# VII. QCD, jets and gluons

## Quantum Chromodynamics (QCD)

❖ *Quantum Chromodynamics (QCD)* is the theory of strong interactions.

→ Interactions are carried out by a massless spin-1 particle – *gauge boson*

→ In quantum electrodynamics (QED) gauge bosons are photons, in QCD – *gluons*

→ Gauge bosons couple to conserved charges: photons in QED – to electric charges (Q), and gluons in QCD – to colour charges: colour hypercharge ($Y^C$) and colour isospin charge ($I_3^C$).

→ The strong interaction acts the same on u,d,s,c,b and t quarks because the strong interaction is **flavour-independent.**

## REMINDER:

In the quark model we have introduced flavour hypercharge (Y) and flavour isospin ($I_3$) in addition to the electric charge (Q).

| | Q | Y | $I_3$ | | Q | Y | $I_3$ |
|---|---|---|---|---|---|---|---|
| d | -1/3 | 1/3 | -1/2 | $\bar{d}$ | 1/3 | -1/3 | 1/2 |
| u | 2/3 | 1/3 | 1/2 | $\bar{u}$ | -2/3 | -1/3 | -1/2 |
| s | -1/3 | -2/3 | 0 | $\bar{s}$ | 1/3 | 2/3 | 0 |
| c | 2/3 | 4/3 | 0 | $\bar{c}$ | -2/3 | -4/3 | 0 |
| b | -1/3 | -2/3 | 0 | $\bar{b}$ | 1/3 | 2/3 | 0 |
| t | 2/3 | 4/3 | 0 | $\bar{t}$ | -2/3 | -4/3 | 0 |

In quantum chromodynamics we introduce in a similar fashion colour hypercharge ($Y^c$) and colour isospin charge ($I_3^c$).

| | $Y^c$ | $I_3^c$ | | $Y^c$ | $I_3^c$ |
|---|---|---|---|---|---|
| r | 1/3 | 1/2 | $\bar{r}$ | -1/3 | -1/2 |
| g | 1/3 | -1/2 | $\bar{g}$ | -1/3 | 1/2 |
| b | -2/3 | 0 | $\bar{b}$ | 2/3 | 0 |

## Gluons carry colour charges themselves !

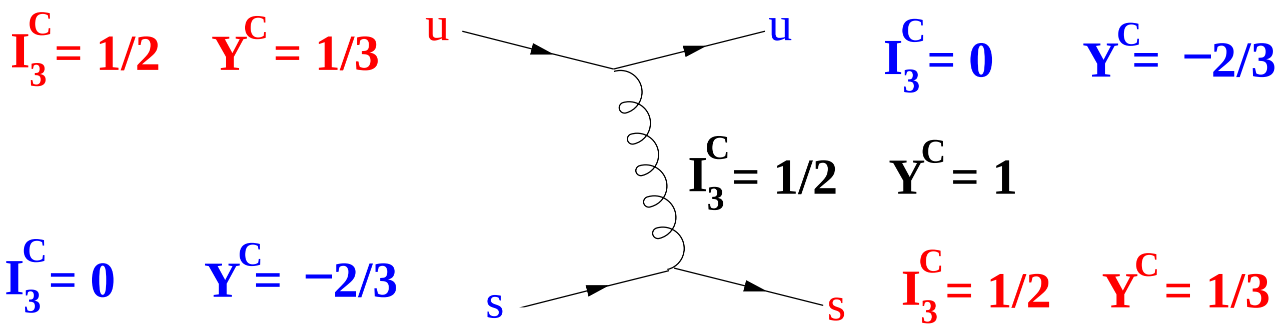



Figure 58: Gluon exchange between quarks.

The colour quantum numbers of the gluon in the figure above are:

$$I_3^C = I_3^C(r) - I_3^C(b) = \frac{1}{2} \tag{86}$$

$$Y^C = Y^C(r) - Y^C(b) = 1$$

**Gluon colour wavefunctions:**

| | | |
|---|---|---|
| $\chi^C_{g1} = r\bar{g}$ | $I_3^C = 1$ | $Y^C = 0$ |
| $\chi^C_{g2} = \bar{r}g$ | $I_3^C = -1$ | $Y^C = 0$ |
| $\chi^C_{g3} = r\bar{b}$ | $I_3^C = 1/2$ | $Y^C = 1$ |
| $\chi^C_{g4} = \bar{r}b$ | $I_3^C = -1/2$ | $Y^C = -1$ |
| $\chi^C_{g5} = g\bar{b}$ | $I_3^C = -1/2$ | $Y^C = 1$ |
| $\chi^C_{g6} = \bar{g}b$ | $I_3^C = 1/2$ | $Y^C = -1$ |
| $\chi^C_{g7} = 1/\sqrt{2}\,(g\bar{g} - \bar{r}r)$ | $I_3^C = 0$ | $Y^C = 0$ |
| $\chi^C_{g8} = 1/\sqrt{6}\,(g\bar{g} - r\bar{r} - 2b\bar{b})$ | $I_3^C = 0$ | $Y^C = 0$ |

❖ Gluons can couple to other gluons since gluons carry colour charge !

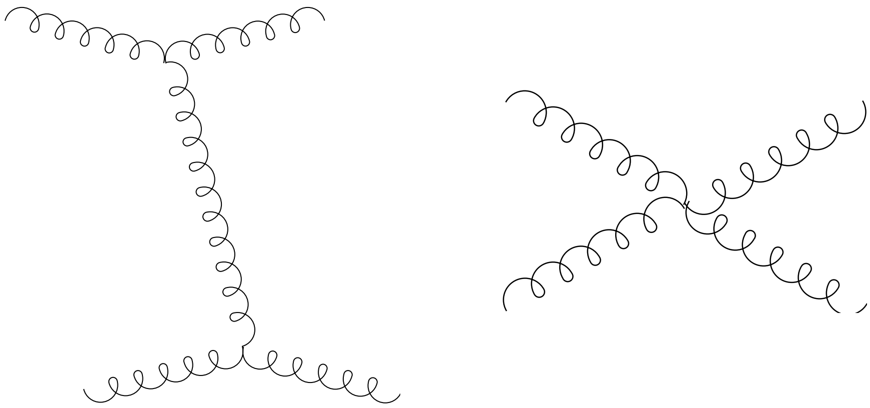


Figure 59: Lowest-order contributions to gluon-gluon scattering.

→ All observed states (all mesons and baryons) have zero colour charge - colour confinement.

→ Gluons does not exist as free particles since they have colour charge.

→ Bound colourless states of gluons are called *glueballs* (not detected experimentally yet).

❖ The principle of asymptotic freedom:

→ At short distances the strong interactions are weaker ⇒ quarks and gluons are essentially free particles ⇒ the interaction can be described by the lowest order diagrams.

→ At large distances the strong interaction gets stronger ⇒ the interaction can be described by high-order diagrams.

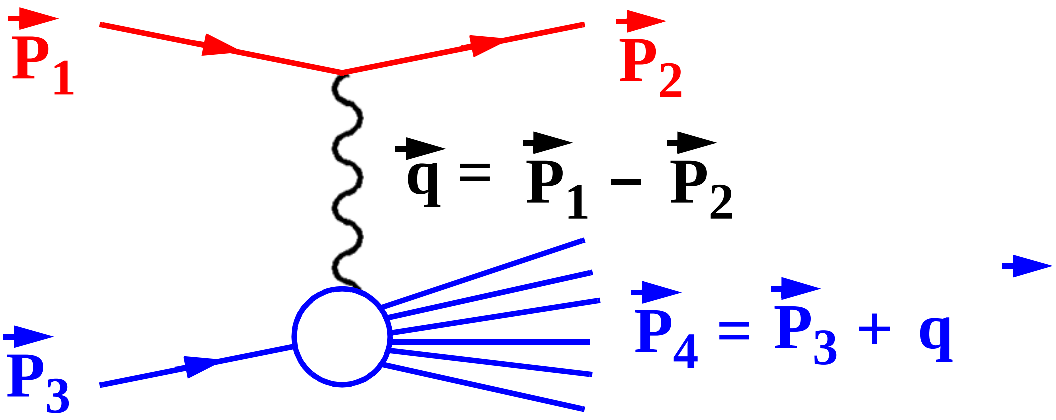
→ The quark-antiquark potential is:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} \qquad (r < 0{,}1\,fm)$$
$$V(r) = \lambda r \qquad (r \geq 1\,fm)$$

where $\alpha_s$ is the strong coupling constant and *r* the distance between the quark and the antiquark.

→ Due to the complexity of high-order diagrams, the very process of confinement can not be calculated analytically ⇒ only numerical models are available.

# VARIABLES




where $\vec{P}$ and $\vec{q}$ are energy-momentum four vectors:

$$\vec{P} = (E, \bar{p}) = (E, p_x, p_y, p_z)$$

$$\vec{q} = (E_q, \bar{q}) = \vec{P}_1 - \vec{P}_2 = (E_1 - E_2, \overline{P}_1 - \overline{P}_2)$$

the momentum and energy transfer is:

$$q = |\bar{q}| = |\overline{P}_1 - \overline{P}_2|$$

$$\nu = E_q = E_1 - E_2$$

(where E and P are in the restframe of particle 3).

The energy-momentum transfer is given by:

$$Q^2 = -\vec{q} \bullet \vec{q} = -(\vec{P}_1 - \vec{P}_2)^2$$

$$Q^2 = \bar{q} \bullet \bar{q} - E_q^2 = (\overline{P}_1 - \overline{P}_2)^2 - (E_1 - E_2)^2$$

This can be regarded as the invariant mass of the exchanged gauge boson since the squared mass of a particle is given by $M^2 = \vec{P} \bullet \vec{P}$

The invariant mass of the hadrons is given by:

$$W^2 = \vec{P}_4 \bullet \vec{P}_4 = (\vec{P}_3 + \vec{q})^2$$

# The strong coupling constant

❖ The strong coupling constant $\alpha_s$ is the analogue in QCD of $\alpha_{em}$ and it is a measure of the strength of the interaction.

→ $\alpha_s$ is not a true constant but a "running constant" since it decreases with increasing $Q^2$.

→ In leading order of QCD, $\alpha_s$ is given by

$$\alpha_s = \frac{12\pi}{(33 - 2N_f)\ln(Q^2/\Lambda^2)} \tag{87}$$

Here $N_f$ is the number of allowed quark flavours, and $\Lambda \approx 0.2$ GeV is the QCD scale parameter which has to be defined experimentally.

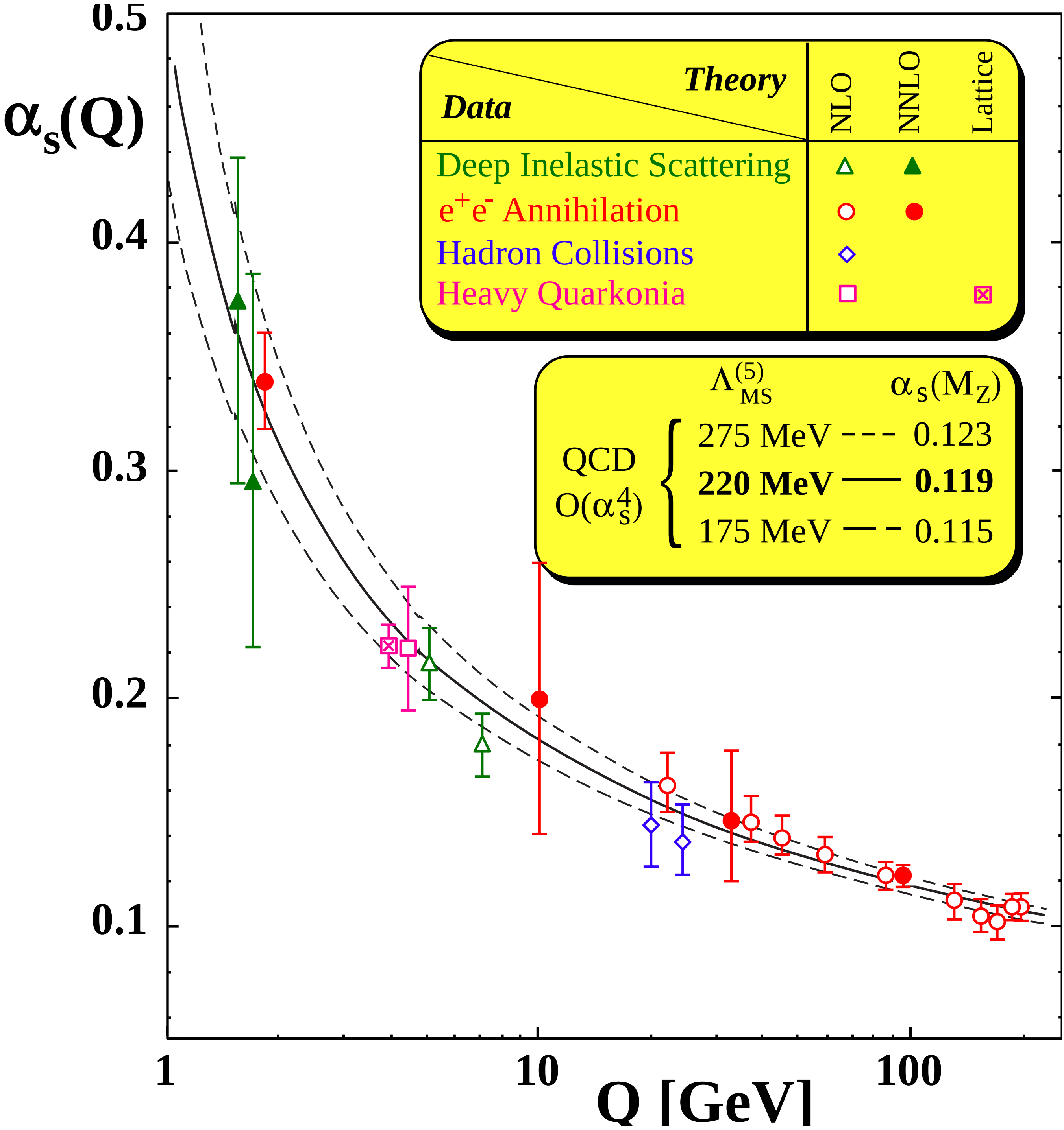
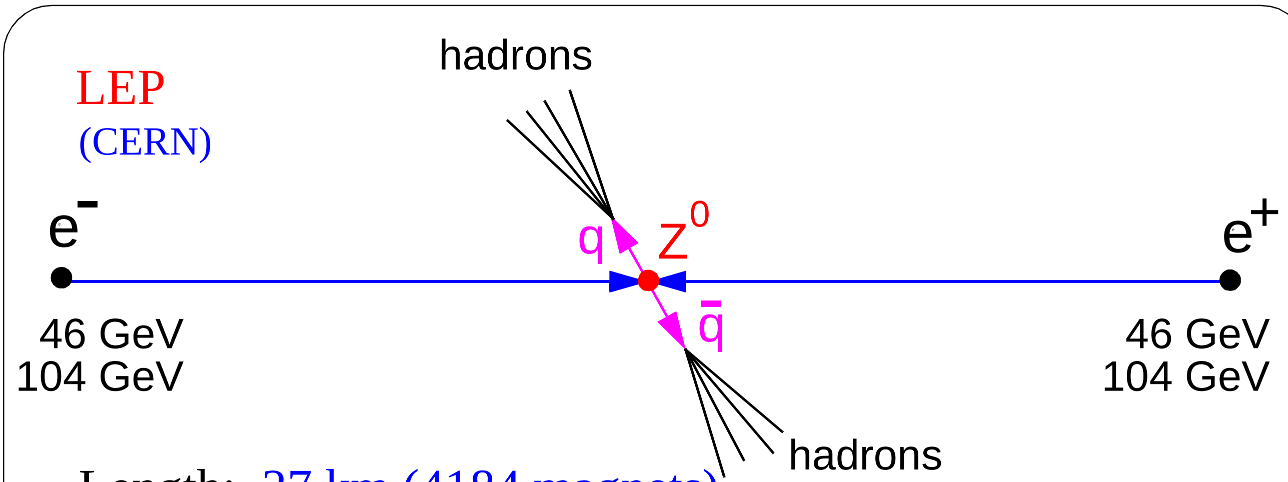
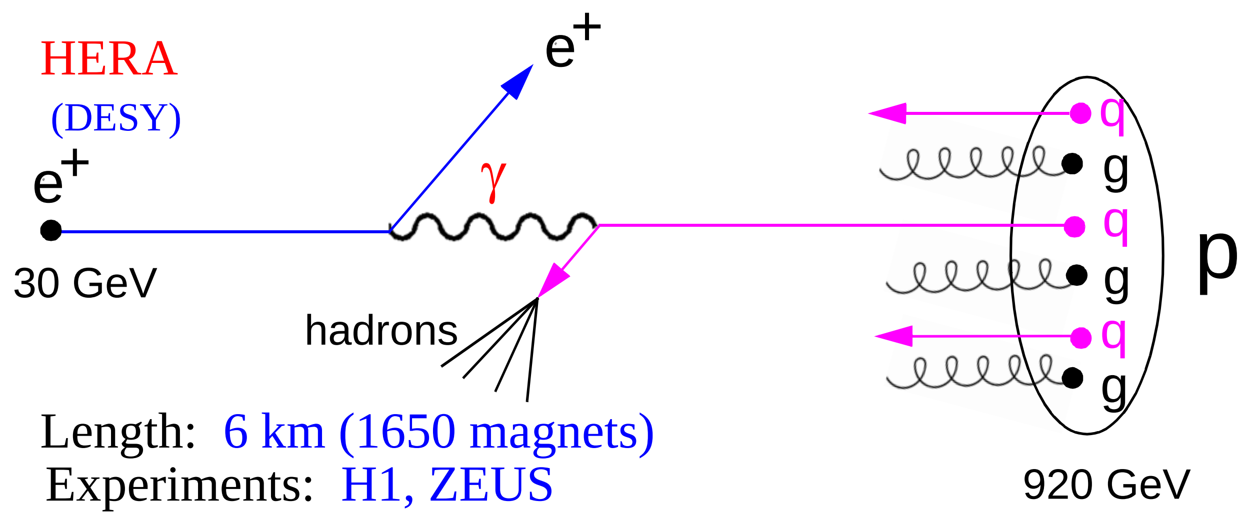



Figure 60: The running of the strong coupling constant.

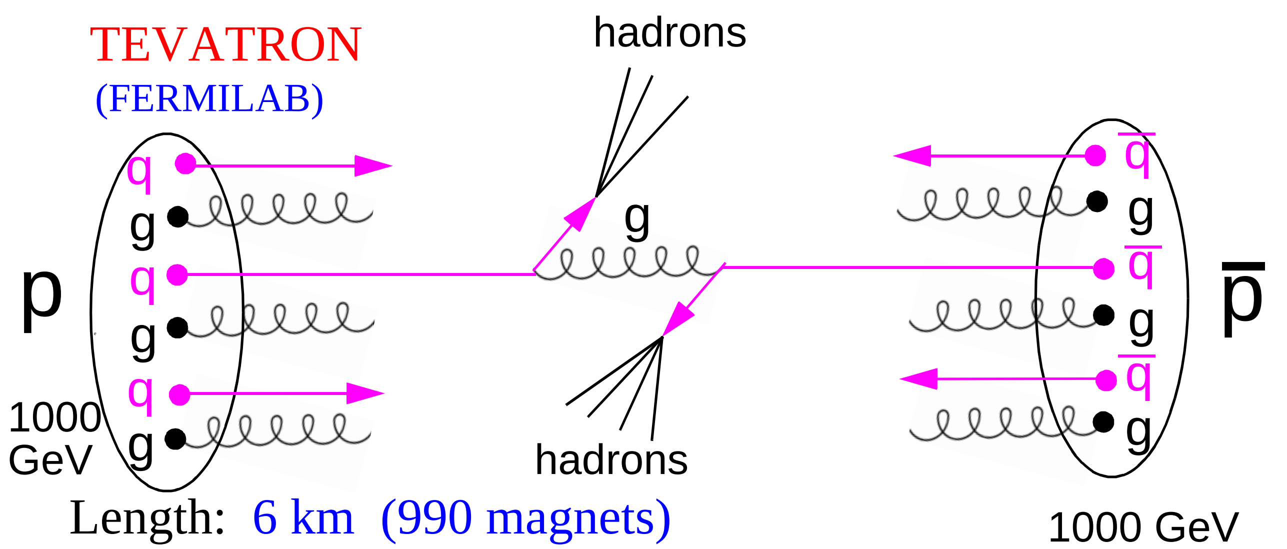
# Different types of accelerators




Length: 27 km (4184 magnets)
Experiments: DELPHI, OPAL, ALEPH, L3




Length: 6 km (1650 magnets)
Experiments: H1, ZEUS




Length: 6 km (990 magnets)
Experiments: CDF, D0

# Electron-positron annihilation

A clean process with which to study QCD is:

$$e^+ + e^- \rightarrow \gamma^* \rightarrow \text{hadrons} \quad (88)$$

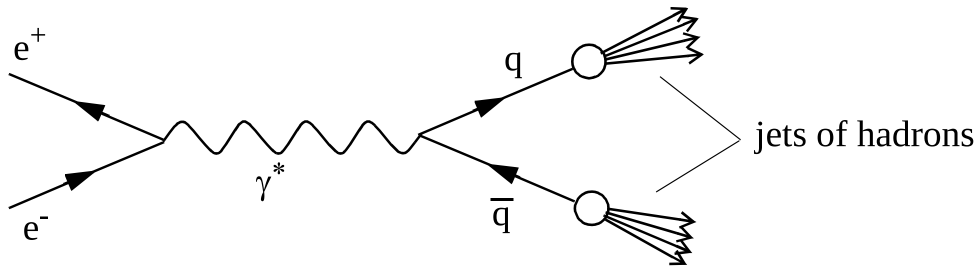



Figure 61: $e^+e^-$ annihilation into hadrons.

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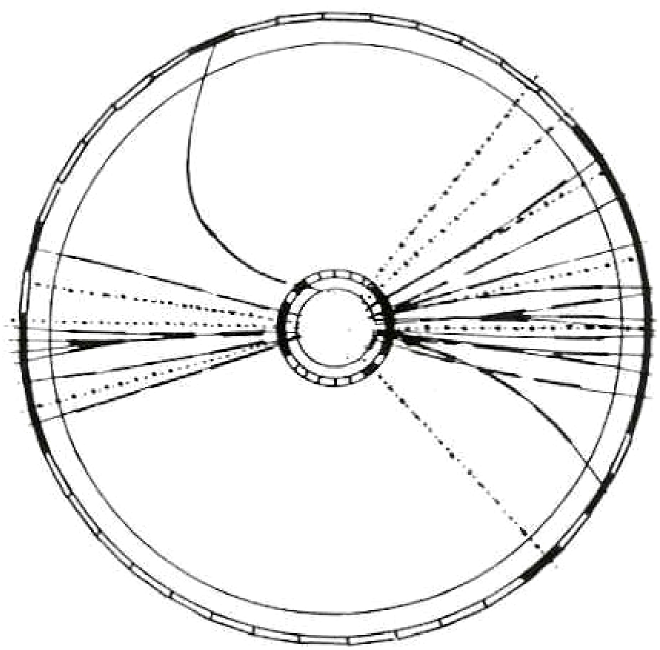


Figure 62: An $e^+e^-$ annihilation event in which two quark jets were created (the event was recorded by the JADE experiment at DESY).

❖ In the lowest order $e^+e^-$ annihilation process, a photon or a $Z^0$ is produced which converts into a quark-antiquark pair.

→ The quark and the antiquark *fragment* into observable hadrons.

→ Since the quark and antiquark momenta are equal and counterparallel, hadrons are produced in two *jets* of equal energies going in opposite direction.

→ The direction of a jet reflects the direction of the corresponding quark.

→ The total cross-section of $e^+e^- \rightarrow hadrons$ is often expressed as:

$$R \equiv \frac{\sigma(e^+e^- \rightarrow hadrons)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

The cross section for muon production and hadron production is the same if the number of quark flavours and colours are taken into account:

$$R = N_c \sum e_q^2$$

→ Here $N_c = 3$ is the number of colours and $e_q$ is the charge of the quarks.

→ If $\sqrt{s} < m_\psi$ then

$$R = N_c(e_u^2 + e_d^2 + e_s^2) = 3\left((-1/3)^2 + (-1/3)^2 + (2/3)^2\right) = 2$$

If $\sqrt{s} < m_\Upsilon$ then $R = N_c(e_u^2 + e_d^2 + e_s^2 + e_c^2) = 10/3$ and

If $\sqrt{s} > m_\Upsilon$ then $R = N_c(e_u^2 + e_d^2 + e_s^2 + e_c^2 + e_b^2) = 11/3$

→ If the radiation of hard gluons is taken into account, an extra factor proportional to $\alpha_s$ has to be added:

$$R = 3\sum_q e_q^2\left(1 + \frac{\alpha_s(Q^2)}{\pi}\right)$$

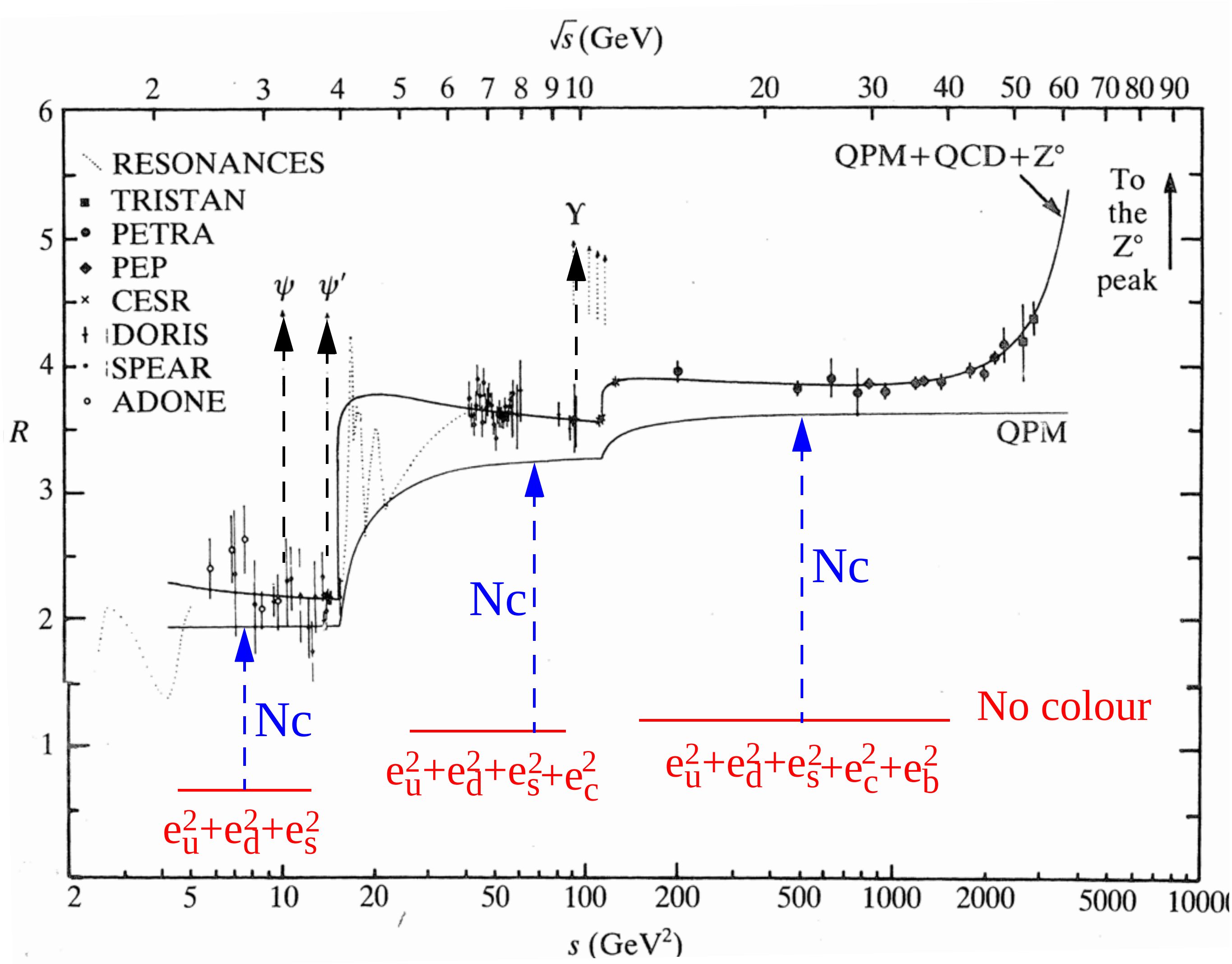



Figure 63: The measured R-value and the predicted R-value for different theoretical assumptions.

❖ A study of the angular distribution of the jets give information about the spin of the quarks.

→ The angular distribution of the process

$e^+ + e^- \rightarrow \gamma^* \rightarrow \mu^+ + \mu^-$ is given by:

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow \mu^+\mu^-) = \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta)$$

where $\theta$ is the production angle with respect to the direction of the colliding electrons.

→ If quarks have spin 1/2 they should have the following angular distribution:

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow q\bar{q}) = N_c e_q^2 \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta)$$

where $e_q$ is the fractional charge of a quark and $N_c = 3$ is the number of colours.

→ If quarks have spin 0 the angular distribution should be:

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow q\bar{q}) = N_c e_q^2 \frac{\pi\alpha^2}{2Q^2}(1 - \cos^2\theta)$$

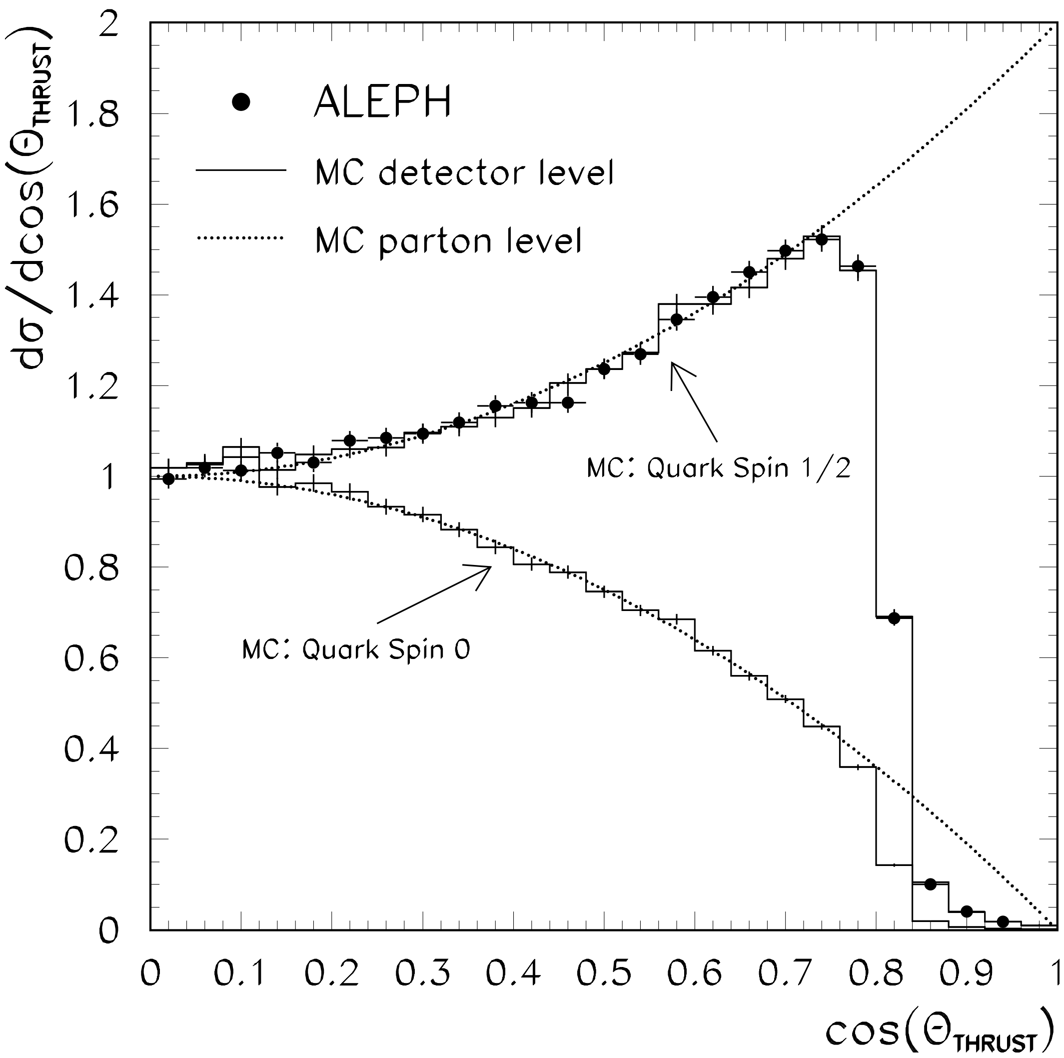
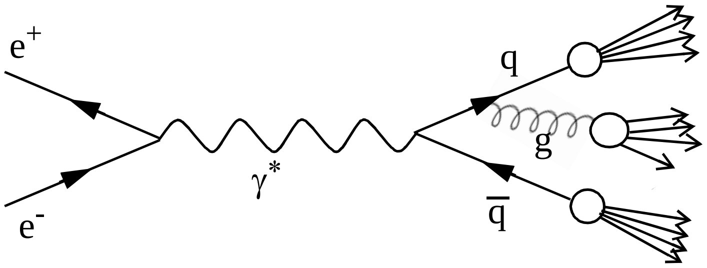



Figure 64: The angular distribution of the quark jet in $e^+e^-$ annihilations, compared with models.

→ The experimentally measured angular dependence of jets is clearly following $(1+\cos^2\theta)$ $\Rightarrow$ jets are associated with spin-1/2 quarks.

→ If a high-momentum (hard) gluon is emitted by a quark or antiquark, it fragments to a jet of its own, which leads to a three-jet event:



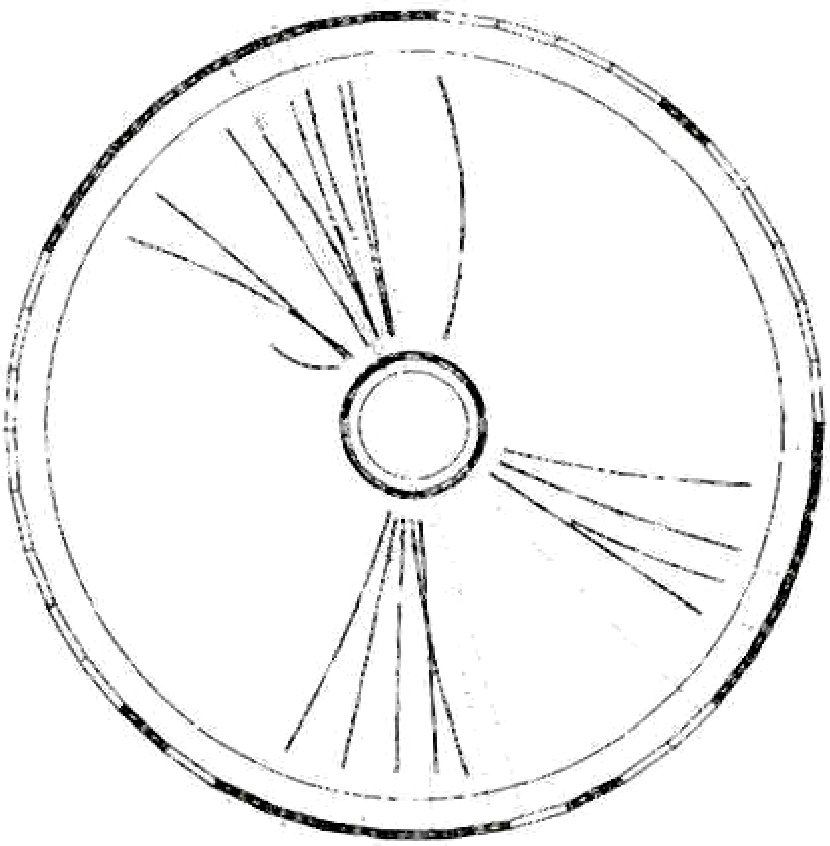



Figure 65: A three-jet event in an $e^+e^-$ annihilation as seen by the JADE experiment.

❖ The probability for a quark to emit a gluon is proportional to $\alpha_S$ and by comparing the rate of two-jet and three-jet events one can determine $\alpha_S$.

→ $\alpha_S = 0.15 \pm 0.03$ for $E_{CM}$=30 to 40 GeV

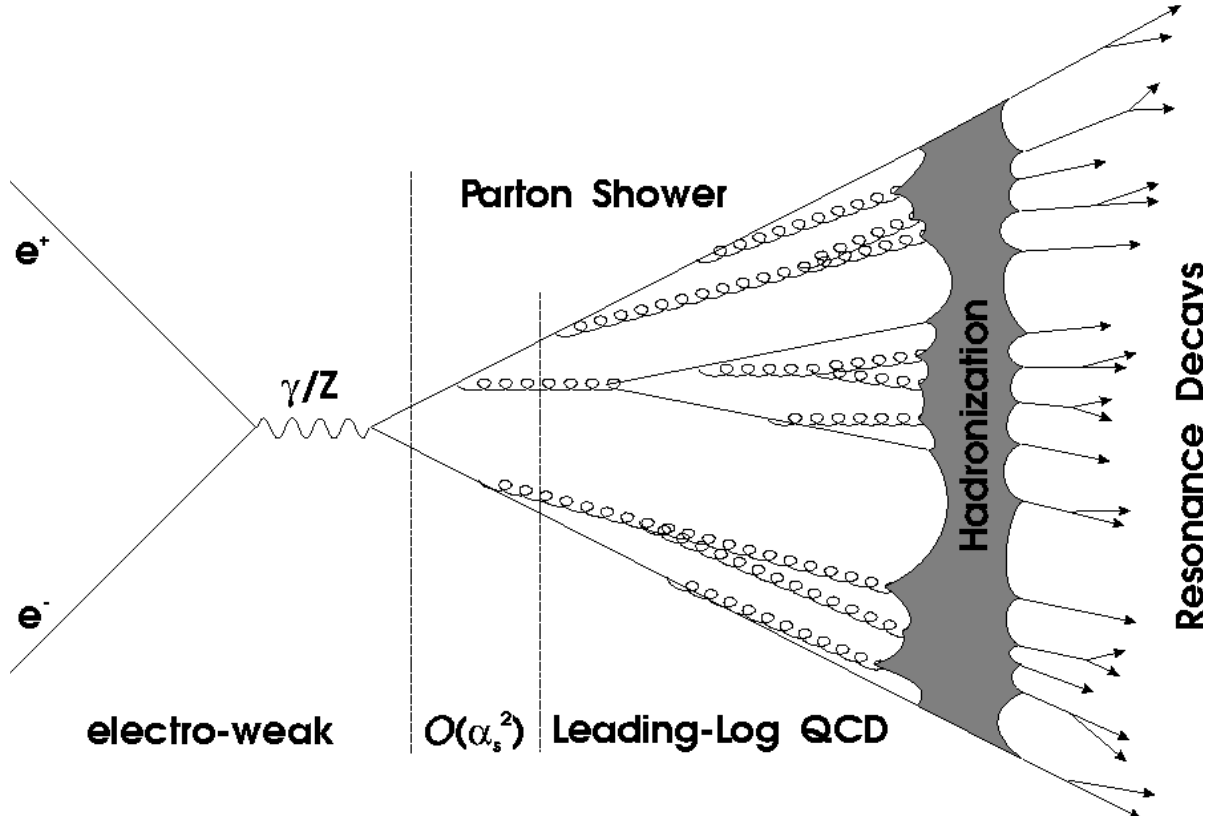
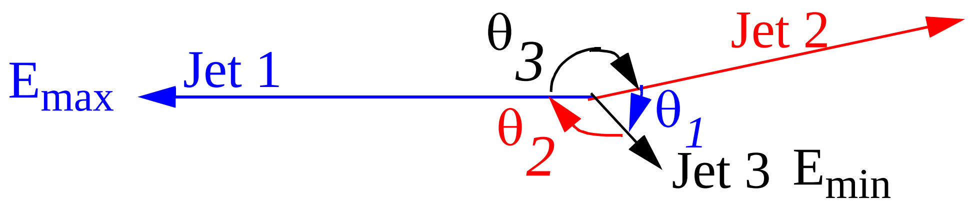



Figure 66: The principal scheme of hadron production in $e^+e^-$ annihilations. Hadronization (= fragmentation) begins at distances of order 1 fm between the partons.

❖ By measuring angular distributions of jets one can confirm models where gluons are spin-1 bosons. This is done by measuring:

$$\cos\phi = \frac{\sin\theta_2 - \sin\theta_3}{\sin\theta_1}$$

where the angles are described below:



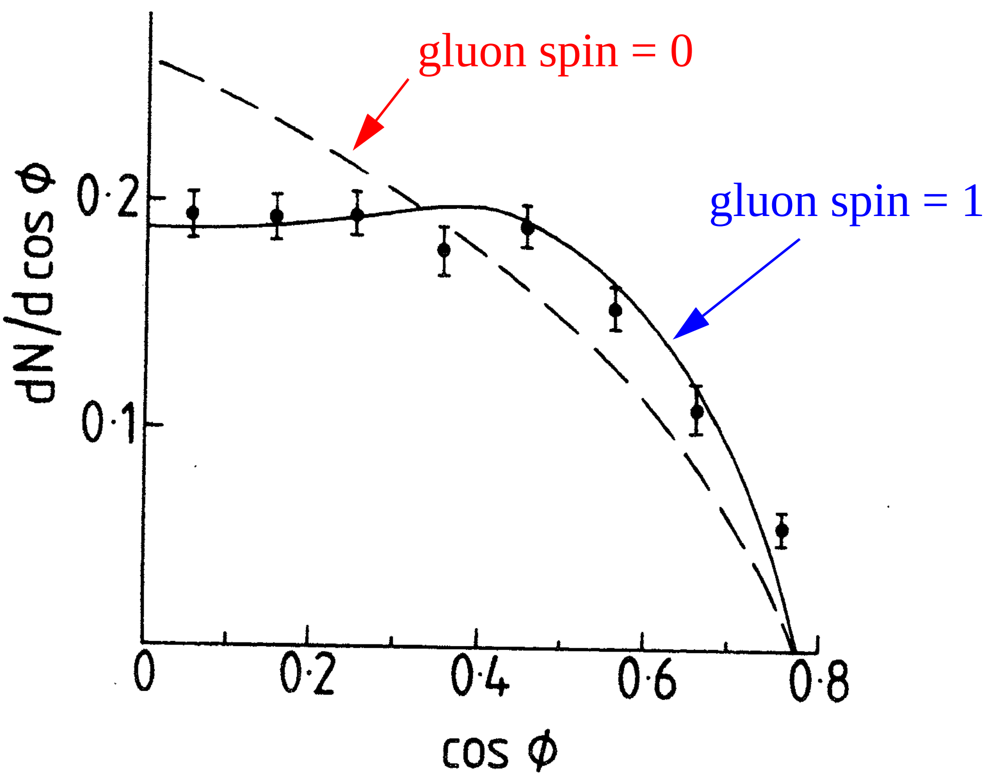




Figure 67: An angular distribution of jets compared to QCD calculations with a spin 0 and a spin 1 gluon.