X. Charge conjugation and parity in weak interactions

REMINDER:

Parity

- The parity transformation is the transformation by reflection:

→ $\vec{x}_i \to \vec{x}'_i = -\vec{x}_i$

A parity operator $\hat{P}$ is defined as

→ $\hat{P}\psi((\vec{x}, t) = p\psi(-\vec{x}, t))$ where p = ±1

Charge conjugation

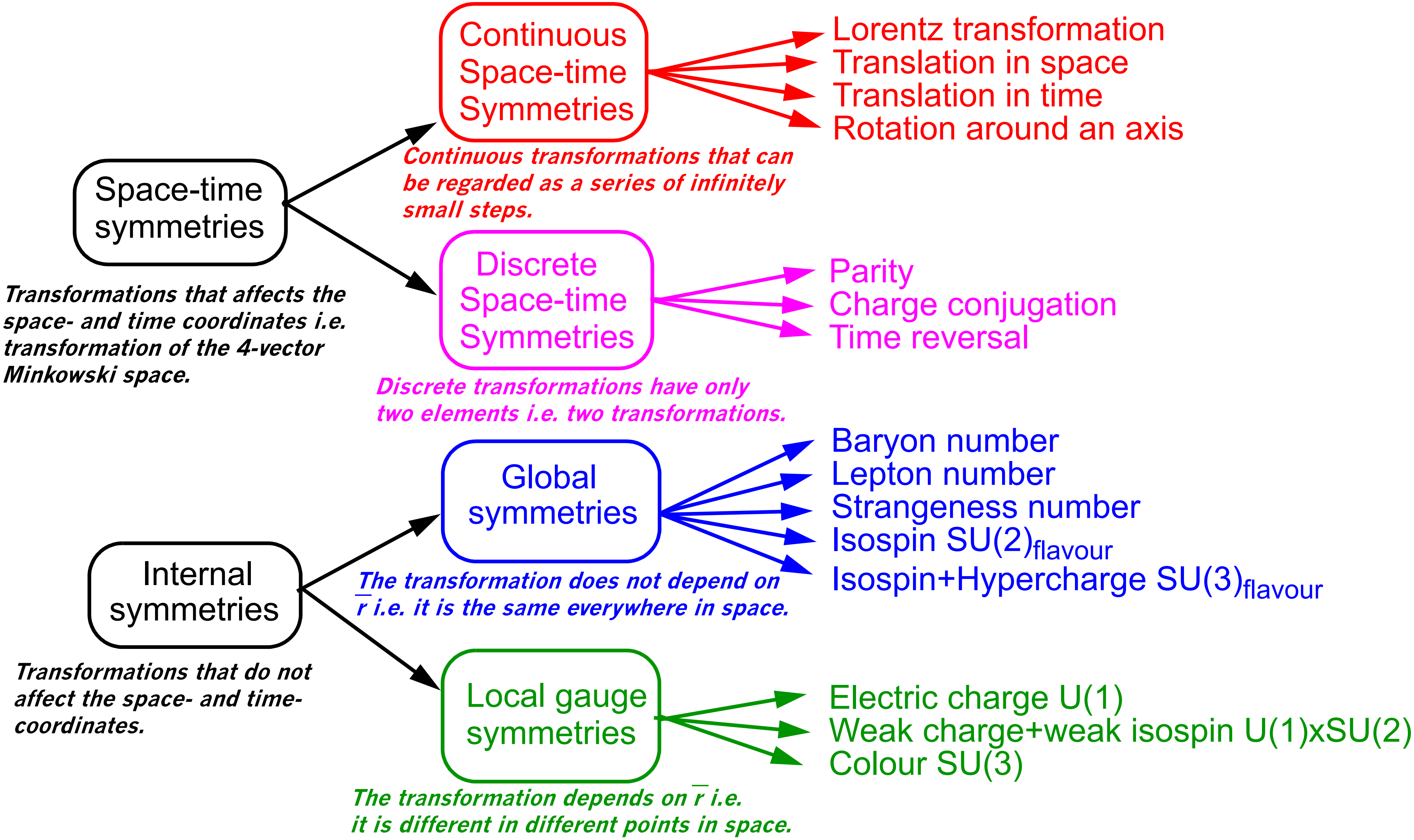
- The charge conjugation replaces particles by their antiparticles, reversing charges and magnetic moments

→ $\hat{C}\Psi_a = c\Psi_{\bar{a}}$ where c = ±1

meaning that from the particle in the initial state we go to the antiparticle in the final state.

Reminder

Symmetries



Space-time symmetries

Transformations that affects the space- and time coordinates i.e. transformation of the 4-vector Minkowski space.

Continuous Space-time Symmetries

- Lorentz transformation
- Translation in space
- Translation in time
- Rotation around an axis

Continuous transformations that can be regarded as a series of infinitely small steps.

Discrete Space-time Symmetries

- Parity
- Charge conjugation
- Time reversal

Discrete transformations have only two elements i.e. two transformations.

Internal symmetries

Transformations that do not affect the space- and time-coordinates.

Global symmetries

- Baryon number
- Lepton number
- Strangeness number
- Isospin $SU(2)_{flavour}$
- Isospin+Hypercharge $SU(3)_{flavour}$

The transformation does not depend on $\overline{r}$ i.e. it is the same everywhere in space.

Local gauge symmetries

- Electric charge U(1)
- Weak charge+weak isospin U(1)xSU(2)
- Colour SU(3)

The transformation depends on $\overline{r}$ i.e. it is different in different points in space.

- While parity is conserved in strong and electromagnetic interactions, it is violated in weak processes:

– 1956: Based on the measurements of Kaon decays, Lee & Yang propose that parity is violated in weak processes:

Two known decays of the K^+ were:

$$K^+ \to \pi^0 + \pi^+ \quad \text{and} \quad K^+ \to \pi^+ + \pi^+ + \pi^-$$

The intrinsic parity of a pion P_π=-1, and for the $\pi^0\pi^+$ and $\pi^+\pi^+\pi^-$ states the parities are

$$P_{\pi\pi} = P_\pi^2(-1)^L = 1$$

$$P_{\pi\pi\pi} = P_\pi^3(-1)^{L_{12}+L_3} = -1$$

- Since the two final states have opposite parities, one of the K^+ decays must **violate** parity!

– 1957: Wu carries out studies of parity violation in β-decay. The ^{60}Co β-decay into $^{60}\mathrm{Ni}^{*} + e^{-} + \bar{\nu}_e$ was studied.

– The ^{60}Co sample was cooled to 0.01 K to prevent thermal disorder.

– The sample was placed in a magnetic field ⇒ the nuclear spins were aligned along the field direction

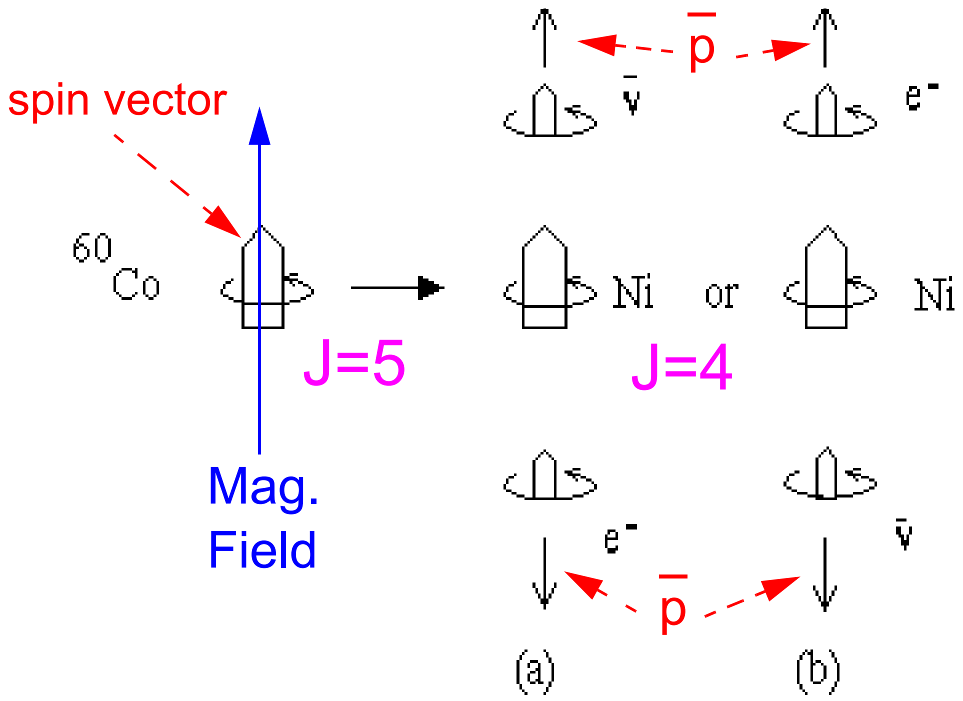


Figure 125: Possible β-decays of ^{60}Co: case (a) is preferred.

– If parity is conserved, processes (a) and (b) must have equal rates.

- *Electrons were emitted predominantly in the direction opposite the ^{60}Co spin*

→ Another case of both parity and C-parity violation was observed in muon decays:

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

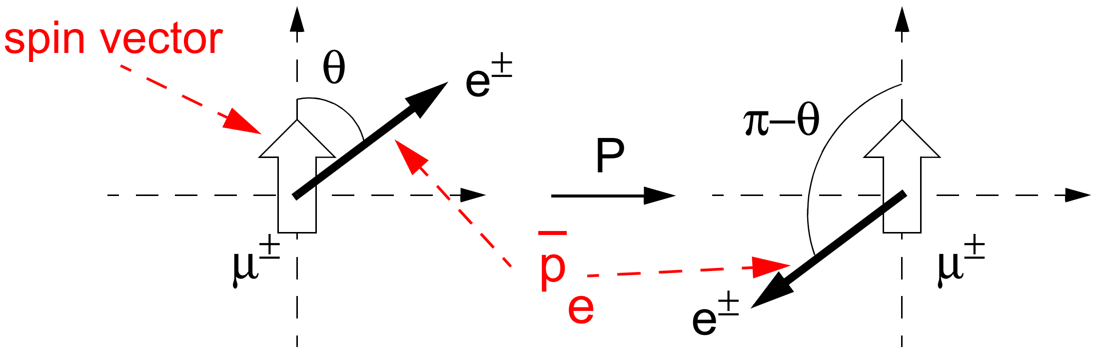


Figure 126: Effect of a parity transformation on the muon decays above

The angular distribution of the electrons (positrons) emitted in μ^- (μ^+) decay are given by

$$\Gamma_{\mu^\pm}(\cos\theta) = \frac{1}{2}\Gamma_\pm\left(1 - \frac{\xi_\pm}{3}\cos\theta\right) \qquad (136)$$

here $\xi_\pm$ are constants – "*asymmetry parameters*", and $\Gamma_\pm$ are total decay rates ⇒ inverse lifetimes

$$\Gamma_\pm = \int_{-1}^{1}\Gamma_\pm(\cos\theta)d\cos\theta \equiv \frac{1}{\tau_\pm} \qquad (137)$$

- If the process is invariant under charge conjugation (C-invariance) ⇒

$$\Gamma_{+} = \Gamma_{-} \qquad \xi_{+} = \xi_{-} \tag{138}$$

(rates and angular distributions are the same for e^- and e^+)

- If the process is P-invariant, then angular distributions in forward and backward directions are the same:

$$\Gamma_{\mu^{\pm}}(\cos\theta) = \Gamma_{\mu^{\pm}}(-\cos\theta) \qquad \xi_{+} = \xi_{-} = 0 \tag{139}$$

- Experimental results:

$$\Gamma_{+} = \Gamma_{-} \qquad \xi_{+} = -\xi_{-} = 1{,}00 \pm 0{,}04 \tag{140}$$

❖ Both C- and P-invariance are violated!

→ However, the combined operation CP is conserved since that requires

$$\Gamma_{\mu^+}(\cos\theta) = \Gamma_{\mu^-}(-\cos\theta) \qquad (141)$$

$$\Gamma_+ = \Gamma_- \qquad \xi_+ = -\xi_- \qquad (142)$$

which is in agreement with the experiments.

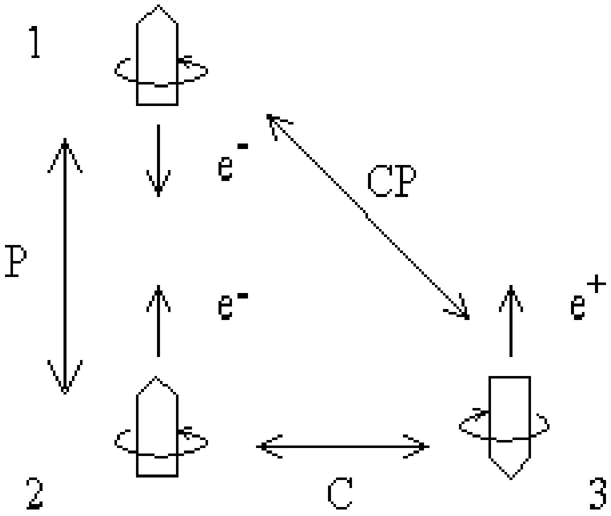


Figure 127: P-, C- and CP-transformation of an electron

❖ The combined transformation CP is a weaker requirement than the individual transformations P and C and it is conserved.

Helicity

helicity – the spin is quantized along the particle´s direction of motion instead of along an arbitrary z-direction

$$\hat{\Lambda} = \frac{\vec{s} \bullet \vec{p}}{|\vec{p}|} \qquad (143)$$

$$\hat{\Lambda}\psi = \lambda\psi$$

The eigenvalues of the helicity operator are $\lambda = -s, -s+1, \ldots, +s$, ⇒ for spin-1/2 particle it can be either -1/2 or 1/2

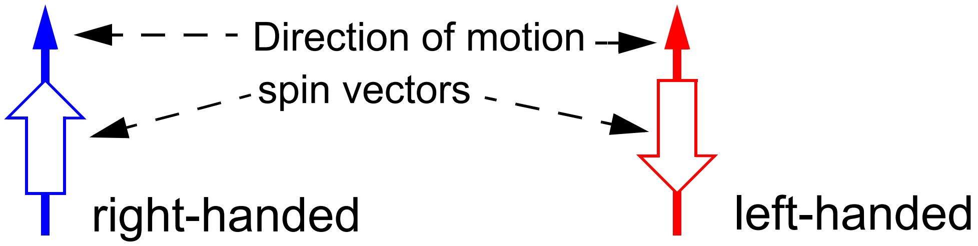


Figure 128: Helicity states of spin-1/2 particle

A particle with $\lambda=+1/2$ is called right handed.

A particle with $\lambda=-1/2$ is called left handed.

A subscript R or L is used to denote if a state is right or left handed e.g. e^-_R and ν_L

- 1958: Goldhaber et al. measured the helicity of the neutrino by studying electron capture in europium:

$$e^- + {}^{152}\mathrm{Eu} \rightarrow {}^{152}\mathrm{Sm}^* + \nu_e \tag{144}$$

$$\hookrightarrow {}^{152}\mathrm{Sm} + \gamma \tag{145}$$

- In this reaction the initial state has zero momentum and ${}^{152}\mathrm{Sm}^*$ and ν_e recoil in opposite directions.

- Events with the γ emitted in the direction of motion of the ${}^{152}\mathrm{Sm}^*$ were selected so that the overall observed reaction was:

$$e^- + {}^{152}\mathrm{Eu}\,(J=0) \rightarrow {}^{152}\mathrm{Sm}(J=0) + \nu_e + \gamma \tag{146}$$

- The spin of the neutrino (+1/2 or -1/2) and the photon (+1 or -1) must add to give the spin of the electron (+1/2 or -1/2).

- The helicity (polarization) of the photons was determined by studying their absorption in magnetized iron.

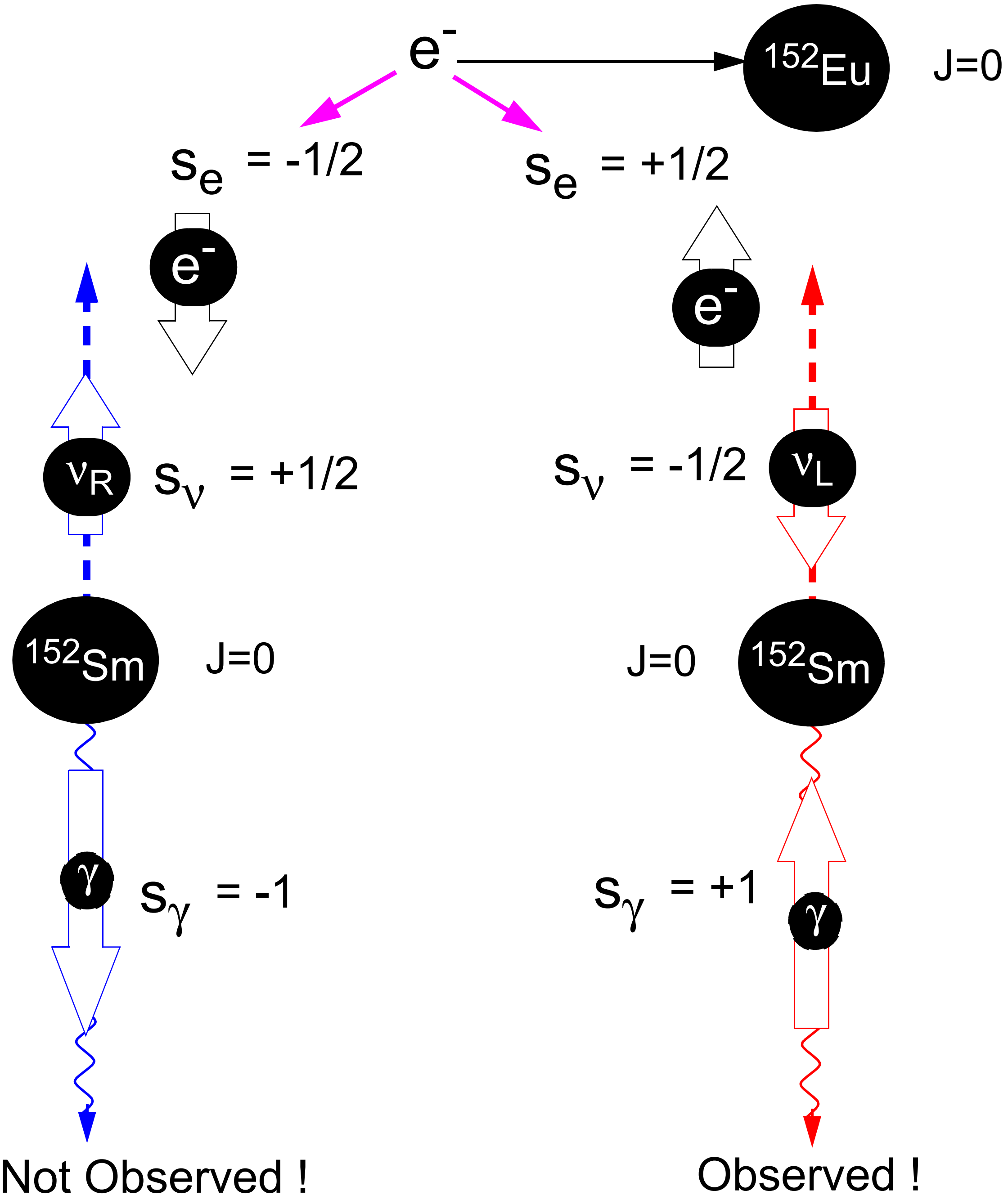


Figure 129: From the helicity of the photons it is possible to determine the helicity of the neutrinos.

❖ From the measured photon helicity it was concluded that neutrinos must be left-handed.

V-A interaction

- V-A interaction theory was introduced by Fermi as an analytic description of spin dependence of charged current interactions.

- It denotes "polar Vector - Axial vector" interaction

– *A Polar vector* is one which direction is reversed by parity transformation e.g. momentum $\vec{p}$

– *An Axial vector* is one which direction is not changed by parity transformation e.g. spin $\vec{s}$ or orbital angular momentum $\vec{L} = \vec{r} \times \vec{p}$

– The weak current has both vector and axial components, hence parity is not conserved in weak interactions

→ Main conclusion: if $v \approx c$, only left-handed fermions ν_L, e^-_L etc. are emitted, and right-handed antifermions.

→ *The very existence of preferred states violates both C- and P- invariance*

- Neutrinos (antineutrinos) are always relativistic and hence always left(right)-handed

- For other fermions, the preferred states are left-handed. Right-handed states are not completely forbidden but suppressed by the factor

$$\left(1-\frac{v}{c}\right)\approx\frac{m^2}{2E^2} \tag{147}$$

Consider the two pion decay modes:

$$\pi^+ \to e^+ + \nu_e \tag{148}$$

$$\pi^+ \to \mu^+ + \nu_\mu \tag{149}$$

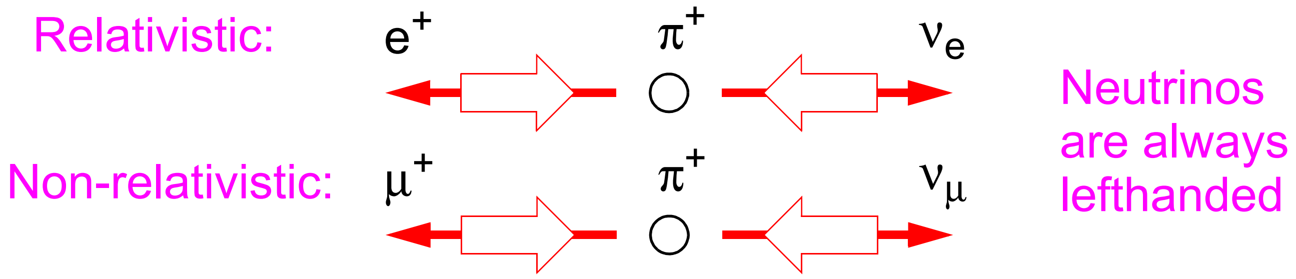


Figure 130: Helicities of leptons emitted in a pion decay

– The π^+ has spin-0 and it is at rest ⇒ the spins of the charged lepton and the neutrino must be opposite.

– The neutrinos are always left-handed ⇒ the charged leptons have to be left-handed as well.

– BUT: the e^+ and the μ^+ should be right-handed since they are anti-fermions.

❖ In these decays the electron will be relativistic but not the muon (due to its large mass).

❖ It follows that the pion to muon decay should be allowed but the pion to positron decay should be suppressed.

– The suppression factor for positrons is expected to be of the order 10^{-5}

The measured ratio:

$$\frac{\Gamma(\pi^+ \to e^+ \nu_e)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} = (1{,}230 \pm 0{,}004) \times 10^{-4} \qquad (150)$$

→ Muons emitted in pion decays are always polarized and this can be used to measure muon decay symmetries by detecting the relativistic electrons in the following decays:

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \qquad (151)$$

$$\hookrightarrow e^- + \bar{\nu}_e + \nu_\mu$$

The electrons are emitted in decays when both the ν_μ and the $\bar{\nu}_e$ are emitted in the direction opposite to the e^-:

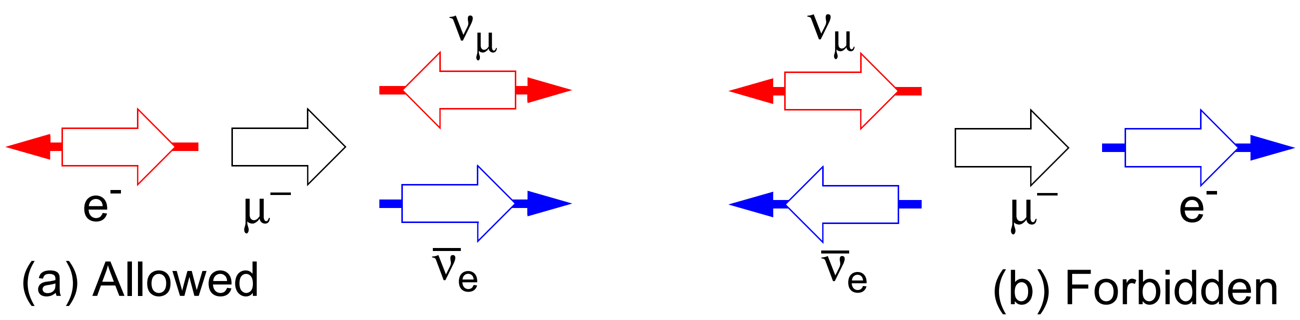


Figure 131: Muon decays with high energy electron emission

– The electron must have a spin parallel to the muon spin ⇒ configuration (a) is strongly preferred ⇒ this is observed experimentally as a forward-backward asymmetry.

Neutral kaons

- It is possible to produce the neutral kaons $K^0 = d\bar{s}$ and $\bar{K}^0 = s\bar{d}$ in πp-collisions. This is a strong interaction process and strangeness has to be conserved:

$$\pi^- + p \rightarrow K^0 + \Lambda$$

s: 0 0 +1 -1

$$\pi^+ + p \rightarrow \bar{K}^0 + p + K^+$$

s: 0 0 -1 0 +1

- The kaons that are produced in this way are pure $K^0 = d\bar{s}$ and $\bar{K}^0 = s\bar{d}$ states.

- However, K^0 and $\bar{K}^0$ can be converted into each other since strangeness is not conserved in weak interactions:

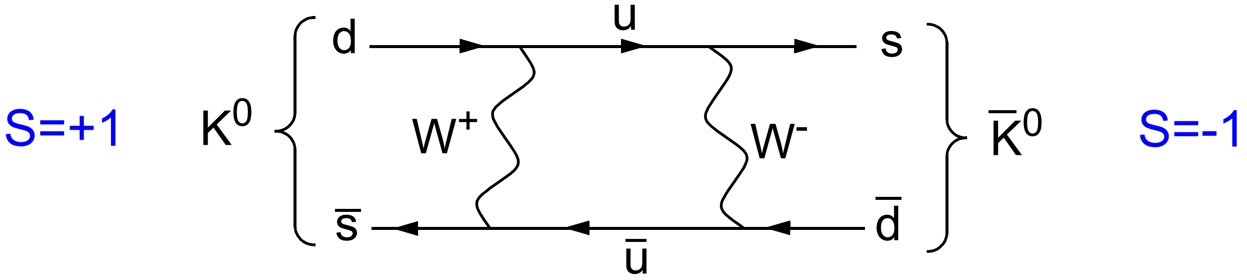


Figure 132: Example of a process converting K^0 to $\bar{K}^0$.

- The observed physical particles are linear combinations of K^0 and $\bar{K}^0$, since there is no conserved quantum number to distinguish them. The phenomenon is called *K^0–$\bar{K}^0$ mixing* .

- We know that neither parity nor charge conjugation are conserved in weak decays. The combined operation CP is, however, almost conserved.

- In this case the CP operators eigenstates can be written as a mixture of K^0 and $\bar{K}^0$:

$$K_1^0 = \frac{1}{\sqrt{2}}\{K^0 + \bar{K}^0\} \qquad (152)$$

$$K_2^0 = \frac{1}{\sqrt{2}}\{K^0 - \bar{K}^0\} \qquad (153)$$

so that

$$\hat{C}PK_1^0 = K_1^0 \ \ and \ \ CPK_2^0 = -K_2^0 \qquad (154)$$

i.e. the CP eigenvalues are cp=+1 for K_1^0 and cp=-1 for K_2^0 and the K_1^0 can therefore only decay to cp-even states while the K_2^0 only to cp-odd states.

Experimentally observed are two types of neutral kaons: K_S^0 ("S" for "short", lifetime $\tau = 0{,}9 \times 10^{-10} s$) and K_L^0 ("long", $\tau = 500 \times 10^{-10} s$).

- Can the K_S^0 be identified with the K_1^0 CP-eigenstate, and the K_L^0 with the K_2^0 ?

→ If CP-invariance holds for neutral kaons, K_S^0 should decay only into states with cp=1 such as 2π-states, and K_L^0 into states with cp=-1 such as 3π-states:

$$K_S^0 \to \pi^+\pi^-, \qquad K_S^0 \to \pi^0\pi^0 \qquad (155)$$

– The parity of a two-pion state is $P = P_\pi^2(-1)^L = 1$

– The C-parity of a $\pi^0\pi^0$ state is $C = (C_{\pi^0})^2 = 1$, and of a $\pi^+\pi^-$ state: $C = (-1)^L = 1$

– i.e. cp= 1 for the $\pi^+\pi^-$ and $\pi^0\pi^0$ states

– i.e. the assumption that $K_S^0 = K_1^0$ seem to be correct.

$$K_L^0 \to \pi^+\pi^-\pi^0, \qquad K_L^0 \to \pi^0\pi^0\pi^0 \qquad (156)$$

– The parity of the 3-π states are -1

– The C-parity of $\pi^0\pi^0\pi^0$ is $C = (C_{\pi^0})^3 = 1$

– The C-parity of $\pi^+\pi^-\pi^0$ is $C = C_{\pi^0}(-1)^{L_{\pi\pi}} = 1$

– i.e. the 3-π final states above have cp=-1

– i.e. the assumption that $K_L^0 = K_2^0$ seem to be correct.

◆

Summary:

The neutral Kaon eigenstates in strong interactions are:

$$K^0 = d\bar{s}$$
$$\bar{K}^0 = s\bar{d}$$

The neutral Kaon eigenstates in weak interactions (if CP is conserved) are:

$$K_s^0 = K_1^0 = \frac{1}{\sqrt{2}}\{K^0 + \bar{K}^0\}$$

$$K_L^0 = K_2^0 = \frac{1}{\sqrt{2}}\{K^0 - \bar{K}^0\}$$

CP-violation

The *CP-violating* decay

$$K_L^0 \rightarrow \pi^+ \pi^- \qquad (157)$$

was first observed in 1964, with a branching ratio of B≈10^{-3}.

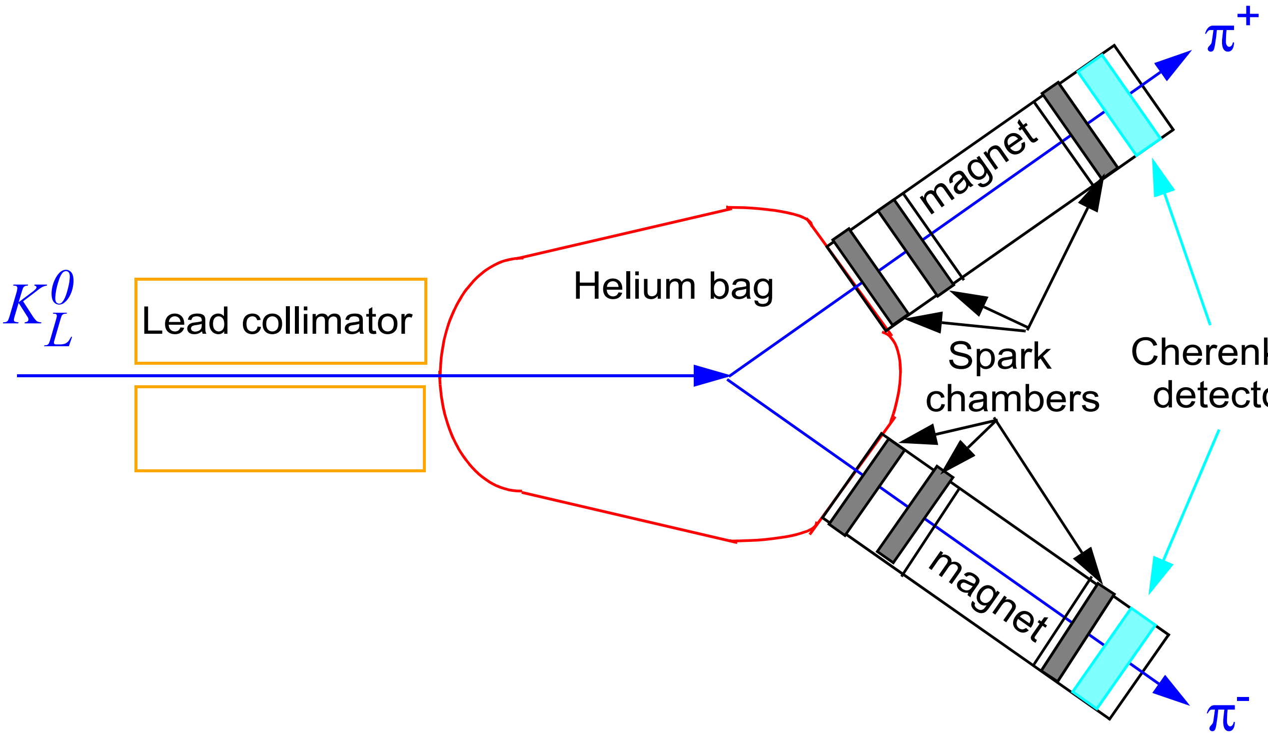


Figure 133: Sketch of the experiment that discovered CP-violation in weak decays.

- In general, the physical states K_S^0 and K_L^0 don’t have to correspond to pure CP-eigenstates K_1^0 and K_2^0. Instead

$$\boxed{\begin{aligned} K_S^0 &= \frac{1}{\sqrt{1+|\varepsilon|^2}}\{K_1^0+\varepsilon K_2^0\} \\ K_L^0 &= \frac{1}{\sqrt{1+|\varepsilon|^2}}\{\varepsilon K_1^0+K_2^0\} \end{aligned}}$$

where ε is a small complex parameter: $|\varepsilon| = 2\times10^{-3}$

- K_S^0 contains mostly K_1^0 but has also a small K_2^0 component while K_L^0 consists mostly of K_2^0 with a small component of K_1^0.

→ Mixing occur also for neutral B-mesons ($B^0 = d\bar{b}$, $\bar{B}^0 = b\bar{d}$, $B_s = s\bar{b}$ and $\bar{B}_s = b\bar{s}$) and for neutral D-mesons ($D^0 = c\bar{u}$ and $\bar{D}^0 = u\bar{c}$).

→ There can be different mechanisms for CP-violation, especially in the B^0-$\bar{B}^0$ systems. Several dedicated experiments have been built to study this system.

Summary

- **Parity and charge conjugation**

  a) Parity is violated in weak processes.

  b) Parity violation was first observed in ^{60}Co-decays.

  c) Muon decays can be used to show that both parity and charged conjugation is violated while the combined CP operation is conserved.

- **Helicity**

  d) Helicity is the spin quantized along the direction of motion.

  e) Neutrinos are left-handed and antineutrinos right-handed.

  f) This was first observed in reactions between electrons and ^{152}Eu atoms.

- **V-A interactions**

  g) While neutrinos are always left-handed other fermions are exclusively left-handed only when they are relativistic.

- **Neutral kaons**

  h) The neutral kaons that are observed experimentally (K^0_S and K^0_L) are due to K^0-$\overline{K}^0$ mixing

  i) CP-violating decays of neutral kaons have been observed with a small branching ratios.