X. Charge conjugation and parity in weak interactions

REMINDER: *Parity*

- The parity transformation is the transformation by reflection:

$$\bar{r} \rightarrow \bar{r}' = -\bar{r} \quad \text{or} \quad x, y, z \rightarrow x', y', z' = -x, -y, -z$$

it reverses the momentum ($\bar{p}$) but not $\bar{L} = \bar{r} \times \bar{p}$ or spin.

A parity operator $\hat{P}$ is defined as

→ $$\hat{P}\psi(\bar{r}, t) = p\psi(-\bar{r}, t)$$ where $p = \pm 1$

Charge conjugation

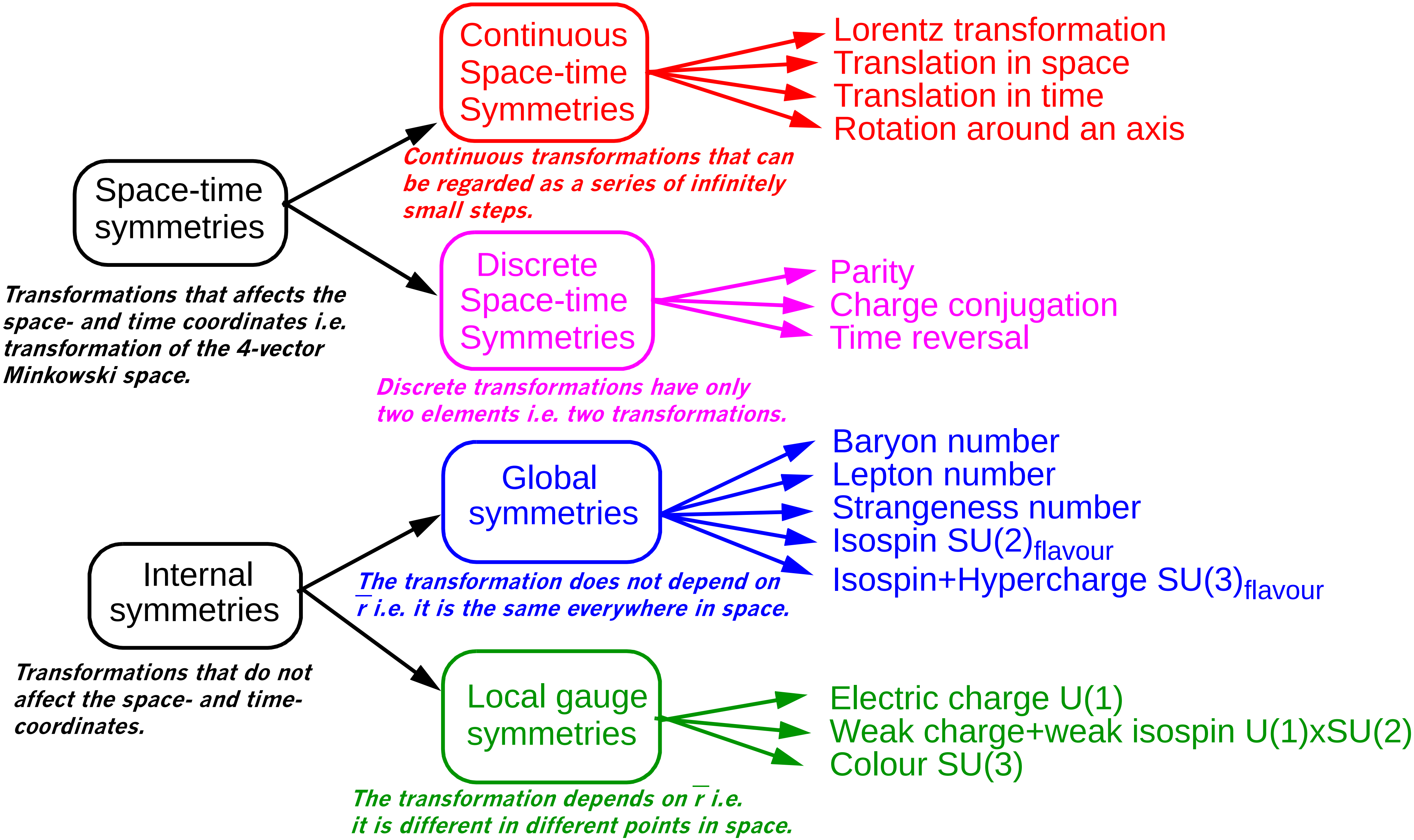
- The charge conjugation replaces particles by their antiparticles, reversing charges and magnetic moments

→ $$\hat{C}\Psi_a = c\Psi_{\bar{a}}$$ where $c = \pm 1$

meaning that from the particle in the initial state we go to the antiparticle in the final state.

Reminder

Symmetries



- While parity is conserved in strong and electromagnetic interactions, it is violated in weak processes:

– 1956: Based on the measurements of Kaon decays, Lee & Yang propose that parity is violated in weak processes:

Two known decays of the K^+ were:

$K^+ \to \pi^0 + \pi^+$ and $K^+ \to \pi^+ + \pi^+ + \pi^-$

The intrinsic parity of a pion $P_\pi = -1$, and for the $\pi^0\pi^+$ and $\pi^+\pi^+\pi^-$ states the parities are

$$P_{\pi\pi} = P_\pi^2(-1)^L = 1 \quad \text{since } L = L_{12} = 0$$

$$P_{\pi\pi\pi} = P_\pi^3(-1)^L = -1 \quad \text{since } L = L_{12} + L_3 = 0$$

- Since the two final states have opposite parities, one of the K^+ decays must **violate** parity!

– 1957: Wu carries out studies of parity violation in β-decay. The ^{60}Co β-decay into $^{60}Ni^{*}+e^{-}+\bar{\nu}_e$ was studied.

– The ^{60}Co sample was cooled to 0.01 K to prevent thermal disorder.

– The sample was placed in a magnetic field ⇒ the nuclear spins were aligned along the field direction

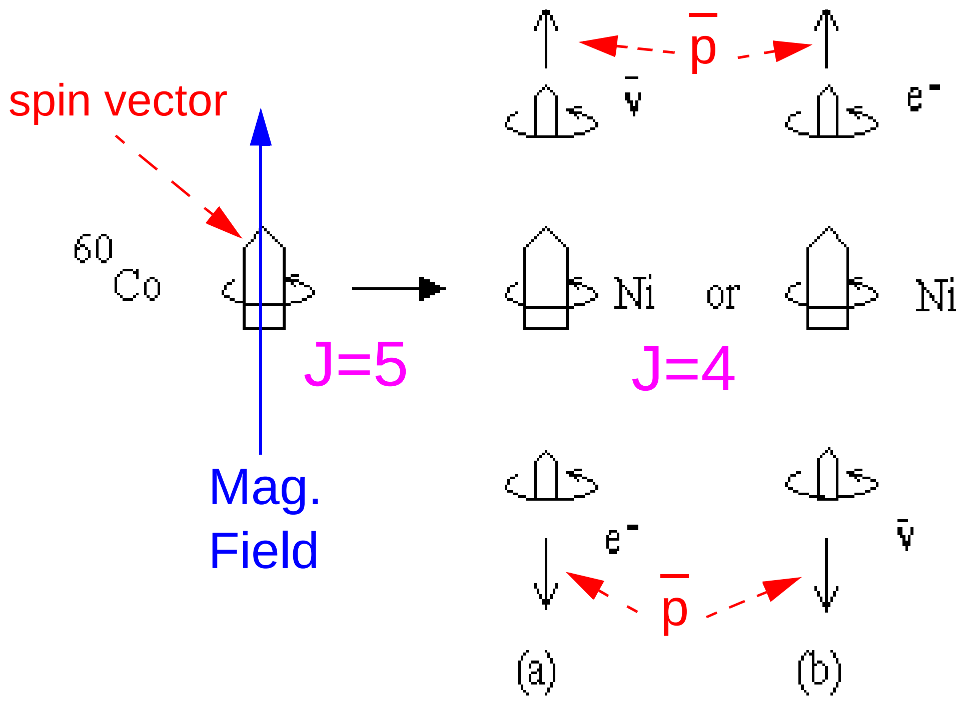


Figure 125: Possible β-decays of ^{60}Co: case (a) is preferred.

– If parity is conserved, processes (a) and (b) must have equal rates.

- *Electrons were emitted predominantly in the direction opposite the ^{60}Co spin*

→ Another case of both parity and C-parity violation was observed in muon decays:

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

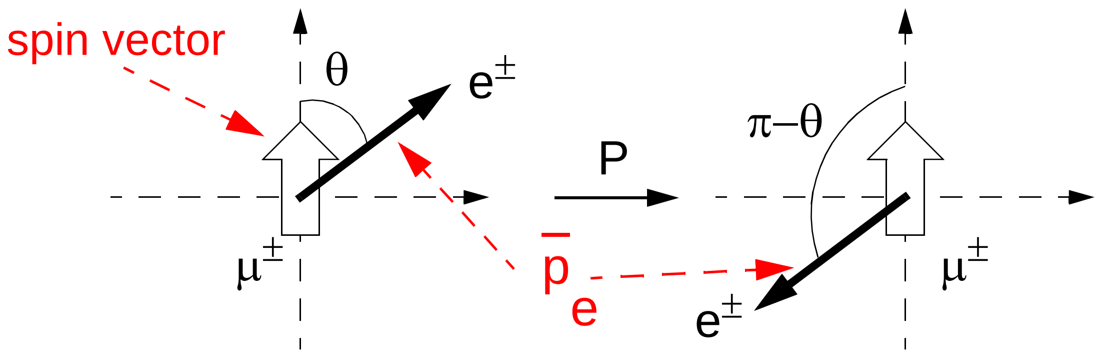


Figure 126: Effect of a parity transformation on the muon decays above

The angular distribution of the electrons (positrons) emitted in μ^- (μ^+) decay are given by

$$\Gamma_{\mu^\pm}(\cos\theta) = \frac{1}{2}\Gamma_\pm\left(1 - \frac{\xi_\pm}{3}\cos\theta\right) \qquad (136)$$

here $\xi_\pm$ are constants – "*asymmetry parameters*", and $\Gamma_\pm$ are total decay rates ⇒ inverse lifetimes

$$\Gamma_\pm = \int_{-1}^{1}\Gamma_\pm(\cos\theta)d\cos\theta \equiv \frac{1}{\tau_\pm} \qquad (137)$$

→ If the process is invariant under charge conjugation (C-invariance) ⇒

$$\Gamma_+ = \Gamma_- \qquad \xi_+ = \xi_- \tag{138}$$

(rates and angular distributions are the same for e^- and e^+)

→ If the process is P-invariant, then angular distributions in forward and backward directions are the same:

$$\Gamma_{\mu^\pm}(\cos\theta) = \Gamma_{\mu^\pm}(-\cos\theta) \qquad \xi_+ = \xi_- = 0 \tag{139}$$

→ Experimental results:

$$\Gamma_+ = \Gamma_- \qquad \xi_+ = -\xi_- = 1{,}00 \pm 0{,}04 \tag{140}$$

❖ Both C- and P-invariance are violated!

→ However, the combined operation CP is conserved since that requires

$$\Gamma_{\mu^+}(\cos\theta) = \Gamma_{\mu^-}(-\cos\theta) \quad (141)$$

$$\Gamma_+ = \Gamma_- \qquad \xi_+ = -\xi_- \quad (142)$$

which is in agreement with the experiments.

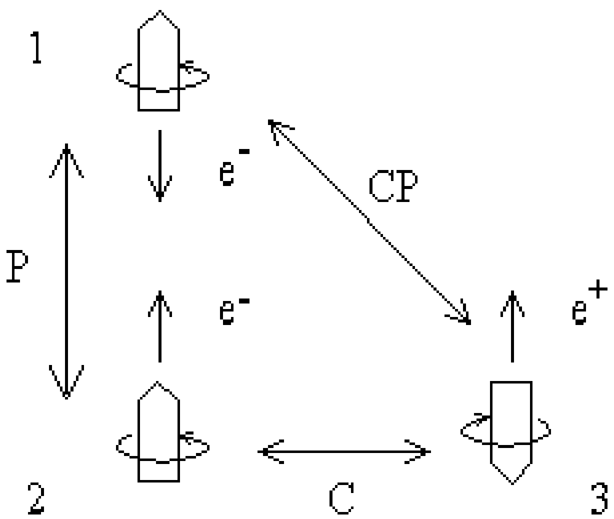


Figure 127: P-, C- and CP-transformation of an electron

❖ The combined transformation CP is a weaker requirement than the individual transformations P and C and it is conserved.

Helicity

helicity – the spin is quantized along the particle´s direction of motion instead of along an arbitrary z-direction

$$\hat{\Lambda} = \frac{\bar{s} \bullet \bar{p}}{|\bar{p}|} \quad (143)$$

$$\hat{\Lambda}\psi = \lambda\psi$$

The eigenvalues of the helicity operator are $\lambda = -s, -s+1, ..., +s$, ⇒ for spin-1/2 particle it can be either -1/2 or 1/2

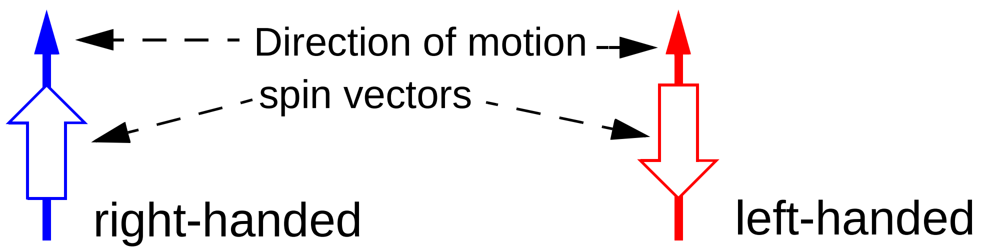


Figure 128: Helicity states of spin-1/2 particle

A particle with $\lambda=+1/2$ is called right handed.

A particle with $\lambda=-1/2$ is called left handed.

A subscript R or L is used to denote if a state is right or left handed e.g. e^-_R and ν_L

- 1958: Goldhaber et al. measured the helicity of the neutrino by studying electron capture in europium:

$$e^- + {}^{152}Eu \rightarrow {}^{152}Sm^* + \nu_e \qquad (144)$$

$$\hookrightarrow {}^{152}Sm + \gamma \qquad (145)$$

- In this reaction the initial state has zero momentum and ${}^{152}Sm^*$ and ν_e recoil in opposite directions.

- Events with the γ emitted in the direction of motion of the ${}^{152}Sm^*$ were selected so that the overall observed reaction was:

$$e^- + {}^{152}Eu\,(J=0) \rightarrow {}^{152}Sm(J=0) + \nu_e + \gamma \qquad (146)$$

- The spin of the neutrino (+1/2 or -1/2) and the photon (+1 or -1) must add to give the spin of the electron (+1/2 or -1/2).

- The helicity (polarization) of the photons was determined by studying their absorption in magnetized iron.

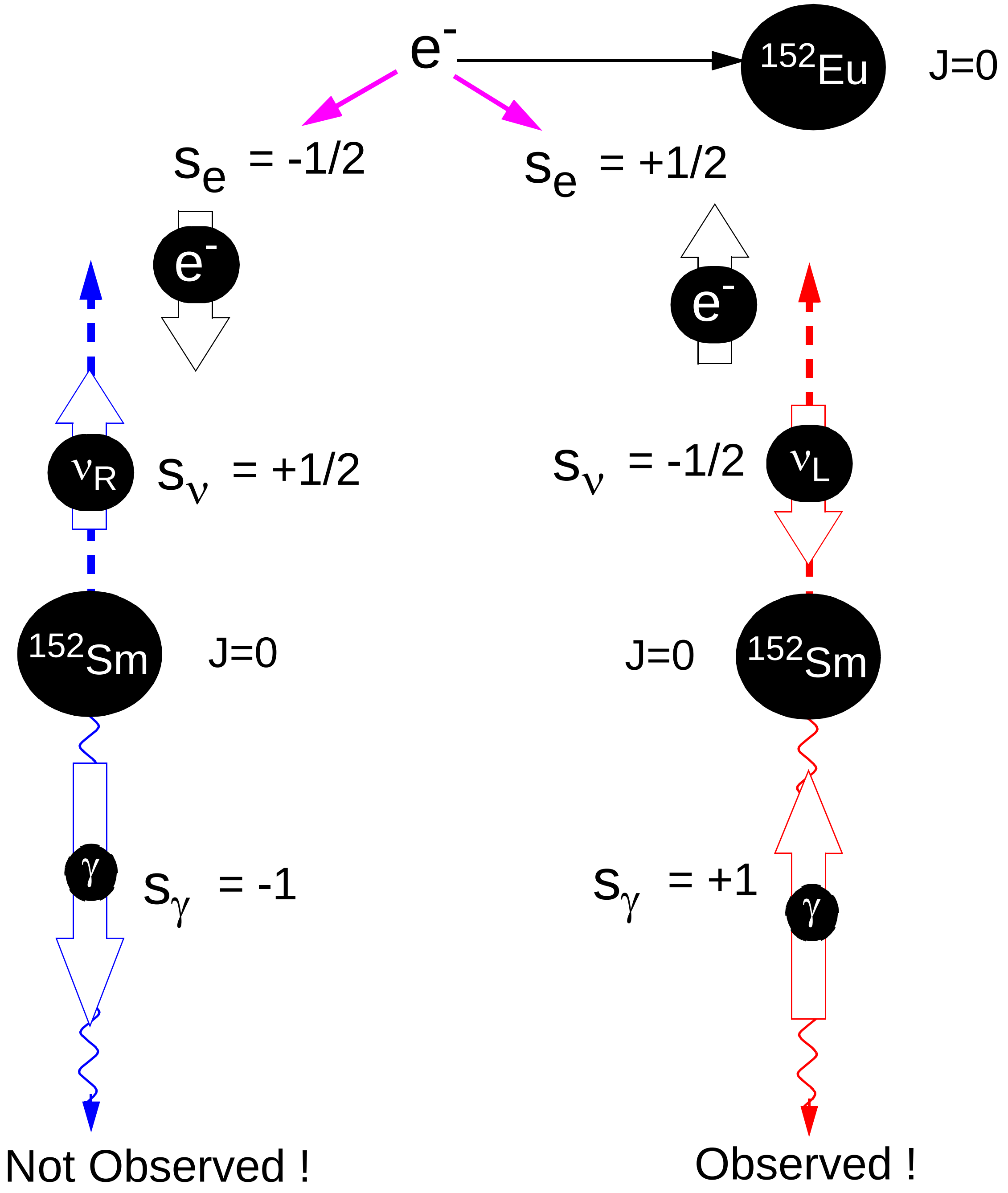


Figure 129: From the helicity of the photons it is possible to determine the helicity of the neutrinos.

- From the measured photon helicity it was concluded that neutrinos must be left-handed.

V-A interaction

- V-A interaction theory was introduced by Fermi as an analytic description of spin dependence of charged current interactions.

- It denotes “polar Vector - Axial vector” interaction

– *A Polar vector* is one which direction is reversed by parity transformation e.g. momentum $\bar{p}$

– *An Axial vector* is one which direction is not changed by parity transformation e.g. spin $\bar{s}$ or orbital angular momentum $\bar{L} = \bar{r} \times \bar{p}$

– The weak current has both vector and axial components, hence parity is not conserved in weak interactions

→ Main conclusion: if $v \approx c$, only left-handed fermions ν_L, e^-_L etc. are emitted, and right-handed antifermions.

→ *The very existence of preferred states violates both C- and P- invariance*

❖ Neutrinos (antineutrinos) are always relativistic and hence always left(right)-handed

❖ For other fermions, the preferred states are left-handed. Right-handed states are not completely forbidden but suppressed by the factor

$$\left(1 - \frac{v}{c}\right) \approx \frac{m^2}{2E^2} \quad (147)$$

Consider the two pion decay modes:

$$\pi^+ \rightarrow e^+ + \nu_e \quad (148)$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \quad (149)$$

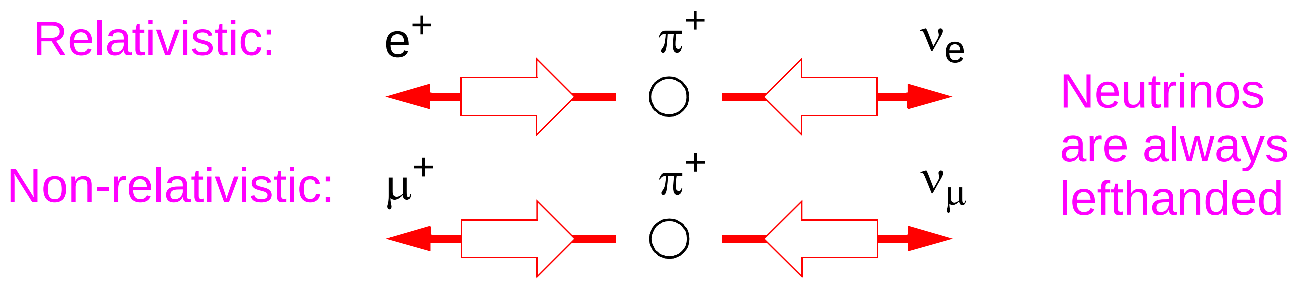


Figure 130: Helicities of leptons emitted in a pion decay

– The π^+ has spin-0 and it is at rest ⇒ the spins of the charged lepton and the neutrino must be opposite.

– The neutrinos are always left-handed $\Rightarrow$ the charged leptons have to be left-handed as well.

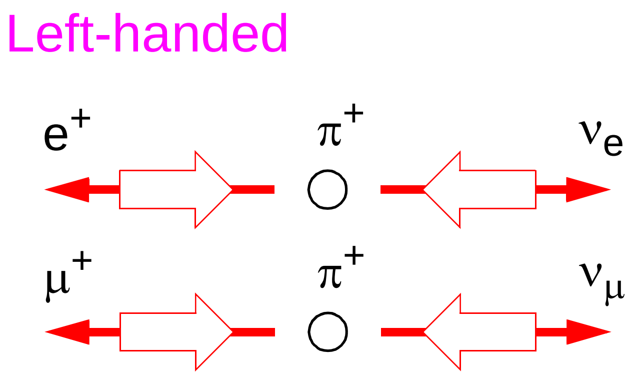
– BUT: the e^+ and the μ^+ should be right-handed since they are anti-fermions.

- In these decays the electron will be relativistic but not the muon (due to its large mass).

- It follows that the pion to muon decay should be allowed but the pion to positron decay should be suppressed.

Should be right-handed since it is relativistic

Can be left-handed since it is non-relativistic



– The suppression factor for positrons is expected to be of the order 10^{-5}.

The measured ratio:

$$\frac{\Gamma(\pi^+ \to e^+\nu_e)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} = (1{,}230 \pm 0{,}004)\times 10^{-4} \qquad (150)$$

❖ Muons emitted in pion decays are always polarized and this can be used to measure muon decay symmetries by detecting the relativistic electrons in the following decays:

$$\pi^- \to \mu^- + \bar{\nu}_\mu \qquad \mu^- \to e^- + \bar{\nu}_e + \nu_\mu \tag{151}$$

The electrons are emitted in decays when both the ν_μ and the $\bar{\nu}_e$ are emitted in the direction opposite to the e^-:

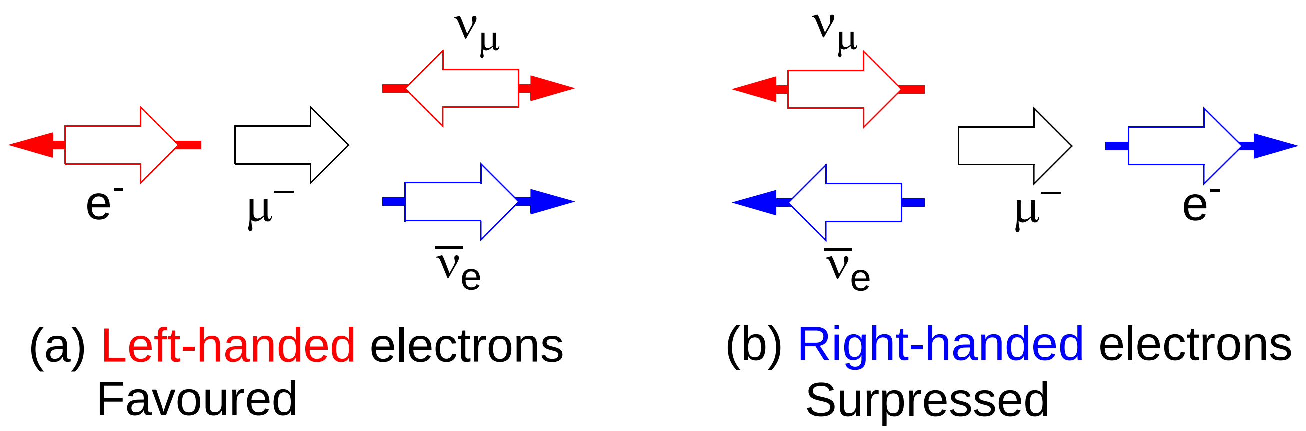


Figure 131: Muon decays with high energy electron emission.

The electron must have a spin parallel to the muon spin ⇒ configuration (a) with left-handed electrons is strongly preferred ⇒ this is observed experimentally as a forward-backward asymmetry.

Neutral kaons

- It is possible to produce the neutral kaons $K^0 = d\bar{s}$ and $\bar{K}^0 = s\bar{d}$ in πp-collisions. This is a strong interaction process and strangeness has to be conserved:

$$\pi^- + p \to K^0 + \Lambda$$

s: 0 0 +1 -1

$$\pi^+ + p \to \bar{K}^0 + p + K^+$$

s: 0 0 -1 0 +1

- The kaons that are produced in this way are pure $K^0 = d\bar{s}$ and $\bar{K}^0 = s\bar{d}$ states.

- However, K^0 and $\bar{K}^0$ can be converted into each other since strangeness is not conserved in weak interactions:

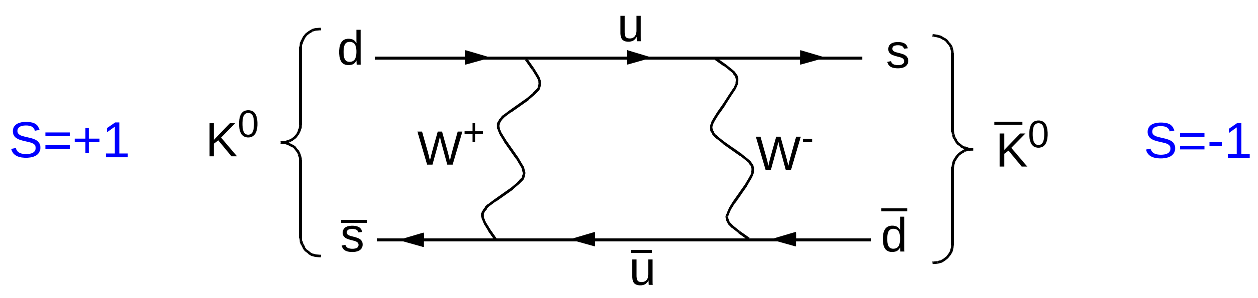


Figure 132: Example of a process converting K^0 to $\bar{K}^0$.

❖ The observed physical particles are linear combinations of K^0 and $\bar{K}^0$, since there is no conserved quantum number to distinguish them. The phenomenon is called *K^0–$\bar{K}^0$ mixing* .

❖ We know that neither parity nor charge conjugation are conserved in weak decays. The combined operation CP is, however, almost conserved.

❖ In this case the CP operators eigenstates can be written as a mixture of K^0 and $\bar{K}^0$:

$$K_1^0 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0) \qquad (152)$$

$$K_2^0 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) \qquad (153)$$

so that

$$\hat{C}PK_1^0 = K_1^0 \text{ and } CPK_2^0 = -K_2^0 \qquad (154)$$

i.e. the CP eigenvalues are cp=+1 for K_1^0 and cp=-1 for K_2^0 and the K_1^0 can therefore only decay to cp-even states while the K_2^0 only to cp-odd states.

Experimentally observed are two types of neutral kaons: K_S^0 ("S" for "short", lifetime $\tau = 0{,}9\times10^{-10}s$) and K_L^0 ("long", $\tau = 500\times10^{-10}s$).

❖ Can the K_S^0 be identified with the K_1^0 CP-eigenstate, and the K_L^0 with the K_2^0 ?

→ If CP-invariance holds for neutral kaons, K_S^0 should decay only into states with cp=1 such as 2π-states, and K_L^0 into states with cp=-1 such as 3π-states:

$$K_S^0 \to \pi^+\pi^-, \qquad K_S^0 \to \pi^0\pi^0 \qquad (155)$$

– The parity of a two-pion state is $P = P_\pi^2(-1)^L = 1$

– The C-parity of a $\pi^0\pi^0$ state is $C = (C_{\pi^0})^2 = 1$, and of a $\pi^+\pi^-$ state: $C = (-1)^L = 1$

– i.e. cp= 1 for the $\pi^+\pi^-$ and $\pi^0\pi^0$ states

– i.e. the assumption that $K_S^0 = K_1^0$ seem to be correct.

$$K_L^0 \to \pi^+\pi^-\pi^0, \qquad K_L^0 \to \pi^0\pi^0\pi^0 \qquad (156)$$

– The parity of the 3-π states are -1

– The C-parity of $\pi^0\pi^0\pi^0$ is $C = (C_{\pi^0})^3 = 1$

– The C-parity of $\pi^+\pi^-\pi^0$ is $C = C_{\pi^0}(-1)^{L_{\pi\pi}} = 1$

– i.e. the 3-π final states above have cp=-1

– i.e. the assumption that $K_L^0 = K_2^0$ seem to be correct.

❖ Summary:

The neutral Kaon eigenstates in strong interactions are:

$$K^0 = d\bar{s}$$
$$\bar{K}^0 = s\bar{d}$$

The neutral Kaon eigenstates in weak interactions (if CP is conserved) are:

$$K_S^0 = K_1^0 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)$$
$$K_L^0 = K_2^0 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0)$$

Strangeness Oscillations

$$\begin{cases} K_S^0 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0) \\ K_L^0 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) \end{cases} \Rightarrow \begin{cases} K^0 = \frac{1}{\sqrt{2}}(K_S^0 + K_L^0) \\ \bar{K}^0 = \frac{1}{\sqrt{2}}(K_S^0 - K_L^0) \end{cases}$$

Kaons created at t=0 can be written as:

$$\begin{cases} K^0(0) = \frac{1}{\sqrt{2}}(K_S^0(0) + K_L^0(0)) \\ \bar{K}^0(0) = \frac{1}{\sqrt{2}}(K_S^0(0) - K_L^0(0)) \end{cases}$$

and the time-depence of the kaon states as:

$$K^0(t) = \frac{1}{\sqrt{2}}\left(e^{-iE_s t} e^{-\Gamma_s t/2} K_s^0(0) + e^{-iE_L t} e^{-\Gamma_L t/2} K_L^0(0)\right)$$

$$\bar{K}^0(t) = \frac{1}{\sqrt{2}}\left(e^{-iE_s t} e^{-\Gamma_s t/2} K_s^0(0) - e^{-iE_L t} e^{-\Gamma_L t/2} K_L^0(0)\right)$$

if the time is measured in the restframe of the kaons then $E_s=m_s$ and $E_L=m_L$ are the masses of the K_s and K_L and Γ_s and Γ_L are the decay rates with $\Gamma=1/\tau$ as usual.

Assume that a pure K^0 beam is produced by the reaction $\pi^- + p \rightarrow K^0 + \Lambda$ at t=0, then

$$K^0(t) = \frac{1}{\sqrt{2}}\left(e^{-iE_s t}e^{-\Gamma_s t/2}K_s^0(0) + e^{-iE_L t}e^{-\Gamma_L t/2}K_L^0(0)\right)$$

$$K_s^0(0) = \frac{1}{\sqrt{2}}(K^0(0) + \bar{K}^0(0))$$

$$K_L^0(0) = \frac{1}{\sqrt{2}}(K^0(0) - \bar{K}^0(0))$$

gives

$$K^0(t) = A(t)K^0(0) + B(t)\bar{K}^0(0)$$ where

$$\begin{cases} A(t) = \frac{1}{2}\left(e^{-(im_s + \Gamma_s/2)t} + e^{-(im_L + \Gamma_L/2)t}\right) \\ B(t) = \frac{1}{2}\left(e^{-(im_s + \Gamma_s/2)t} - e^{-(im_L + \Gamma_L/2)t}\right) \end{cases}$$

If the intensities of the K^0 and $\bar{K}^0$ is measured by strong interaction processes such as $\bar{K}^0 + p \rightarrow \pi^+ + \Lambda$ then one will find that they are

$$I(K^0) = |A(t)|^2 = \frac{1}{4}(e^{-t\Gamma_s} + e^{-t\Gamma_L} + 2e^{-t(\Gamma_s + \Gamma_L)/2}\cos(\Delta m t))$$

$$I(\bar{K}^0) = |B(t)|^2 = \frac{1}{4}(e^{-t\Gamma_s} + e^{-t\Gamma_L} - 2e^{-t(\Gamma_s + \Gamma_L)/2}\cos(\Delta m t))$$

where $\Delta m = m_s - m_L = 3.5 \times 10^{-6}$ eV

This phenomena when a K^0 beam partly turns into a $\overline{K}^0$ beam is called strangeness oscillations.

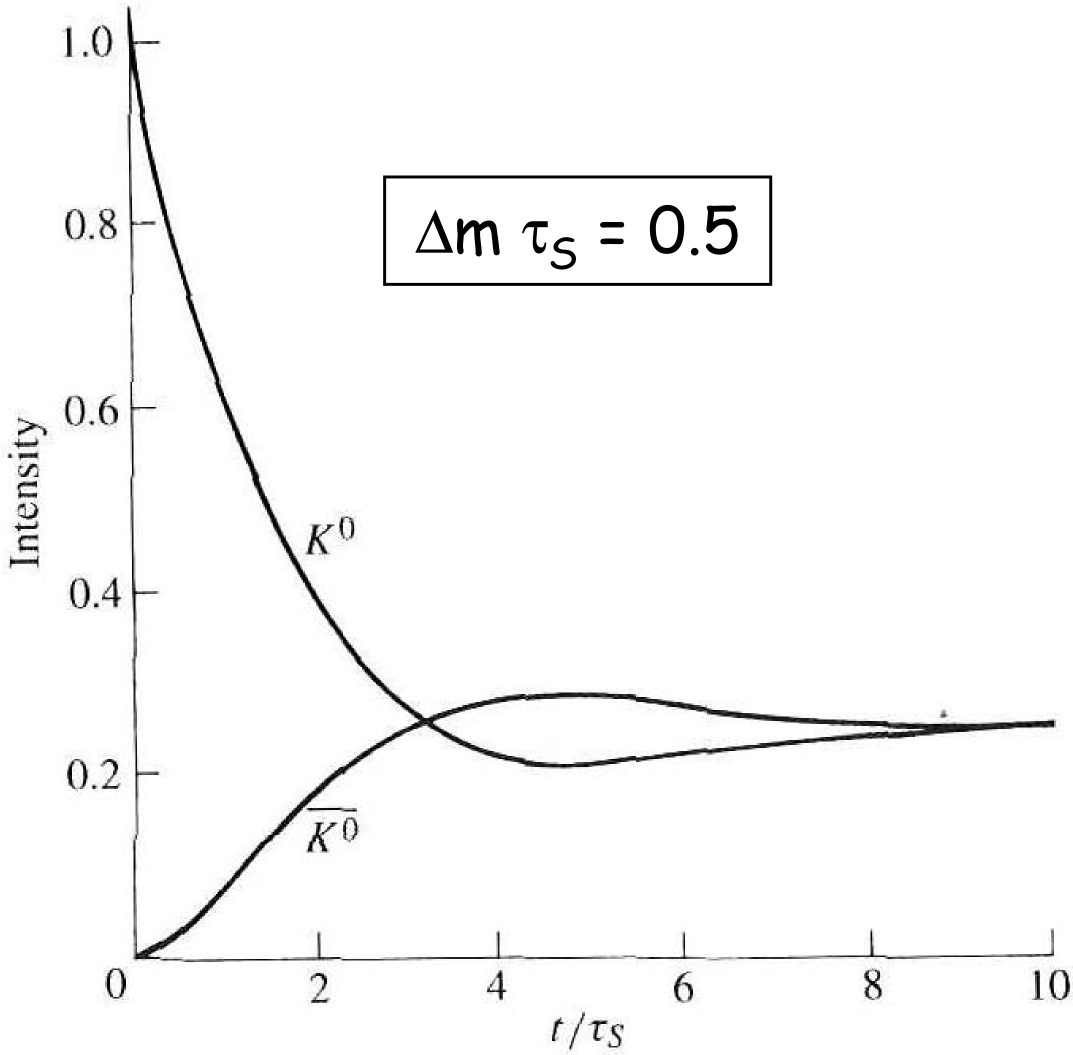


Figure 133: Strangeness oscillations: The intensity as a function of time of K^0 and $\overline{K}^0$ in a beam that was produced as pure K^0.

CP-Violation

The *CP-violating* decay

$$K_L^0 \to \pi^+\pi^- \qquad (157)$$

was first observed in 1964, with a branching ratio of $B \approx 10^{-3}$.

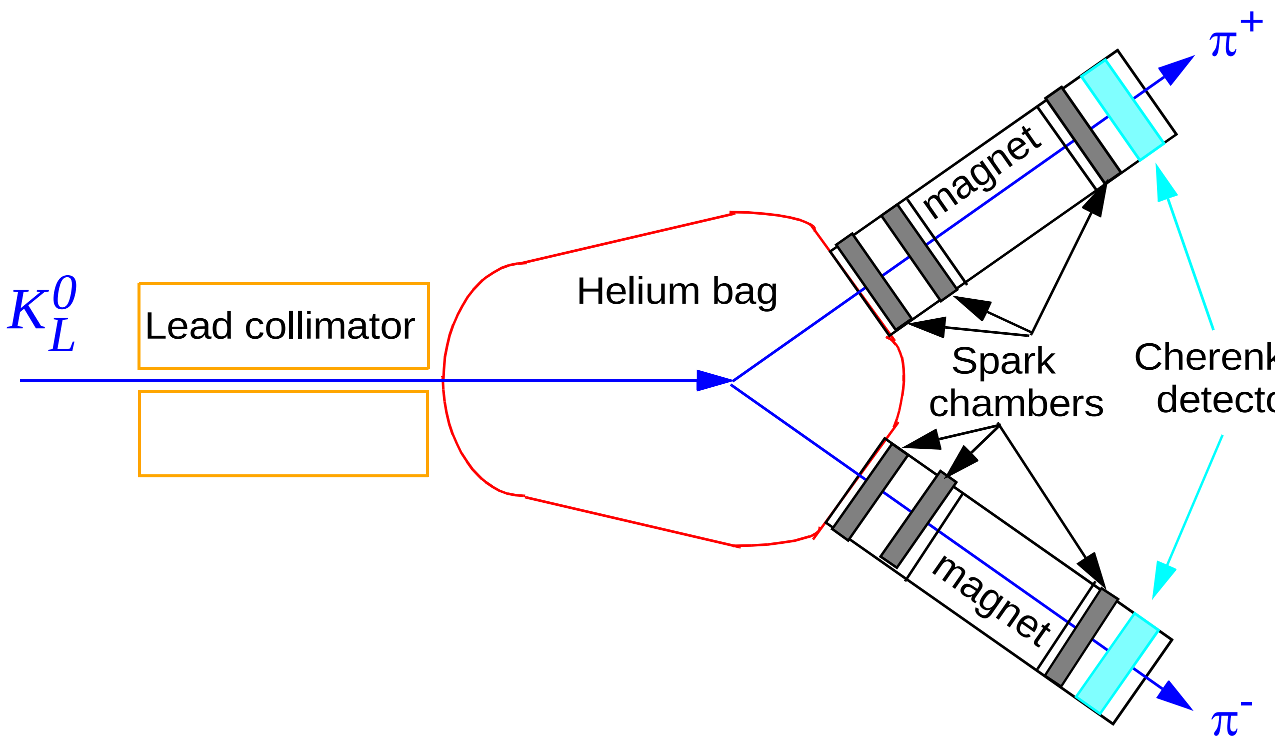


Figure 134: Sketch of the experiment that discovered CP-violation in weak decays.

❖ In general, the physical states K_S^0 and K_L^0 don't have to correspond to pure CP-eigenstates K_1^0 and K_2^0. Instead

$$K_S^0 = \frac{1}{\sqrt{1+|\varepsilon|^2}}(K_1^0 + \varepsilon K_2^0)$$

$$K_L^0 = \frac{1}{\sqrt{1+|\varepsilon|^2}}(\varepsilon K_1^0 + K_2^0)$$

where ε is a small complex parameter: $|\varepsilon| = 2\times 10^{-3}$

❖ K_S^0 contains mostly K_1^0 but has also a small K_2^0 component while K_L^0 consists mostly of K_2^0 with a small component of K_1^0.

→ Mixing occur also for neutral B-mesons ($B^0 = d\bar{b}$, $\bar{B}^0 = \bar{d}b$, $B_s = s\bar{b}$ and $\bar{B}_s = \bar{s}b$) and for neutral D-mesons ($D^0 = c\bar{u}$ and $\bar{D}^0 = \bar{c}u$).

→ There can be different mechanisms for CP-violation, especially in the B^0-$\bar{B}^0$ systems. Several dedicated experiments have been built to study this system.

Summary

• Parity and charge conjugation

a) Parity is violated in weak processes.

b) Parity violation was first observed in ^{60}Co-decays.

c) Muon decays can be used to show that both parity and charged conjugation is violated while the combined CP operation is conserved.

• Helicity

d) Helicity is the spin quantized along the direction of motion.

e) Neutrinos are left-handed and antineutrinos right-handed.

f) This was first observed in reactions between electrons and ^{152}Eu atoms.

• V-A interactions

g) While neutrinos are always left-handed other fermions are exclusively left-handed only when they are relativistic.

• Neutral kaons

h) The neutral kaons decays that are observed experimentally (K^0_S and K^0_L) are due to K^0-$\overline{K}^0$ mixing.

i) If a pure beam of K^0 is produced one can later detect $\overline{K}^0$ in the beam (by strong interaction hyperon production). The phenomena is called strangeness oscillations.

j) CP-violating decays of neutral kaons have been observed with a small branching ratios.