Elastic electron-proton scattering

- Beams of leptons are good tools for investigating the properties of hadrons since leptons have no substructure.

→ Elastic lepton-hadron scattering can be used to measure the size of the hadron.

→ Elastic scattering means that the same type of particles goes into and comes out of the scattering process.

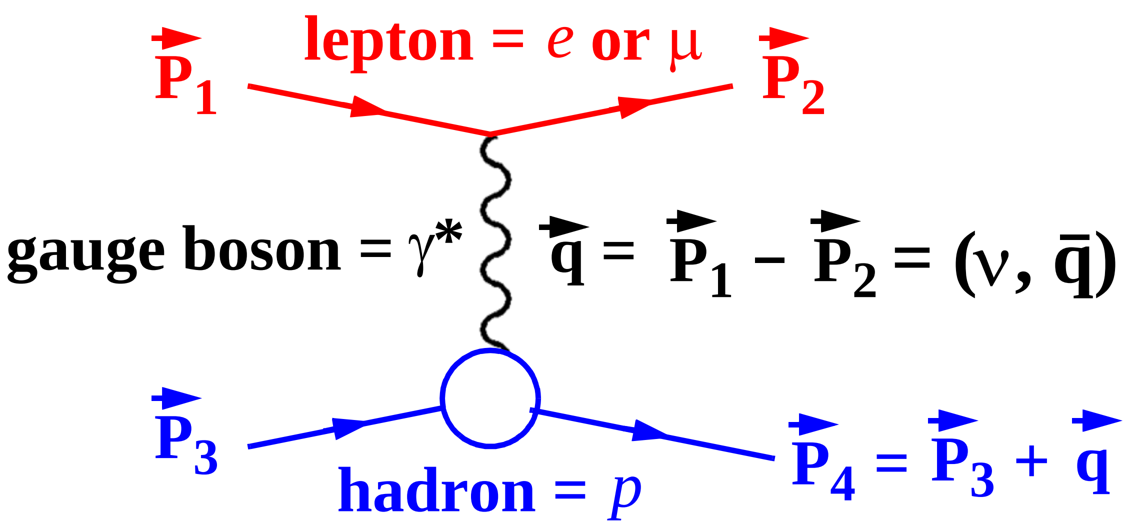
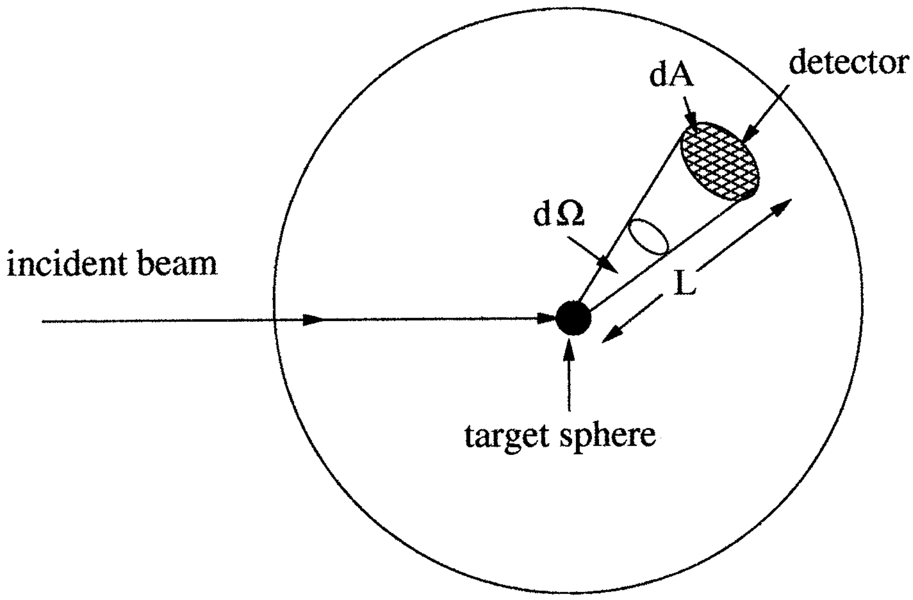


Figure 68: The dominant one-photon exchange mechanism in elastic lepton-proton scattering.

❖ The angular distribution of the particles emerging from a scattering reaction is given by the differential cross-section

$$\frac{d\sigma(\theta,\varphi)}{d\Omega} \quad \text{where} \quad d\Omega = \sin\theta d\theta d\varphi$$



$$d\Omega = \frac{dA}{L^2}$$

Figure 69: The definition of the solid angle $d\Omega$ in scattering experiments.

→ The total cross section of the reaction is obtained by integrating the differential cross section:

$$\sigma = \int \frac{d\sigma(\theta,\varphi)}{d\Omega} d\Omega = \int_0^{\pi}\int_0^{2\pi} \frac{d\sigma(\theta,\varphi)}{d\Omega} \sin\theta d\theta d\varphi$$

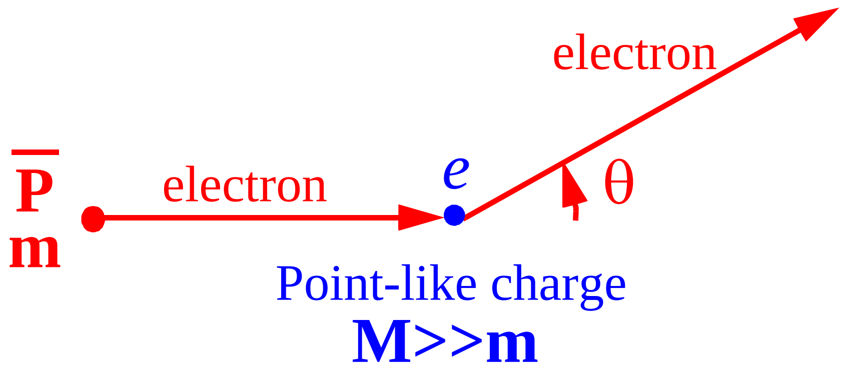


Figure 70: The scattering of an electron on a static point-like electrical charge.

→ The angular distribution of a relativistic electron of momentum p which is scattered by a point-like static electric charge e is described by the Mott scattering formula:

$$\left(\frac{d\sigma}{d\Omega}\right)_M = \frac{\alpha^2}{4p^4 \sin^4\left(\frac{\theta}{2}\right)}\left(m^2 + p^2 \cos^2\frac{\theta}{2}\right)$$

In the low energy limit $p \ll m$, the Mott scattering formula is reduced to the non-relativistic Rutherford scattering formula:

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{m^2\alpha^2}{4p^4 \sin^4\left(\frac{\theta}{2}\right)} \qquad \text{where } \alpha = \frac{e^2}{4\pi}$$

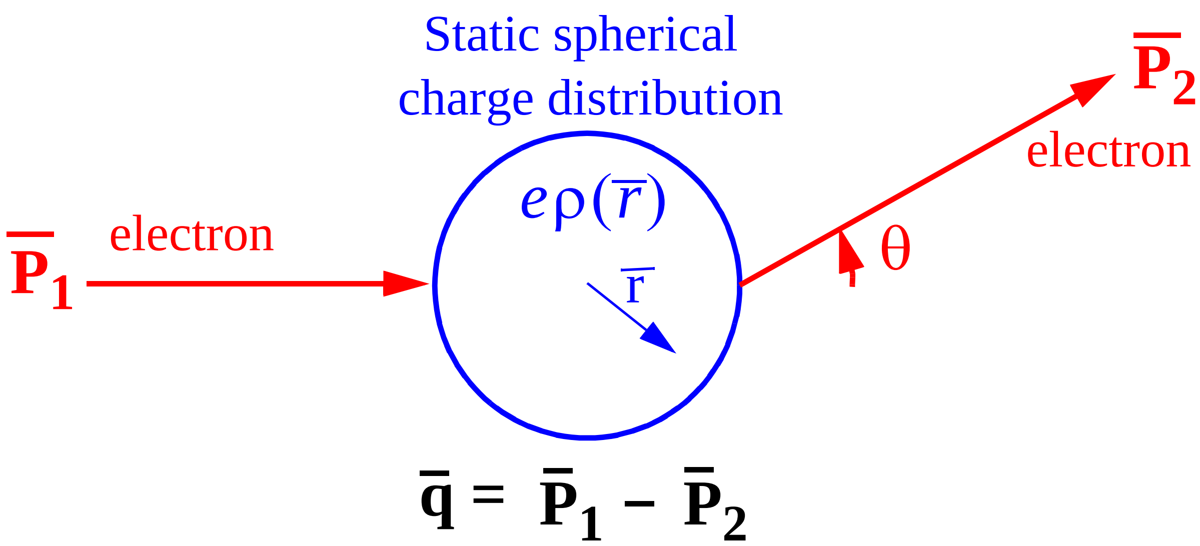


Figure 71: The scattering of an electron on a static spherical charge distribution.

If the electric charge is not point-like, but it is spread out with a spherically symmetric density distribution, i.e., $e \to e\rho(r)$, where $\rho(r)$ is normalized:

$$\int \rho(r) d^3\bar{x} = 1$$

then the Rutherford scattering formula has to be modified by an electric form factor $G^2_E(q^2)$:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_R G_E^2(q^2) \tag{89}$$

→ The electric form factor is the Fourier transform of the charge distribution with respect to the momentum transfer $\bar{q}$:

$$G_E(q^2) = \int \rho(r) e^{i\bar{q}\cdot\bar{x}} d^3\bar{x} \tag{90}$$

– For $q = 0$, $G_E(0) = 1$ (low momentum transfer)

– For $q^2 \to \infty$, $G_E(q^2) \to 0$ (large momentum transfer)

→ Measurements of the cross-section can be used to determine the form-factor and hence the charge distribution.

The mean quadratic charge radius is for example given by

$$r_E^2 = \int r^2 \rho(r) d^3\bar{x} = -6 \left.\frac{dG_E(q^2)}{dq^2}\right|_{q^2 = 0} \tag{91}$$

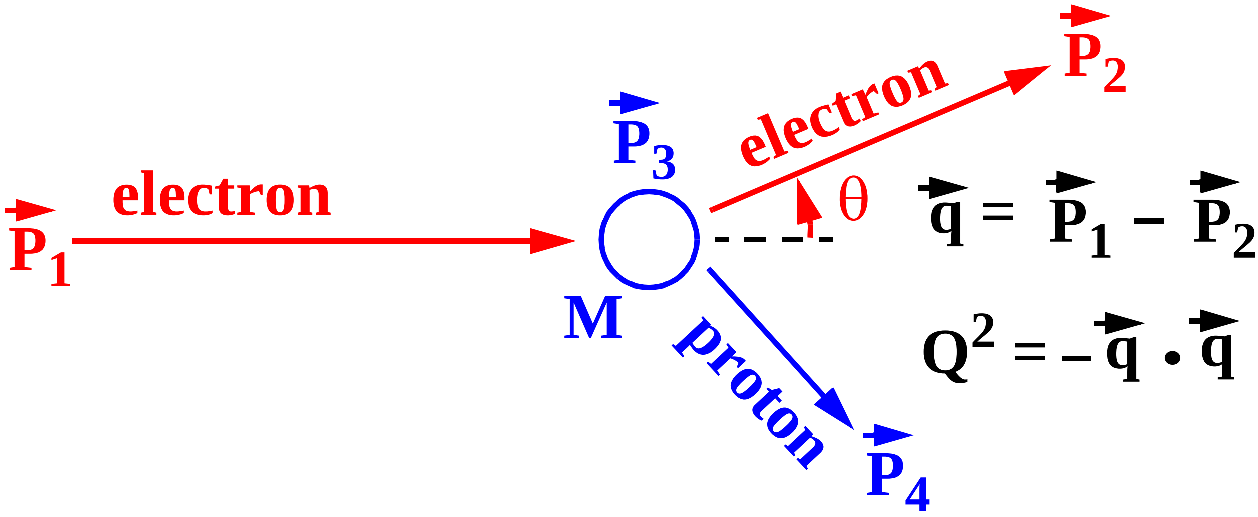


Figure 72: Elastic electron-proton scattering when the recoil energy of the proton is taken into account.

→ Scattering of electrons on protons depend not only on the electric form factor G_E but also on a magnetic form factor G_M which is associated with the magnetic moment distribution.

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_M \times \left(G_1(Q^2)\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}G_2(Q^2)\sin^2\frac{\theta}{2}\right)$$

$$G_1(Q^2) = \frac{G_E^2 + \frac{Q^2}{4M^2}G_M^2}{1 + \frac{Q^2}{4M^2}} \qquad G_2(Q^2) = G_M^2$$

→ Measurement of the form factors are conveniently divided into three Q^2 regions:

1) low $Q^2 \Rightarrow Q \ll M \Rightarrow G_E$ dominates the cross-section and r_E can be precisely measured:

$$r_E = 0{,}85 \pm 0{,}02\ fm \tag{92}$$

2) An intermediate range: $0.02 \le Q^2 \le 3\ \mathrm{GeV}^2 \Rightarrow$ both G_E and G_M give sizeable contribution $\Rightarrow$ the result can be given by the parameterization:

$$G_E(Q^2) \approx \frac{G_M(Q^2)}{\mu_p} \approx \left(\frac{\beta^2}{\beta^2 + Q^2}\right)^2 \tag{93}$$

with β^2=0.84 GeV

3) high $Q^2 > 3\ \mathrm{GeV}^2 \Rightarrow G_M$ dominates the cross section:

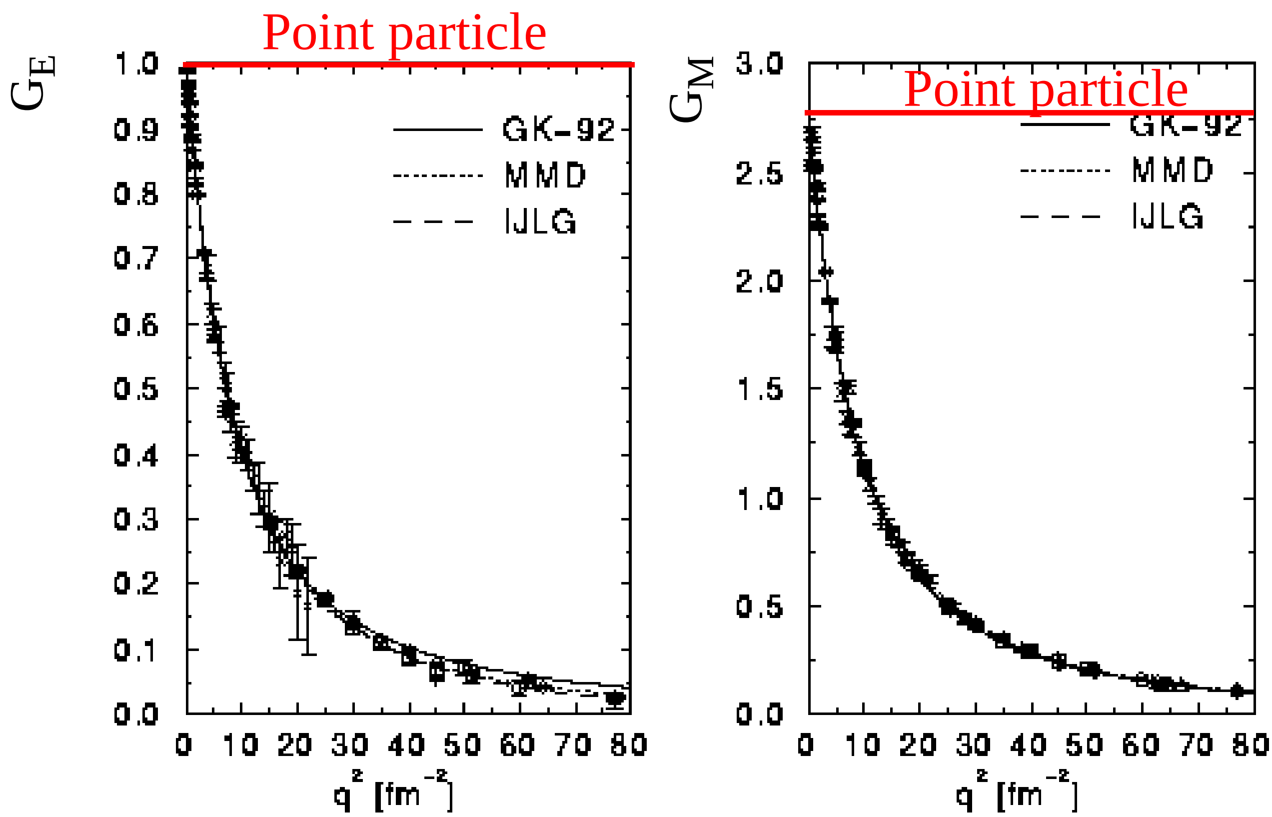


Figure 73: Electric and magnetic proton form-factors, compared with different parameterizations.

The form factors are normalized so that

$$G_E(0) = \text{total charge} = 1 \ (p)$$
$$= 0 \ (n)$$
$$G_M(0) = \text{magnetic moment} = \mu_p = +2.79 \ (p)$$
$$= \mu_n = -1.91 \ (n)$$

→ If the proton is a point particle then

$$\mathbf{G_E(Q^2) = 1} \ \text{and} \ \mathbf{G_M(Q^2) = \mathit{2{,}79}}$$

Inelastic lepton-proton scattering

❖ Inelastic electron-proton scattering can be used to probe the proton structure and gave the first evidence for the existence of quarks.

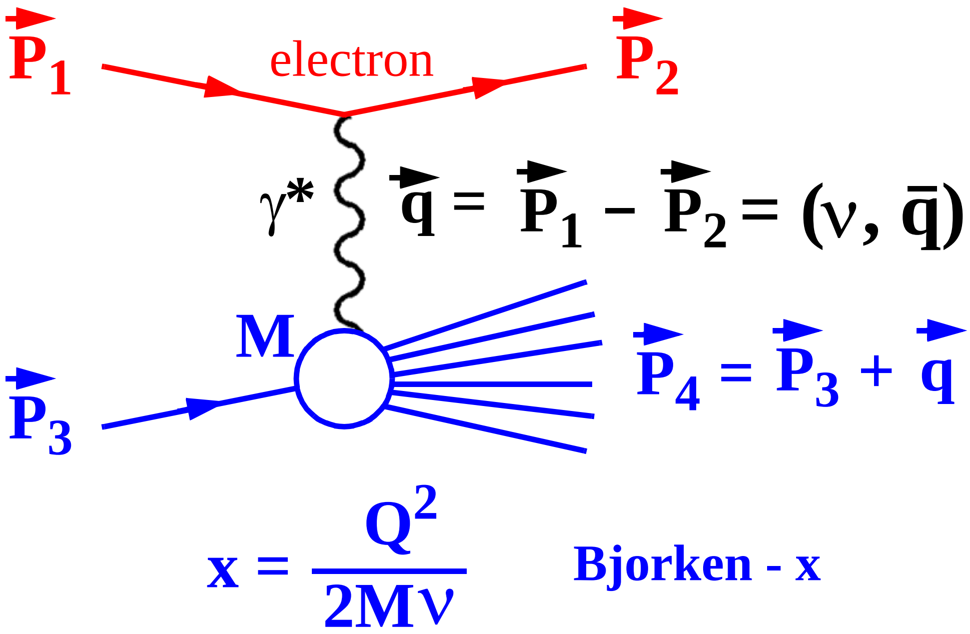


Figure 74: One-photon exchange in inelastic lepton-proton scattering.

→ In inelastic lepton-proton scattering a new dimensionless variable called the Bjorken scaling variable x is introduced where $0<x<1$.

→ The differential cross section for inelastic electron-proton scattering can be written as:

$$\frac{d\sigma}{dE_2 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)} \bullet \frac{1}{\nu} \bullet$$

$$\bullet \left[F_2(x, Q^2) \cos^2\left(\frac{\theta}{2}\right) + \frac{Q^2}{xM^2} F_1(x, Q^2) \sin^2\left(\frac{\theta}{2}\right) \right]$$

→ The two dimensionless structure functions $F_1(x,Q^2)$ and $F_2(x,Q^2)$ parameterize the photon-proton interaction in the same way as $G_1(Q^2)$ and $G_2(Q^2)$ in elastic scattering.

→ Bjorken scaling or scale invariance:

$$F_{1,2}(x, Q^2) \approx F_{1,2}(x)$$

i.e. for $Q \gg M$, structure functions are almost independent of Q^2. If all particle masses, energies and momenta are multiplied by a scale factor, structure functions at any given *x* remain unchanged.

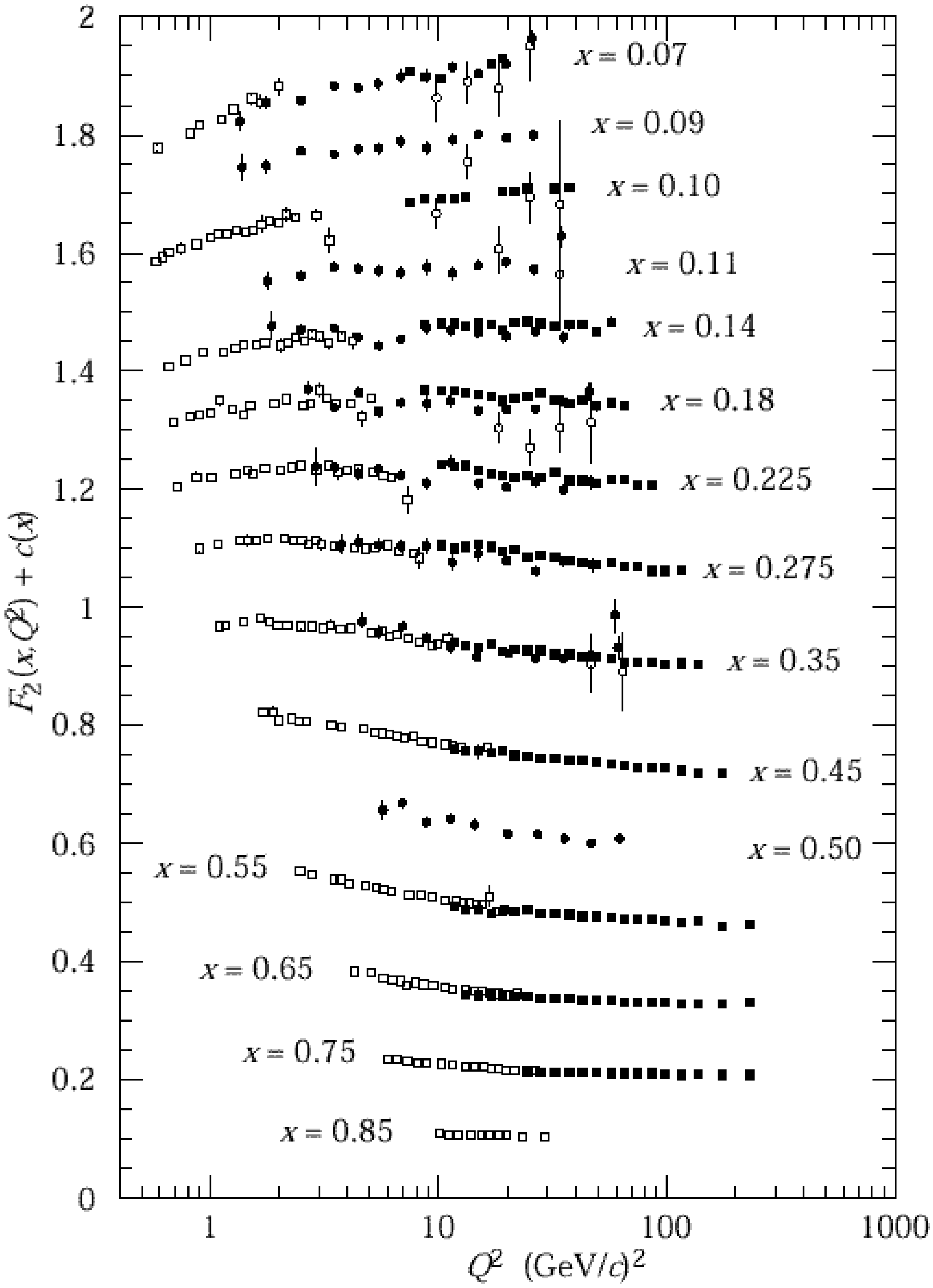


Figure 75: The measured structure function F_2 (compilation of data from different experiments).

→ The first observation of scale invariance in inelastic scattering was observed at SLAC in 1969 and was later interpreted as the first evidence for quarks.

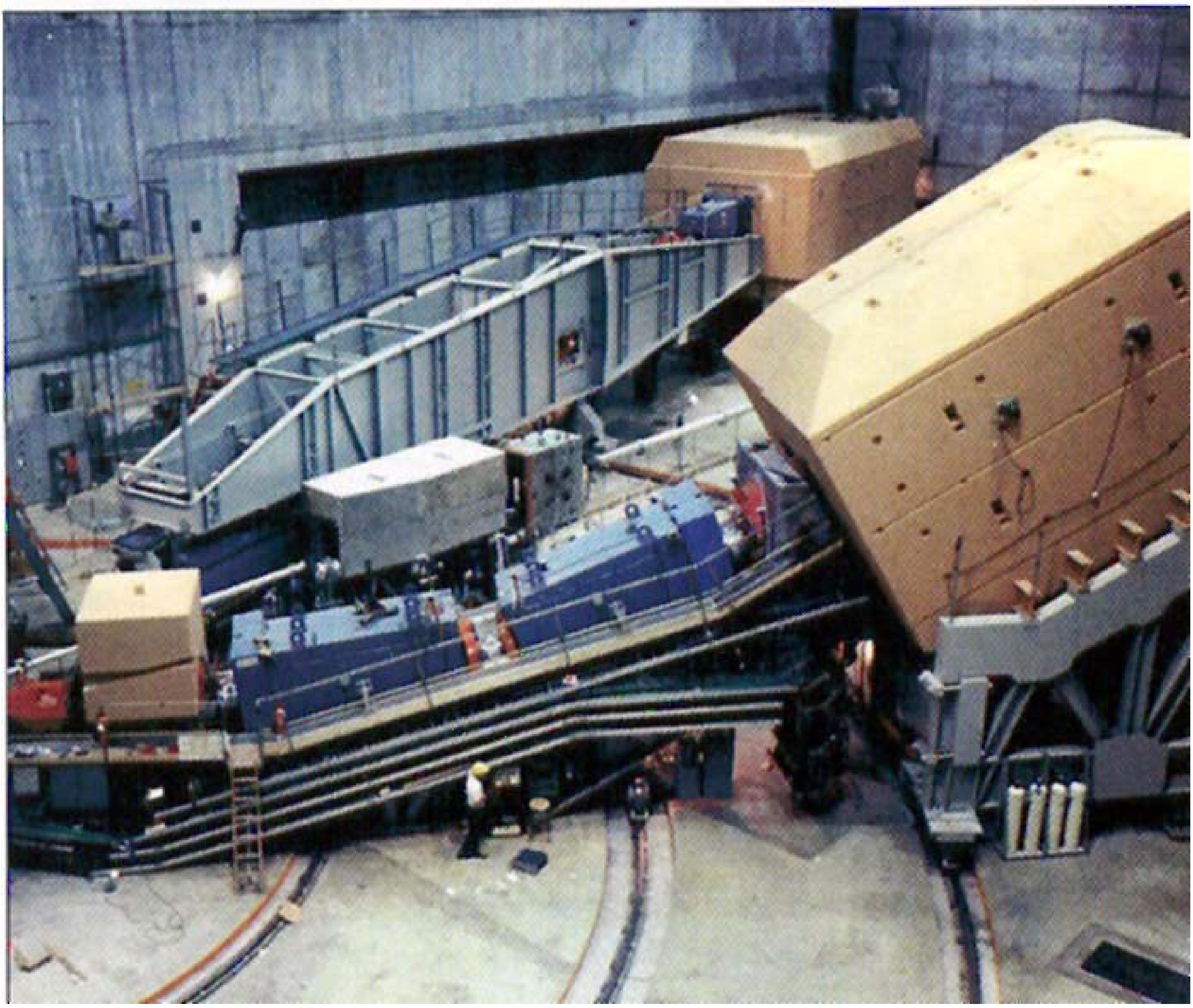
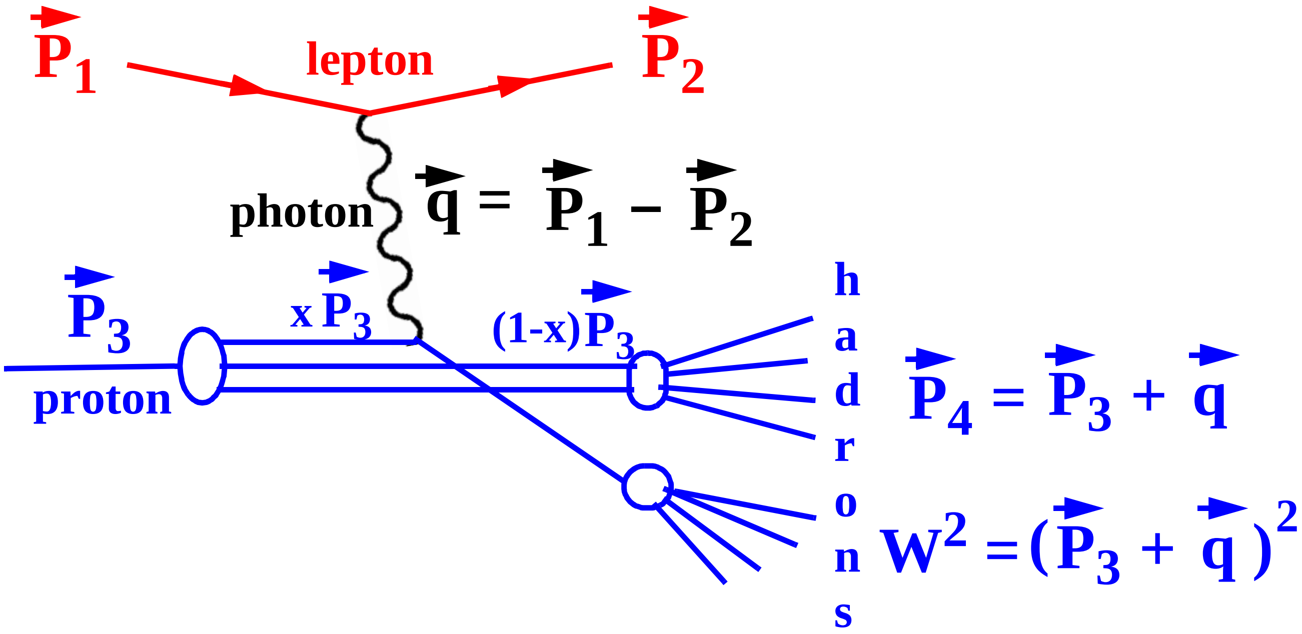


Figure 76: Two spectrometers in SLAC´s End Station A that were used to discover quarks in the late 1960s.

Deep inelastic electron-proton scattering.

❖ In the parton model the scale invariance is explained by scattering on point-like constituents (partons) in the proton.



→ We now know that the partons in the parton model are identical to the quarks in the quark model.

→ The parton model is valid if the target proton has a sufficiently large momentum, so that the fraction of the proton momentum carried by the struck quark is given by Bjorken x.

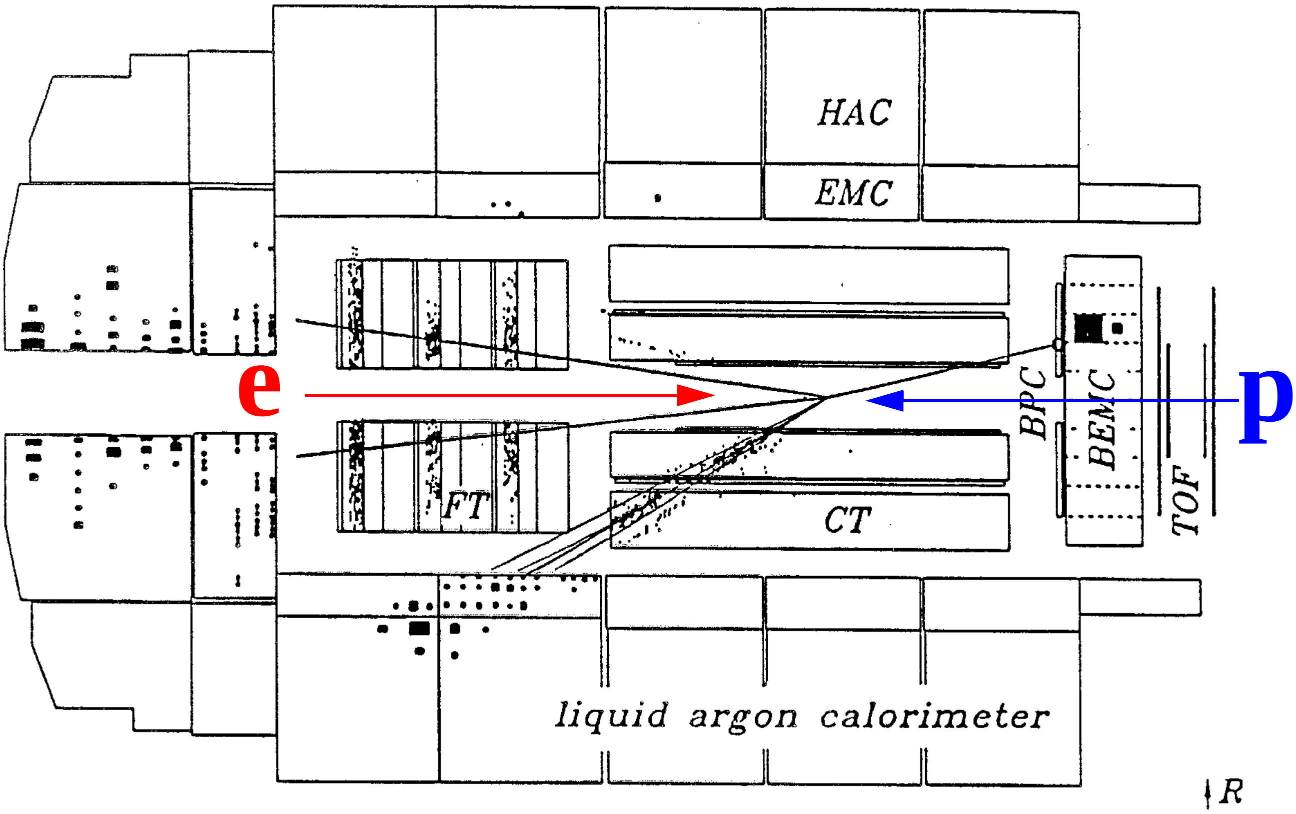


Figure 77: A computer reconstruction of a deep inelastic electron-proton scattering event recorded by the H1 experiment at DESY.

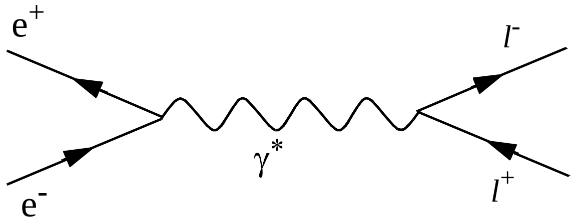
→ In the parton model, the structure function F_1 depends on the spin of the partons:

$$F_1(x, Q^2) = 0 \qquad \text{(spin-0)}$$

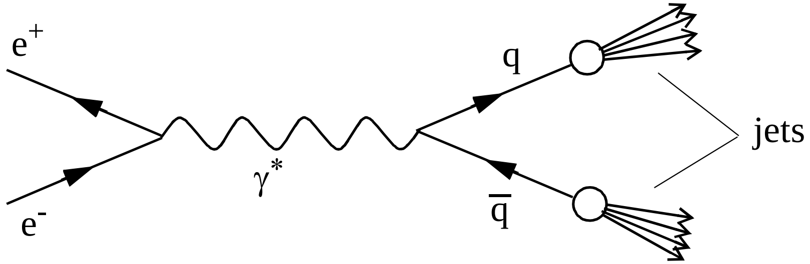
$$2xF_1(x, Q^2) = F_2(x, Q^2) \qquad \text{(spin-1/2)}$$

The data favours the second relation (called the Callan-Gross relation) i.e. quarks have spin 1/2.

ELECTRON-POSITRON INTERACTIONS

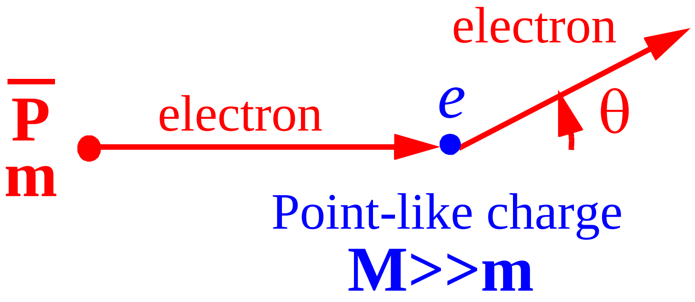


$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to l^+l^-) = \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta)$$

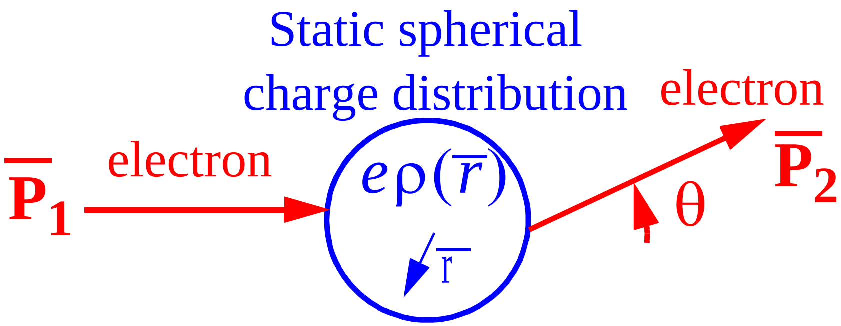


$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to q\bar{q}) = N_c e_q^2 \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta)$$

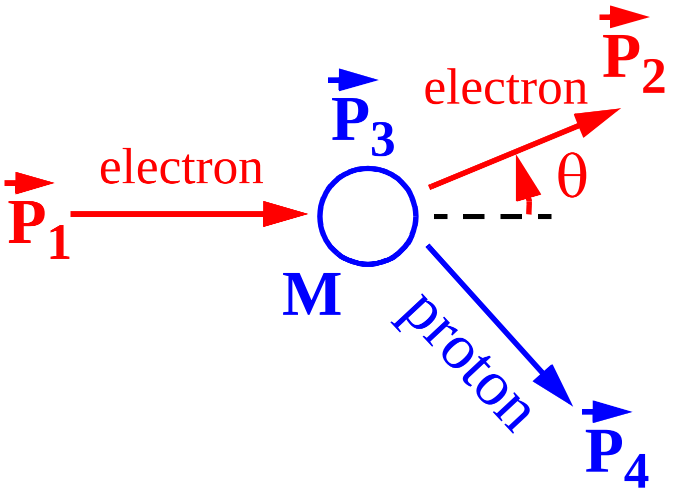
ELASTIC ELECTRON-PROTON SCATTERING



$$\left(\frac{d\sigma}{d\Omega}\right)_M = \underbrace{\frac{\alpha^2}{4p^4 \sin^4\left(\frac{\theta}{2}\right)}}\left(m^2 + p^2\cos^2\frac{\theta}{2}\right)$$



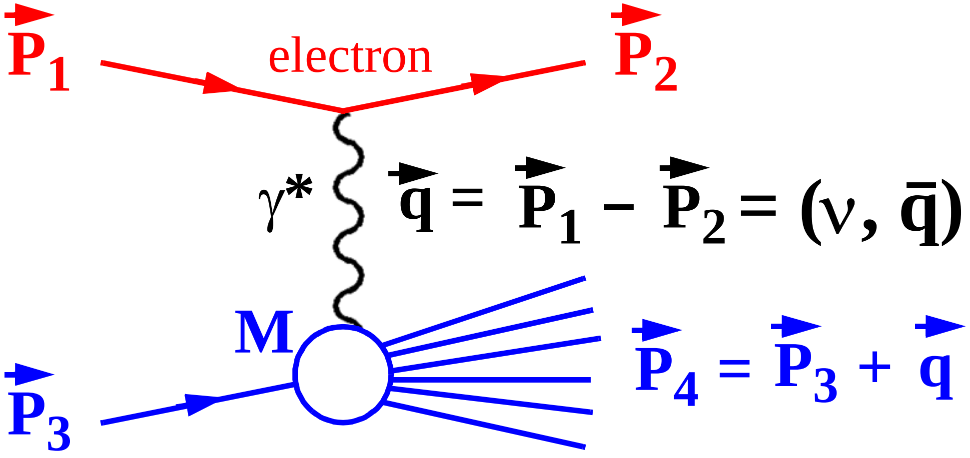
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_R G_E^2(q^2)$$

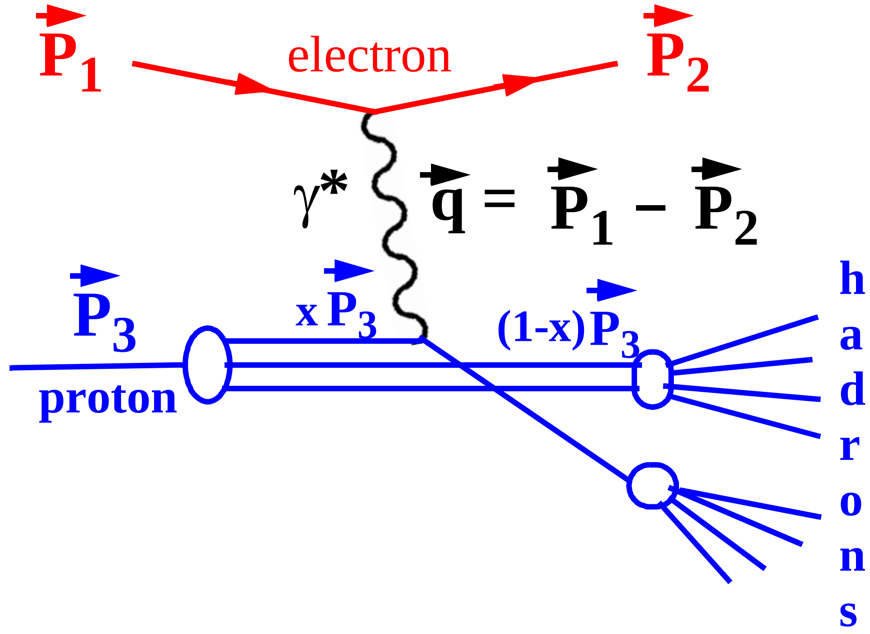


$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_M \mathrm{x} \left(G_1(Q^2)\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}G_2(Q^2)\sin^2\frac{\theta}{2}\right)$$

$$G_1\left(Q^2\right) = \frac{G_E^2 + \frac{Q^2}{4M^2}G_M^2}{1 + \frac{Q^2}{4M^2}} \qquad G_2\left(Q^2\right) = G_M^2$$

INELASTIC ELECTRON-PROTON SCATTERING





$$Q^2 = -\vec{q} \cdot \vec{q}$$

$$x = \frac{Q^2}{2M\nu}$$ **Bjorken - x**

$$\frac{d\sigma}{dE_2 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)} \cdot \frac{1}{\nu} \cdot \left[F_2(x, Q^2)\cos^2\left(\frac{\theta}{2}\right) + \frac{Q^2}{xM^2} F_1(x, Q^2)\sin^2\left(\frac{\theta}{2}\right) \right]$$

Summary

• Quantum Chromodynamics

a) The gauge bosons in QCD are called gluons and are spin 1 particles.

b) The charge in QCD is called colour and gluons carry colour charge but not electric charge.

c) The strong interaction is flavour independent.

d) Colour confinement means that a particle with a colour charge (such as a gluon or a quark) cannot exist as a free particle.

• The strong coupling constant.

e) The strong coupling constant α_s gives the strength of the strong interaction.

f) α_s is not a true constant since it depends on Q^2.

• Electron-positron interactions.

g) Quarks are seen as jets of hadrons in electron-positron interactions.

h) The measured cross section ratio R can only be explained if there are 3 colours.

i) A measurement of the angular distribution of jets in two-jet events show that the quark is a spin 1/2 particle.

j) Three-jet events can be used to measure α_s and to show that the gluon is a spin 1 particle.

• Elastic electron-proton scattering.

k) Elastic electron-proton scattering can be used to measure the size of the proton.

l) Scattering of electrons on protons depends on an electric and a magnetic form factor.

m) The measurement of these form factors show that the proton is not a point particle.

• Inelastic lepton-proton scattering.

n) Inelastic scattering of electrons on protons depends on two structure functions F_1 and F_2.

o) Scale invariance means that these structure functions are almost independent on Q^2. The scale invariance of F_2 is evidence for the existence of quarks in the proton.

• Deep inelastic electron-proton scattering.

p) The measurement of F_1 show that the quarks have to be spin 1/2 particles.