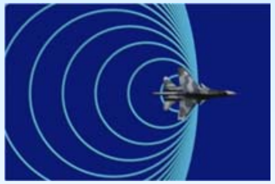
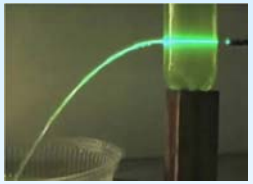
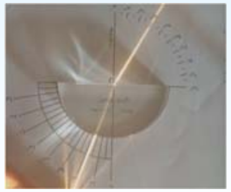
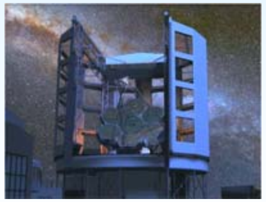
Vågrörelselära och optik

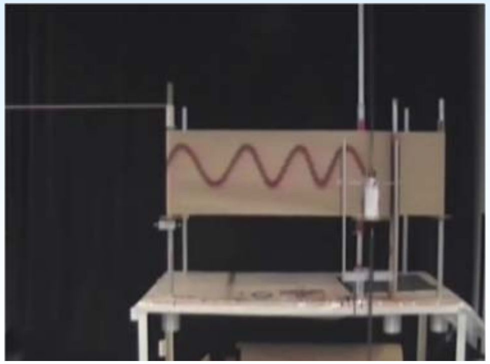




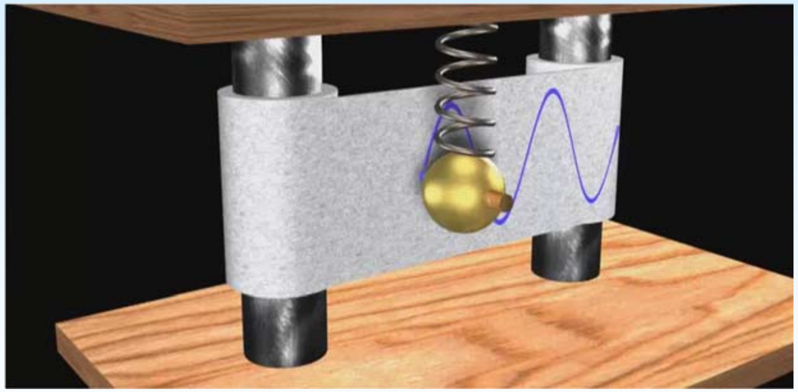
Kapitel 14 – Harmonisk oscillator

Harmonic oscillation: Experiment

Experiment to find a mathematical description of harmonic oscillation

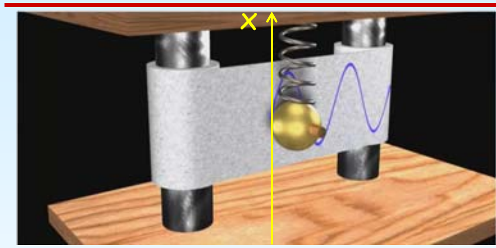


Harmonic oscillation: Experiment



Conclusion: Harmonic oscillation can be described by the function:
$x = A \sin(Bt + C)$
where t is time and A, B and C are constants describing the motion.

Harmonic oscillation: Function



$X = A \sin(Bt + C)$
or
$X = A \cos(Bt + C - \pi/2)$

x : Vertical displacement. Unit: meters

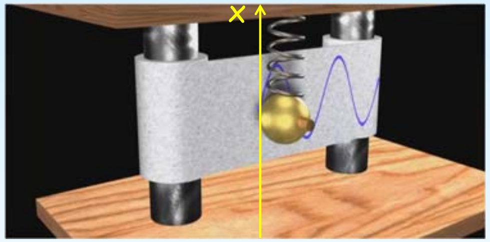
t : Time. Unit: seconds

A : Amplitude (maximum movement). Unit: meters

B = ω : Angular frequency (number of oscillations per second times 2π). Unit: Radians per second

C = ϕ : Phase angle that determines position at time = 0. Unit: radians

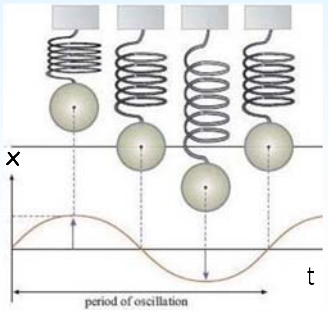
Harmonic oscillation: f and T



$X = A \sin(\omega t + \phi')$

or

$X = A \cos(\omega t + \phi)$



T: Period = The time it takes for the weight to go up and down. Unit: seconds

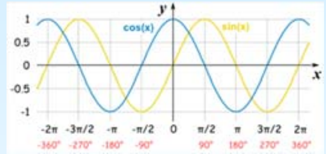
f: Frequency = The number of periods per second. Unit: 1/Seconds

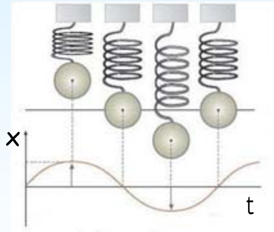
$f = 1 / T$ $\omega = 2\pi f$

Formelsamling

Harmonic oscillation: Phase angle

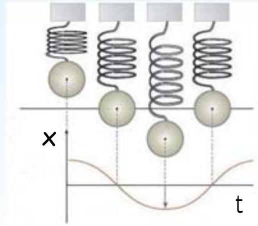
The phase angle (ϕ) determines the position at time = 0 since then $x = A\sin(\phi')$ or $x = A\cos(\phi)$





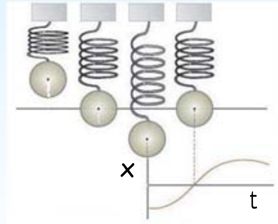
$X = A \sin(\omega t)$

$X = A \cos(\omega t - \pi/2)$



$X = A \cos(\omega t)$

$X = A \sin(\omega t + \pi/2)$



$X = A \cos(\omega t + \pi)$

$X = A \sin(\omega t - \pi/2)$

Harmonic oscillation: velocity & acceleration

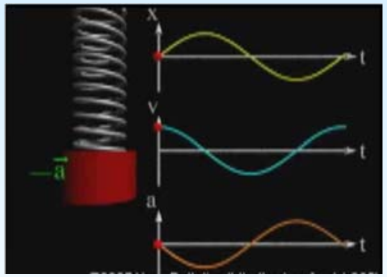
We now have a mathematical description of the displacement.

What is the velocity and acceleration ?

$$v(t) = \frac{dx}{dt}$$

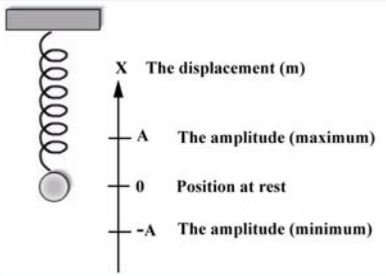
$$a(t) = \frac{dv}{dt}$$

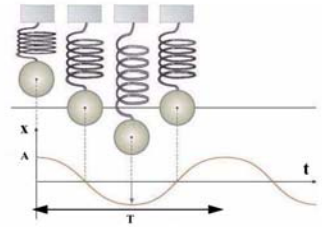
Harmonic oscillation: velocity & acceleration



Displacement:	$x = A\sin(\omega t)$ →	$x_{max} = A$
Velocity: $v = \frac{dx}{dt}$	$v = \omega A \cos(\omega t)$ →	$v_{max} = \omega A$
Acceleration: $a = \frac{dv}{dt}$	$a = -\omega^2 A \sin(\omega t)$ →	$a_{max} = \omega^2 A$

Harmonic oscillation: Summary





- **x** The displacement (m)
- **A** The amplitude (m)
- **t** Time (s)
- **T** Period (s)
- **f** Frequency (Hz) = 1 / T
- **ω** Angular Frequency (Hz) = 2π / T = 2 π f

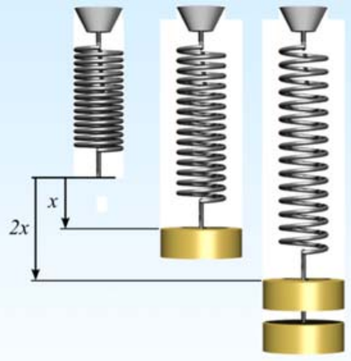
$x = A\cos(\omega t+\phi)$ →	$x_{max} = A$	$x_{min} = -A$
$v = -\omega A\sin(\omega t+\phi)$ →	$v_{max} = \omega A$	$v_{min} = -\omega A$
$a = -\omega^2 A\cos(\omega t+\phi)$ →	$a_{max} = \omega^2 A$	$a_{min} = -\omega^2 A$

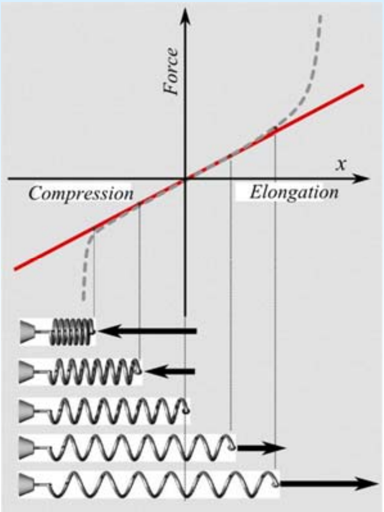
Harmonic oscillation: The spring

Properties of a spring

Hookes law & Forces

Harmonic oscillation: The spring





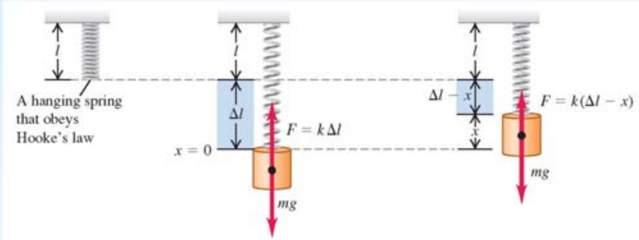
Hooke's law for a spring

$$F = -kX$$

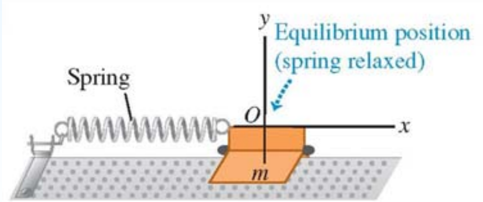
Formelsamling

Harmonic oscillation: The spring

Gravity will stretch the spring to a new eqilibrium position.



This is not the case when the spring is horizonthal.



However, the oscillations will be the same.

Harmonic oscillation: Forces

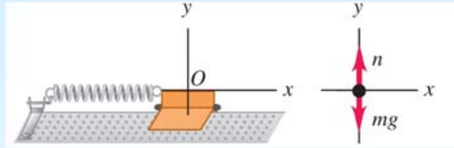
Newton's first law of motion: A body acted on by no net force moves with constant velocity (which may be zero) and zero acceleration.

Newton's second law of motion: If a net external force acts on a body, the body accelerates. The direction of acceleration is the same as the direction of the net force. The mass of the body times the acceleration of the body equals the net force vector.

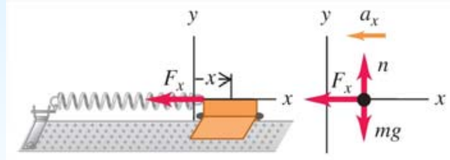
$$\sum \vec{F} = m\vec{a} \qquad \text{(Newton's second law of motion)}$$

Harmonic oscillation: Forces

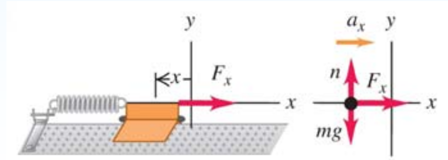
$x = 0 \quad F_{total} = 0 \quad a_x = 0$



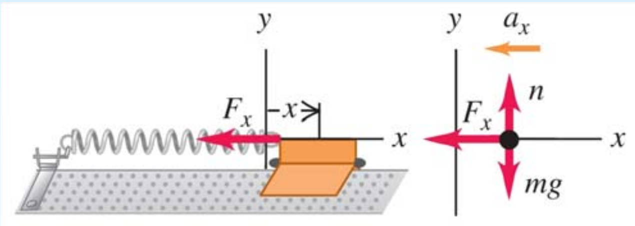
$x > 0 \quad F_{total} < 0 \quad a_x < 0$



$x < 0 \quad F_{total} > 0 \quad a_x > 0$



Harmonic oscillation: Forces



$F_x = -kx$ (restoring force exerted by an ideal spring)

$\sum \vec{F} = m\vec{a}$ (Newton's second law of motion)

$$a_x = \frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad \text{(simple harmonic motion)}$$

Harmonic oscillation: Forces

Old formulas:

$x = A\cos(\omega t + \phi)$

$v = -\omega A\sin(\omega t + \phi)$

$a = -\omega^2 A\cos(\omega t + \phi)$

$a_x = -\omega^2 x$

New formula: $a_x = \frac{d^2x}{dt^2} = -\frac{k}{m}x$ (simple harmonic motion)

Combine: $-\omega^2 x = -\frac{k}{m}x$

Formelsamling $\omega = \sqrt{\frac{k}{m}}$

The frequency depends on the spring constant and the mass

Harmonic oscillation: Forces

An alternative way to look at it:

$$a_x = \frac{d^2x}{dt^2} = -\frac{k}{m}x \quad \text{(simple harmonic motion)}$$

$$\frac{\partial^2 x}{\partial t^2} + \frac{k}{m}x = 0$$

This is a differential equation with the following solution:

$$x = A\cos(\omega t + \varphi)$$

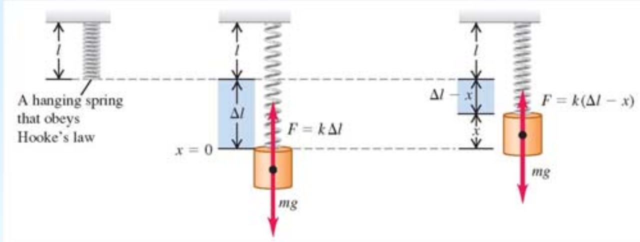
$$\omega = \sqrt{\frac{k}{m}}$$

$$-\omega^2 A\cos(\omega t + \varphi) + \frac{k}{m}A\cos(\omega t + \varphi) = 0$$

$$-\omega^2 A\cos(\omega t + \varphi) + \omega^2 A\cos(\omega t + \varphi) = 0$$

Harmonic oscillation: Forces

Gravity will stretch the spring to a new eqilibrium position.



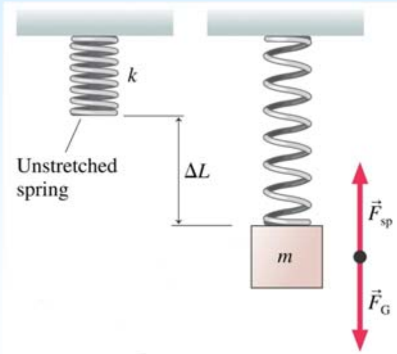
Harmonic oscillation: Forces

The mass hangs in the spring without oscillations:

$$\vec{F}_{total} = \vec{F}_{sp} - \vec{F}_G = k\Delta L - mg$$

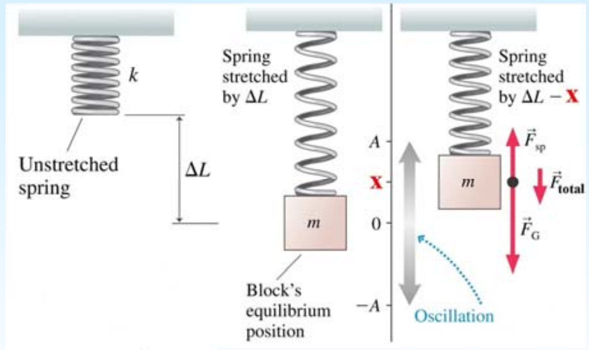
$$\vec{F}_{total} = m\vec{a} = 0$$

$$\Delta L = \frac{mg}{k}$$



Harmonic oscillation: Forces

The mass hangs in the spring and oscillates:



$$\vec{F}_{total} = \vec{F}_{sp} - \vec{F}_G = k(\Delta L - x) - mg$$

Harmonic oscillation: Forces

Spring at rest: $\Delta L = \frac{mg}{k}$

$$\vec{F}_{total} = \vec{F}_{sp} - \vec{F}_G = k(\Delta L - x) - mg$$

$$\vec{F}_{total} = \vec{F}_{sp} - \vec{F}_G = -kx$$

Newton's second law:

$$\vec{F}_{total} = m\vec{a} \neq 0$$

$$-kx = m\vec{a} = m\frac{\partial^2 x}{\partial t^2} \Rightarrow \frac{\partial^2 x}{\partial t^2} + \frac{k}{m}x = 0$$

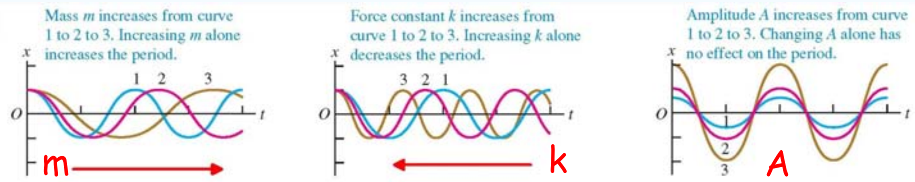
This is a differential equation with the following solution: $x = A\cos(\omega t + \varphi)$ $\omega = \sqrt{\frac{k}{m}}$

Harmonic oscillation: Frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{(simple harmonic motion)}$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \quad \text{(simple harmonic motion)}$$

Note: f and T depends only on k and m but not on the amplitude !



Harmonic oscillation: Summary Forces

Hooke's law for a spring

$F = -kX$

$\Delta L = \frac{mg}{k}$

$\vec{F}_{total} = \vec{F}_{sp} - \vec{F}_G = k(\Delta L - x) - mg$

The differential equation describing the motion:

$$\frac{\partial^2 x}{\partial t^2} + \frac{k}{m}x = 0 \Rightarrow x = A\cos(\omega t + \varphi), \quad \omega = \sqrt{\frac{k}{m}}$$

Formelsamling

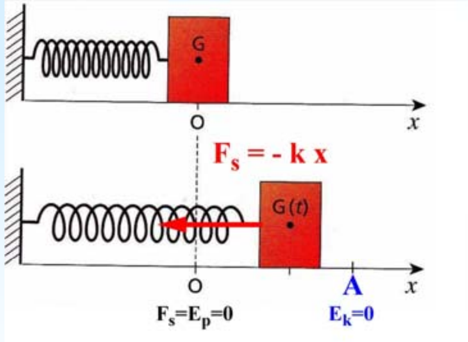
$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{(simple harmonic motion)}$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \quad \text{(simple harmonic motion)}$$

Harmonic oscillation: Energy

Energy in harmonic oscillation

Harmonic oscillation: Energy

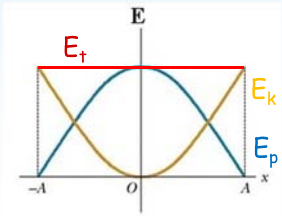


Kinetic energy: $E_k = \frac{mv^2}{2}$

Potential energy: $E_p = \frac{kx^2}{2}$

Total energy: $E_t = E_k + E_p = \frac{kA^2}{2}$ ($E_k = 0$ for $x = A$)

The total mechanical energy is constant



Harmonic oscillation: Energy

$x = A\cos(\omega t+\phi)$

$v = -\omega A\sin(\omega t+\phi)$

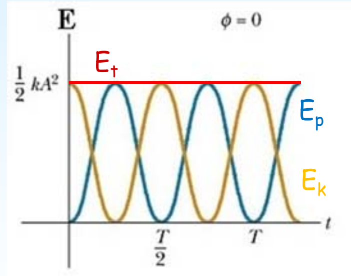
$$\omega = \sqrt{\frac{k}{m}}$$

$$E_p = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t+\phi)$$

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\sin^2(\omega t+\phi)$$

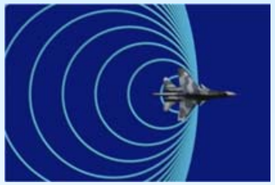
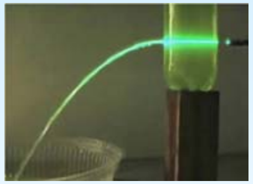
$$E_k = \frac{1}{2}kA^2\sin^2(\omega t+\phi)$$

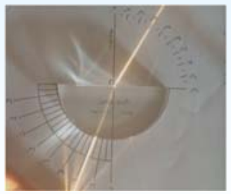
$$E_t = E_k + E_p$$
$$= \frac{1}{2}kA^2\left(\cos^2(\omega t+\phi)+\sin^2(\omega t+\phi)\right)$$
$$= \frac{1}{2}kA^2$$

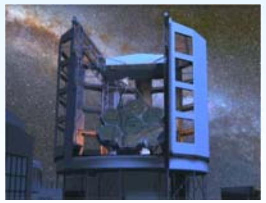


$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = konst.$$

Vågrörelselära och optik





Kapitel 15 – Mekaniska vågor

Mechanical waves: Transverse waves

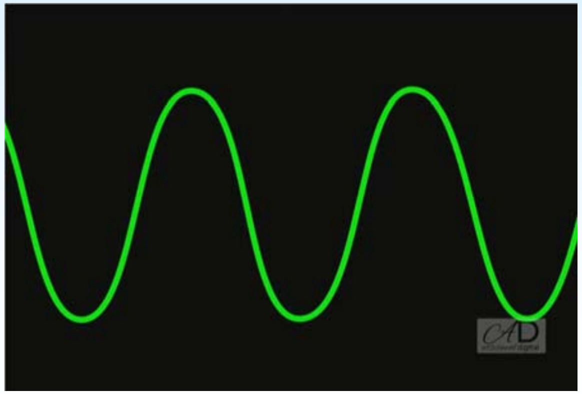
Transverse waves

Mechanical waves: Transverse waves

Transverse wave: The medium moves transverse to the wave direction.

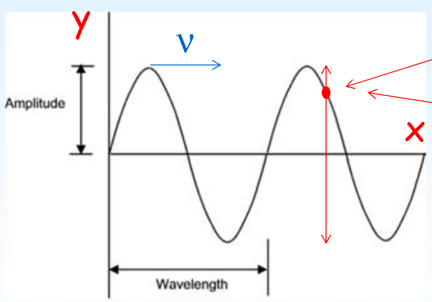
Mechanical waves: Transverse waves

A sinusoidal transverse wave is when the waves have a periodic sinus shape.

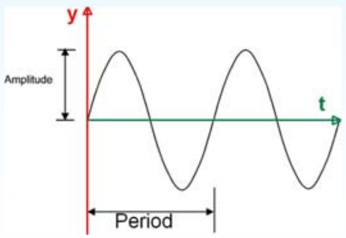


Mechanical waves: Transverse waves

Transversal sinusoidal wave:

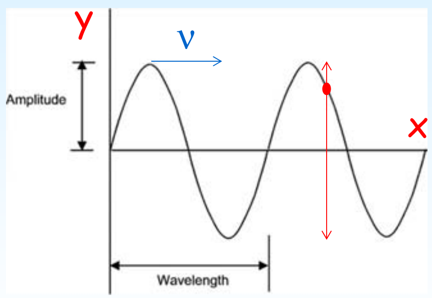


Every point on the wave moves up and down like an harmonic oscillator with the period T.



Mechanical waves: Transverse waves

Definitions:



A: Amplitude (m)

T: Period (s)

λ: Wavelength (m)

v: Wave speed (m/s) = λ / T

f: Frequency (Hz) = $1 / T$

ω: Angular frequency (radians/s) = $2 \pi f$

k: Wave number (radians/m) = $2 \pi / \lambda$

Mechanical waves: Longitudinal waves

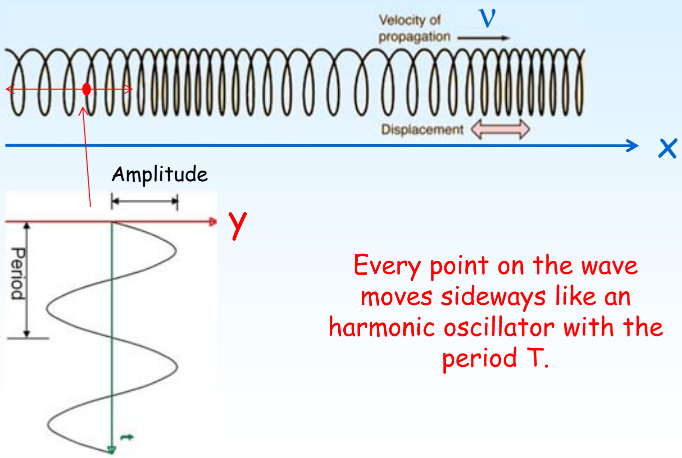
Longitudinal waves

Mechanical waves: Longitudinal waves

Longitudinal wave: The medium moves in the wave direction.

Mechanical waves

Longitudinal sinusoidal wave



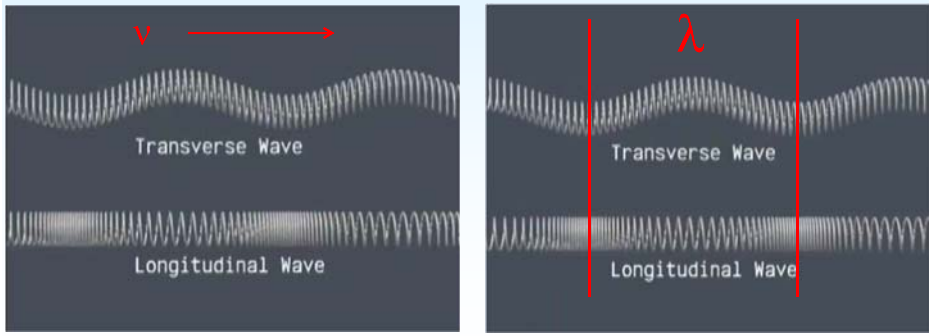
Every point on the wave moves sideways like an harmonic oscillator with the period T.

Mechanical waves: Longitudinal waves

What is the wavelength (λ) for a sinusoidal wave ?

What is the wave speed (v) ?

$$v = \lambda / T$$

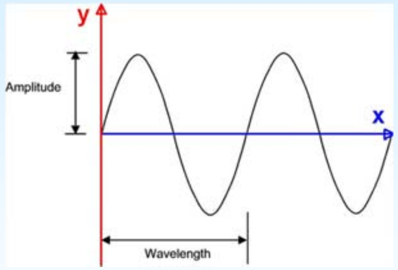


Mechanical waves: The wavefunction

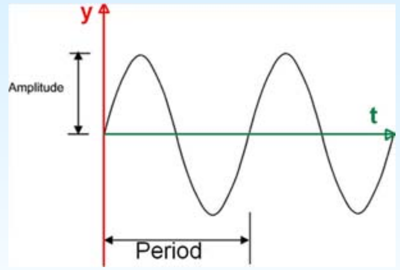
The wavefunction

Mechanical waves: The wavefunction

The height of the wave as a function of distance x



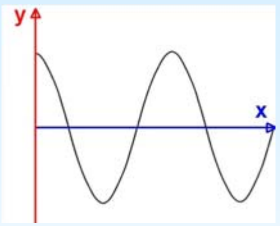
The height of the wave as a function of time t



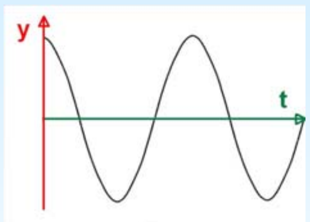
Wavefunction y(x,t):

Function that describs the height of the wave as a function of time and distance

Mechanical waves: The wavefunction



$y(x, t = 0) = A\cos kx$



$y(x = 0, t) = A\cos \omega t$

$y(x, t) = A\cos(kx - \omega t)$ (sinusoidal wave moving in $+x$-direction)

+ if moving in the –x direction

Mechanical waves: The wavefunction

$y(x, t) = A\cos(kx - \omega t)$ (sinusoidal wave moving in $+x$-direction)

Amplitude: A

Wavenumber: $k = \frac{2\pi}{\lambda}$

Formelsamling

$v = \lambda / T$
$f = 1 / T$

Angular frequency: $\omega = \frac{2\pi}{T}$

$v = \lambda / T = (2\pi/k) / (2\pi/\omega) = \omega / k$

Mechanical waves: Summary

The wavefunction: $y(x,t) = A\cos(kx - \omega t)$

Velocity and acceleration up and down:

$$v_y(x,t) = \frac{\partial y(x,t)}{\partial t} = \omega A \sin(kx - \omega t)$$

$$a_y(x,t) = \frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 A\cos(kx - \omega t) = -\omega^2 y(x,t)$$

The wave equation: $\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$ Formelsamling

$v = \lambda / T = (2\pi/k) / (2\pi/\omega) = \omega / k$

Mechanical waves: Wave speed

Wave speed and the string characteristics

Mechanical waves: Wave speed

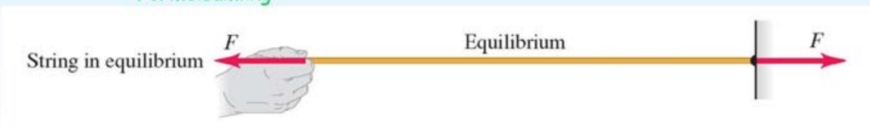
The wave speed in a string depends on two things:

$$v = \sqrt{\frac{F}{\mu}}$$

F: Force (or string trension)

μ: String mass per unit length

Formelsamling



More generally:

$$v = \sqrt{\frac{\text{Restoring force returning the system to equilibrium}}{\text{Inertia resisting the return to equilibrium}}}$$

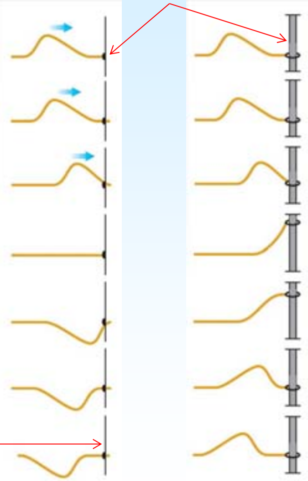
Mechanical waves: Reflections

Reflections

Mechanical waves: Reflections

Reflections of a wave

Boundary conditions



The support provides an opposite force which produces and inverted wave.

Mechanical waves: Reflections

The wavefunction of two waves is typically the sum of the individual wavefunctions.

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

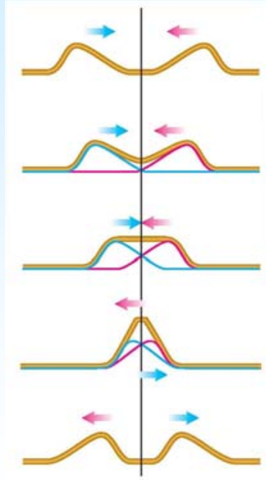
This is called the principle of superposition.

This is true if the wave equations for the waves are linear (they contain the function y(x,t) only to the first power).

For example can sinusoidal waves be superimposed like this because their wave equation

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

is linear.

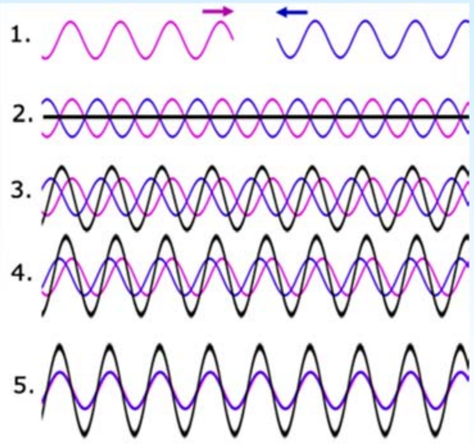


Mechanical waves: Standing waves

Standing waves

Mechanical waves: Standing waves

Mechanical waves: Standing waves



$y_1(x,t) = -A\cos(kx + \omega t)$

$y_2(x,t) = A\cos(kx - \omega t)$

Different times

$$y(x,t) = y_1(x,t) + y_2(x,t) = A[-\cos(kx + \omega t) + \cos(kx - \omega t)]$$

Mechanical waves: Standing waves

Wavefunction from superposition of two waves:

$$y(x,t) = y_1(x,t) + y_2(x,t) = A[-\cos(kx + \omega t) + \cos(kx - \omega t)]$$

Trigonometrical relationship: $\cos(a \mp b) = \cos a \cos b \pm \sin a \sin b$

Wavefunction: $y(x,t) = y_1(x,t) + y_2(x,t) = 2A \sin kx \, \sin \omega t$

Nodes are given by sin(kx) = 0

$kx = 0, \pi, 2\pi, 3\pi, \ldots$

$k = 2\pi/\lambda$

$$x = 0, \frac{\pi}{k}, \frac{2\pi}{k}, \frac{3\pi}{k}, \ldots$$

$$= 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \ldots$$

Mechanical waves: Standing waves

Wavefunction:

$y(x,t) = 2A \sin(kx) \sin(\omega t)$

Velocity:

$$v_y(x,t) = \frac{\partial y(x,t)}{\partial t} \Rightarrow v_y(x,t) = 2A\omega \sin(kx) \cos(\omega t)$$

Acceleration:

$$a_y(x,t) = \frac{\partial v_y(x,t)}{\partial t} = \frac{\partial^2 y(x,t)}{\partial t^2} \Rightarrow a_y(x,t) = -2A\omega^2 \sin(kx) \sin(\omega t)$$

Mechanical waves: Stringed instrument

Stringed instrument

Mechanical waves: Stringed instrument

Instrument with strings of length L has nodes at both ends.

$$L = n\frac{\lambda}{2} \quad (n = 1, 2, 3, \ldots)$$

$$\lambda_n = \frac{2L}{n} \quad (n = 1, 2, 3, \ldots)$$

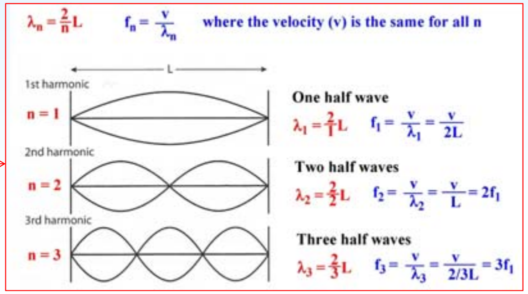
Nodes when sin(kx) = 0: $x = 0, \frac{\pi}{k}, \frac{2\pi}{k}, \frac{3\pi}{k}, \ldots = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \ldots$

Formelsamling

$\lambda = v / f$

$$f_n = n\frac{v}{2L} = nf_1 \quad (n = 1, 2, 3, \ldots)$$

f_1, f_2, f_3.... Harmonic frequencies
f_1: Fundamental frequency
f_2, f_3, f_4.... Overtones



Mechanical waves: Stringed instrument

$f_1 = v/2L$

Formelsamling

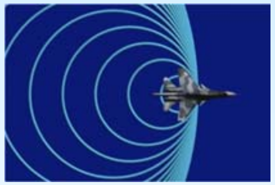
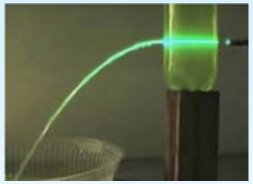
$v = \sqrt{F/\mu}$

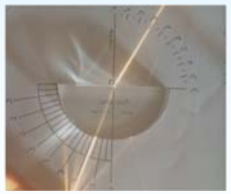
$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

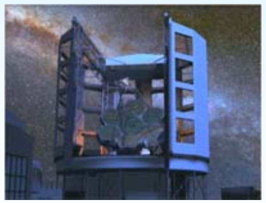
Long string: Low frequency
Thick string: Low frequency
Large tension: High frequency

A stringed instrument does not produce only harmonic frequencies but a superposition of many normal modes.

Vågrörelselära och optik





Kapitel 16 - Ljud

Sound & Pressure

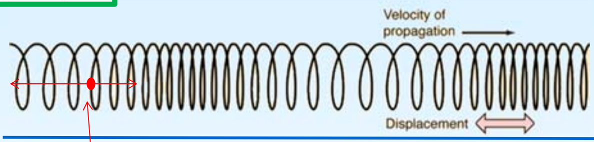
Sound as pressure waves

Sound & Pressure

$f_1 = v/2L$

Longitudinal sinusoidal wave

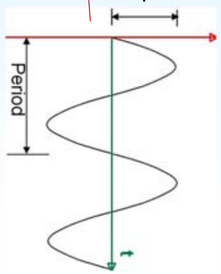
Formelsamling



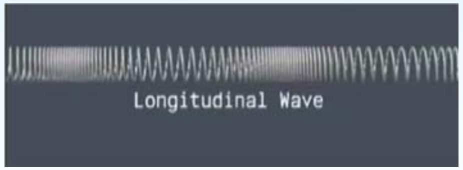
$v = f \cdot \lambda = \frac{\omega}{k}$

x

Amplitude



y

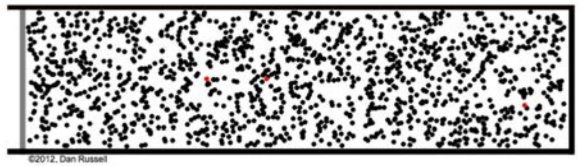


$$y(x,t) = A\cos(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

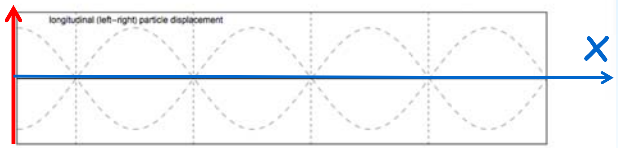
Sound & Pressure

Piston moving in and out:



Air molecule movement:

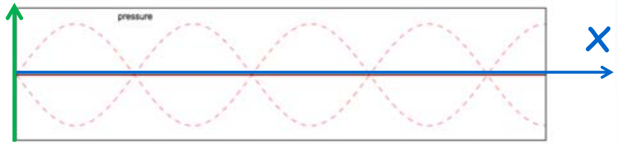
y



x

Pressure:

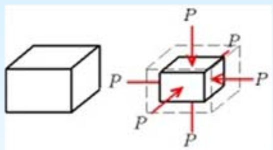
p



x

Sound & Pressure

Bulk modulus



The bulk modulus measures a mediums resistance to uniform compression:

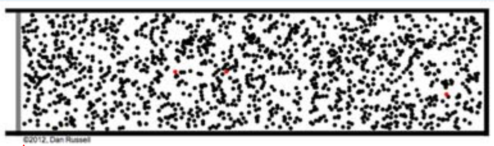
$B = -V\frac{\Delta p}{\Delta V}$ → **Pressure change** / **Volyme change**

The change in pressure after a change of volume:

$\Delta p = -B\,\Delta V/V$

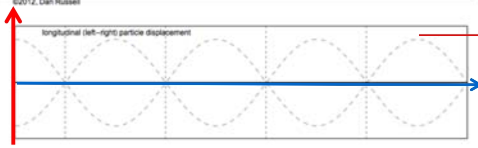
Pressure increase: $\Delta p > 0$ and $\Delta V < 0$

Sound & Pressure



$\Delta p = -B\,\Delta V/V$

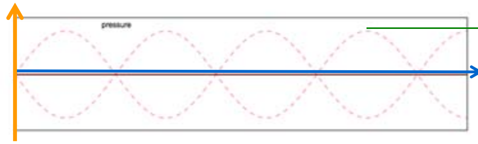
y



x

$$y(x,t) = A\cos(kx - \omega t)$$

p



x

$$p(x,t) = BkA\sin(kx - \omega t)$$

$p_{max} = BkA = \rho\omega v A$

$p_{\max} = BkA$

Sound - velocity

The velocity of sound waves

Sound - velocity

General: $v = \sqrt{\frac{\text{Restoring force returning the system to equilibrium}}{\text{Inertia resisting the return to equilibrium}}}$

String: $v = \sqrt{\frac{F}{\mu}}$ — F: String tension, μ: Mass per unit length

Liquid: $v = \sqrt{\frac{B}{\rho}}$ — B: Bulk modulus, ρ: Density

Solid: $v = \sqrt{\frac{Y}{\rho}}$ — Y: Young's module, ρ: Density

Gas: $v = \sqrt{\frac{B}{\rho}}$ — B: Bulk modulus, ρ: Density

Formelsamling

Mechanical waves: Power & Intensity

Power of mechanical waves on strings

Mechanical waves: Power

The power in general: $P = \vec{F} \cdot \vec{v}$ (instantaneous rate at which force $\vec{F}$ does work on a particle)

Wave power (P): $P(x,t) = F_y(x,t)v_y(x,t)$

y is the only direction where the velocity is not zero

The instantaneous rate at which energy is transfered along the wave.

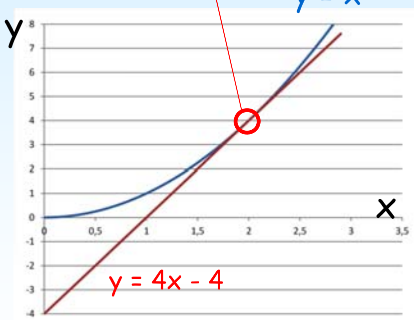
Unit: W or J/s

Mechanical waves: Power

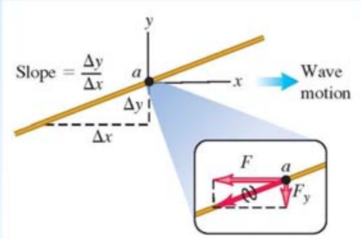
$y = x^2$

$\frac{dy}{dx} = 2x$

$\frac{dy}{dx} = 4 \quad for\ x = 2$



The derivative gives the slope of the tangent.



The ratio of the force in the y-direction to the force in the x-direction is the slope of the string:

$$Slope = \frac{\Delta y}{\Delta x} = \frac{F_y}{F} = \frac{dy}{dx}$$

$$F_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}$$

Mechanical waves: Power

The power in general: $P = \vec{F} \cdot \vec{v}$ (instantaneous rate at which force $\vec{F}$ does work on a particle)

Wave power (P): $P(x,t) = F_y(x,t)v_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t}$

The instantaneous rate at which energy is transfered along the wave.

Unit: W or J/s

Mechanical waves: Power

$$P(x,t) = F_y(x,t)v_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t}$$

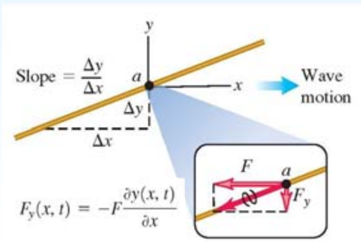
$$y(x,t) = A\cos(kx - \omega t)$$

$$\frac{\partial y(x,t)}{\partial x} = -kA\sin(kx - \omega t)$$

$$\frac{\partial y(x,t)}{\partial t} = \omega A\sin(kx - \omega t)$$

The wave power:

$$P(x,t) = Fk\omega A^2\sin^2(kx - \omega t)$$



$$F_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}$$

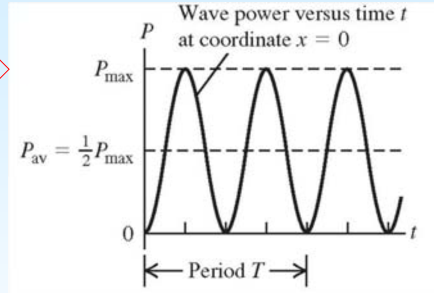
Mechanical waves: Power

The wave power:

$$P(x,t) = Fk\omega A^2\sin^2(kx - \omega t)$$

$$P_{max} = Fk\omega A^2 = \sqrt{\mu F}\omega^2 A^2$$

$$P_{av} = \frac{1}{2}Fk\omega A^2 = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$$



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$$v = \sqrt{\frac{F}{\mu}} \qquad v = \frac{\omega}{k} \qquad k = \frac{\omega}{\sqrt{\frac{F}{\mu}}}$$

Sound – power & intensity

The power of sound

Sound – power

The wave power: $P(x,t) = F_y(x,t)v_y(x,t)$

The pressure function:

$p(x,t) = BkA\sin(kx - \omega t)$

Pressure is equal to force per unit area

The wave function:

$y(x,t) = A\cos(kx - \omega t)$

$v_y(x,t) = \frac{\partial y(x,t)}{\partial t} = \omega A\sin(kx - \omega t)$

The wave power per unit area: $P(x,t)/Area = B\omega kA^2 sin^2(kx - \omega t)$

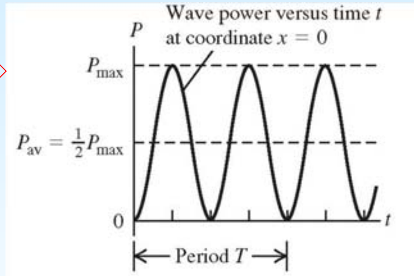
Sound – power

The wave power:

$P(x,t)/Area = B\omega kA^2 sin^2(kx - \omega t)$

$P_{max}/Area = B\omega kA^2 = \sqrt{\rho B}\omega^2 A^2$

$P_{av}/Area = \frac{1}{2}B\omega kA^2 = \frac{1}{2}\sqrt{\rho B}\omega^2 A^2$



$v = \sqrt{\frac{B}{\rho}}$

$v = \omega/k$

$k = \omega / \sqrt{\frac{B}{\rho}}$

Sound – power

Power in general: $P = \vec{F} \cdot \vec{v}$ (instantaneous rate at which force $\vec{F}$ does work on a particle)

Wave power - string:

$P(x,t) = Fk\omega A^2 \sin^2(kx - \omega t)$

$P_{max} = Fk\omega A^2 = \sqrt{\mu F}\omega^2 A^2$

$P_{av} = \frac{1}{2}Fk\omega A^2 = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$

Wave power - sound:

$P(x,t)/Area = B\omega kA^2 sin^2(kx - \omega t)$

$P_{max}/Area = B\omega kA^2 = \sqrt{\rho B}\omega^2 A^2$

$P_{av}/Area = \frac{1}{2}B\omega kA^2 = \frac{1}{2}\sqrt{\rho B}\omega^2 A^2$

Sound - Intensity

Intensity of sound

Sound - Intensity

The power in general: $P = \vec{F} \cdot \vec{v}$ (instantaneous rate at which force $\vec{F}$ does work on a particle)

Wave power (P):
The instantaneous rate at which energy is transfered along the wave.

Unit: W or J/s

$$P(x,t) = F_y(x,t)v_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t}$$

Wave intensity (I):
Average power per unit area through a surface perpendicular to the wave direction.

Unit: W/m^2

$$I = P_{av} / Area$$

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$$I = \frac{Effekt}{Area}$$

Sound - Intensity

$$I = P_{av}/Area = \frac{1}{2}B\omega kA^2 = \frac{1}{2}\sqrt{\rho B}\omega^2 A^2$$

The pressure function:

$$p(x,t) = BkA\sin(kx - \omega t)$$

The pressure amplitude:

$$p_{max} = BkA$$

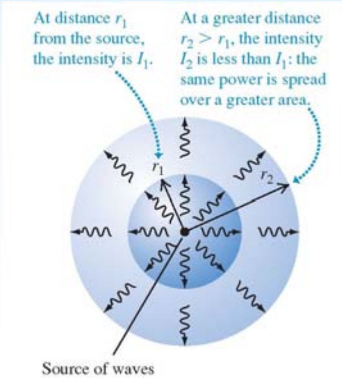
$$I = \frac{p_{max}^2}{2\sqrt{\rho B}}$$

The intensity is proportional to the square of the pressure amplitude

$$p_{max} = BkA = \rho\omega vA \qquad I = \frac{1}{2}\rho(\omega A)^2 v = \frac{1}{2}\sqrt{\rho B}(\omega A)^2 = \frac{p_{max}^2}{2\rho v} = \frac{p_{max}^2}{2\sqrt{\rho B}}$$

Mechanical waves: Power & Intensity

Wave intensity (I): The rate at which energy is transported by a wave through a surface perpendicular to the wave direction per unit surface area (average power per unit area). Unit: W/m^2



The intensity through a sphere with radius r_1

$$I_1 = \frac{P}{4\pi r_1^2}$$

If there is no loss of power:

$$4\pi r_1^2 I_1 = 4\pi r_2^2 I_2$$

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad \text{(inverse-square law for intensity)}$$

Sound - Decibel

The decibel scale of the intensity

Sound - Decibel

Intensity in the unit of decibel (dB)

$$\beta = (10\,\text{dB}) \log \frac{I}{I_0} \quad \text{(definition of sound intensity level)}$$

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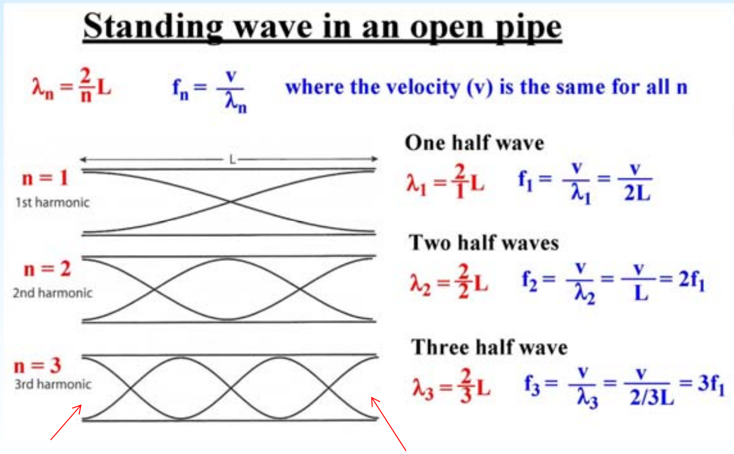
$I_0 = 10^{-12}\ W/m^2$ is a reference intensity
It is roughfly the threshold of human hearing

β = 0 dB for $I = I_0$
β = 120 dB for $I = 1\ W/m^2$

Sound – Standing waves

Sound and standing waves

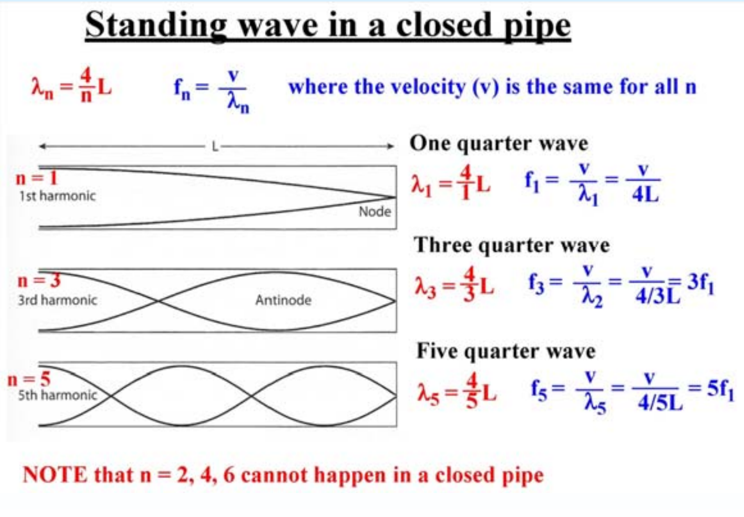
Sound – Standing waves



Antinode Antinode

Sound – Standing waves

Here the pressure is atmospheric giving displacement antinode (pressure node)



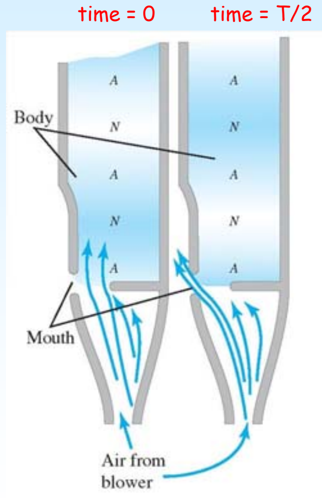
Sound – Standing waves

Organpipe: Airflow from below.

Standing wave: If the airspeed and pipelengths are choosen correctly.

Mouth: Pipe is open at the bottom and gives a pressure node (displacement antinode).

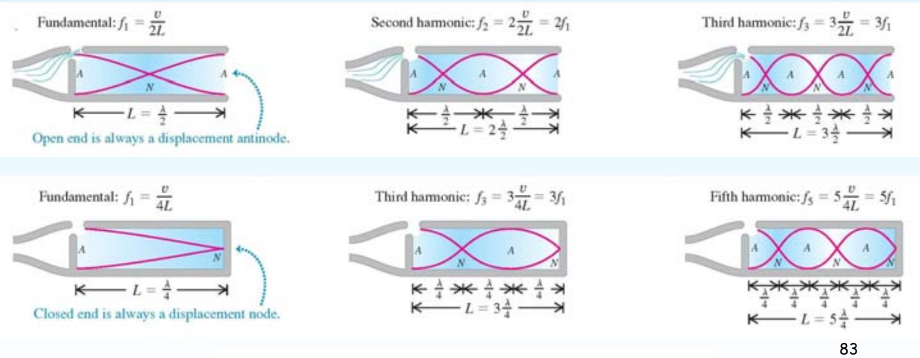
Airflow: Depending on time the air flow will either go into the pipe or out through the mouth.



Sound – Standing waves

An organ pipe can be open-open or open-closed.

Remember: The distance between two nodes is λ/2



$$f_n = \frac{nv}{2L} \qquad f_n = \frac{nv}{4L} \quad (n\ udda)$$

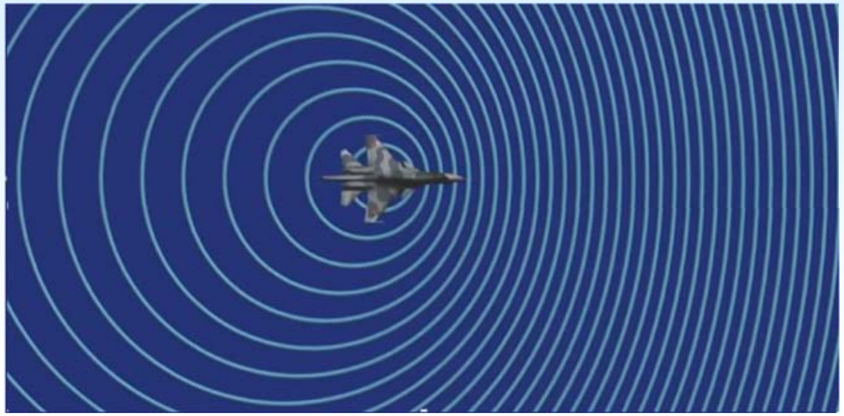
Formelsamling

Sound – Doppler effect

The Doppler effect

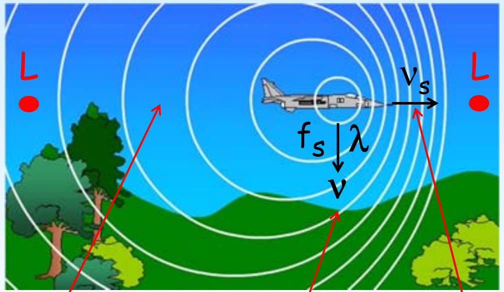
Sound – Doppler effect

Doppler effect



Sound – Doppler effect

The time for a sound wave to reach a listener (L) gets longer if the source (S) is moving away.



The time for a sound wave to reach a listener (L) gets shorter if the source is moving closer.

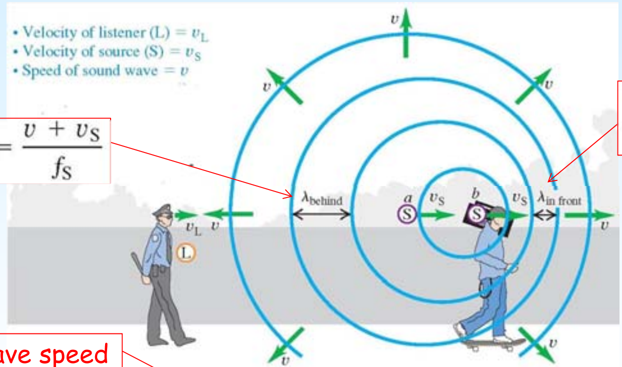
λ_{behind} longer

$\lambda_{in\ front}$ shorter

$$\lambda_{behind} = \frac{v + v_S}{f_S} \qquad \lambda = \frac{v}{f_S} \qquad \lambda_{in\ front} = \frac{v - v_S}{f_S}$$

Sound – Doppler effect

What if the listener is also moving ?



$$\lambda_{behind} = \frac{v + v_S}{f_S} \qquad \lambda_{in\ front} = \frac{v - v_S}{f_S}$$

The wave speed relative to L is $v + v_L$

$$f_L = \frac{v + v_L}{\lambda_{behind}} = \frac{v + v_L}{(v + v_S)/f_S} = \frac{v + v_L}{v + v_S} f_S$$

change in frequency

Sound – Doppler effect

$$f_L = \frac{v+v_L}{v+v_S} f_S$$

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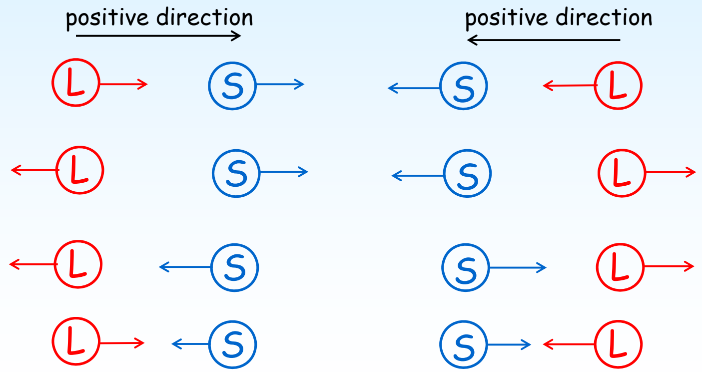
always works if the positive direction is defined as going from the listener to the source.



$$f_L = \frac{v+v_L}{v+v_S} f_S$$

$$f_L = \frac{v-v_L}{v+v_S} f_S$$

$$f_L = \frac{v-v_L}{v-v_S} f_S$$

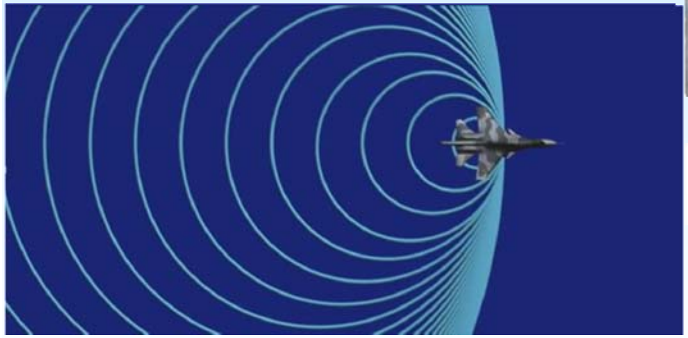
$$f_L = \frac{v+v_L}{v-v_S} f_S$$

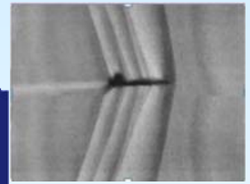
Sound – shockwave

Shockwave

Sound – shockwave

Shock waves





$$\lambda_{\text{in front}} = \frac{v - v_S}{f}$$

ν: Speed of sound
ν_s: Speed of the plane

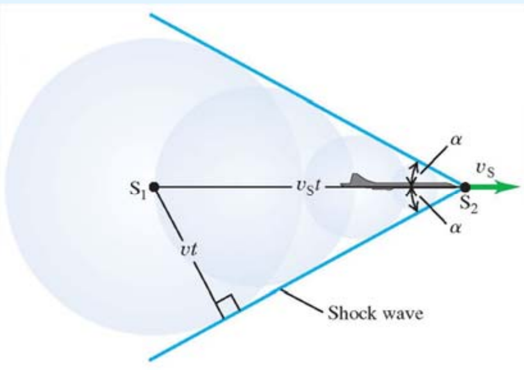
$\nu_s > \nu$ Shockwave is created (not only when $\nu_s = \nu$)

$\nu_s > \nu$ No sound in front of the plane

Sound

A conical shock wave is produced if a plane flies faster than the speed of sound.

A series of circular wave crests from the plane interfere constructively along a line that is given by an angle α.



ν: Speed of sound
ν_s: Speed of the plane

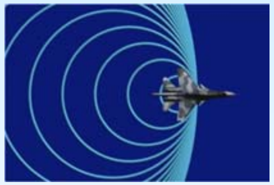
Speed of the plane in Mach number:

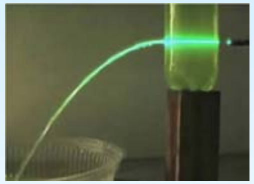
$N_M = \nu_s/\nu$

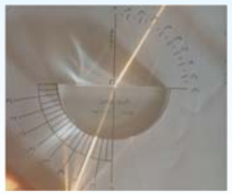
$$\sin\alpha = \frac{vt}{v_S t} = \frac{v}{v_S}$$

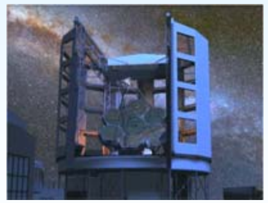
Formelsamling

Vågrörelselära och optik









Kapitel 32 – Elektromagnetiska vågor

Electromagnetic waves
Maxwell's equations

The implications of Maxwell's Equations for magnetic and electric fields:

1. A static electric field can exist in the absence of a magnetic field e.g. a capacitor with a static charge has an electric field without a magnetic field.

2. A constant magnetic field can exist without an electric field e.g. a conductor with constant current has a magnetic field without an electric field.

3. Where electric fields are time-variable, a non-zero magnetic field must exist.

4. Where magnetic fields are time-variable, a non-zero electric field must exist

5. Magnetic fields can be generated by permanent magnets, by an electric current or by a changing electric field.

6. Magnetic monopoles cannot exist. All lines of magnetic flux are closed loops.

Electromagnetic waves
Maxwell's equations

The speed of light from Maxwell's equations

$E = cB$ from Faraday´s law

$E = B / (\varepsilon_0 \mu_0 c)$ from Ampere´s law

ε_0 is the permittivity in vacuum $= 8.85 \times 10^{-12}$ F/m

μ_0 is the permeability in vacuum $= 1.26 \times 10^{-6}$ N/A^2

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 \text{ m/s}$$

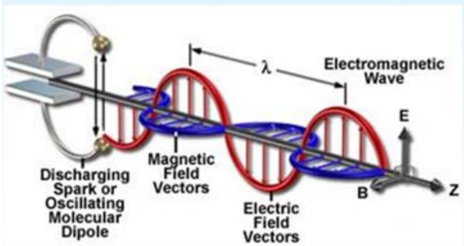
Formelsamling

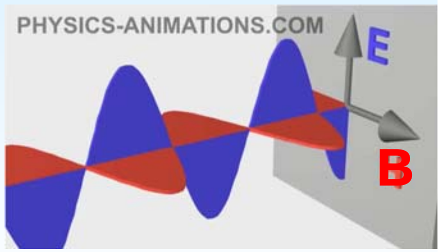
$$\vec{E} = c\vec{B} \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Permittivity: A mediums ability to form an electric field in itself.
Permeability: A mediums ability to form a magnetic field in itself.

Electromagnetic waves
Maxwell's equations

The electromagnetic wave



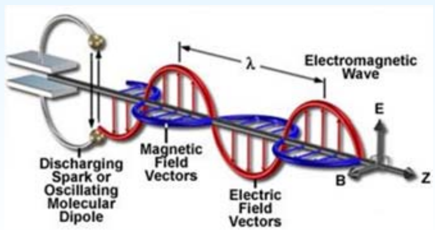


Electromagnetic waves
Maxwell's equations

Electromagnetic waves are produced by the vibration of charged particles.

An **electromagnetic wave** is a wave that is capable of transmitting its energy through **a vacuum**.

The propagation of an electromagnetic wave, which has been generated by a discharging capacitor or an oscillating molecular dipole.



The field is strongest at 90 degrees to the moving charge and zero in the direction of the moving charge.

As the current oscillates up and down in the spark gap a magnetic field is created that oscillates in a horizontal plane.

The changing magnetic field, in turn, induces an electric field so that a series of electrical and magnetic oscillations combine to produce a formation that propagates as an electromagnetic wave.

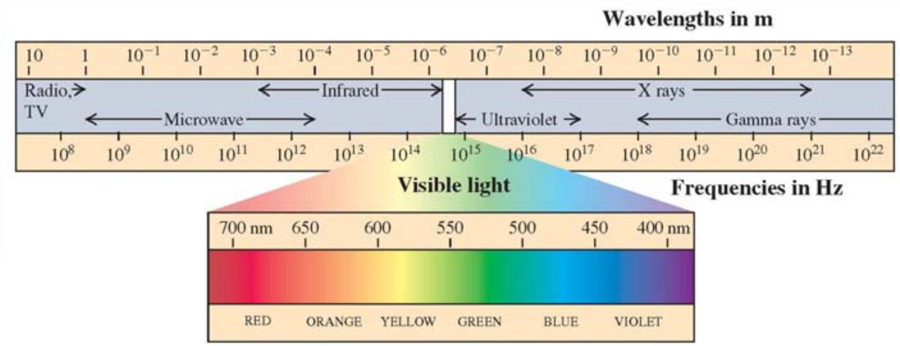
Electromagnetic waves

Electromagnetic waves

Electromagnetic waves

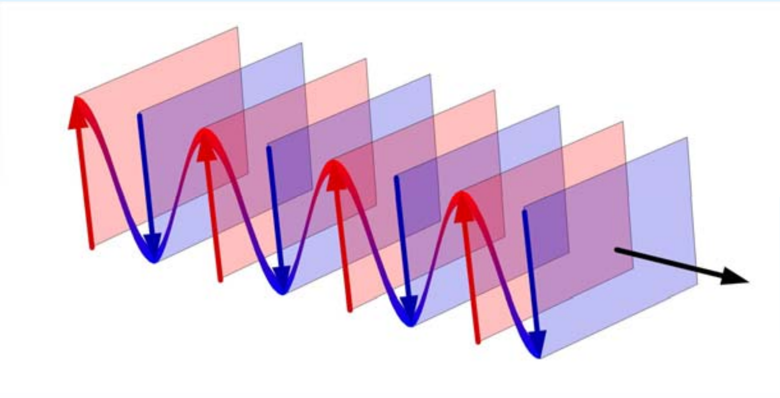
The electromagnetic spectrum

$\lambda = c / f$



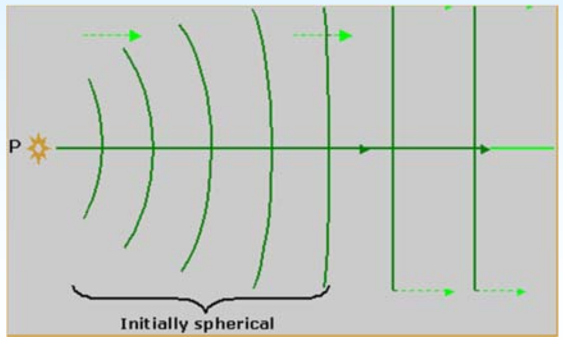
Electromagnetic waves

Wavefronts: surfaces with constant phase



Electromagnetic waves

Wavefronts depends on the distance to the source



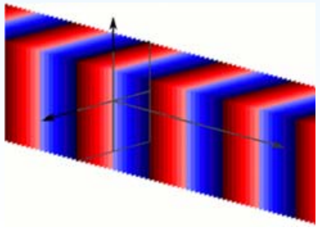
Electromagnetic waves

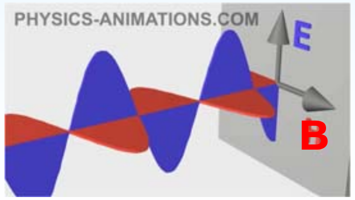
A **plane wave** is a constant-frequency wave whose wavefronts are infinite parallel planes of constant peak-to-peak amplitude normal to the phase velocity vector.

At a particular point and time all E and B vectors in the plane have the same magnitude.

No true plane waves exist since only a plane wave of infinite extent will propagate as a plane wave. However, many waves are approximately plane waves in a localized region of space.

In a plane electromagnetic wave the E and B fields are perpendicular to the direction of propagation so it is a transverse wave.



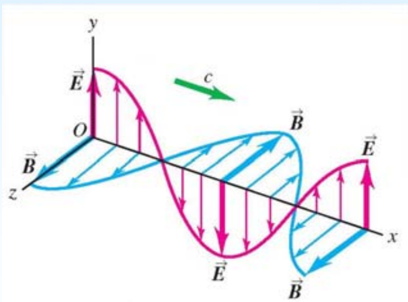


Electromagnetic waves
The wave function

The wavefunction

Electromagnetic waves
The wave function

The electromagnetic wavefunction



$$\vec{E}(x,t) = \hat{j}E_{max}\cos(kx - \omega t)$$
$$\vec{B}(x,t) = \hat{k}B_{max}\cos(kx - \omega t)$$

not the same k

Electromagnetic waves
The wave function

$$\vec{E}(x,t) = \hat{j}E_{max}\cos(kx - \omega t)$$
$$\vec{B}(x,t) = \hat{k}B_{max}\cos(kx - \omega t)$$

Amplitude: $E_{max} = c\,B_{max}$

Wavenumber: $k = \frac{2\pi}{\lambda}$

$c = \lambda / T$
$f = 1 / T$

Angular frequency: $\omega = \frac{2\pi}{T}$

$c = \lambda / T = (2\pi/k) / (2\pi/\omega) = \omega / k$

Compare wavefunctions

Mechanical waves
Formelsamling

$$y(x,t) = A\cos(kx - \omega t)$$

Amplitude: A

Wavenumber: $k = \frac{2\pi}{\lambda}$

Angular frequency: $\omega = \frac{2\pi}{T}$

$v = \lambda / T = \omega / k$

Electromagnetic waves

$$\vec{E}(x,t) = \hat{j}E_{max}\cos(kx - \omega t)$$
$$\vec{B}(x,t) = \hat{k}B_{max}\cos(kx - \omega t)$$

Amplitude: $E_{max} = c\, B_{max}$

Wavenumber: $k = \frac{2\pi}{\lambda}$

Angular frequency: $\omega = \frac{2\pi}{T}$

$c = \lambda / T = \omega / k$

Electromagnetic waves
The wave function

In a dielectric medium the speed of light is smaller than c !

Electromagnetic waves in matter:

$c \rightarrow v$
$\mu_0 \rightarrow \mu$
$\varepsilon_0 \rightarrow \varepsilon$

$c = \frac{1}{\sqrt{\epsilon_0\mu_0}} \rightarrow v = \frac{1}{\sqrt{\epsilon\mu}}$

Dielectric constant
$K = \varepsilon / \varepsilon_0$

Relative permeability
$K_m = \mu / \mu_0$

Electromagnetic waves
The wave function

Electromagnetic wave in vacuum

E = c B	from Faraday´s law
$E = B / (\varepsilon_0 \mu_0 c)$	from Ampere´s law

$c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$

Electromagnetic wave in matter

E = v B	from Faraday´s law
$E = B / (\varepsilon \mu v)$	from Ampere´s law

$v = \frac{1}{\sqrt{\epsilon\mu}}$

Permettivity Permability

$$\frac{c}{v} = n = \frac{\frac{1}{\sqrt{\epsilon_0\mu_0}}}{\frac{1}{\sqrt{\epsilon\mu}}} = \sqrt{KK_m} \cong \sqrt{K}$$

Refractive index Dielectric constant Relative permeability

$K = \varepsilon/\varepsilon_0$ $K_m = \mu/\mu_0$

Electromagnetic waves
Power & Intensity

Power & Intensity

Mechanical waves: Power & Intensity

The power in general: $P = \vec{F} \cdot \vec{v}$ (instantaneous rate at which force $\vec{F}$ does work on a particle)

Wave power (P):

The instantaneous rate at which energy is transfered along the wave.

Unit: W or J/s

Wave intensity (I):

Average power per unit area through a surface perpendicular to the wave direction.

Unit: W/m²

Electromagnetic waves Power & Intensity

Total energy density (u):
Energy per unit volume due to an electric and magnetic field.
Unit: J/m³

Power (P):
The instantaneous rate at which energy is transfered along a wave.
Unit: W or J/s

The Poynting vector ($\bar{S}$):
Energy transferred per unit time per unit area = Power per unit area.
Unit: W/m²

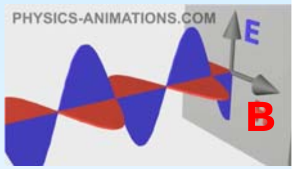
Intensity (I):
Average power per unit area through a surface perpendicular to the wave direction = the average value of S.
Unit: W/m²

Electromagnetic waves Power & Intensity

The total energy density (energy per unit volume) due to an electric and magnetic field is

$$u_E = \frac{1}{2}\epsilon_0 E^2 \qquad u_B = \frac{B^2}{2\mu_0}$$

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2$$



E = c B from Faraday´s law
E = B / ($\varepsilon_0 \mu_0$ c) from Ampere´s law

+ $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$ ⇒ $B^2 = \varepsilon_0 \mu_0 E^2$

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}(\sqrt{\epsilon_0\mu_0}E)^2 = \epsilon_0 E^2$$

where $E(x,t) = E_{max}\cos(kx - \omega t)$

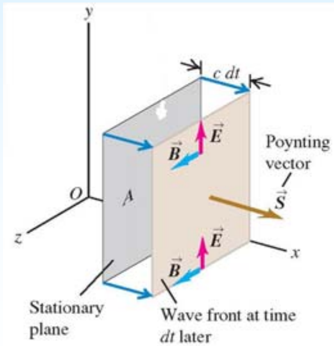
Energy E-field   Energy B-field

Conclusions: The electric and magnetic fields carry the same amount of energy.
The energy density varies with position and time.

Electromagnetic waves Power & Intensity

Energy transfer = energy transferred per unit time per unit area.

S = Power per unit area = Energy transfer = Energy flow



$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \quad \text{(Poynting vector in vacuum)}$$

Formelsamling

$$S_x(x,t) = \frac{E_{max}B_{max}}{\mu_0}\cos^2(kx - \omega t)$$

Amplitude = maximum energy transfer

Electromagnetic waves Power & Intensity

Intensity = the average value of S

$$S_x(x,t) = \frac{E_{max}B_{max}}{\mu_0}\cos^2(kx - \omega t)$$

The average of $\cos^2(x) = 1/2$

$$I = S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}{}^2}{2\mu_0 c}$$

$E = c B$

$$S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}{}^2}{2\mu_0 c} = \frac{1}{2}\varepsilon_0 c E_{max}{}^2$$

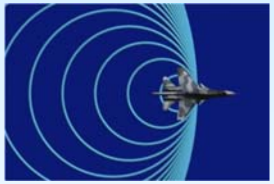
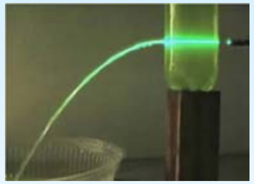
Formelsamling

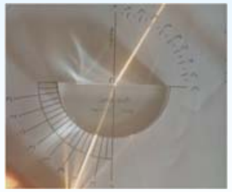
Electromagnetic waves in matter:

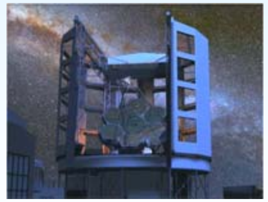
$\mu_0 \rightarrow \mu$

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}} \longrightarrow v = \frac{1}{\sqrt{\epsilon\mu}}$$

Vågrörelselära och optik





Kapitel 33 - Ljus

The nature of light

Source of electromagnetic radiation
is
electric charges in accelerated motion

Thermal radiation:
Thermal motions of molecules create electromagnetic radiation.

Lamp:
A current heats the filament which then sends out thermal radiation with many wavelengths.

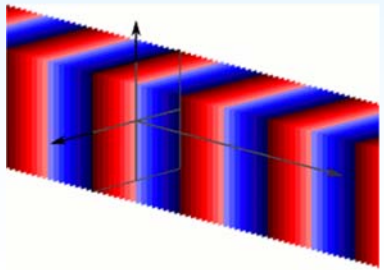
Laser:
Atoms emits light coherently giving (almost) monocromatic radiation.

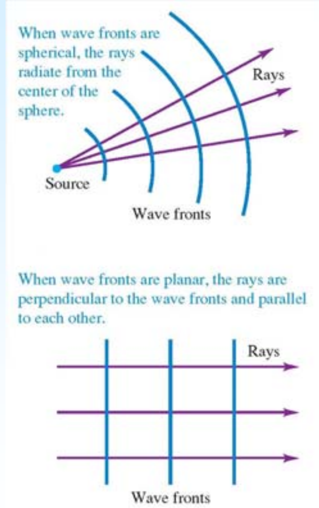
The nature of light

Wave front: surface with constant phase.

Plane wave: is a wave whose wave fronts are infinite parallel planes.

Ray: an imaginary line along the direction of the wave's propagation.



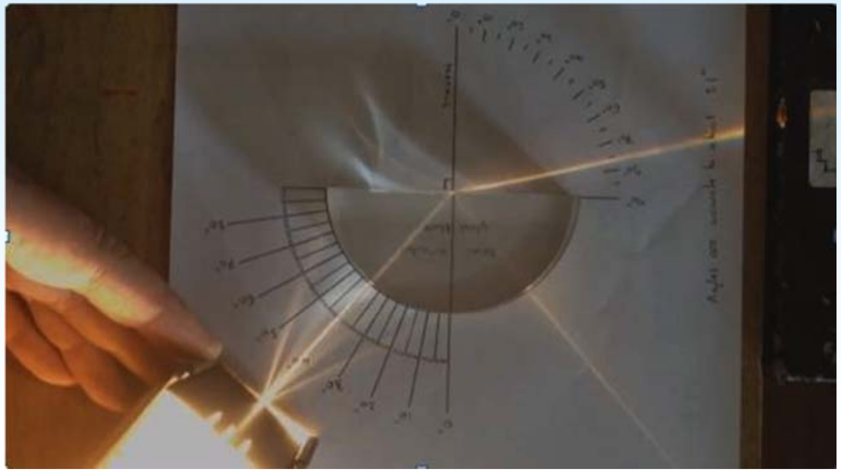


The nature of light

Reflection and refraction

The nature of light

Reflection & Refraction



The nature of light

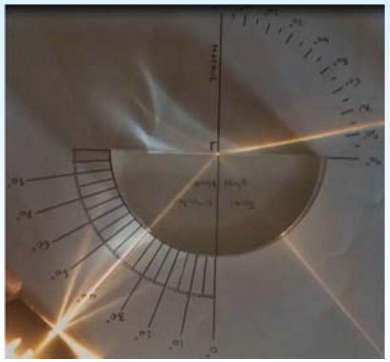
Conclusions:

At the surface between air and glass the angle is always 90 degrees and then the reflected and refracted light is also at 90 degrees.

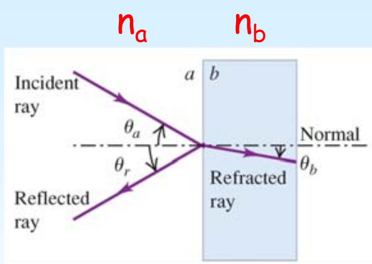
At the surface between glass and air some of the light is reflected and some is refracted.

The angle of reflection is the same as the incident angle.

The angle of refraction is larger than the incident angle.



The nature of light



$$n = \frac{c}{v} \quad \text{(index of refraction)}$$

n = 1 in vacuum
n > 1 in a material

The plane of incident:
The plane of the incident ray and the normal to the surface.

The reflected and refracted rays are in the plane of incident.

$$\theta_r = \theta_a \quad \text{(law of reflection)}$$

Snell's law:

$$n_a \sin\theta_a = n_b \sin\theta_b \quad \text{(law of refraction)}$$

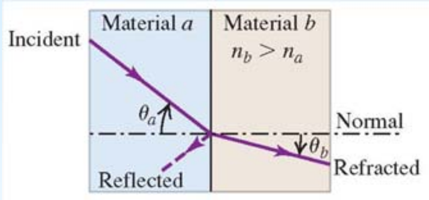
Formelsamling

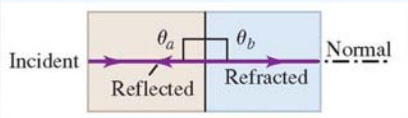
The nature of light

n_a < n_b

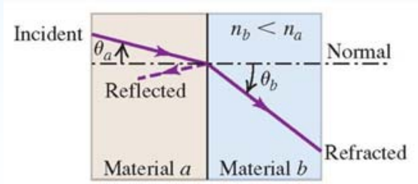
Snell's law:

$n_a \sin\theta_a = n_b \sin\theta_b$ (law of refraction)





n_a > n_b



Rule:
Large n ➡ Small angle

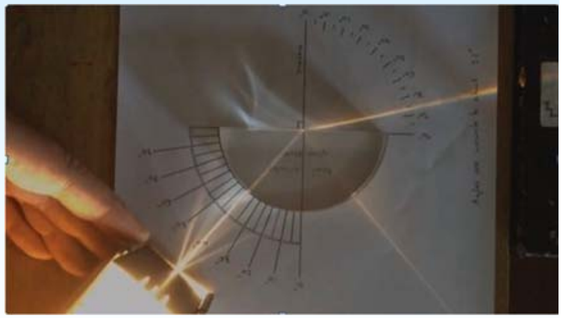
The nature of light

Light intensity

The nature of light

Intensity

The intensity of the reflected light increases from almost 0% at θ = 0° to 100% at θ = 90°.



The intensity of the reflected light also depends on n and on polarization of the incoming light.

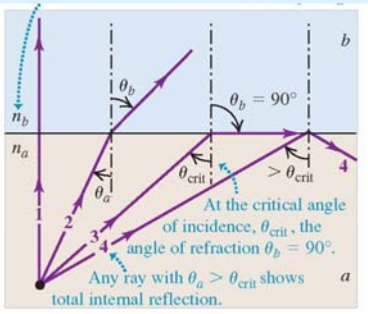
The sum of the intensity of the reflected and refracted light is equal to the intensity of the incoming light.

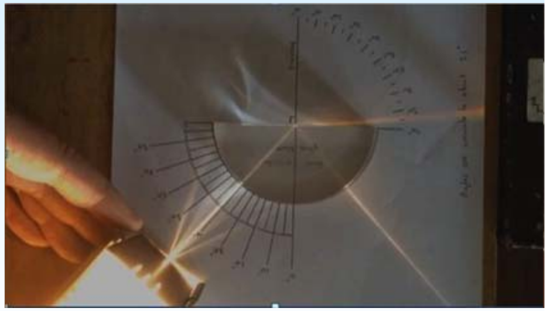
The nature of light

Total internal reflection

Total Internal Reflection

when light goes to a medium with smaller n



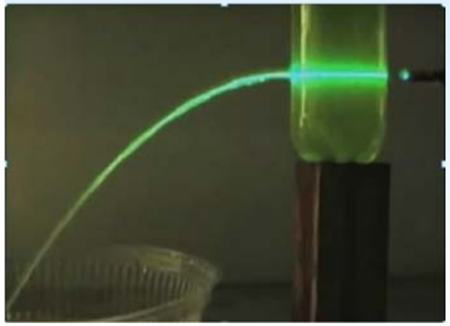


90°

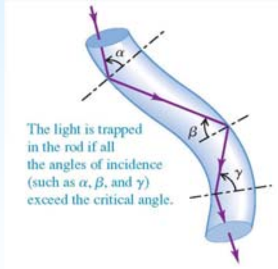
$$n_a \sin\theta_a = n_b \sin\theta_b$$

$$\sin\theta_{crit} = \frac{n_b}{n_a} \quad \text{(critical angle for total internal reflection)}$$

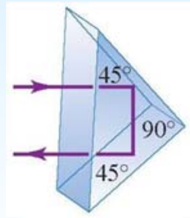
Total Internal Reflection



optical fiber



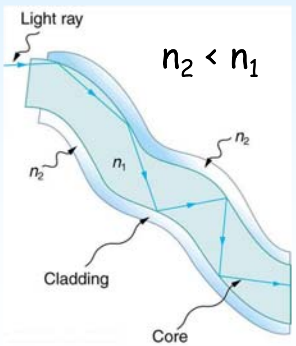
Porro prism



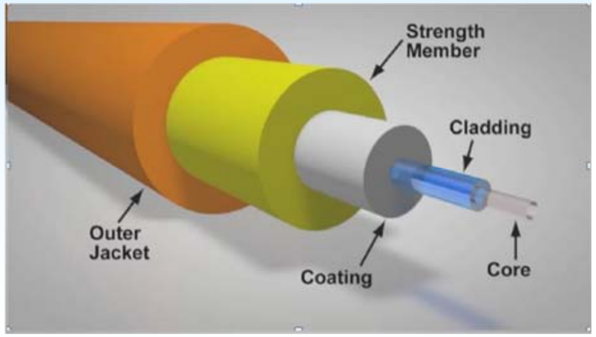
$\theta_{crit} = 41.1°$

Optical fibers

Principle



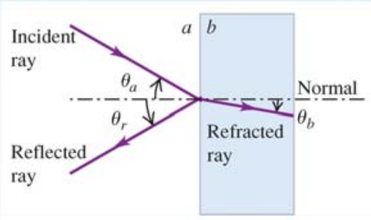
Structure



Dependency on frequency and wavelength

The nature of light

n_a n_b



$n = \frac{c}{v}$ (index of refraction)

n = 1 in vacuum
n > 1 in a material

Frequency and wavelength

v: The speed is larger in a material with a small n.

f: The frequency does not depend on n.

λ: The wavelength is longer in a material with a small n.

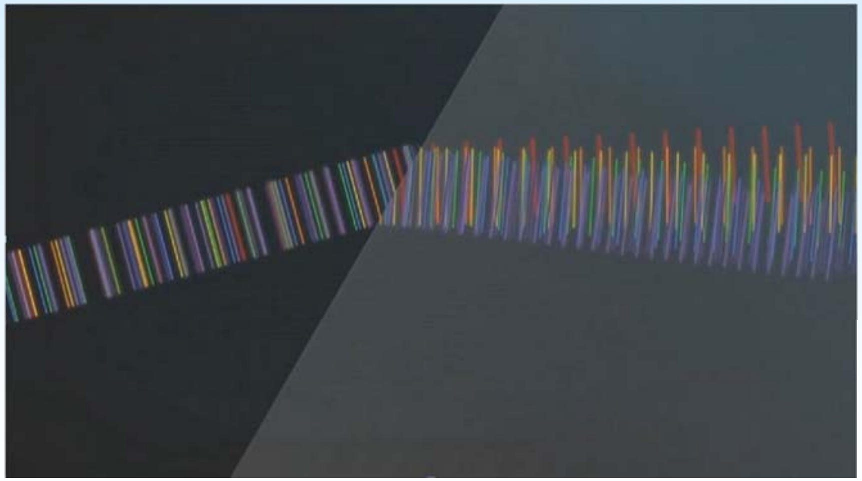
$\lambda = v / f$ n > 1
$\lambda_0 = c / f$ n = 1

$\lambda = \lambda_0 / n$

The nature of light

Dispersion

The nature of light



The nature of light

How is this possible ?

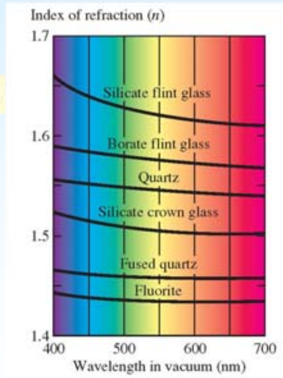
Dispersion

$n_a \sin\theta_a = n_b \sin\theta_b$ (law of refraction)

Answer: n must depend on λ !

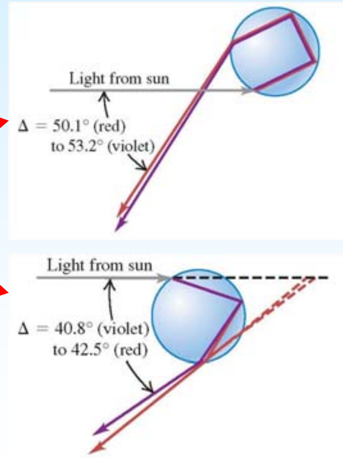
n = c / v

so the speed in a material must then depend on λ

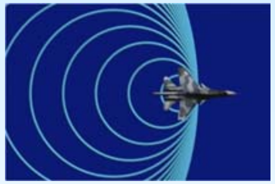


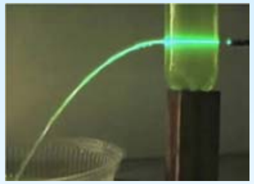
The nature of light

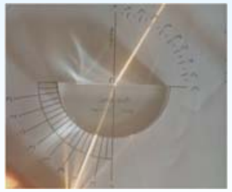
Rainbow

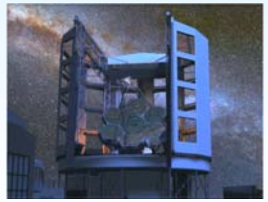


Vågrörelselära och optik









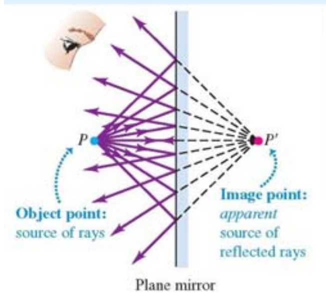
Kapitel 34 - Optik

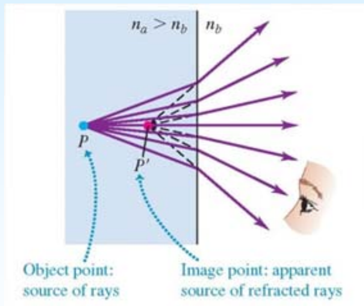
Geometrical optics

Mirrors

Geometrical optics

Virtual Images: outgoing rays diverge



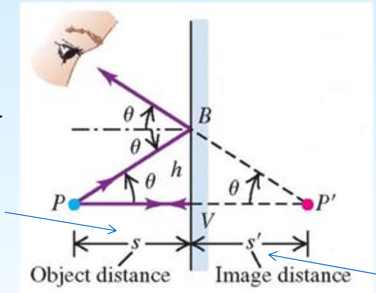


Real Images: outgoing rays converge to an image that can be shown on a screen

Geometrical optics

Point object

positive



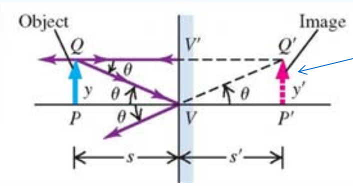
negative

Sign rules:

Object distance (s) – positive if same side as incoming light.

Image distance (s') – positive if same side as outgoing light.

Extended object



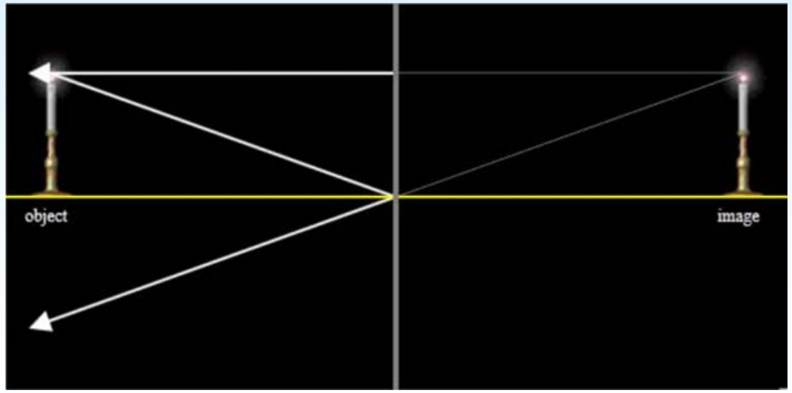
Virtual image

$$m = \frac{y'}{y} \quad \text{(lateral magnification)}$$

Formelsamling

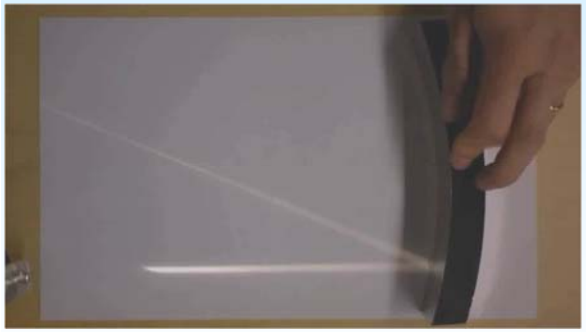
Geometrical optics

Flat mirror

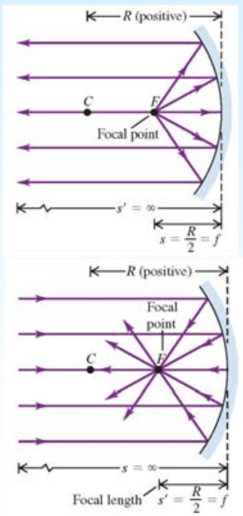


Geometrical optics

Spherical mirror

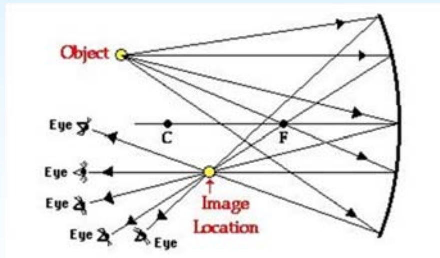


$$f = \frac{R}{2}$$

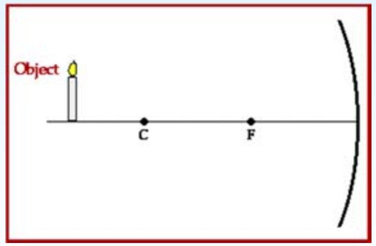


Geometrical optics

An infinite number of rays can be drawn from an object to its image.



But only two rays are needed to determine the location of the image.

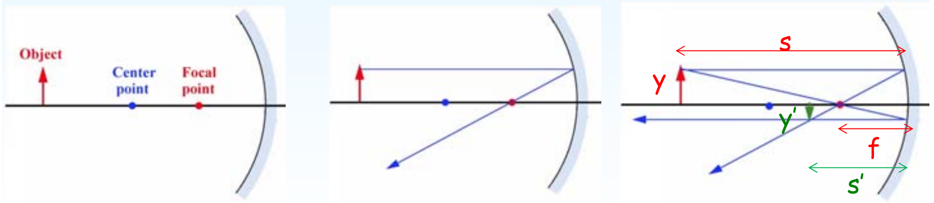


Geometrical optics

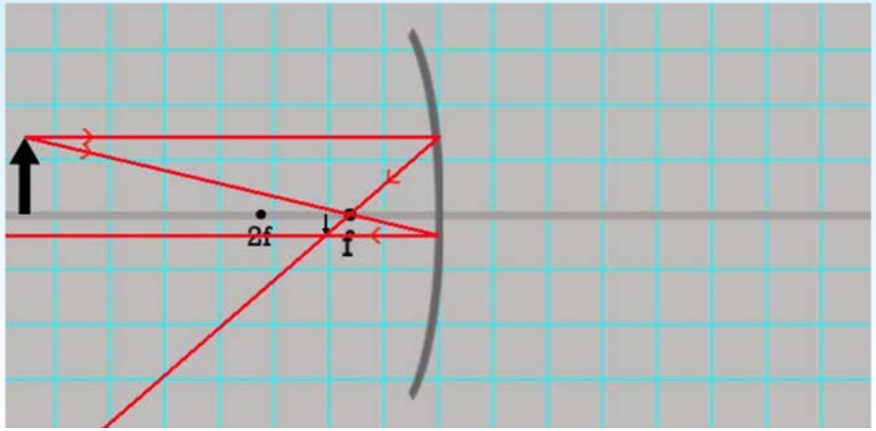
How to find the image in a concave mirror

The bottom of the object is on the optical axis and so the bottom of the image will also be on the optical axis.

The top of the image can be found with any two rays. Use for example two rays that goes through the focal point.



Geometrical optics



http://simbucket.com/lensesandmirrors/

Geometrical optics

Summary spherical mirrors

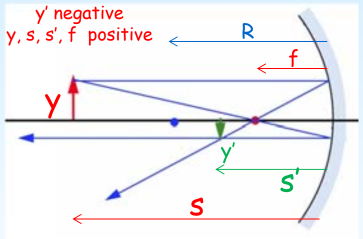
Sign rules:

Object distance (s) – positive if same side as incoming light.

Image distance (s') – positive if same side as outgoing light.

Radius of curvature (R) – positive if center is on same side as outgoing light.

Magnification (m) – positive if direction of object and image is the same.



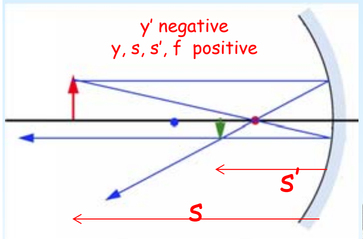
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$f = \frac{R}{2}$$

Formelsamling

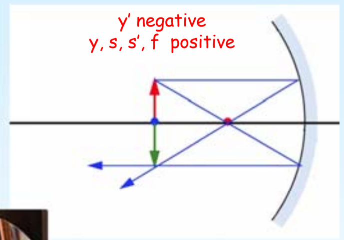
$$m = \frac{y'}{y} = -\frac{s'}{s}$$

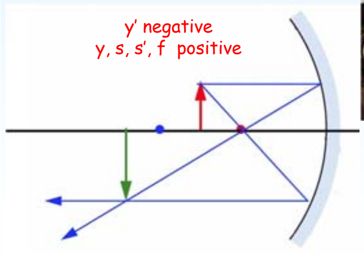
Geometrical optics

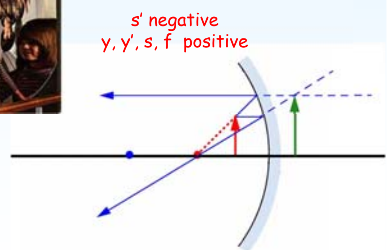


$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

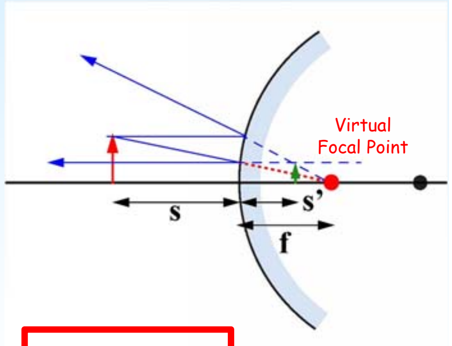






Geometrical optics

Convex mirrors

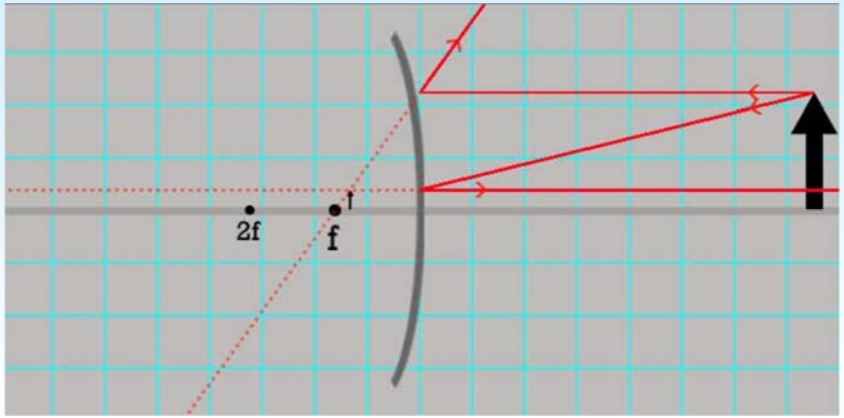


$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

$$m=\frac{y'}{y}=-\frac{s'}{s}$$

s', f negative
y, y', s positive

Geometrical optics

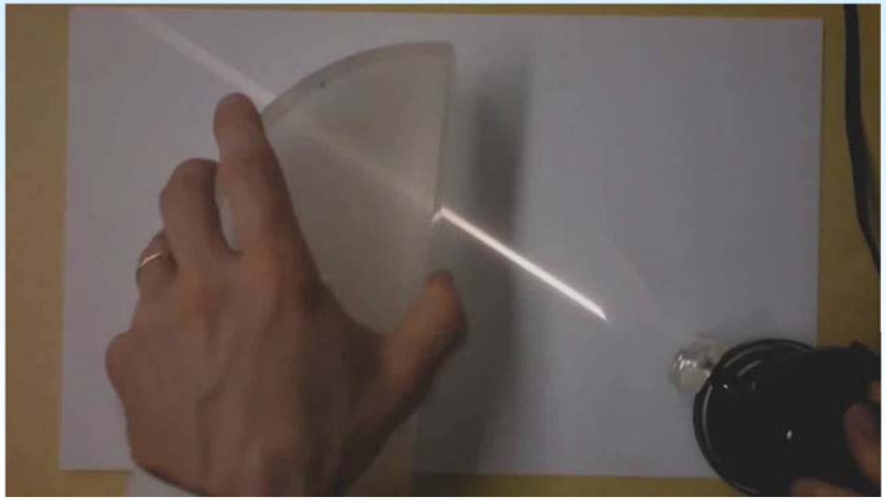


http://simbucket.com/lensesandmirrors/

Geometrical optics

Spherical surface

Geometrical optics



Geometrical optics

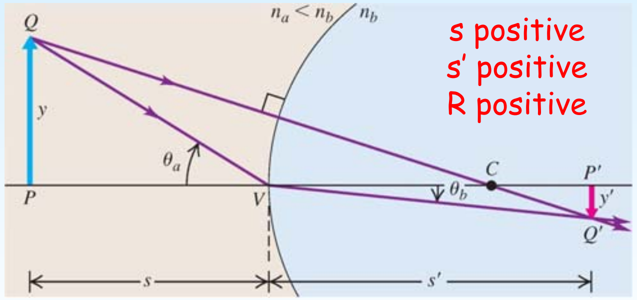
Spherical surface -Summary

Sign rules:

Object distance (s) – positive if same side as incoming light.

Image distance (s') – positive if same side as outgoing light.

Radius of curvature (R) – positive if center is on same side as outgoing light.



s positive
s' positive
R positive

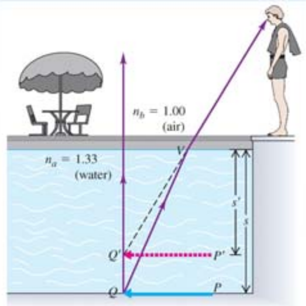
Formelsamling

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s}$$

Geometrical optics

Special case: flat surface



$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} = 0$$

∞

$n_a/s = -n_b/s'$

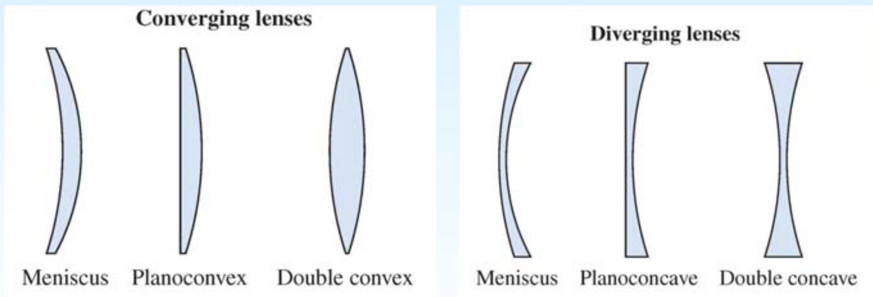
$-s'/s = n_b/n_a$

Geometrical optics

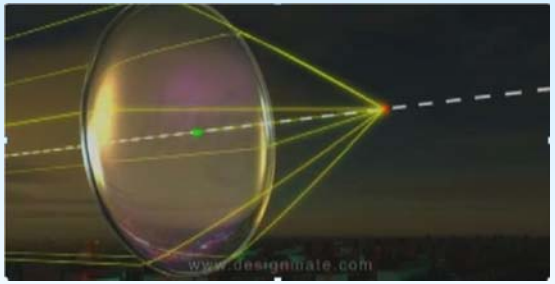
Lenses

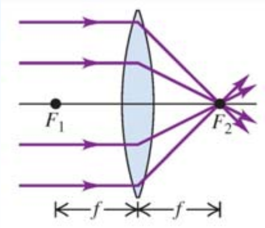
Geometrical optics

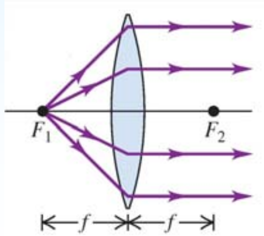
Different type of lenses



Geometrical optics

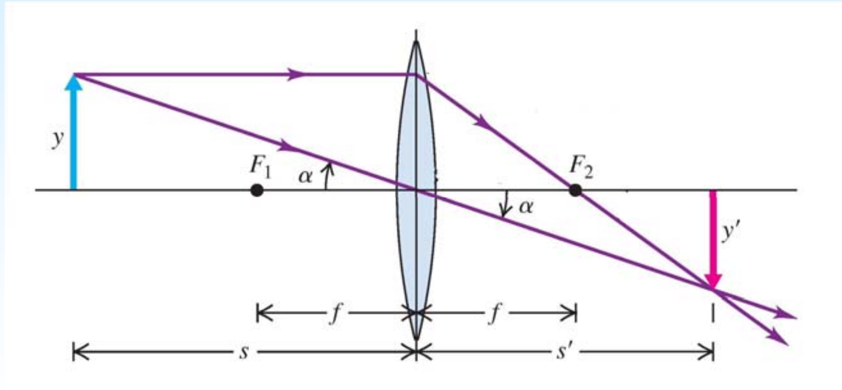




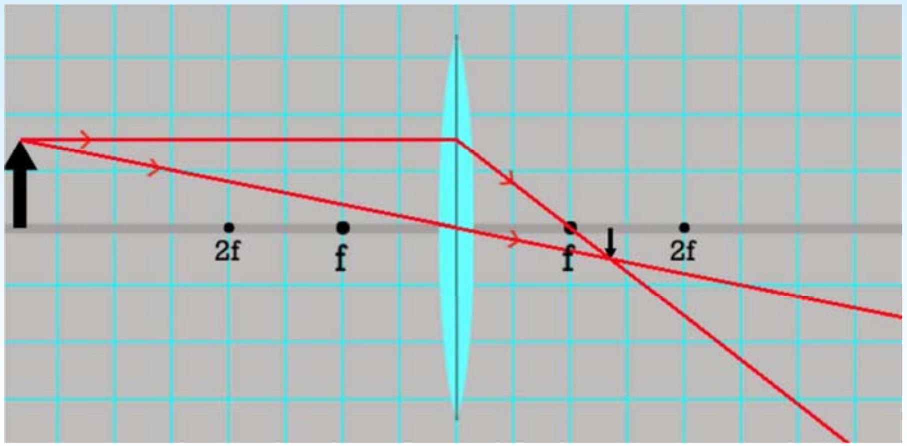


Geometrical optics

Useful rays



Geometrical optics



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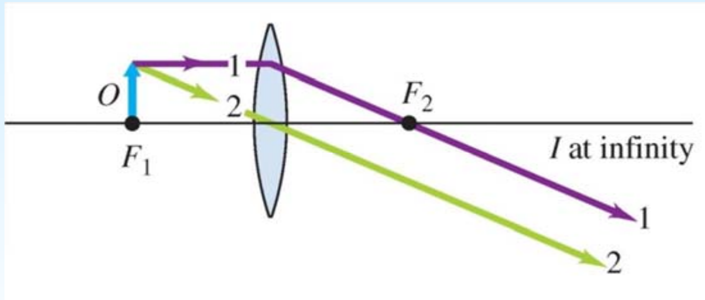
Geometrical optics



An object placed at the focal point appear to be at infinity

Geometrical optics

Sign rules:

Object distance (s) – positive if same side as incoming light.

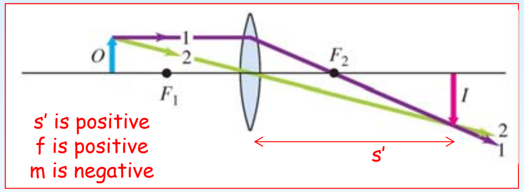
Image distance (s') – positive if same side as outgoing light.

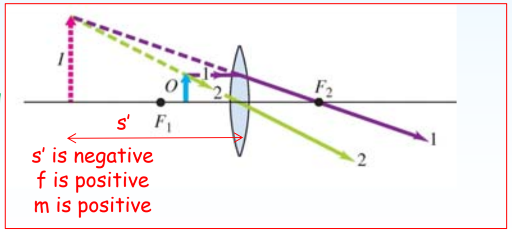
Focal length (f) – positive for converging lenses (convex lenses)

$$\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$$ Formelsamling

$$m=\frac{y'}{y}=-\frac{s'}{s}$$

Convex lenses -Summary





Geometrical optics

Gauss' formula

Formelsamling

$$\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$$

$$m=\frac{y'}{y}=-\frac{s'}{s}$$

Newton's formula

$$f=\frac{s\,s'}{s+s'}$$

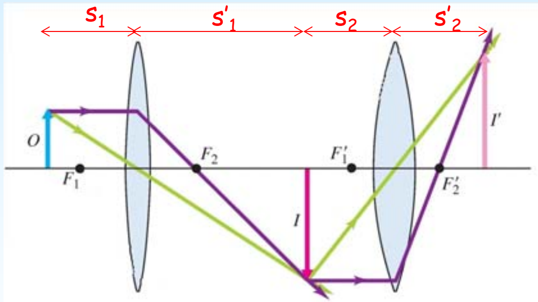
$$s=\frac{s'f}{s'-f}$$

$$s'=\frac{sf}{s-f}$$

$$m=-\frac{f}{s-f}$$

Geometrical optics

Two lenses



$$\frac{1}{f_1}=\frac{1}{s_1}+\frac{1}{s'_1} \qquad \frac{1}{f_2}=\frac{1}{s_2}+\frac{1}{s'_2}$$

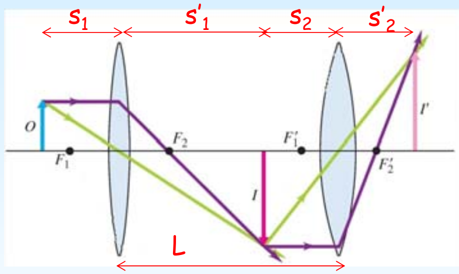
$$m_1=-\frac{s'_1}{s_1} \qquad m_2=-\frac{s'_2}{s_2} \quad \Rightarrow \quad m=\frac{I'}{O}=m_1m_2=\frac{s'_1}{s_1}\frac{s'_2}{s_2}$$

Geometrical optics

EXAMPLE
Known: s_1, f_1, f_2 and L
Calculate s'_2 and m

$$s=\frac{s'f}{s'-f} \qquad s'=\frac{sf}{s-f} \qquad m=\frac{y'}{y}=-\frac{s'}{s}$$

$$L=s'_1+s_2$$



$$s_2=L-s'_1=L-\frac{s_1f_1}{s_1-f_1}$$

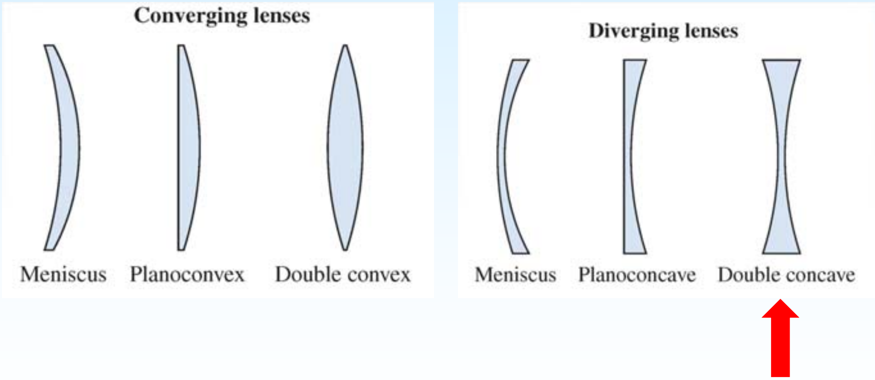
$$s'_2=\frac{s_2f_2}{s_2-f_2}=\frac{Lf_2-\frac{s_1f_1f_2}{s_1-f_1}}{L-\frac{s_1f_1}{s_1-f_1}-f_2}$$

$$m=\frac{I'}{O}=m_1m_2=\frac{s'_1}{s_1}\frac{s'_2}{s_2}=\frac{f_1}{s_1-f_1}\frac{s'_2}{L-s'_1}$$

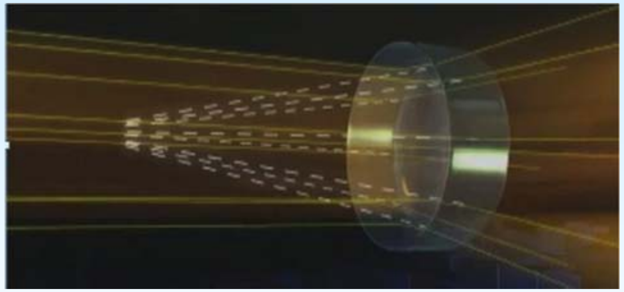
$$m=\frac{I'}{O}=\frac{f_1}{s_1-f_1}\frac{s'_2}{L-\frac{s_1f_1}{s_1-f_1}}=\frac{f_1}{(s_1-f_1)L-s_1f_1}s'_2$$

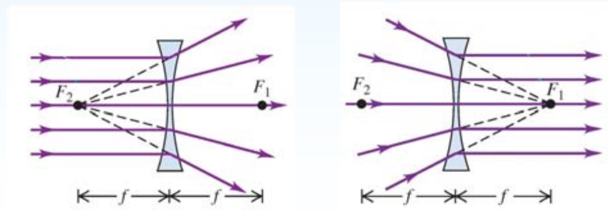
Geometrical optics

Lenses

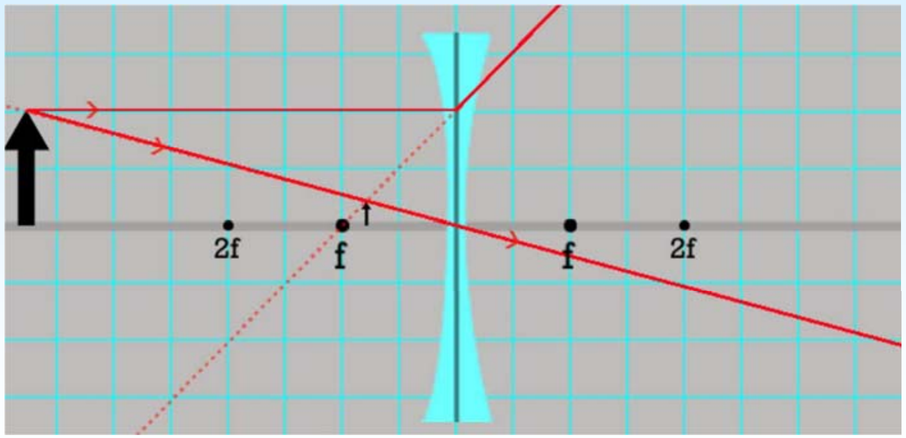


Geometrical optics





Geometrical optics



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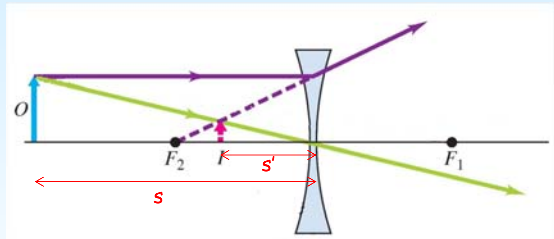
Geometrical optics

Lens formula for concave lenses



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

f is negative for diverging lenses

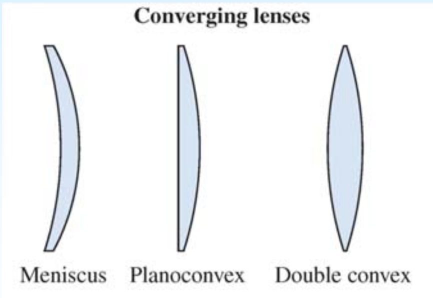
s' is negative for diverging lenses

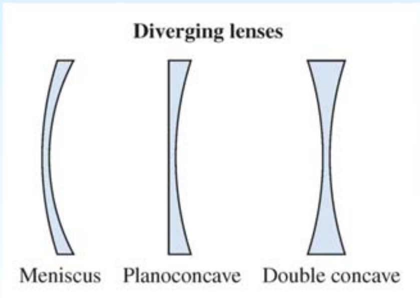
$$m = -\frac{s'}{s}$$

m is positive

Geometrical optics

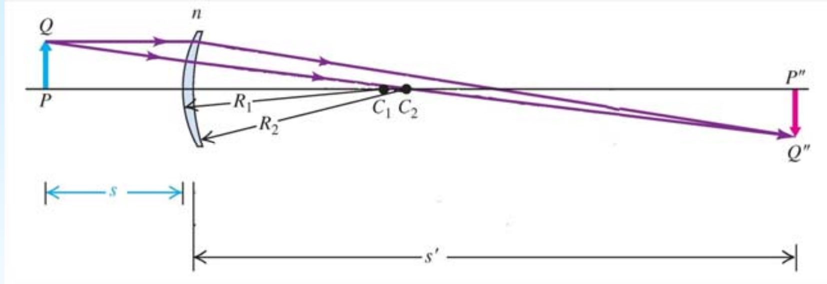
Lenses





Rule:
A lens that is thicker at the center than the edges is converging (positive f)
A lens that is thinner at the center than the edges is diverging (negative f)

Geometrical optics



The lensmaker's equation

$$\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Formelsamling

Geometrical optics

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad m = -\frac{s'}{s} \qquad m = \frac{y'}{y}$$

Sign rule: Radius of curvature – positive if center is on same side as outgoing light.

f = positive R_1 = positive R_2 = positive   s' = positive or negative

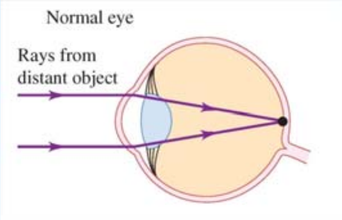
f = positive R_1 = positive R_2 = negative   s' = positive or negative

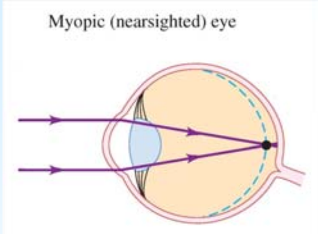
f = negative R_1 = negative R_2 = positive   s' = negative

Geometrical optics

The eye

Geometrical optics



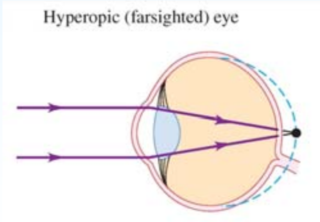


Near point: Closest distance to the eye at which people can see clear (7cm at age 10 to 40cm at age 50 for normal eye).

Normal reading distance: Assumed to be 25 cm when designing correction lenses.

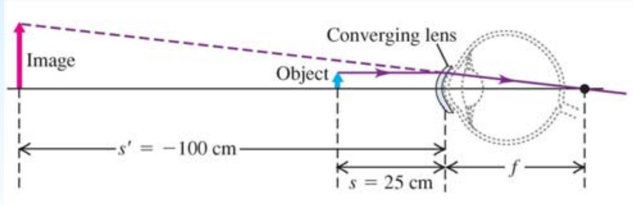
Lenses for corrections are given in diopter.

Lens power = 1/f (unit diopter = m^{-1})



Geometrical optics

The near point of a certain hyperopic eye is 100 cm in front of the eye. Find the focal length and power of the contact lens that will permit the wearer to see clearly an object that is 25 cm in front of the eye.

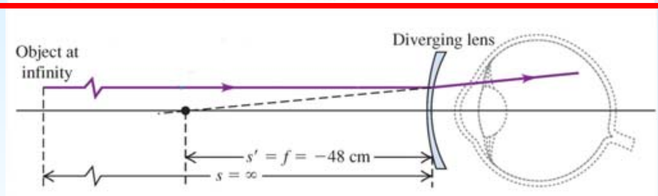


When the person puts an object at s = 25 cm from the correcting lens we want the image to end up at s' = 100 cm because this is the nearest point the eye can see sharply.

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{+25 \text{ cm}} + \frac{1}{-100 \text{ cm}}$$
$$f = +33 \text{ cm}$$

Geometrical optics

The far point of a certain myopic eye is 50 cm in front of the eye. Find the focal length and power of the eyeglass lens that will permit the wearer to see clearly an object at infinity. Assume that the lens is worn 2 cm in front of the eye.



The lens should move the actual far point from 50 cm to infinity.
The correcting lens should therefore have s = infinity for s' = 50-2 = 48 cm.

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty} + \frac{1}{-48 \text{ cm}}$$
$$f = -48 \text{ cm}$$

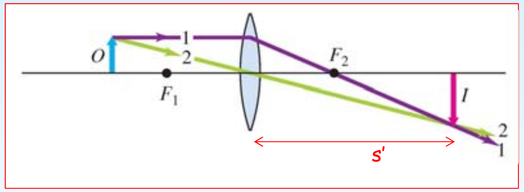
Geometrical optics

The magnifying glass

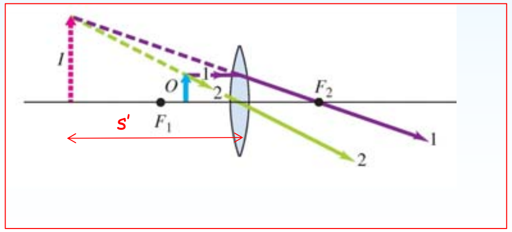
Geometrical optics

A magnifying glass is a convex lens.

If you hold a magnifying glass far away from the eye (arms lengths distance) you can see a magnified and up-side down image.

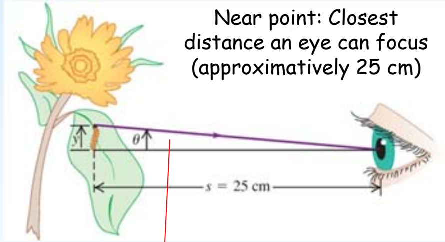


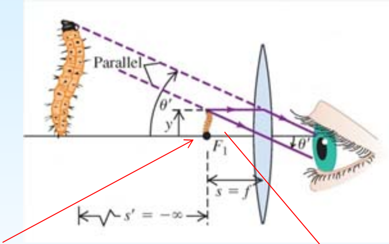
The normal use of a magnifying glass is to put the object between the focal point and the lens to get a magnified up-right image.



Geometrical optics

The magnifying glass





$$\theta = \frac{y}{25\ \text{cm}}$$

When the object is at the focal point one uses angular magnification (M) instead of lateral magification (m).

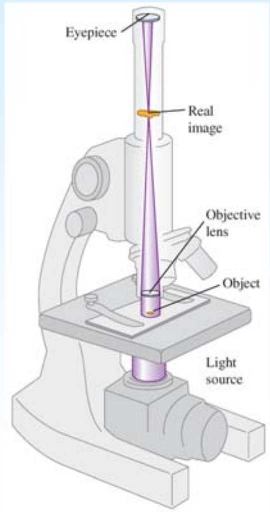
$$\theta' = \frac{y}{f}$$

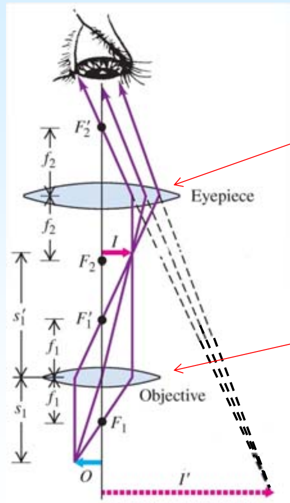
$$M = \frac{\theta'}{\theta} = \frac{y/f}{y/25\ \text{cm}} = \frac{25\ \text{cm}}{f} \quad \text{(angular magnification for a simple magnifier)}$$

Geometrical optics

The microscope

Geometrical optics

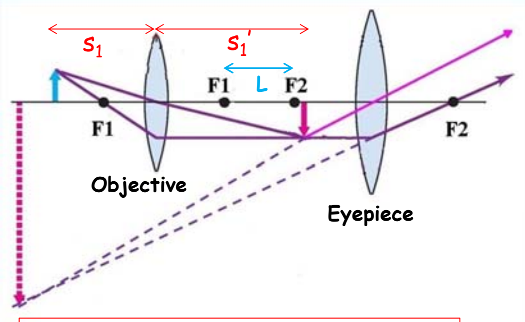




Magnifying glass (f is a couple of cm)

Creates magnified image close to the focal point of the eye piece (f < 1 cm)

Geometrical optics



Eyepiece

Angular magnification of magnifying glass

$M = \frac{\sigma}{f}$ where $\sigma = 25$ cm

Objective

$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \Rightarrow s = \frac{s'f}{s'-f}$ $\quad s' \approx f + L$

$m = -\frac{s'}{s} = -\frac{s'-f}{f} \approx \frac{f+L-f}{f} = \frac{L}{f}$

Microscope

Magnification

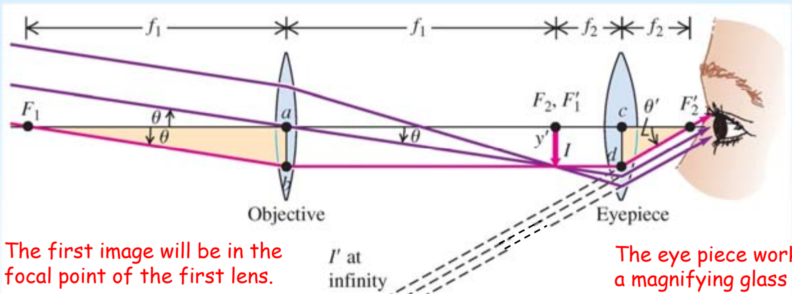
$$M = m_1 M_2 = -\frac{s'_1}{s_1}\frac{\sigma}{f_2} = -\frac{L\sigma}{f_1 f_2}$$

σ is the nearpoint which is typically 25 cm

Geometrical optics

The telescope

Geometrical optics



The first image will be in the focal point of the first lens.

The eye piece works as a magnifying glass with I in its focal point.

$\tan(\theta) = \theta = \frac{-y'}{f_1}$ $\quad \tan(\theta') = \theta' = \frac{y'}{f_2}$

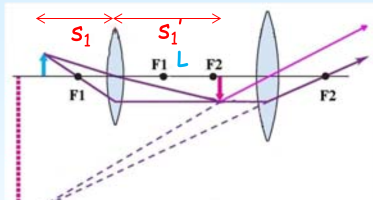
The angular magnification of a telescope is defined as the ratio of the angle of the image to that of the incoming light.

$$M = \frac{\theta'}{\theta} = -\frac{y'/f_2}{y'/f_1} = -\frac{f_1}{f_2}$$

Geometrical optics

Comparing microscopes with telescopes

Object at a close distance

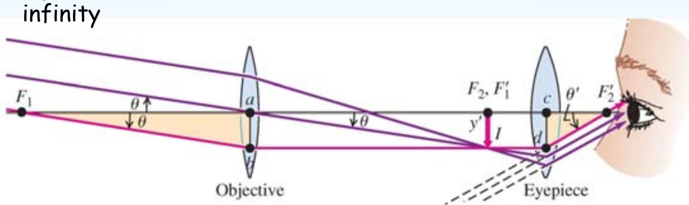


Small f_1 & Small f_2

$$M = m_1 M_2 = -\frac{s'_1}{s_1}\frac{\sigma}{f_2} = -\frac{L\sigma}{f_1 f_2}$$

σ is the nearpoint which is typically 25 cm

Object at infinity

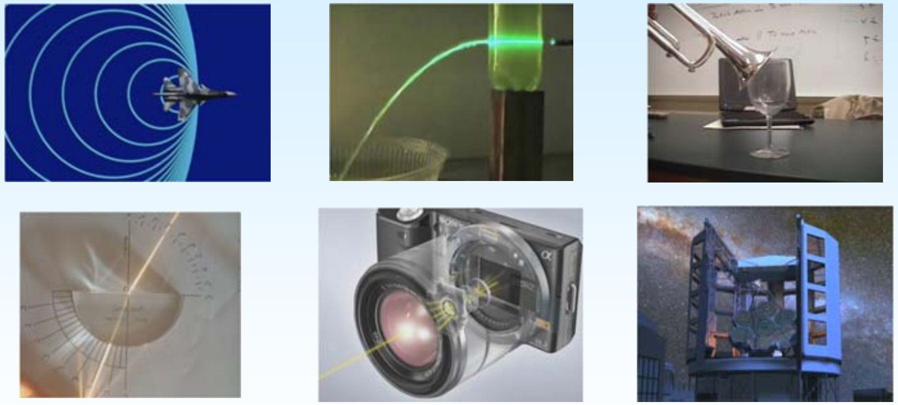


Large f_1 & Small f_2

Formelsamling

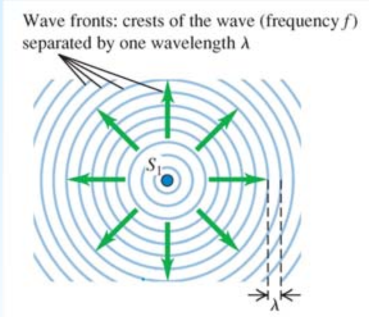
$$M = -\frac{f_1}{f_2}$$

Vågrörelselära och optik



Kapitel 35 - Interferens

Interference



Interference: Wave overlap in space

Coherent sources: Same frequency (or wavelength) and constant phase relationship (not necessarily in phase).

The principle of superposition states:

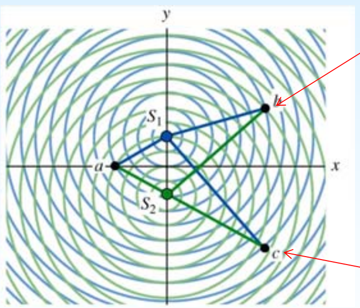
When two or more waves overlap, the resultant displacement at any point and at any instant is found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.

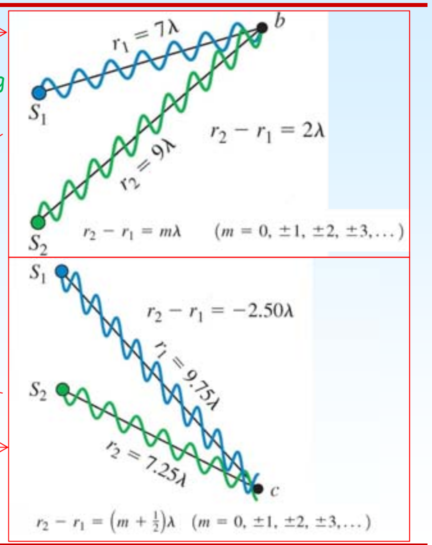
Interference

Contructive interference

$$\delta = r_2 - r_1 = m\lambda$$

Formelsamling





Destructive interference

$$\delta = r_2 - r_1 = \left(m + \tfrac{1}{2}\right)\lambda$$

Interference

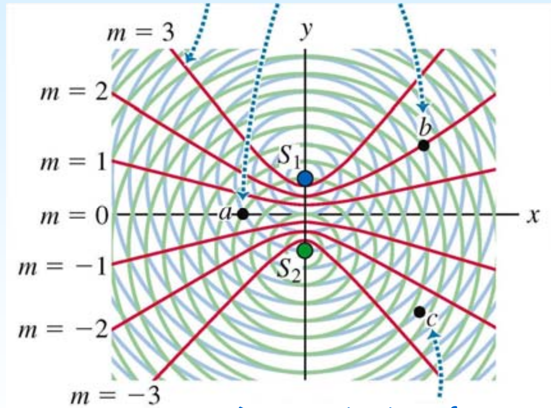
Antinodal curves = Contructive interference

A path difference of one wavelength corresponds to a phase difference of 2π

$$\frac{\phi}{2\pi} = \frac{r_2 - r_1}{\lambda}$$

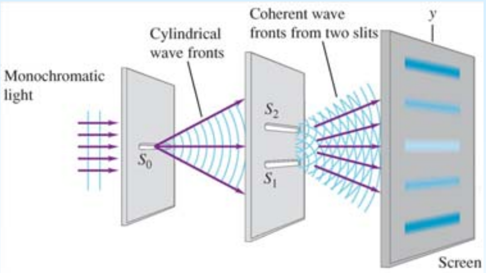
Formelsamling

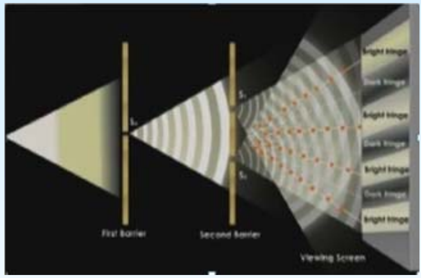
Contructive interference

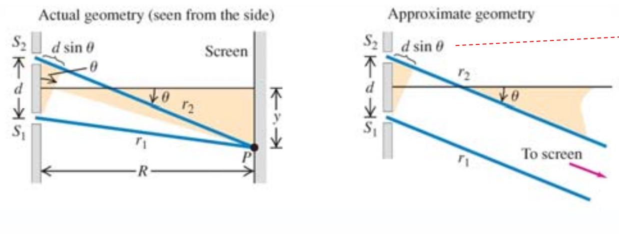


Destructive interference

Interference







$$\delta = r_2 - r_1 = d\,sin(\theta)$$

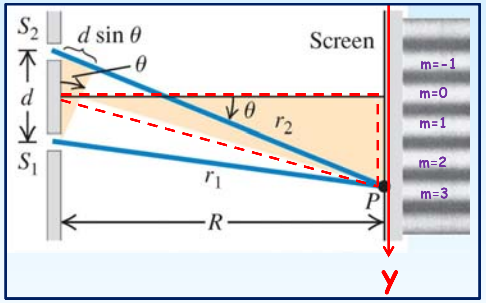
Contructive

$$d \sin\theta = m\lambda$$

Destructive

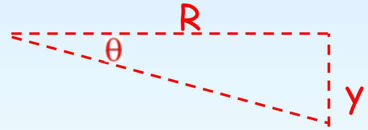
$$d \sin\theta = \left(m + \tfrac{1}{2}\right)\lambda$$

Interference



Geometry:

$$r_2 - r_1 = d\ \sin(\theta) \approx d\ \theta$$



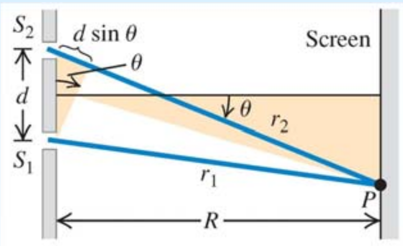
$$y = R\tan(\theta) \approx R\,\theta \approx R\,(r_2 - r_1) / d$$

Contructive interference:

$$r_2 - r_1 = m\,\lambda$$

$$y_m = R\frac{m\lambda}{d}$$

Interference



Path difference

$$r_2 - r_1 = d\sin\theta$$

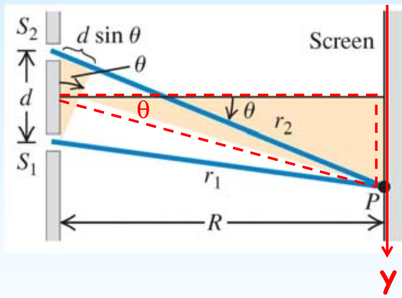
A path difference of one wavelength corresponds to a phase difference of 2π

$$\frac{\phi}{2\pi} = \frac{r_2 - r_1}{\lambda}$$

$$\phi = \frac{2\pi d}{\lambda}\sin\theta$$

Interference

Introduce y in the formula



$$\phi = \frac{2\pi d}{\lambda}\sin\theta$$

$$\tan(\theta) = y / R \approx \sin(\theta)$$

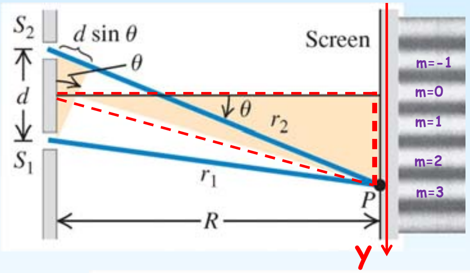
small θ

$$\phi = \frac{2\pi d}{\lambda}\sin\theta \approx \frac{2\pi d y}{\lambda R}$$

Formelsamling

$$\phi = \frac{2\pi\delta}{\lambda}$$

Interference

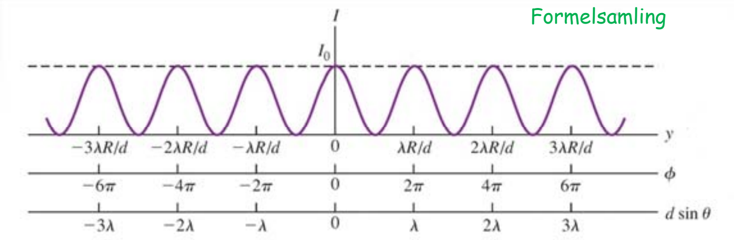


Intensity:

$$I = I_0 \cos^2 \frac{\phi}{2} = I_0 \cos^2\left(\frac{\pi d y}{\lambda R}\right)$$

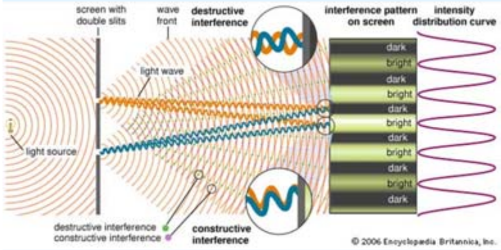
$I = I_0 \cos^2 \frac{\phi}{2}$ $\quad I_0$ = intensiteten rakt fram

Formelsamling



Interference

Summary



Contructive interference:

$$r_2 - r_1 = d \sin(\theta) = m\,\lambda$$

$$y_m \approx m \cdot (R\,\lambda / d)$$

Intensity:

$$I = I_0 \cos^2 \frac{\phi}{2}$$

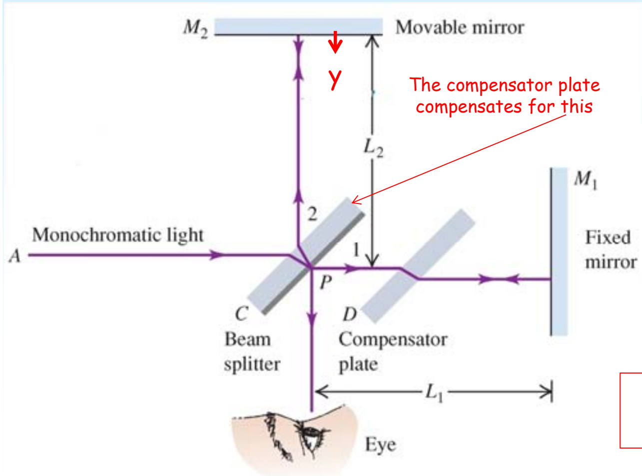
$$\phi = \frac{2\pi d}{\lambda} \sin\theta \approx \frac{2\pi d y}{\lambda R}$$

Interference

The Michelson Interferometer

Interference

The Michelson Interferometer



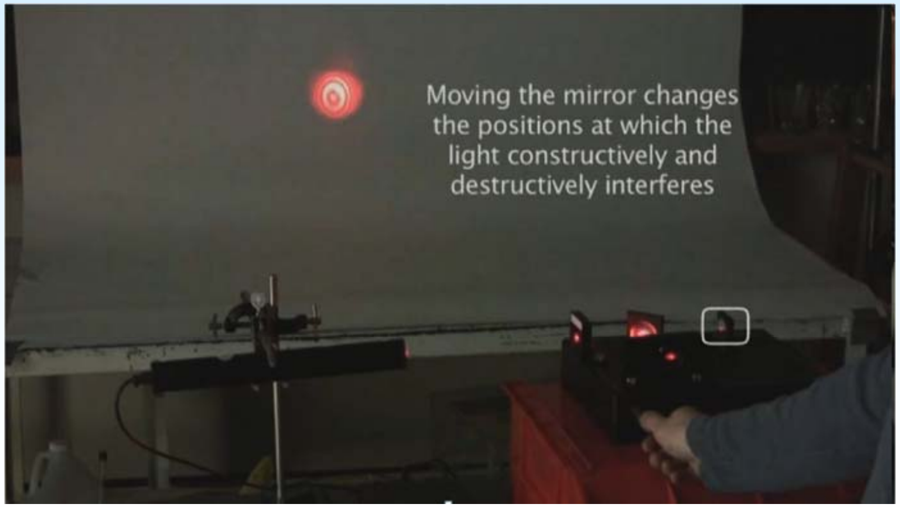
The observer will see an **interference pattern** with rings.

The **fringes** in the pattern will **move** when the mirror is moved.

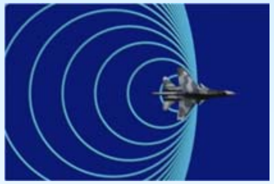
The number of fringes (m) can be used to calculate y or λ

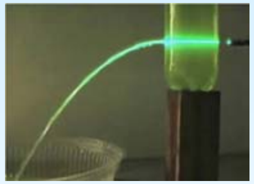
$$y = m\frac{\lambda}{2} \qquad \lambda = \frac{2y}{m}$$

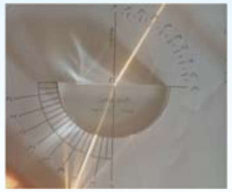
Interference

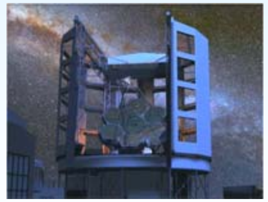


Vågrörelselära och optik



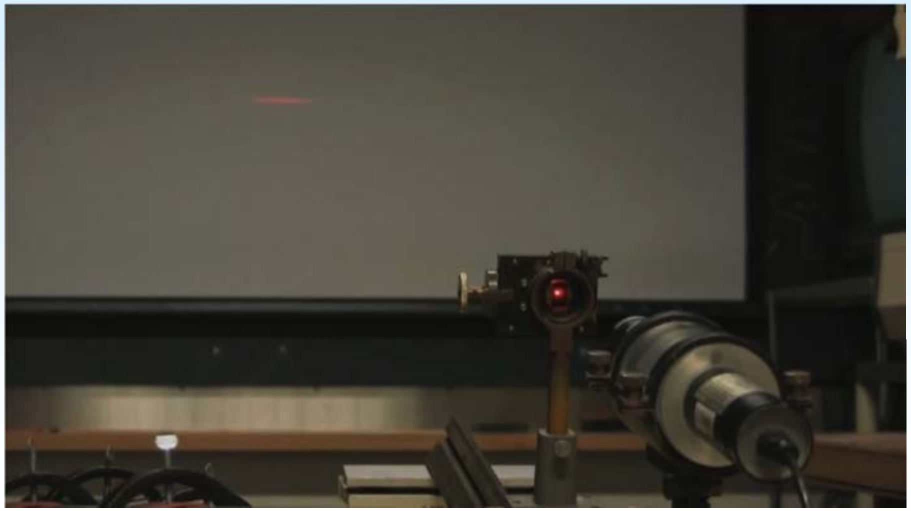






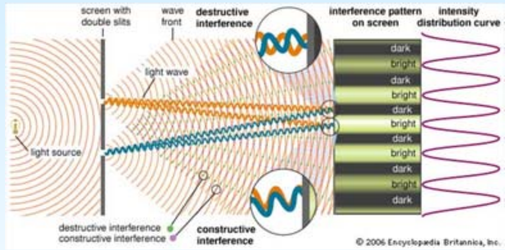
Kapitel 36 - Diffraktion

Diffraction

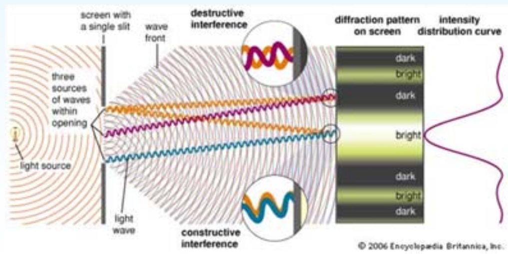


Diffraction

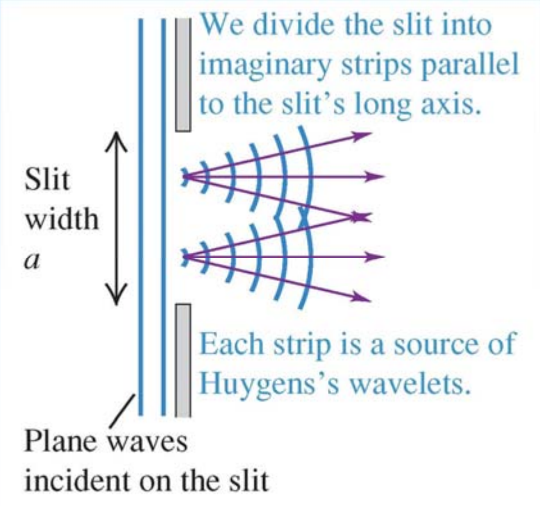
Interference: Double slit experiment



Diffraction: single slit experiment

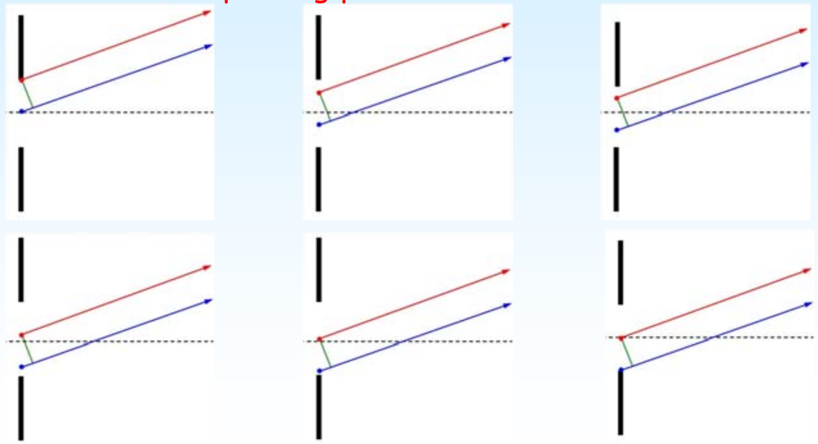


Diffraction

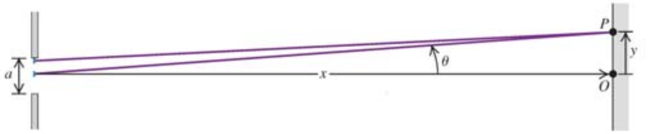


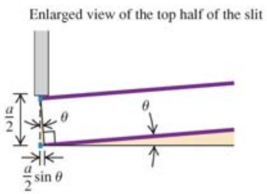
Diffraction

For every point in the top half of the slit there is a corresponding point in the bottom half.



Diffraction





Formelsamling

$$a \sin\theta = m\lambda \quad (m \neq 0)$$

Destructive Interference:

$$\frac{a}{2}\sin\theta = \pm\frac{\lambda}{2}\,\mathbf{m}$$

Geometry:

$$\tan(\theta) = y / x$$

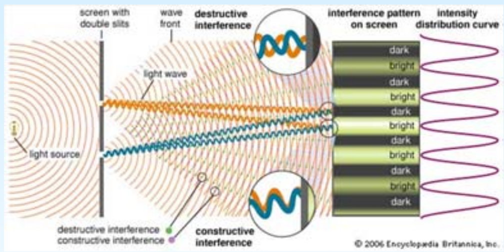
Small angles:

$$\tan(\theta) \approx \theta$$
$$\sin(\theta) \approx \theta$$

$$y_m = x\frac{m\lambda}{a} \quad (\text{for } y_m \ll x)$$

$$m = \pm 1, \ \pm 2,$$

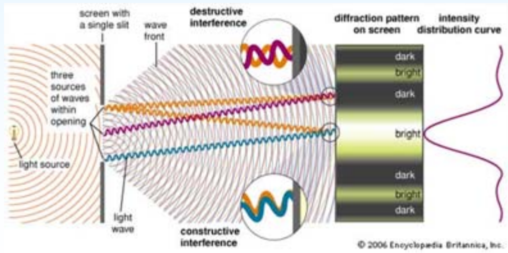
Diffraction



Bright bands:

$$y_m = R\frac{m\lambda}{d}$$

$$m = \mathbf{0}, \ \pm 1, \ \pm 2,$$

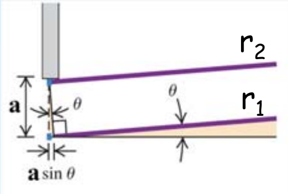


Dark bands:

$$y_m = x\frac{m\lambda}{a}$$

$$m = \pm 1, \ \pm 2,$$

Diffraction



Path difference:
$r_2 - r_1 = a \sin(\theta)$

A path difference of one wavelength corresponds to a phase difference of 2π

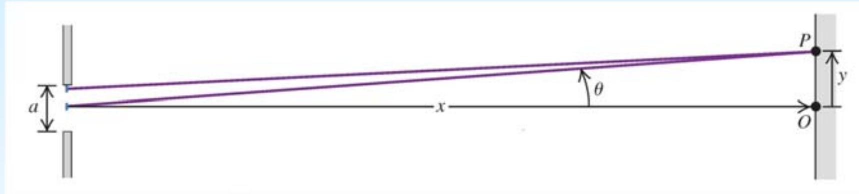
$$\frac{\beta}{2\pi} = \frac{r_2 - r_1}{\lambda}$$

r_2-r_1 is the path difference between a ray at the top and bottom of the slit.

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

Formelsamling

Diffraction



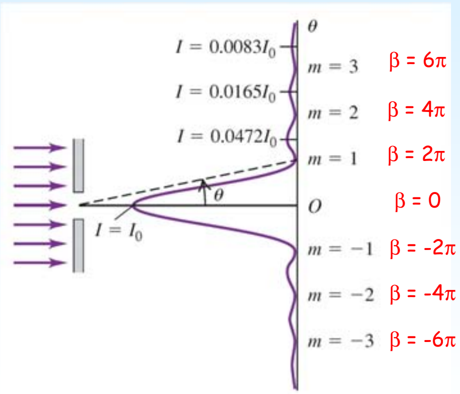
$\tan(\theta) = y / x \approx \sin(\theta)$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta \approx \frac{2\pi a y}{\lambda x}$$

Diffraction

Intensity



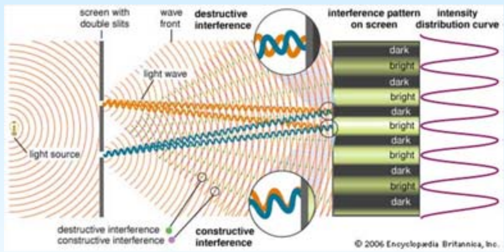
$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

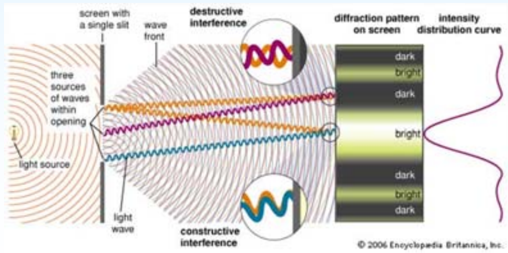
where

$$\beta = \frac{2\pi}{\lambda} a \sin\theta \approx \frac{2\pi a y}{\lambda x}$$

Formelsamling

Summary





Intensity:

$$I = I_0 \cos^2 \frac{\phi}{2}$$

$$\phi = \frac{2\pi d}{\lambda} \sin\theta$$

$\tan(\theta) = y / R \approx \sin(\theta)$

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

$\tan(\theta) = y / x \approx \sin(\theta)$

Two broad slits

In the analysis of interference from two slits it was assumed that they were very narrow. What if they are broad ?

Two narrow slits:

$$I = I_0 \cos^2 \frac{\phi}{2}$$

One broad slit:

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

Two broad slits:

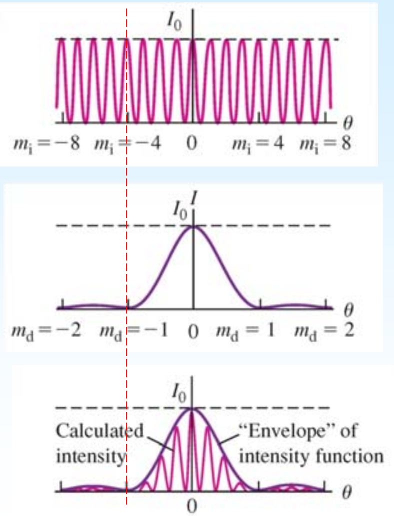
$$I = I_0 \cos^2 \frac{\phi}{2} \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

where

$$\phi = \frac{2\pi d}{\lambda} \sin\theta$$

$$\beta = \frac{2\pi a}{\lambda} \sin\theta$$

Formelsamling



Two narrow slits:

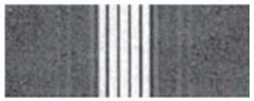
$$I = I_0 \cos^2 \frac{\phi}{2}$$

One broad slit:

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

Two broad slits:

$$I = I_0 \cos^2 \frac{\phi}{2} \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

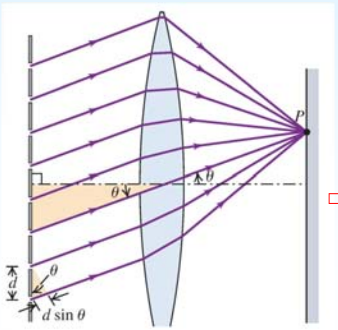


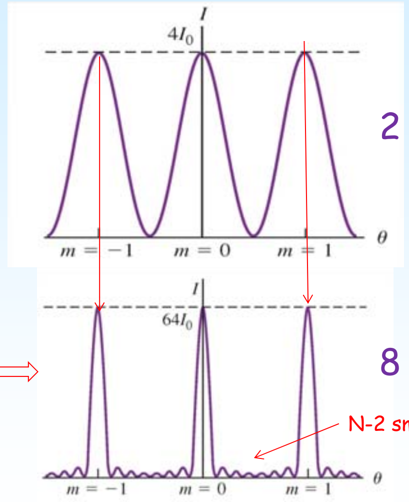
Multiple slits

Diffraction

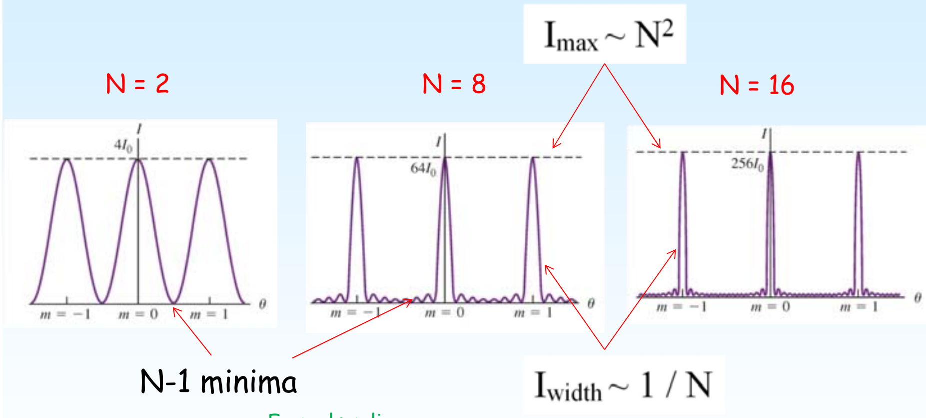
The path difference between adjacent slits that gives maximum intensity with many slits is always:

$$d \sin\theta = m\lambda \qquad (m = 0, \pm1, \pm2, \dots)$$





Diffraction



Formelsamling

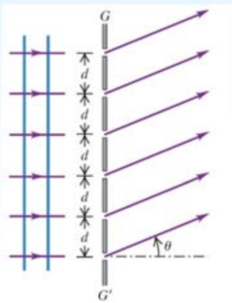
Principal maxima: $d \sin\theta = m\lambda \qquad (m = 0, \pm1, \pm2, \pm3, \dots)$

Diffraction

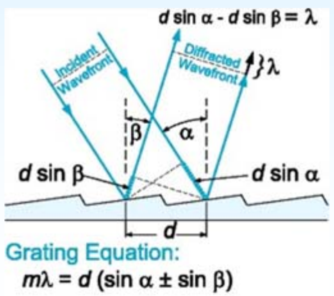
In diffraction grating one uses devices with thousands of slits or reflecting surfaces.

This gives very narrow principal maximum that can be used to determine the wavelength of light.

Transmission grating



Reflection grating

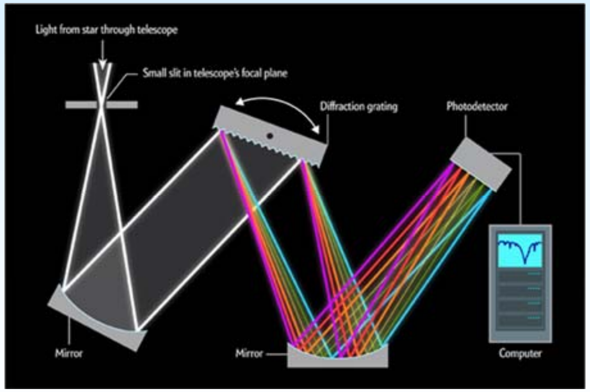


Diffraction

Spectrometers

Diffraction

Spectrometer for astronomy



Light incident on a grating is dispursed into a spectrum. The angles of deviations of the maxima are measured to calculate the wave length.

Diffraction

Chromatic resolving power:

The minimum wavelength difference ($\Delta\lambda$) that can be distinguished by a spectrograph.

$$R = \frac{\lambda}{\Delta\lambda} \quad \text{(chromatic resolving power)}$$

$$R = \frac{\lambda}{\Delta\lambda} = Nm$$

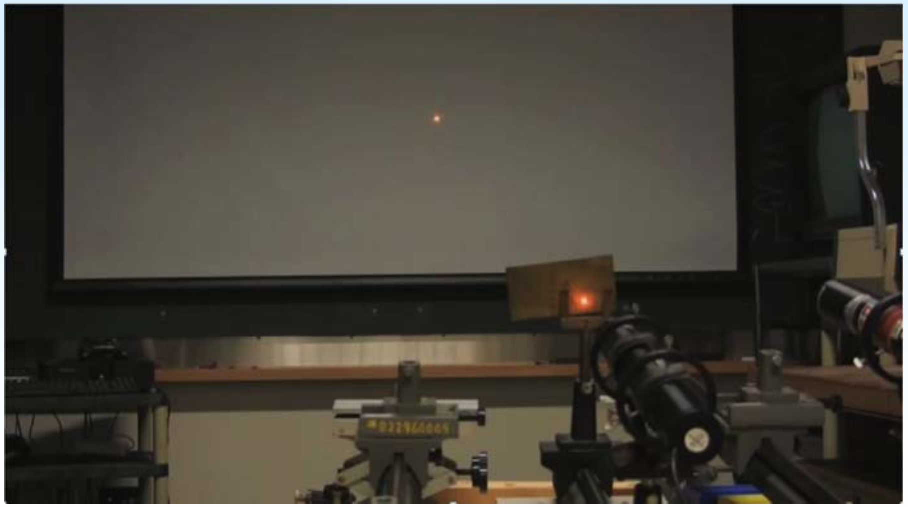
Formelsamling

R is higher for many slits and higher orders !

Diffraction

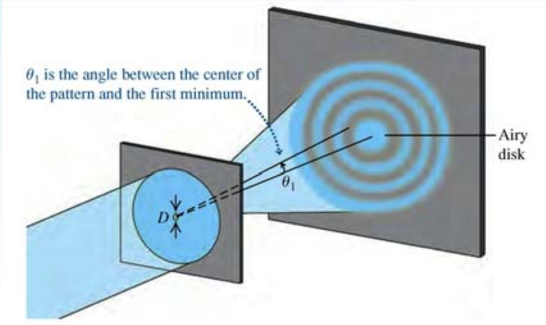
Pinhole diffraction

Diffraction



Diffraction

Diffraction limits the angular resolution of optical intruments.

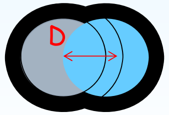


$\sin\theta_1 = 1.22\frac{\lambda}{D}$ (diffraction by a circular aperture)

Diffraction

Rayleigh's criterion:
Two point objects can be resolved by an optical system if their angular separation is larger than θ_1 where $\sin\theta_1 = 1.22\frac{\lambda}{D}$

The limit for two objects to be resolved is when the center of one diffraction pattern is in the first minimum of the other.



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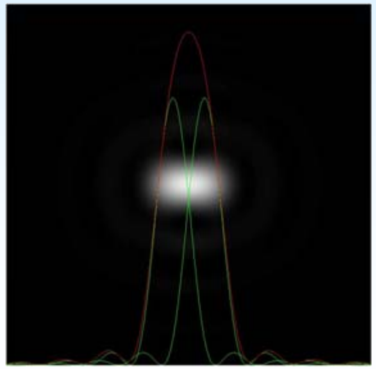
$\theta_c = \frac{1.22 \cdot \lambda}{d}$ (runt hål)



The End