Interna Globala Symmetrier

Lepton tal → se boken 2.1

Baryon tal → se boken 5.1

Laddning → se boken 2.2, 5.1

Kvark kvanttal → se boken 2.2, 5.1

SU(2) Isospin → se boken 5.2

SU(3) Isospin + Hypercharge → se boken 5.2, 6.2

Lepton talet

	$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$	$\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$	$\begin{pmatrix} e^+ \\ \bar{\nu}_e \end{pmatrix}$	$\begin{pmatrix} \mu^+ \\ \bar{\nu}_\mu \end{pmatrix}$	$\begin{pmatrix} \tau^+ \\ \bar{\nu}_\tau \end{pmatrix}$
$L_e =$	1	0	0	−1	0	0
$L_\mu =$	0	1	0	0	−1	0
$L_\tau =$	0	0	1	0	0	−1

- L_e, L_μ och L_τ är var och för sig konserverade i svag och elektromagnetisk växelverkan. (e, μ och τ kan inte växelverka starkt och ν_e, ν_μ och ν_τ kan inte växelverka starkt eller elektromagnetiskt).

- Leptontalen är additiva kvanttal.

- För hadroner gäller att $L_e = L_\mu = L_\tau = 0$.

- Exempel:

$$\left.\begin{array}{lccccccc} & \mu^+ & \rightarrow & e^+ & + & \nu_e & + & \bar{\nu}_\mu \\ L_e: & 0 & & -1 & + & 1 & + & 0 \\ L_\mu: & -1 & & 0 & + & 0 & + & -1 \end{array}\right\}$$

Le och Lμ konserverade

$$\left.\begin{array}{lccccccc} & \mu^+ & \rightarrow & e^- & + & e^+ & + & e^- \\ L_e: & 0 & & 1 & + & -1 & + & 1 \\ L_\mu: & -1 & & 0 & + & 0 & + & 0 \end{array}\right\}$$

Le och Lμ ej konserverade
Detta sönderfall sker ej.

Baryon tal

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \quad \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} \begin{pmatrix} \bar{c} \\ \bar{s} \end{pmatrix} \begin{pmatrix} \bar{t} \\ \bar{b} \end{pmatrix}$$

$$B = \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3} \quad -\frac{1}{3} \quad -\frac{1}{3} \quad -\frac{1}{3}$$

- Baryontalet är konserverat i alla typer av växelverkan.

- Leptoner $\longrightarrow$ $B=0$

  Hadroner $\to$ Mesoner $\longrightarrow$ $B=0$

  Hadroner $\to$ Baryoner $\longrightarrow$ $B=1$

- Baryontalet är additivt.

- Exempel:

$$p + p \longrightarrow p + n + \pi^+$$

$$B: \quad 1 + 1 \qquad 1 + 1 + 0$$

OK

$$p + p \longrightarrow p + \pi^0 + \pi^+$$

$$B: \quad 1 + 1 \qquad 1 + 0 + 0$$

✓

Laddning

$$\begin{pmatrix} u^{+\frac{2}{3}} \\ d^{-\frac{1}{3}} \end{pmatrix} \begin{pmatrix} c^{+\frac{2}{3}} \\ s^{-\frac{1}{3}} \end{pmatrix} \begin{pmatrix} t^{+\frac{2}{3}} \\ b^{-\frac{1}{3}} \end{pmatrix} \quad \begin{pmatrix} \bar{u}^{-\frac{2}{3}} \\ \bar{d}^{+\frac{1}{3}} \end{pmatrix} \begin{pmatrix} \bar{c}^{-\frac{2}{3}} \\ \bar{s}^{+\frac{1}{3}} \end{pmatrix} \begin{pmatrix} \bar{t}^{-\frac{2}{3}} \\ \bar{b}^{+\frac{1}{3}} \end{pmatrix}$$

$$\begin{pmatrix} \nu_e^{0} \\ e^{-1} \end{pmatrix} \begin{pmatrix} \nu_\mu^{0} \\ \mu^{-1} \end{pmatrix} \begin{pmatrix} \nu_\tau^{0} \\ \tau^{-1} \end{pmatrix} \quad \begin{pmatrix} \bar{\nu}_e^{0} \\ e^{+1} \end{pmatrix} \begin{pmatrix} \bar{\nu}_\mu^{0} \\ \mu^{+1} \end{pmatrix} \begin{pmatrix} \bar{\nu}_\tau^{0} \\ \tau^{+1} \end{pmatrix}$$

- Mesoner ($q\bar{q}$) har laddningen $-1, 0, +1$
- Baryoner (qqq) har laddningen $-1, 0, +1, +2$
  Antibaryoner ($\bar{q}\bar{q}\bar{q}$) har laddningen $-2, -1, 0, +1$
- Laddningskvanttalet är additivt.
- Laddningskvanttalet är bevarat i all växelverkan

Exempel:

$$\begin{array}{lllr} & p + p & \longrightarrow p + n + \pi^{+} & \\ Q: & 1 + 1 & \quad\; 1 + 0 + 1 & \text{OK} \end{array}$$

$$\begin{array}{lllr} & p + p & \longrightarrow p + n + \pi^{+} + \pi^{-} & \\ Q: & 1 + 1 & \quad\; 1 + 0 + 1 + (-1) & \checkmark \\ (B: & 1 + 1 & \quad\; 1 + 1 + 0 + 0 & \text{OK}) \end{array}$$

Kvark kvanttal

	d	$\bar{d}$	u	$\bar{u}$	s	$\bar{s}$	c	$\bar{c}$	b	$\bar{b}$	t	$\bar{t}$
S:	0	0	0	0	−1	+1	0	0	0	0	0	0
C:	0	0	0	0	0	0	+1	−1	0	0	0	0
b:	0	0	0	0	0	0	0	0	−1	+1	0	0
t:	0	0	0	0	0	0	0	0	0	0	+1	−1

- Kvarkkvanttalen är inte bevarade i svag växelverkan
- Leptonerna har $s = c = b = t = 0$.
- Kvark kvanttalet är additivt

Exempel:

$$\pi^- + p \rightarrow K^+ + \pi^- + \Lambda^0$$

	π^-	p	K^+	π^-	Λ^0	
S:	0	0	+1	0	−1	OK
B:	0	1	0	0	1	OK
Q:	−1	+1	+1	−1	0	OK

$$K^+ \rightarrow \begin{cases} \mu^+ \nu & 64\% \\ \pi^+ \pi^0 & 21\% \\ \pi^+ \pi^+ \pi^- & 6\% \end{cases}$$

} Svaga sönderfall ty s är ej bevarat

Enbart svaga sönderfall $\Rightarrow$ Lång livstid

Flavour SU(2)

- Anta att det bara finns u, $\bar{u}$, d och $\bar{d}$ kvarkar.
- Inför isospin med $\hat{\vec{I}} = (\hat{I}_1, \hat{I}_2, \hat{I}_3)$ i direkt analogi med vanligt spin.
- Låt u och d kvarkar vara egentillstånd till isospin operatorn med samma I-kvanttal men olika I_3 kvanttal.

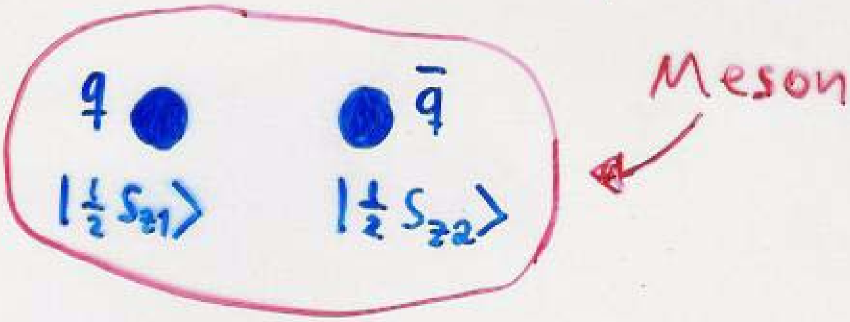
$$\left.\begin{aligned} \hat{\vec{I}}^2 |u\rangle &= I(I+1)|u\rangle = \tfrac{3}{4}|u\rangle \\ \hat{I}_3 |u\rangle &= I_3 |u\rangle \quad = \tfrac{1}{2}|u\rangle \end{aligned}\right\} \quad |u\rangle = \overset{I\ \ I_3}{\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle}$$

$$\left.\begin{aligned} \hat{\vec{I}}^2 |d\rangle &= I(I+1)|d\rangle = \tfrac{3}{4}|d\rangle \\ \hat{I}_3 |d\rangle &= I_3 |d\rangle \quad = -\tfrac{1}{2}|d\rangle \end{aligned}\right\} \quad |d\rangle = \left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle$$

$$\left.\begin{aligned} \hat{\vec{I}}^2 |\bar{u}\rangle &= I(I+1)|\bar{u}\rangle = \tfrac{3}{4}|\bar{u}\rangle \\ \hat{I}_3 |\bar{u}\rangle &= I_3 |\bar{u}\rangle \quad = -\tfrac{1}{2}|\bar{u}\rangle \end{aligned}\right\} \quad |\bar{u}\rangle = \overset{\text{konvention}}{-}\left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle$$

$$\left.\begin{aligned} \hat{\vec{I}}^2 |\bar{d}\rangle &= I(I+1)|\bar{d}\rangle = \tfrac{3}{4}|\bar{d}\rangle \\ \hat{I}_3 |\bar{d}\rangle &= I_3 |\bar{d}\rangle \quad = \tfrac{1}{2}|\bar{d}\rangle \end{aligned}\right\} \quad |\bar{d}\rangle = \left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle$$

Addition av spinkvanttal



Vad är det totala spinnet för mesonen?

$$S = s_1 + s_2, \ldots, |s_1 - s_2| = \begin{cases} 0 & S_z = 0 \\ 1 & S_z = -1, 0, 1 \end{cases}$$

(ty $S_z = -S \ldots +S$)

Det vill säga mesonen kan befinna sig i två spintillstånd med $S = 0$ och $S = 1$.

Hur kan man skriva den totala spinvågfunktionen som en kombination av kvarkarnas spinvågfunktioner?

$$\left.\begin{cases} S = 1 \\ S_z = -1 \quad \Rightarrow \quad s_{z1} = -\frac{1}{2} \quad s_{z2} = -\frac{1}{2} \end{cases}\right\} \Rightarrow |1\,{-1}\rangle = |\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle |\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle$$

$$\left.\begin{cases} S = 1 \\ S_z = 1 \quad \Rightarrow \quad s_{z1} = \frac{1}{2} \quad s_{z2} = \frac{1}{2} \end{cases}\right\} \Rightarrow |1\,1\rangle = |\tfrac{1}{2}\,\tfrac{1}{2}\rangle |\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$\begin{cases} S = 1 \\ S_z = 0 \end{cases} \Rightarrow \begin{cases} s_{z1} = \frac{1}{2} \\ s_{z2} = -\frac{1}{2} \end{cases} \text{ eller } \begin{cases} s_{z1} = -\frac{1}{2} \\ s_{z2} = \frac{1}{2} \end{cases}$$

$$\Rightarrow |1\,0\rangle = A|\tfrac{1}{2}\,\tfrac{1}{2}\rangle |\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle + B|\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle |\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$\begin{cases} S=0 \\ S_z=0 \end{cases} \Rightarrow \begin{cases} s_{z1}=\frac{1}{2} \\ s_{z2}=-\frac{1}{2} \end{cases} \text{ eller } \begin{cases} s_{z1}=-\frac{1}{2} \\ s_{z2}=\frac{1}{2} \end{cases}$$

$$\Rightarrow \quad |0\ 0\rangle = A\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle\left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle + B\left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle$$

Normering av vågfunktionerna ger:

$$S=1 \begin{cases} |1\ 0\rangle = \frac{1}{\sqrt{2}}\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle\left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle + \frac{1}{\sqrt{2}}\left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle \\ |1\ 1\rangle = \left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle \\ |1\ -1\rangle = \left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle\left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle \end{cases}$$

$$S=0 \left\{ |0\ 0\rangle = \frac{1}{\sqrt{2}}\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle\left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle - \frac{1}{\sqrt{2}}\left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle \right.$$

Eller med andra beteckningar:

$$S=1 \begin{cases} |1\ 0\rangle = \frac{1}{\sqrt{2}}\alpha\beta + \frac{1}{\sqrt{2}}\beta\alpha \\ |1\ 1\rangle = \alpha\alpha \\ |1\ -1\rangle = \beta\beta \end{cases}$$

$$S=0 \left\{ |0\ 0\rangle = \frac{1}{\sqrt{2}}\alpha\beta - \frac{1}{\sqrt{2}}\beta\alpha \right.$$

CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND D FUNCTIONS

Note: A $\sqrt{\ }$ is to be understood over every coefficient, e.g., for −8/15 read $-\sqrt{8/15}$.

Notation:

	J	J ...
	M	M ...
m_1 m_2		
m_1 m_2	Coefficients	

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

1/2 × 1/2

m_1 m_2	J=1, M=+1
+1/2 +1/2	1

m_1 m_2	J=1, M=0	J=0, M=0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	J=1, M=−1
−1/2 −1/2	1

1 × 1/2

m_1 m_2	J=3/2, M=+3/2
+1 +1/2	1

m_1 m_2	J=3/2, M=+1/2	J=1/2, M=+1/2
+1 −1/2	1/3	2/3
0 +1/2	2/3	−1/3

m_1 m_2	J=3/2, M=−1/2	J=1/2, M=−1/2
0 −1/2	2/3	1/3
−1 +1/2	1/3	−2/3

m_1 m_2	J=3/2, M=−3/2
−1 −1/2	1

2 × 1/2

m_1 m_2	J=5/2, M=+5/2
+2 +1/2	1

m_1 m_2	J=5/2, M=+3/2	J=3/2, M=+3/2
+2 −1/2	1/5	4/5
+1 +1/2	4/5	−1/5

m_1 m_2	J=5/2, M=+1/2	J=3/2, M=+1/2
+1 −1/2	2/5	3/5
0 +1/2	3/5	−2/5

m_1 m_2	J=5/2, M=−1/2	J=3/2, M=−1/2
0 −1/2	3/5	2/5
−1 +1/2	2/5	−3/5

m_1 m_2	J=5/2, M=−3/2	J=3/2, M=−3/2
−1 −1/2	4/5	1/5
−2 +1/2	1/5	−4/5

m_1 m_2	J=5/2, M=−5/2
−2 −1/2	1

3/2 × 1/2

m_1 m_2	J=2, M=+2
+3/2 +1/2	1

m_1 m_2	J=2, M=+1	J=1, M=+1
+3/2 −1/2	1/4	3/4
+1/2 +1/2	3/4	−1/4

m_1 m_2	J=2, M=0	J=1, M=0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	J=2, M=−1	J=1, M=−1
−1/2 −1/2	3/4	1/4
−3/2 +1/2	1/4	−3/4

m_1 m_2	J=2, M=−2
−3/2 −1/2	1

2 × 1

m_1 m_2	J=3, M=+3
+2 +1	1

m_1 m_2	J=3, M=+2	J=2, M=+2
+2 0	1/3	2/3
+1 +1	2/3	−1/3

m_1 m_2	J=3, M=+1	J=2, M=+1	J=1, M=+1
+2 −1	1/15	1/3	3/5
+1 0	8/15	1/6	−3/10
0 +1	6/15	−1/2	1/10

m_1 m_2	J=3, M=0	J=2, M=0	J=1, M=0
+1 −1	1/5	1/2	3/10
0 0	3/5	0	−2/5
−1 +1	1/5	−1/2	3/10

m_1 m_2	J=3, M=−1	J=2, M=−1	J=1, M=−1
0 −1	6/15	1/2	1/10
−1 0	8/15	−1/6	−3/10
−2 +1	1/15	−1/3	3/5

m_1 m_2	J=3, M=−2	J=2, M=−2
−1 −1	2/3	1/3
−2 0	1/3	−2/3

m_1 m_2	J=3, M=−3
−2 −1	1

1 × 1

m_1 m_2	J=2, M=+2
+1 +1	1

m_1 m_2	J=2, M=+1	J=1, M=+1
+1 0	1/2	1/2
0 +1	1/2	−1/2

m_1 m_2	J=2, M=0	J=1, M=0	J=0, M=0
+1 −1	1/6	1/2	1/3
0 0	2/3	0	−1/3
−1 +1	1/6	−1/2	1/3

m_1 m_2	J=2, M=−1	J=1, M=−1
0 −1	1/2	1/2
−1 0	1/2	−1/2

m_1 m_2	J=2, M=−2
−1 −1	1

3/2 × 1

m_1 m_2	J=5/2, M=+5/2
+3/2 +1	1

m_1 m_2	J=5/2, M=+3/2	J=3/2, M=+3/2
+3/2 0	2/5	3/5
+1/2 +1	3/5	−2/5

m_1 m_2	J=5/2, M=+1/2	J=3/2, M=+1/2	J=1/2, M=+1/2
+3/2 −1	1/10	2/5	1/2
+1/2 0	3/5	1/15	−1/3
−1/2 +1	3/10	−8/15	1/6

m_1 m_2	J=5/2, M=−1/2	J=3/2, M=−1/2	J=1/2, M=−1/2
+1/2 −1	3/10	8/15	1/6
−1/2 0	3/5	−1/15	−1/3
−3/2 +1	1/10	−2/5	1/2

m_1 m_2	J=5/2, M=−3/2	J=3/2, M=−3/2
−1/2 −1	3/5	2/5
−3/2 0	2/5	−3/5

m_1 m_2	J=5/2, M=−5/2
−3/2 −1	1

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

3/2 × 3/2

m_1 m_2	J=3, M=+3
+3/2 +3/2	1

m_1 m_2	J=3, M=+2	J=2, M=+2
+3/2 +1/2	1/2	1/2
+1/2 +3/2	1/2	−1/2

m_1 m_2	J=3, M=+1	J=2, M=+1	J=1, M=+1
+3/2 −1/2	1/5	1/2	3/10
+1/2 +1/2	3/5	0	−2/5
−1/2 +3/2	1/5	−1/2	3/10

m_1 m_2	J=3, M=0	J=2, M=0	J=1, M=0	J=0, M=0
+3/2 −3/2	1/20	1/4	9/20	1/4
+1/2 −1/2	9/20	1/4	−1/20	−1/4
−1/2 +1/2	9/20	−1/4	−1/20	1/4
−3/2 +3/2	1/20	−1/4	9/20	−1/4

m_1 m_2	J=3, M=−1	J=2, M=−1	J=1, M=−1
+1/2 −3/2	1/5	1/2	3/10
−1/2 −1/2	3/5	0	−2/5
−3/2 +1/2	1/5	−1/2	3/10

m_1 m_2	J=3, M=−2	J=2, M=−2
−1/2 −3/2	1/2	1/2
−3/2 −1/2	1/2	−1/2

m_1 m_2	J=3, M=−3
−3/2 −3/2	1

$$d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^1_{1,-1} = \frac{1-\cos\theta}{2} \qquad d^1_{0,0} = \cos\theta$$

2 × 3/2

m_1 m_2	J=7/2, M=+7/2
+2 +3/2	1

m_1 m_2	J=7/2, M=+5/2	J=5/2, M=+5/2
+2 +1/2	3/7	4/7
+1 +3/2	4/7	−3/7

m_1 m_2	J=7/2, M=+3/2	J=5/2, M=+3/2	J=3/2, M=+3/2
+2 −1/2	1/7	16/35	2/5
+1 +1/2	4/7	1/35	−2/5
0 +3/2	2/7	−18/35	1/5

m_1 m_2	J=7/2, M=+1/2	J=5/2, M=+1/2	J=3/2, M=+1/2	J=1/2, M=+1/2
+2 −3/2	1/35	6/35	2/5	2/5
+1 −1/2	12/35	5/14	0	−3/10
0 +1/2	18/35	−3/35	−1/5	1/5
−1 +3/2	4/35	−27/70	2/5	−1/10

m_1 m_2	J=7/2, M=−1/2	J=5/2, M=−1/2	J=3/2, M=−1/2	J=1/2, M=−1/2
+1 −3/2	4/35	27/70	2/5	1/10
0 −1/2	18/35	3/35	−1/5	−1/5
−1 +1/2	12/35	−5/14	0	3/10
−2 +3/2	1/35	−6/35	2/5	−2/5

m_1 m_2	J=7/2, M=−3/2	J=5/2, M=−3/2	J=3/2, M=−3/2
0 −3/2	2/7	18/35	1/5
−1 −1/2	4/7	−1/35	−2/5
−2 +1/2	1/7	−16/35	2/5

m_1 m_2	J=7/2, M=−5/2	J=5/2, M=−5/2
−1 −3/2	4/7	3/7
−2 −1/2	3/7	−4/7

m_1 m_2	J=7/2, M=−7/2
−2 −3/2	1

2 × 2

m_1 m_2	J=4, M=+4
+2 +2	1

m_1 m_2	J=4, M=+3	J=3, M=+3
+2 +1	1/2	1/2
+1 +2	1/2	−1/2

m_1 m_2	J=4, M=+2	J=3, M=+2	J=2, M=+2
+2 0	3/14	1/2	2/7
+1 +1	4/7	0	−3/7
0 +2	3/14	−1/2	2/7

m_1 m_2	J=4, M=+1	J=3, M=+1	J=2, M=+1	J=1, M=+1
+2 −1	1/14	3/10	3/7	1/5
+1 0	3/7	1/5	−1/14	−3/10
0 +1	3/7	−1/5	−1/14	3/10
−1 +2	1/14	−3/10	3/7	−1/5

m_1 m_2	J=4, M=0	J=3, M=0	J=2, M=0	J=1, M=0	J=0, M=0
+2 −2	1/70	1/10	2/7	2/5	1/5
+1 −1	8/35	2/5	1/14	−1/10	−1/5
0 0	18/35	0	−2/7	0	1/5
−1 +1	8/35	−2/5	1/14	1/10	−1/5
−2 +2	1/70	−1/10	2/7	−2/5	1/5

m_1 m_2	J=4, M=−1	J=3, M=−1	J=2, M=−1	J=1, M=−1
+1 −2	1/14	3/10	3/7	1/5
0 −1	3/7	1/5	−1/14	−3/10
−1 0	3/7	−1/5	−1/14	3/10
−2 +1	1/14	−3/10	3/7	−1/5

m_1 m_2	J=4, M=−2	J=3, M=−2	J=2, M=−2
0 −2	3/14	1/2	2/7
−1 −1	4/7	0	−3/7
−2 0	3/14	−1/2	2/7

m_1 m_2	J=4, M=−3	J=3, M=−3
−1 −2	1/2	1/2
−2 −1	1/2	−1/2

m_1 m_2	J=4, M=−4
−2 −2	1

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

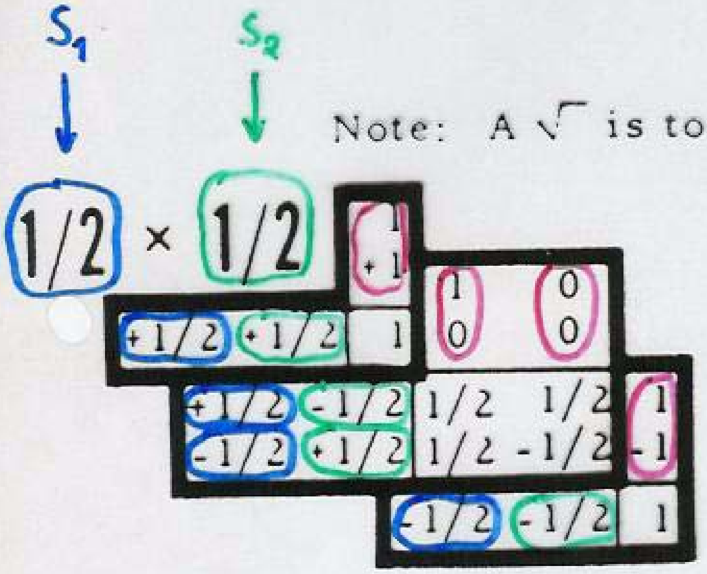
$$d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

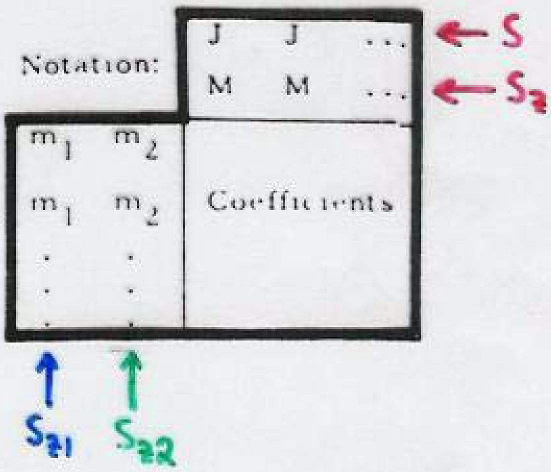
$$d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

ign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of* omic Spectra, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and ohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The signs and umbers in the current tables have been calculated by computer programs written independently by Cohen and at LBL. (Table extended pril 1974.)

Clebsch-Gordan koefficienter

Note: A $\sqrt{\ }$ is to be understood over every coefficient





$$|1\,1\rangle = 1\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$|1\,0\rangle = \tfrac{1}{\sqrt{2}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle + \tfrac{1}{\sqrt{2}}\,|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$|0\,0\rangle = \tfrac{1}{\sqrt{2}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle - \tfrac{1}{\sqrt{2}}\,|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

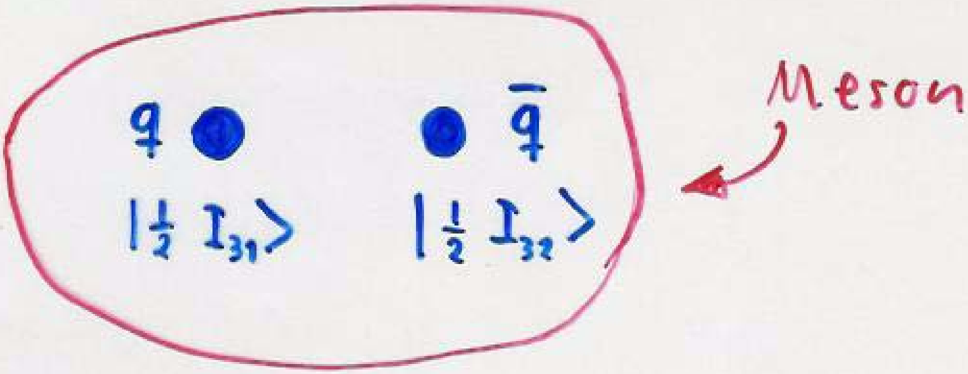
$$|1\,-1\rangle = 1\,|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle$$

$$\left|\tfrac{3}{2}\ \tfrac{3}{2}\right\rangle = |1\ \ 1\rangle\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}|1\ 1\rangle\left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\ |1\ 0\rangle\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\,|1\ 1\rangle\left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\ |1\ 0\rangle\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle$$

Addition av isospinkvanttal



Vad är det totala isospinnet för mesonen?

$$I = I_1 + I_2 \ldots\ldots |I_1 - I_2| = \begin{cases} 0 & I_3 = 0 \\ 1 & I_3 = -1, 0, 1 \end{cases}$$

Det vill säga mesonen kan befinna sig i två isospinn tillstånd med $I=0$ och $I=1$.

Om $S=0$ gäller att

$I=0 \Rightarrow \eta$-mesonen (i en modell med bara u och d)

$I=1 \Rightarrow \pi$-mesonen

Om $S=1$ gäller att

$I=0 \Rightarrow \omega$-mesonen (i en modell med bara u och d)

$I=1 \Rightarrow \rho$-mesonen

<u>Våg funktioner</u>

			Pseudoscalar mesons $S=0$	Vector mesons $S=1$
$I=1$ SU(2) triplet	$\lvert 1\,0\rangle = \frac{1}{\sqrt{2}}\left(\lvert \tfrac{1}{2}\,\tfrac{1}{2}\rangle \lvert \tfrac{1}{2}\,-\tfrac{1}{2}\rangle + \lvert \tfrac{1}{2}\,-\tfrac{1}{2}\rangle \lvert \tfrac{1}{2}\,\tfrac{1}{2}\rangle\right)$	$= \frac{1}{\sqrt{2}}\left(\bar{d}d - \bar{u}u\right)$	π^0	ρ^0
	$\lvert 1\,1\rangle = \lvert \tfrac{1}{2}\,\tfrac{1}{2}\rangle \lvert \tfrac{1}{2}\,\tfrac{1}{2}\rangle$	$= u\bar{d}$	π^+	ρ^+
	$\lvert 1\,-1\rangle = \lvert \tfrac{1}{2}\,\tfrac{1}{2}\rangle \lvert \tfrac{1}{2}\,-\tfrac{1}{2}\rangle$	$= -\bar{u}d$	π^-	ρ^-
$I=0$ SU(2) singlet	$\lvert 0\,0\rangle = \frac{1}{\sqrt{2}}\left(\lvert \tfrac{1}{2}\,\tfrac{1}{2}\rangle \lvert \tfrac{1}{2}\,-\tfrac{1}{2}\rangle - \lvert \tfrac{1}{2}\,-\tfrac{1}{2}\rangle \lvert \tfrac{1}{2}\,\tfrac{1}{2}\rangle\right)$	$= \frac{1}{\sqrt{2}}\left(\bar{d}d + \bar{u}u\right)$	η	ω

Flavour SU(3)

- Anta att det bara finns u, $\bar{u}$, d, $\bar{d}$, s och $\bar{s}$ kvarkar.
- Inför **isospin** med $\hat{\vec{I}} = (\hat{I}_1, \hat{I}_2, \hat{I}_3)$ och **hypercharge** $\hat{Y}$.
- Kvarkarna är då egentillstånd till $\hat{\vec{I}}^2$, $\hat{I}_3$ och $\hat{Y}$.

$$\begin{array}{ll} & \qquad\qquad\quad I \;\; I_3 \;\; Y \\ \left.\begin{aligned} \hat{\vec{I}}^2 |u\rangle &= I(I+1)|u\rangle = \tfrac{3}{4}|u\rangle \\ \hat{I}_3 |u\rangle &= I_3 |u\rangle = \tfrac{1}{2}|u\rangle \\ \hat{Y} |u\rangle &= Y |u\rangle = \tfrac{1}{3}|u\rangle \end{aligned}\right\} & |u\rangle = |\tfrac{1}{2}\;\tfrac{1}{2}\;\tfrac{1}{3}\rangle \end{array}$$

$$\left.\begin{aligned} \hat{\vec{I}}^2 |\bar{u}\rangle &= I(I+1)|\bar{u}\rangle = \tfrac{3}{4}|u\rangle \\ I_3 |\bar{u}\rangle &= I_3 |\bar{u}\rangle = -\tfrac{1}{2}|u\rangle \\ \hat{Y} |\bar{u}\rangle &= Y |\bar{u}\rangle = -\tfrac{1}{3}|u\rangle \end{aligned}\right\} \quad |\bar{u}\rangle = -|\tfrac{1}{2}\;-\tfrac{1}{2}\;-\tfrac{1}{3}\rangle$$

$$\left.\begin{aligned} \hat{\vec{I}}^2 |d\rangle &= I(I+1)|d\rangle = \tfrac{3}{4}|d\rangle \\ \hat{I}_3 |d\rangle &= I_3 |d\rangle = -\tfrac{1}{2}|d\rangle \\ \hat{Y} |d\rangle &= Y |d\rangle = \tfrac{1}{3}|d\rangle \end{aligned}\right\} \quad |d\rangle = |\tfrac{1}{2}\;-\tfrac{1}{2}\;\tfrac{1}{3}\rangle$$

$$\left.\begin{aligned} \hat{\vec{I}}^2 |\bar{d}\rangle &= I(I+1)|\bar{d}\rangle = \tfrac{3}{4}|\bar{d}\rangle \\ \hat{I}_3 |\bar{d}\rangle &= I_3 |\bar{d}\rangle = \tfrac{1}{2}|\bar{d}\rangle \\ \hat{Y} |\bar{d}\rangle &= Y |\bar{d}\rangle = -\tfrac{1}{3}|\bar{d}\rangle \end{aligned}\right\} \quad |\bar{d}\rangle = |\tfrac{1}{2}\;\tfrac{1}{2}\;-\tfrac{1}{3}\rangle$$

$$\left.\begin{array}{lll}\hat{\vec{I}}^2\,|s\rangle = I(I+1)\,|s\rangle & = 0\,|s\rangle \\ \hat{I}_3\,|s\rangle = I_3\,|s\rangle & = 0\,|s\rangle \\ \hat{Y}\,|s\rangle = Y\,|s\rangle & = -\frac{2}{3}\,|s\rangle\end{array}\right\}\quad |s\rangle = |0\ 0\ -\tfrac{2}{3}\rangle$$

$$\left.\begin{array}{lll}\hat{\vec{I}}^2\,|\bar{s}\rangle = I(I+1)\,|\bar{s}\rangle & = 0\,|\bar{s}\rangle \\ \hat{I}_3\,|\bar{s}\rangle = I_3\,|\bar{s}\rangle & = 0\,|\bar{s}\rangle \\ \hat{Y}\,|\bar{s}\rangle = Y\,|\bar{s}\rangle & = +\frac{2}{3}\,|\bar{s}\rangle\end{array}\right\}\quad |\bar{s}\rangle = |0\ 0\ +\tfrac{2}{3}\rangle$$

Flavour kvanttal

	I	I_3	Y
u	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$
$\bar{u}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$
d	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$
$\bar{d}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{3}$
s	0	0	$-\frac{2}{3}$
$\bar{s}$	0	0	$+\frac{2}{3}$

Vågfunktioner		Pseudoscalar mesons $S=0$	Vector mesons $S=1$
	$\lvert I\ I_3\ Y \rangle$		
SU(3) octet	$\lvert 1\,0\,0 \rangle = \frac{1}{\sqrt{2}}(\bar{d}d - \bar{u}u)$	π^0	ρ^0
	$\lvert 1\,1\,0 \rangle = u\bar{d}$	π^+	ρ^+
	$\lvert 1\,{-1}\,0 \rangle = -\bar{u}d$	π^-	ρ^-
	$\lvert \frac{1}{2}\,\frac{1}{2}\,1 \rangle = u\bar{s}$	K^+	K^{*+}
	$\lvert \frac{1}{2}\,{-\frac{1}{2}}\,1 \rangle = d\bar{s}$	K^0	K^{*0}
	$\lvert \frac{1}{2}\,{-\frac{1}{2}}\,{-1} \rangle = \bar{u}s$	K^-	K^{*-}
	$\lvert \frac{1}{2}\,\frac{1}{2}\,{-1} \rangle = \bar{d}s$	$\bar{K}^0$	$\bar{K}^{*0}$
	$\lvert 0\,0\,0 \rangle = \frac{1}{\sqrt{6}}(d\bar{d} - u\bar{u} - 2s\bar{s})$	η_8	ω_8
SU(3) singlet	$\lvert 0\,0\,0 \rangle = \frac{1}{\sqrt{3}}(d\bar{d} + u\bar{u} + s\bar{s})$	η_0	ω_0

η_8, η_0, ω_8 och ω_0 är inte tillstånd som observeras i naturen. De partiklar som observeras $(\eta, \eta', \varphi, \omega)$ är linjär kombinationer av η_8, η_0, ω_8 och ω_0. Man brukar säga att symmetrin inte är exakt utan bruten.

$$\begin{cases} \eta = \eta_8 \cos\theta - \eta_0 \sin\theta \\ \eta' = \eta_8 \sin\theta + \eta_0 \cos\theta \end{cases} \quad \text{där } \theta \approx 11^\circ$$

$$\begin{cases} \varphi = \omega_8 \cos\theta' - \omega_0 \sin\theta' \approx s\bar{s} \\ \omega = \omega_8 \sin\theta' + \omega_0 \cos\theta' \approx \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \end{cases}$$

där $\theta' \approx 35^\circ$