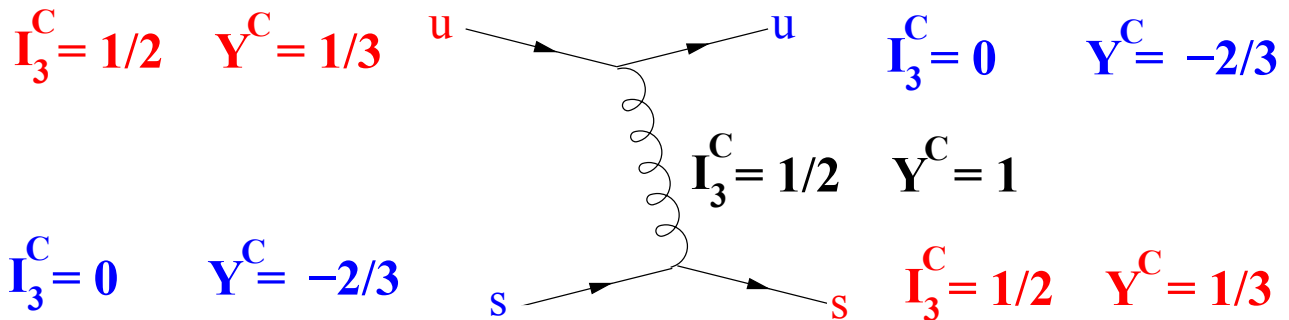


Figure 58: Gluon exchange between quarks.

The colour quantum numbers of the gluon in the figure above are:

$$I_3^C = I_3^C(r) - I_3^C(b) = \frac{1}{2} \tag{86}$$

$$Y^C = Y^C(r) - Y^C(b) = 1$$

Gluon colour wavefunctions:

$\chi_{g1}^C = r \bar{g}$	$I_3^C = 1$	$Y^C = 0$
$\chi_{g2}^C = \bar{r} g$	$I_3^C = -1$	$Y^C = 0$
$\chi_{g3}^C = r \bar{b}$	$I_3^C = 1/2$	$Y^C = 1$
$\chi_{g4}^C = \bar{r} b$	$I_3^C = -1/2$	$Y^C = -1$
$\chi_{g5}^C = g \bar{b}$	$I_3^C = -1/2$	$Y^C = 1$
$\chi_{g6}^C = \bar{g} b$	$I_3^C = 1/2$	$Y^C = -1$
$\chi_{g7}^C = 1/\sqrt{2} (g \bar{g} - \bar{r} r)$	$I_3^C = 0$	$Y^C = 0$
$\chi_{g8}^C = 1/\sqrt{6} (g \bar{g} - r \bar{r} - 2 b \bar{b})$	$I_3^C = 0$	$Y^C = 0$

❖ **Gluons can couple to other gluons** since gluons carry colour charge !

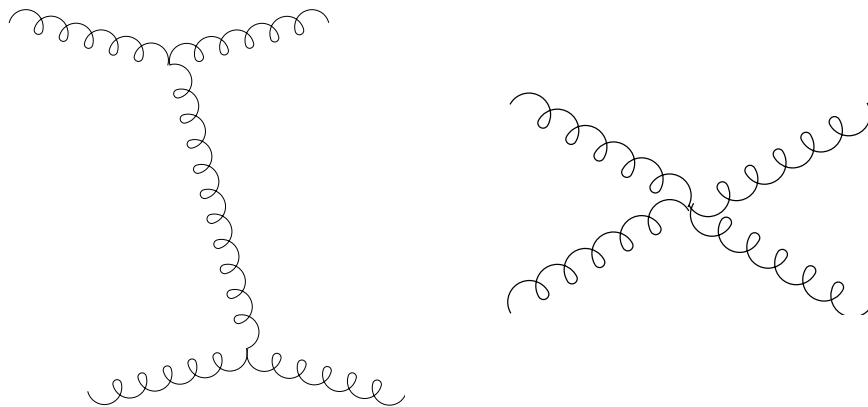


Figure 59: Lowest-order contributions to gluon-gluon scattering.

- All observed states have zero colour charge - **colour confinement**.
- Gluons does **not** exist as **free particles** since they have colour charge.
- Bound colourless states of gluons are called **glueballs** (not detected experimentally yet).



The principle of **asymptotic freedom**:

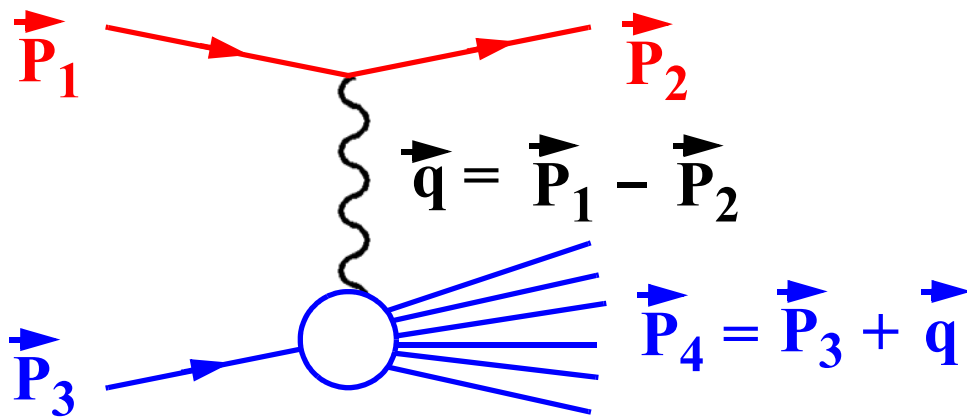
- At **short distances** the strong interactions are weaker \Rightarrow **quarks** and **gluons** are essentially **free** particles \Rightarrow the interaction can be described by the lowest order diagrams.
- At **large distances** the strong interaction gets stronger \Rightarrow the interaction can be described by high-order diagrams.
- The **quark-antiquark potential** is:

$$V(r) = -\frac{4}{3} \frac{\alpha_s}{r} \quad (r < 0, 1 \text{ fm})$$

$$V(r) = \lambda r \quad (r \geq 1 \text{ fm})$$

where α_s is the strong coupling constant and r the distance between the quark and the antiquark.

- Due to the complexity of high-order diagrams, the very process of **confinement** can not be calculated analytically \Rightarrow only **numerical models** are available.



where \vec{P} and \vec{q} are energy-momentum four vectors:

$$\vec{P} = (E, \vec{p}) = (E, p_x, p_y, p_z)$$

$$\vec{q} = (E_q, \vec{q}) = \vec{P}_1 - \vec{P}_2 = (E_1 - E_2, \vec{P}_1 - \vec{P}_2)$$

the momentum and energy transfer is:

$$q = |\vec{q}| = |\vec{P}_1 - \vec{P}_2|$$

$$v = E_q = E_1 - E_2$$

(where E and P are in the restframe of particle 3).

The energy-momentum transfer is given by:

$$Q^2 = -\vec{q} \cdot \vec{q} = -(\vec{P}_1 - \vec{P}_2)^2$$

$$Q^2 = \vec{q} \cdot \vec{q} - E_q^2 = (\vec{P}_1 - \vec{P}_2)^2 - (E_1 - E_2)^2$$

This can be regarded as the invariant mass of the exchanged gauge boson since the squared mass of

a particle is given by $M^2 = \vec{P} \cdot \vec{P}$

The invariant mass of the hadrons is given by:

$$W^2 = \vec{P}_4 \cdot \vec{P}_4 = (\vec{P}_3 + \vec{q})^2$$

The strong coupling constant

❖ The strong coupling constant α_s is the analogue in QCD of α_{em} and it is a measure of the strength of the interaction.

→ α_s is not a true constant but a “**running constant**” since it decreases with increasing Q^2 .

→ In leading order of QCD, α_s is given by

$$\alpha_s = \frac{12\pi}{(33 - 2N_f) \ln(Q^2 / \Lambda^2)} \quad (87)$$

Here N_f is the number of allowed quark flavours, and $\Lambda \approx 0.2$ GeV is the QCD scale parameter which has to be defined experimentally.

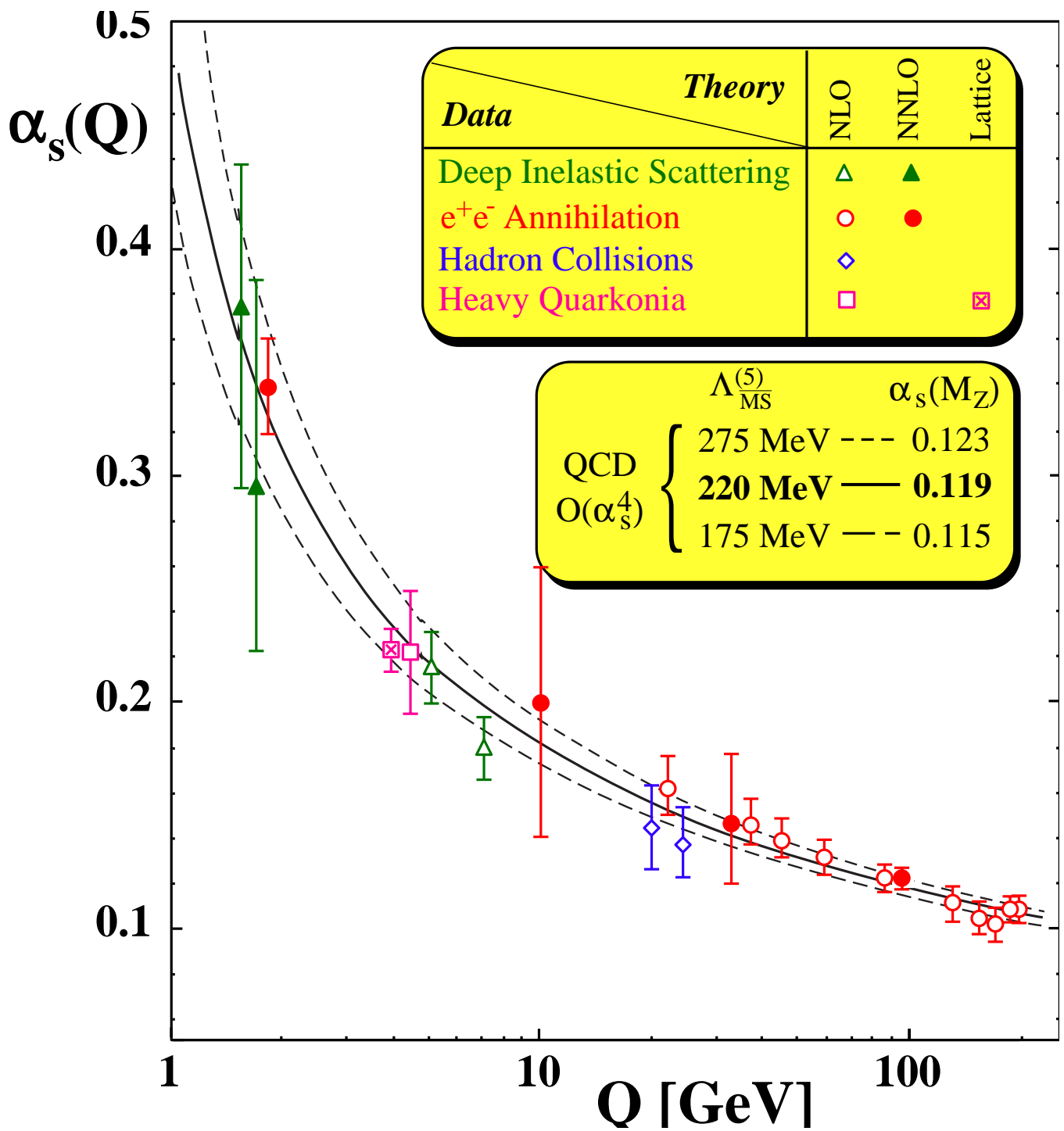


Figure 60: The running of the strong coupling constant.

Electron-positron annihilation

A clean process with which to study QCD is:

$$e^+ + e^- \rightarrow \gamma^* \rightarrow \text{hadrons} \tag{88}$$

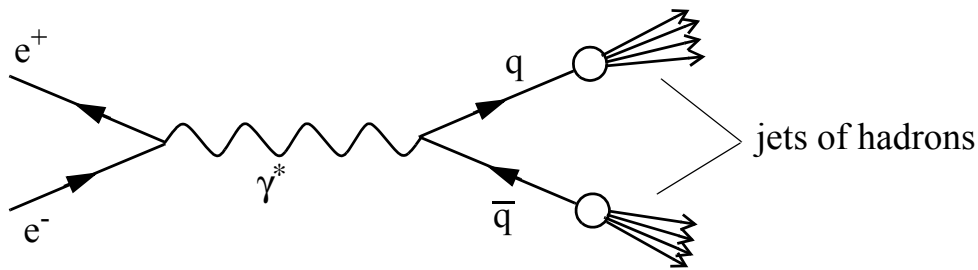


Figure 61: e^+e^- annihilation into hadrons.

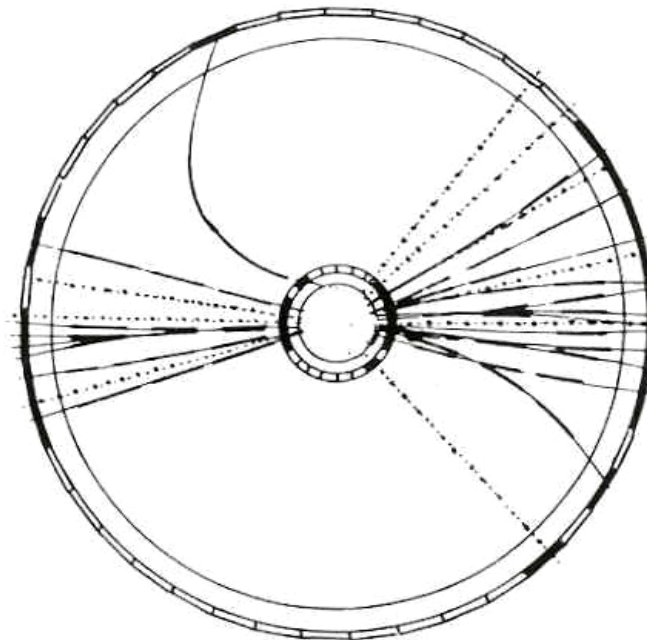


Figure 62: An e^+e^- annihilation event in which two quark jets were created (the event was recorded by the JADE experiment at DESY).

- ❖ In the lowest order e^+e^- annihilation process, a **photon or a Z^0** is produced which converts into a **quark-antiquark** pair.
- The quark and the antiquark **fragment** into observable hadrons.
- Since the quark and antiquark momenta are equal and counterparallel, hadrons are produced in two **jets** of equal energies going in opposite direction.
- The **direction** of a jet reflects the direction of the corresponding quark.

→ The **total cross-section** of $e^+e^- \rightarrow \text{hadrons}$ is often expressed as:

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

The cross section for muon production and hadron production is the same if the number of quark flavours and colours are taken into account:

$$R = N_c \sum e_q^2$$

→ Here $N_c = 3$ is the **number of colours** and e_q is the **charge** of the quarks.

→ If $\sqrt{s} < m_\psi$ then

$$R = N_c(e_u^2 + e_d^2 + e_s^2) = 3 \left((-1/3)^2 + (-1/3)^2 + (2/3)^2 \right) = 2$$

If $\sqrt{s} < m_\gamma$ then $R = N_c(e_u^2 + e_d^2 + e_s^2 + e_c^2) = 10/3$ and

If $\sqrt{s} > m_\gamma$ then $R = N_c(e_u^2 + e_d^2 + e_s^2 + e_c^2 + e_b^2) = 11/3$

→ If the **radiation of hard gluons** is taken into account, an extra factor proportional to α_s has to be added:

$$R = 3 \sum_q e_q^2 \left(1 + \frac{\alpha_s(Q^2)}{\pi} \right)$$

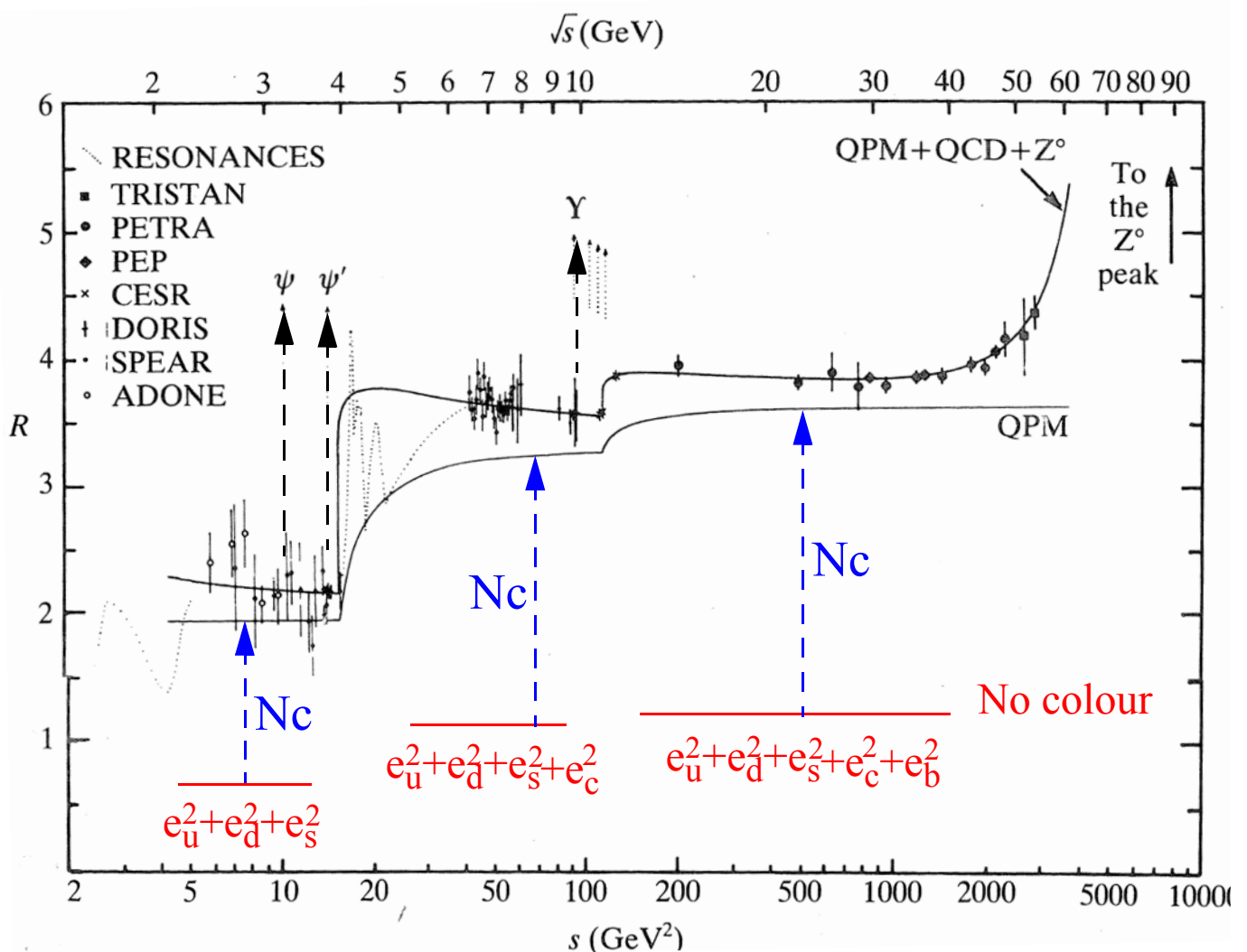


Figure 63: The measured R-value and the predicted R-value for different theoretical assumptions.

❖ A study of the angular distribution of the jets give information about the **spin of the quarks**.

→ The angular distribution of the process

$e^+ + e^- \rightarrow \gamma^* \rightarrow \mu^+ + \mu^-$ is given by:

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow \mu^+\mu^-) = \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta)$$

where θ is the production angle with respect to the direction of the colliding electrons.

→ If **quarks** have **spin 1/2** they should have the following angular distribution:

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow q\bar{q}) = N_c e_q^2 \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta)$$

where e_q is the fractional charge of a quark and $N_c = 3$ is the number of colours.

→ If **quarks** have **spin 0** the angular distribution should be:

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow q\bar{q}) = N_c e_q^2 \frac{\pi\alpha^2}{2Q^2}(1 - \cos^2\theta)$$

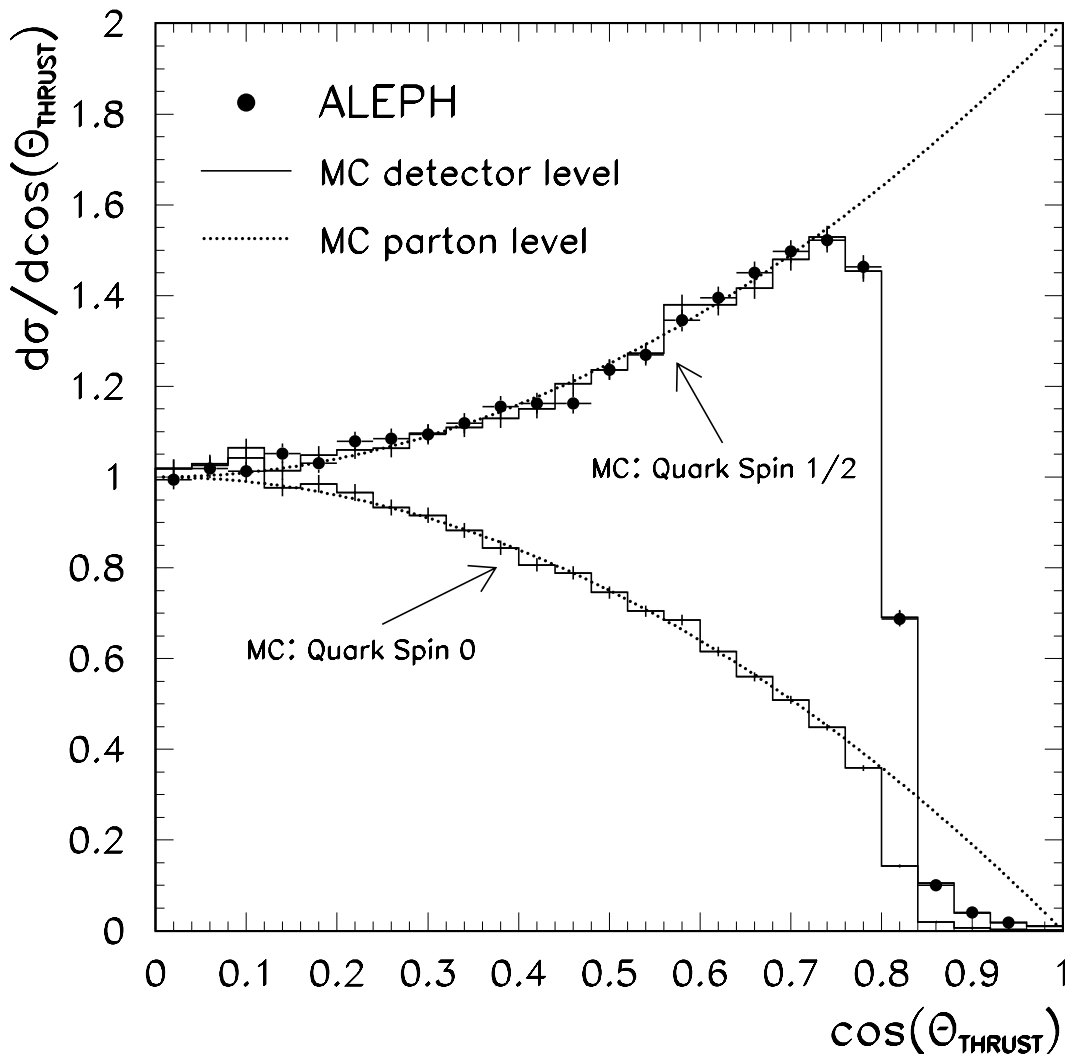


Figure 64: The angular distribution of the quark jet in e^+e^- annihilations, compared with models.

→ The experimentally measured angular dependence of jets is clearly following $(1+\cos^2\theta)$
 ⇒ jets are associated with spin-1/2 quarks.

→ If a high-momentum (hard) gluon is emitted by a quark or antiquark, it fragments to a jet of its own, which leads to a **three-jet event**:

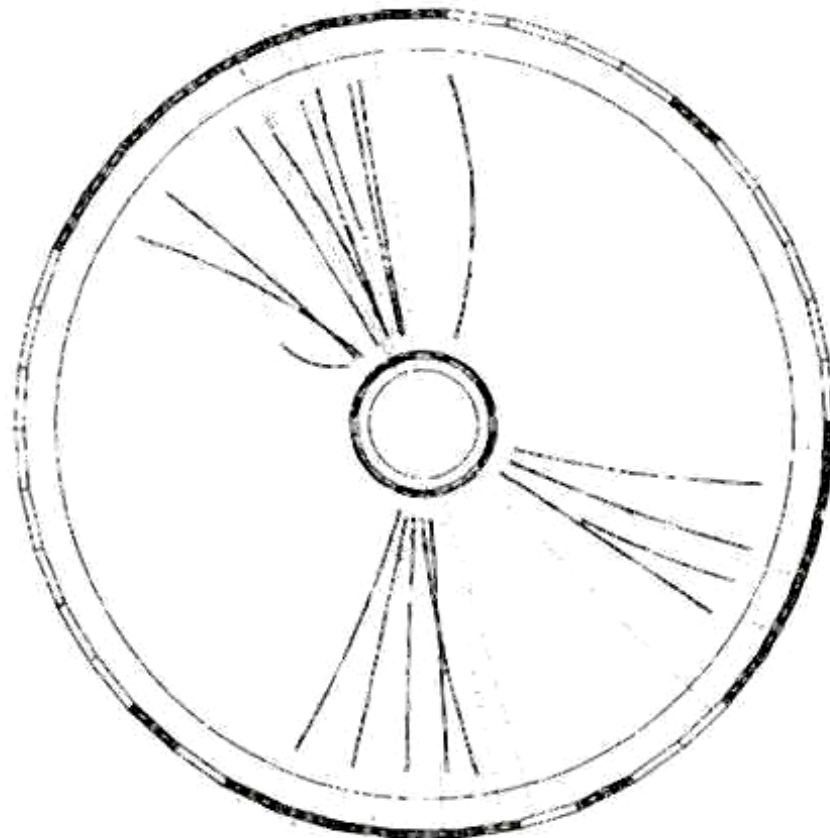
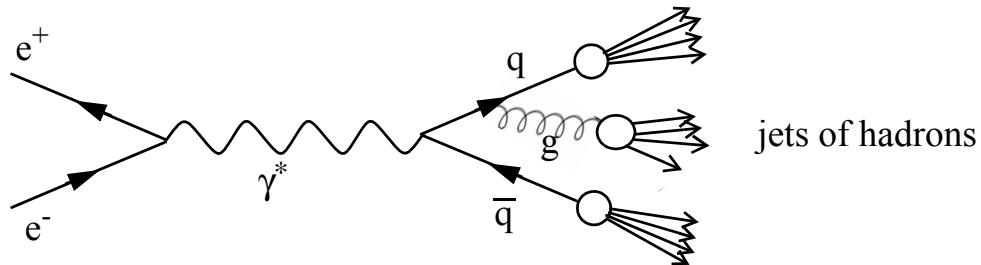


Figure 65: A three-jet event in an e^+e^- annihilation as seen by the JADE experiment.

❖ The **probability** for a quark to emit a **gluon** is proportional to α_s and by comparing the rate of two-jet and three-jet events one can determine α_s .

➔ $\alpha_s = 0.15 \pm 0.03$ for $E_{CM} = 30$ to 40 GeV

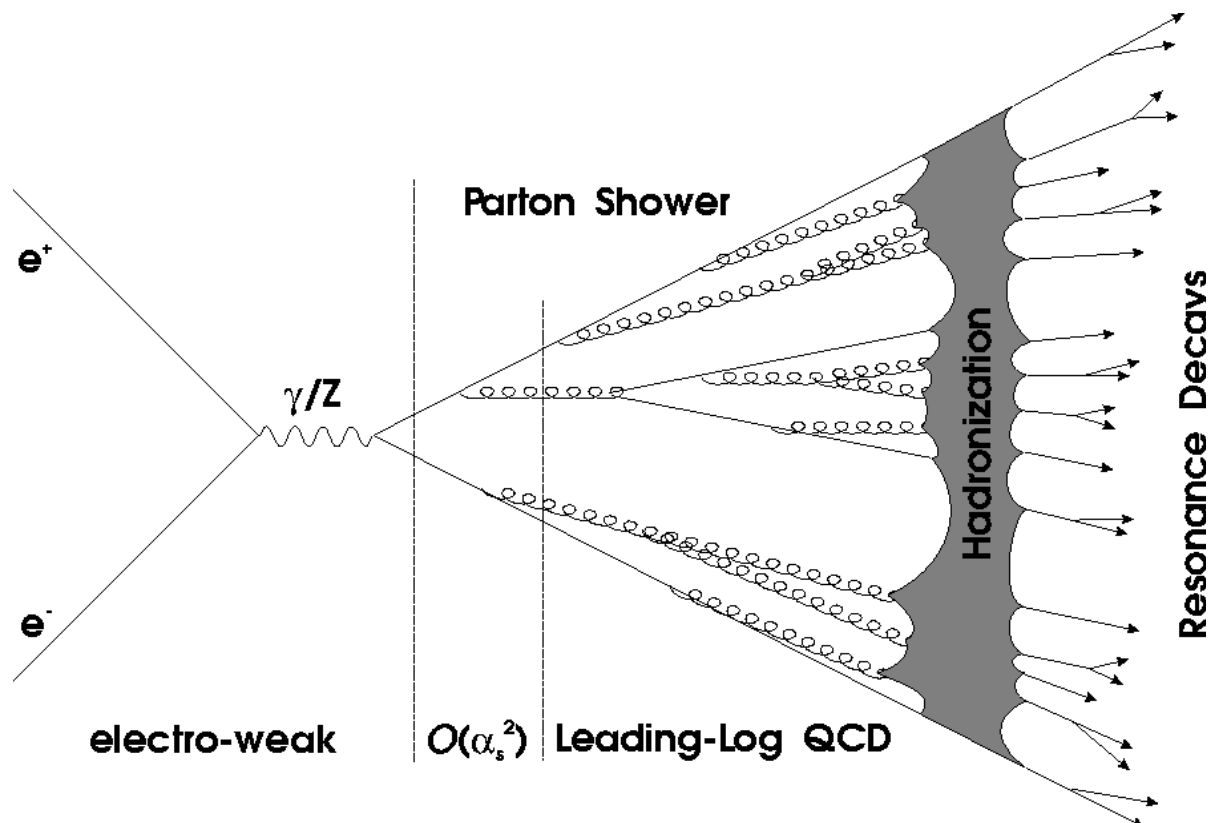


Figure 66: The principal scheme of hadron production in e^+e^- annihilations. Hadronization (= fragmentation) begins at distances of order 1 fm between the partons.

❖ By measuring angular distributions of jets one can confirm models where **gluons** are **spin-1** bosons. This is done by measuring:

$$\cos \phi = \frac{\sin \theta_2 - \sin \theta_3}{\sin \theta_1}$$

where the angles are described below:

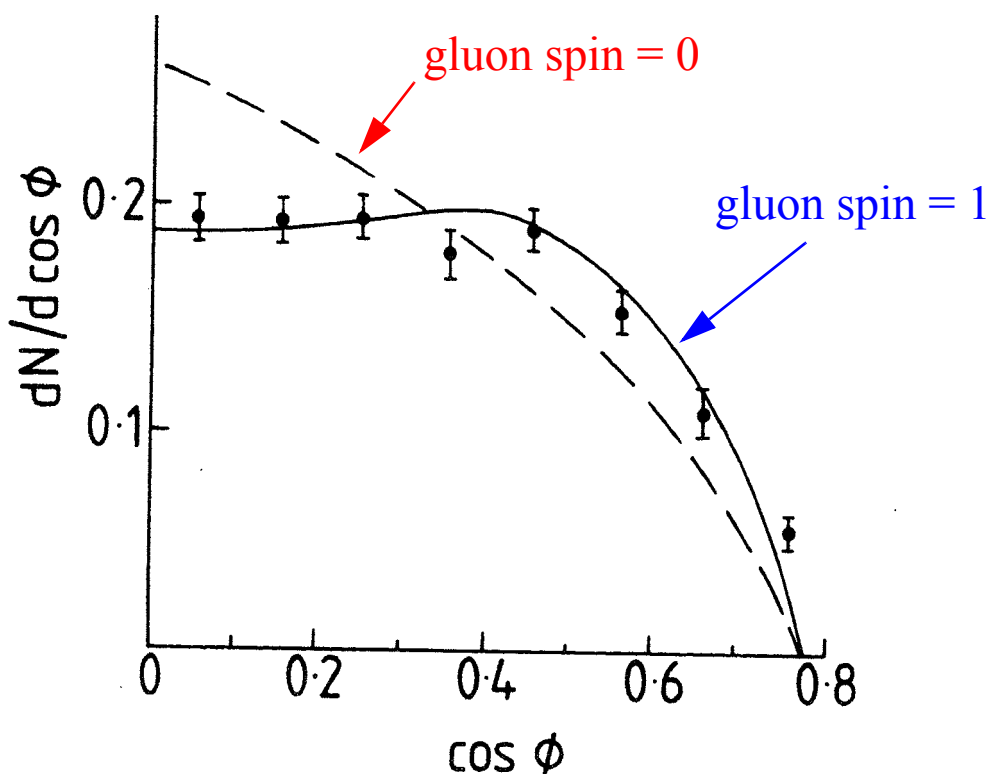
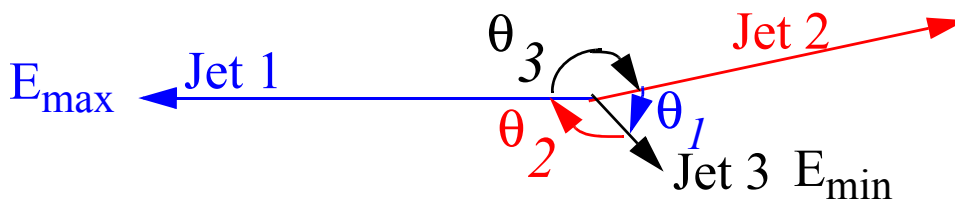


Figure 67: An angular distribution of jets compared to QCD calculations with a spin 0 and a spin 1 gluon.