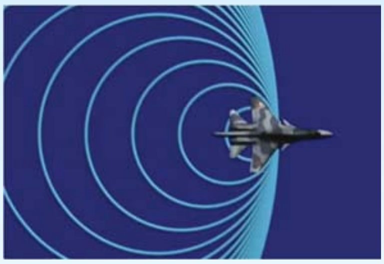
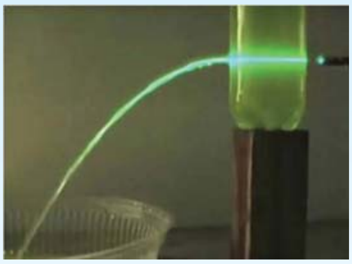
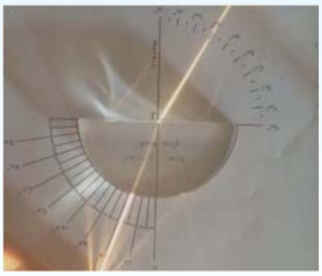
Vågrörelselära och optik







Kapitel 36 - Diffraktion

Vågrörelselära och optik

Kurslitteratur: University Physics by Young & Friedman

Harmonisk oscillator:	Kapitel 14.1 – 14.4
Mekaniska vågor:	Kapitel 15.1 – 15.8
Ljud och hörande:	Kapitel 16.1 – 16.9
Elektromagnetiska vågor:	Kapitel 32.1 & 32.3 & 32.4
Ljusets natur:	Kapitel 33.1 – 33.4 & 33.7
Stråloptik:	Kapitel 34.1 – 34.8
Interferens:	Kapitel 35.1 – 35.5
Diffraktion:	Kapitel 36.1 - 36.5 & 36.7

Vågrörelselära och optik

Tid	Må 02-nov	Ti 03-nov	On 04-nov	To 05-nov	Fr 06-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 14	Kvantfysik (A)	Våglära/optik (A)	Kvantfysik (A)
10-12	Intro period 2 (A) Informationssökning (A)	Kvantfysik (A)	Våglära/optik kap 14+15 ÄFYA11 (L218)	Kvantfysik (A)	Kvantfysik (A) kap 15
13-15	Utvärdering (A) 12-13	Övningar Optik&Våg (L218-19)	SI gp6-10 (L219) ÄFYA11 (L218)	SI gp11-15 (L219)	Övningar Optik&Våg (L218-19)
15-17					

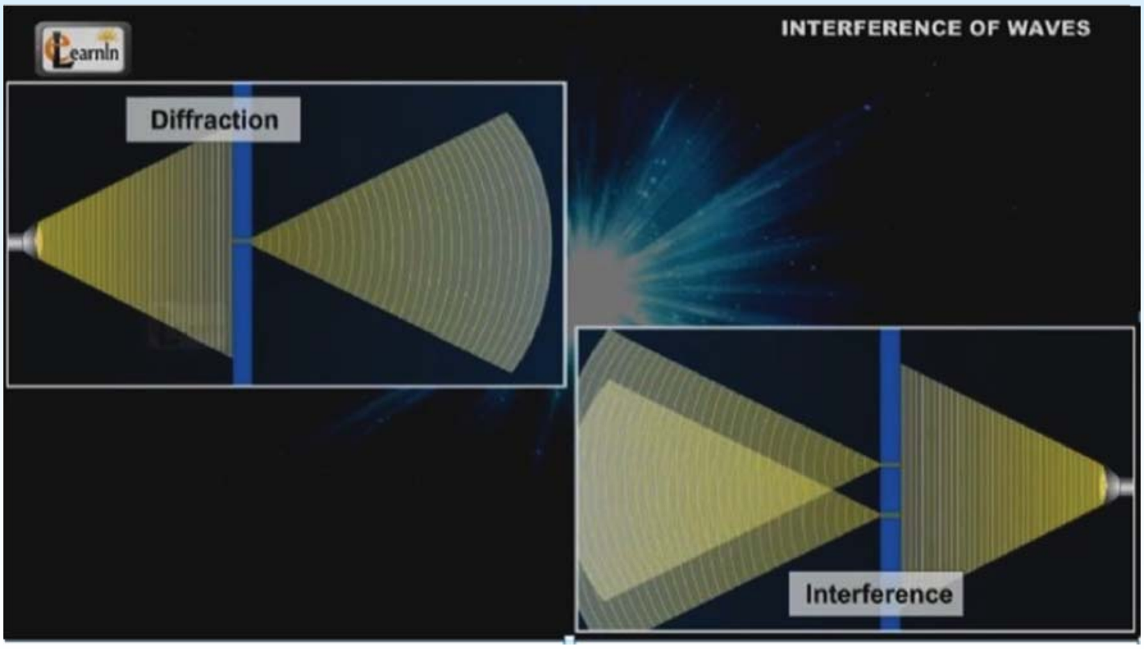
Tid	Må 09-nov	Ti 10-nov	On 11-nov	To 12-nov	Fr 13-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 16	Våglära/optik (A) kap 16+32	Kvantfysik (A)	Kvantfysik (A)
10-12	Våglära/optik kap 15+16 ÄFYA11 (L218)	Kvantfysik (A)	Kvantfysik (A)	Våglära/optik (A) kap 32+33	Våglära/optik (A) kap 33
13-15	SI gp1-5 (L219) ÄFYA11 (L218)	Övningar Optik&Våg (L218-19)	ÄFYA11 (l218) SI gp6-10 (L219)	SI gp1-5 (L218) SI gp11-15 (L219)	Övningar Optik&Våg (L218-19)
15-17					

Tid	Må 16-nov	Ti 17-nov	On 18-nov	To 19-nov	Fr 20-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 34	Kvantfysik (A)	Våglära/optik (A) kap 35 ÄFYA11 (L218)	Våglära/optik (A) kap 36
10-12	Våglära/optik (A) kap 34	Kvantfysik (A)	Våglära/optik (A) kap 34+35	Våglära/optik (A) kap 36	Kvantfysik (A)
13-15	SI gp6-10 (L219)	Övningar Optik&Våg (L218-19)	Seminar.gen.gång (A) 12-13 SI gp1-5 (L218) 13-15 SI gp11-15 (L219) 13-15	Labbintroduktion (A) O2-O3, K1-K3	Övningar Optik&Våg (L218-19)
15-17					

Diffraction

What is diffraction ?

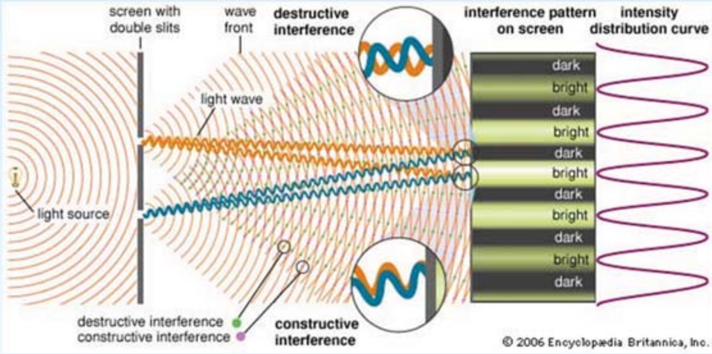
Diffraction



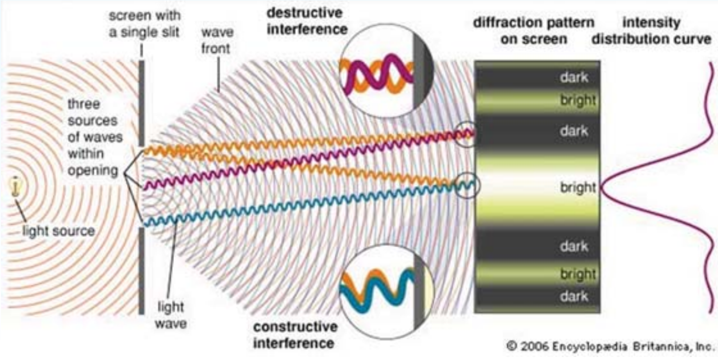
Diffraction

Diffraction

Interference: Double slit experiment

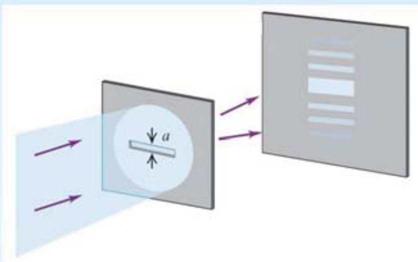


Diffraction: single slit experiment

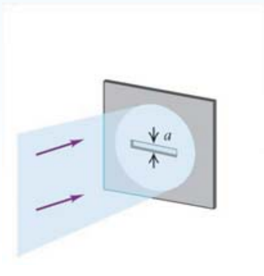


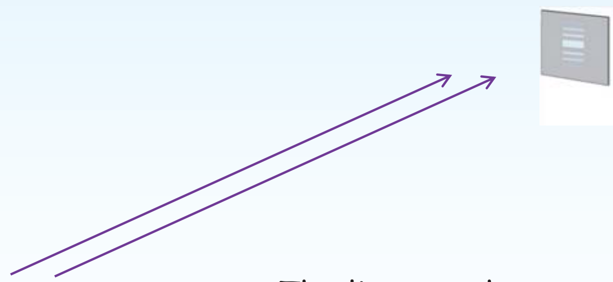
Diffraction

Fresnel diffraction or near-field diffraction.



Frauenhofer diffraction or far-field diffraction.





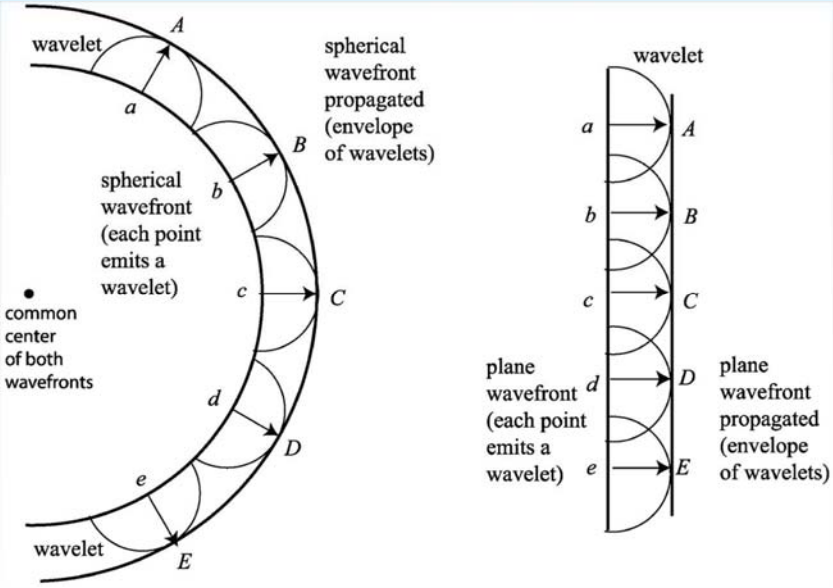
The lines to the screen are assumed to be parallel

Diffraction

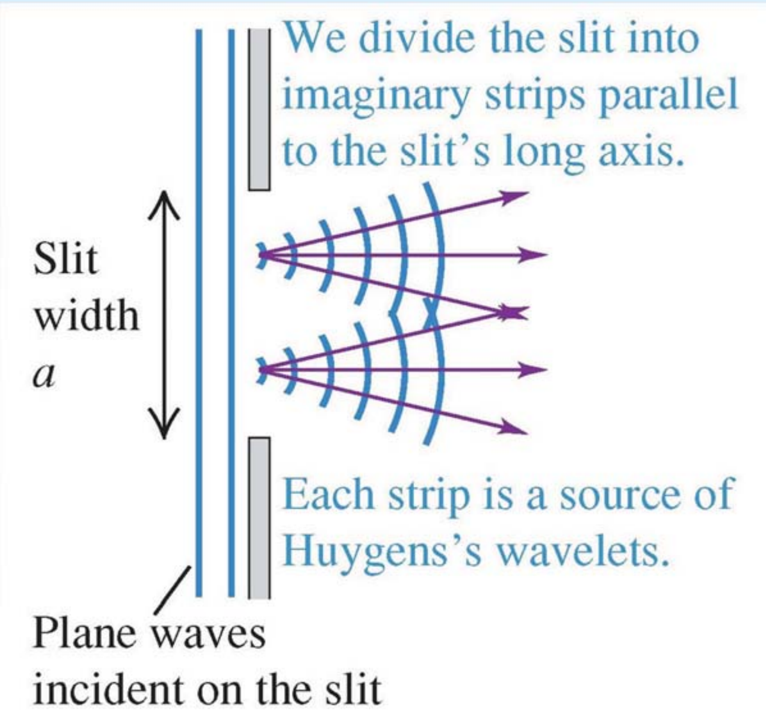
Huygen's principle

Each point in a wavefront is regarded as a new source of secondary wavelets.

All the combined circles (wavelets) from all the points add up to create the new wavefronts.

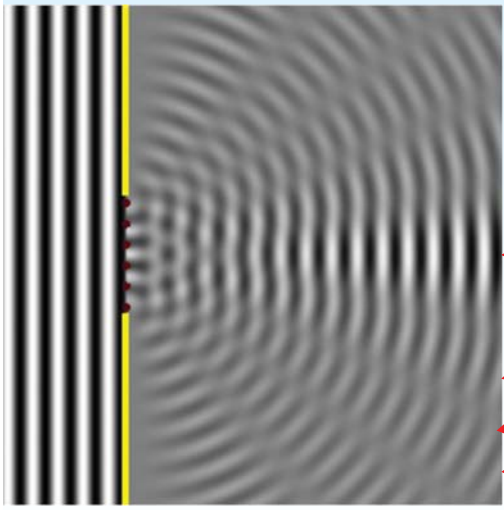


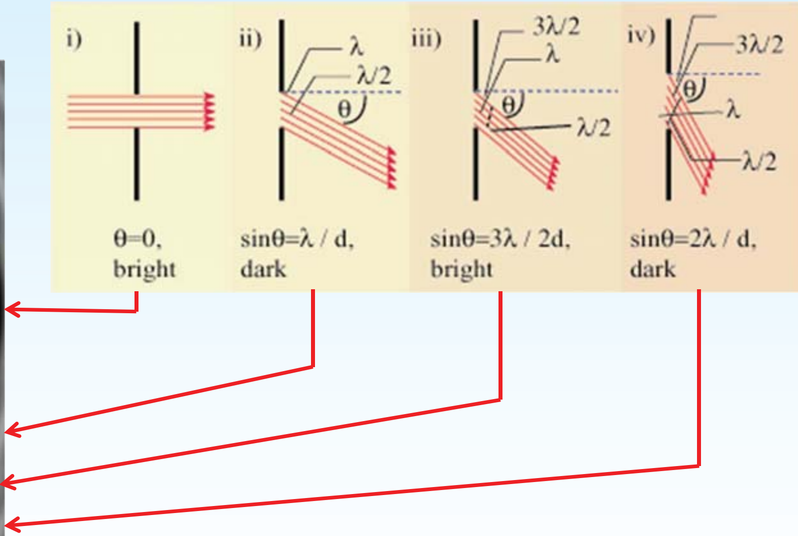
Diffraction



Diffraction

Interference from many points in a slit





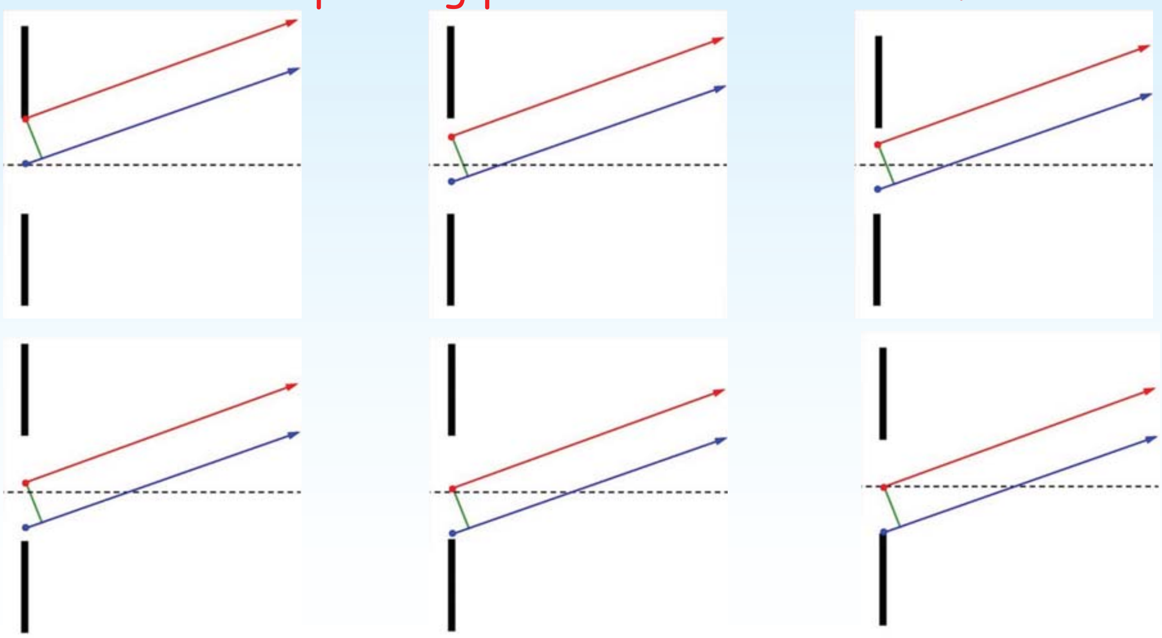
$d \sin(\theta) = m\lambda$ - destructive interference

$m = 1, 2, 3$ ....

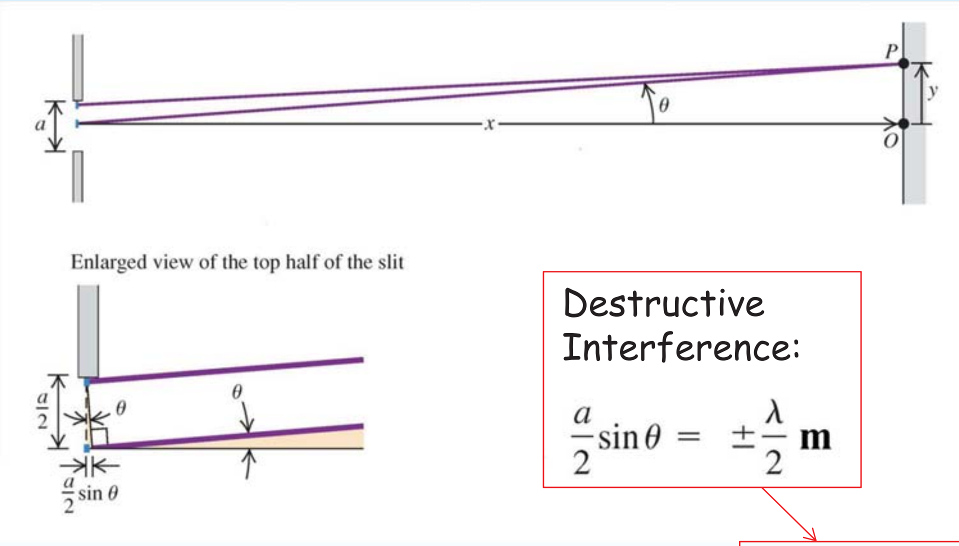
d: width of the slit

Diffraction

For every point in the top half of the slit there is a corresponding point in the bottom half.



Diffraction



Geometry:

$\tan(\theta) = y / x$

Destructive Interference:

$$\frac{a}{2}\sin\theta = \pm\frac{\lambda}{2}\,\mathbf{m}$$

Small angles:

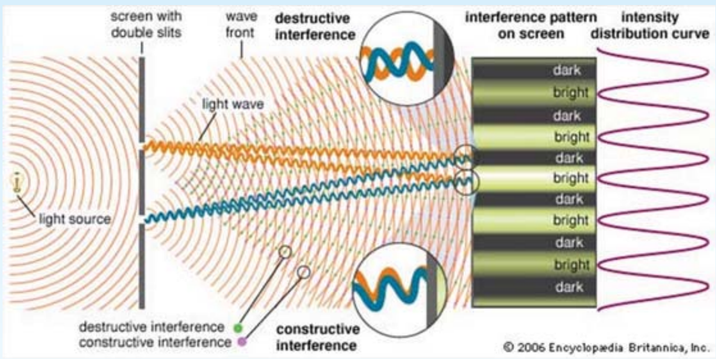
$\tan(\theta) \approx \theta$

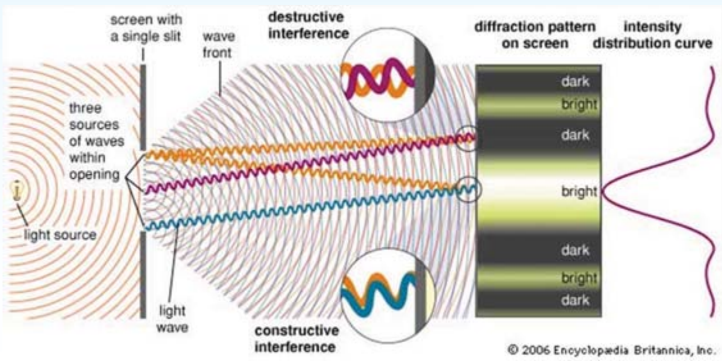
$\sin(\theta) \approx \theta$

$$y_m = x\frac{m\lambda}{a} \qquad (\text{for } y_m \ll x)$$

$$m = \pm 1, \ \pm 2,$$

Diffraction





Bright bands:

$$y_m = R\frac{m\lambda}{d}$$

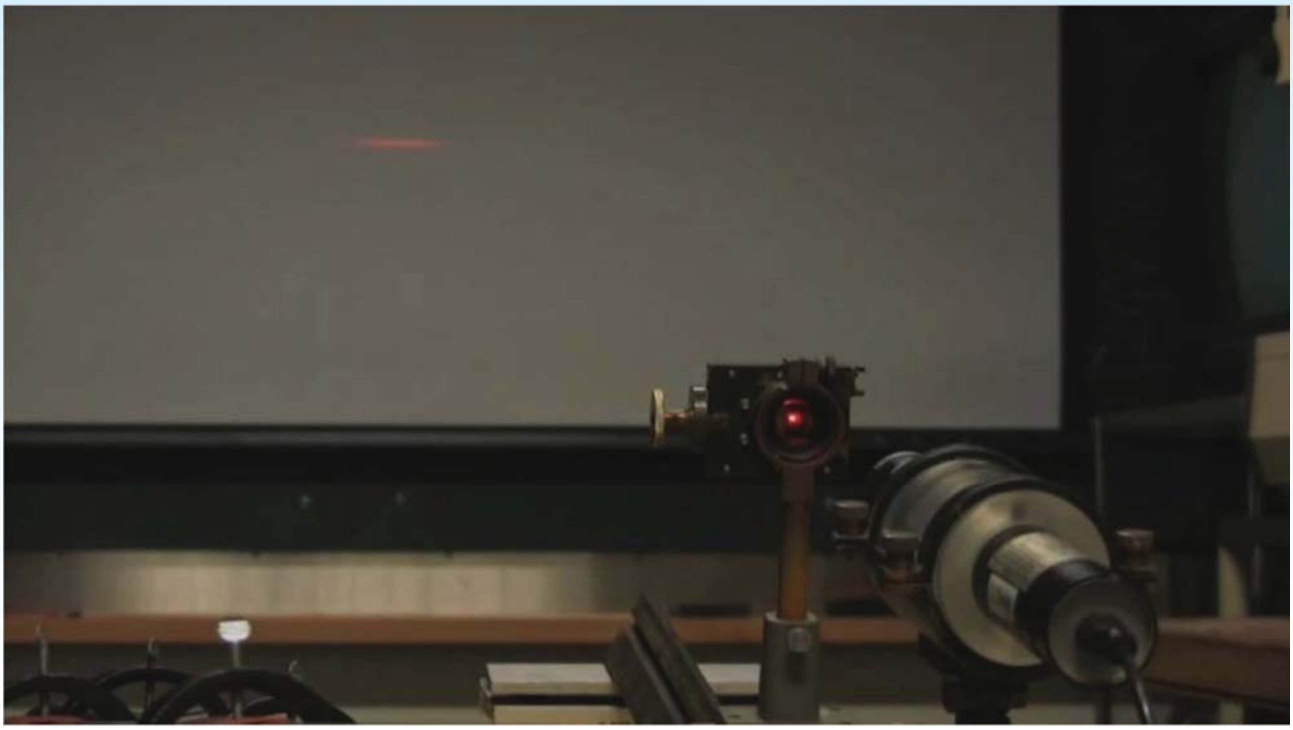
$$m = \mathbf{0}, \ \pm 1, \ \pm 2,$$

Dark bands:

$$y_m = x\frac{m\lambda}{a}$$

$$m = \pm 1, \ \pm 2,$$

Diffraction

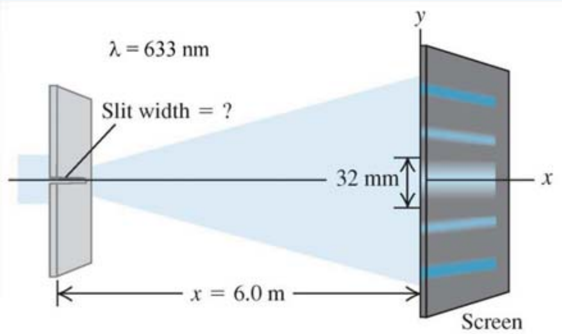


Diffraction

Problem solving

Diffraction

You pass 633-nm laser light through a narrow slit and observe the diffraction pattern on a screen 6.0 m away. The distance on the screen between the centers of the first minima on either side of the central bright fringe is 32 mm How wide is the slit?



$$y_m = x\frac{m\lambda}{a}$$

$$y = (32\text{ mm})/2 = 16\text{ mm}$$

$$a = \frac{x\lambda}{y} = \frac{(6.0\text{ m})(633 \times 10^{-9}\text{ m})}{16 \times 10^{-3}\text{ m}} = 2.4 \times 10^{-4}\text{ m} = 0.24\text{ mm}$$

Diffraction

Intensity for single slit diffraction

Diffraction

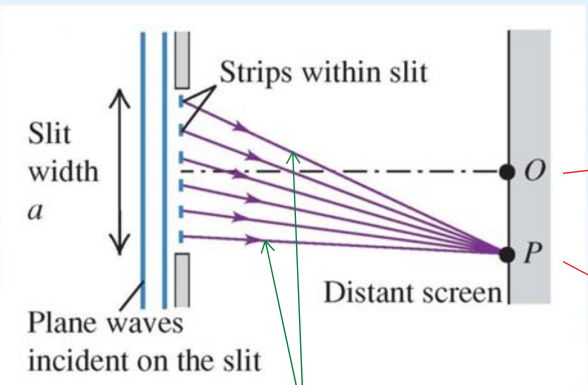
The intensity of light (I) is proportional to the square of the amplitude of the total electric field (E_p)

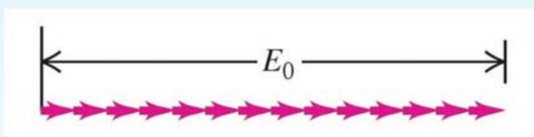
$$I \sim E_p^2$$

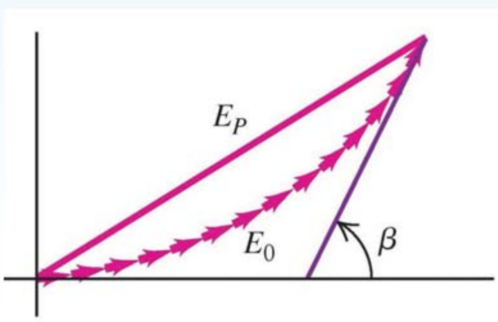
So what is E_p ?

Diffraction

Assume many small phasors with a total length E_0 are giving the total electric field strength (E_p)



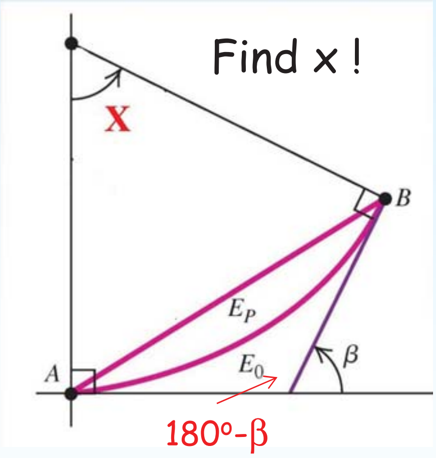




β is the phase difference between a ray at the top and bottom of the slit.

Diffraction

Step 1

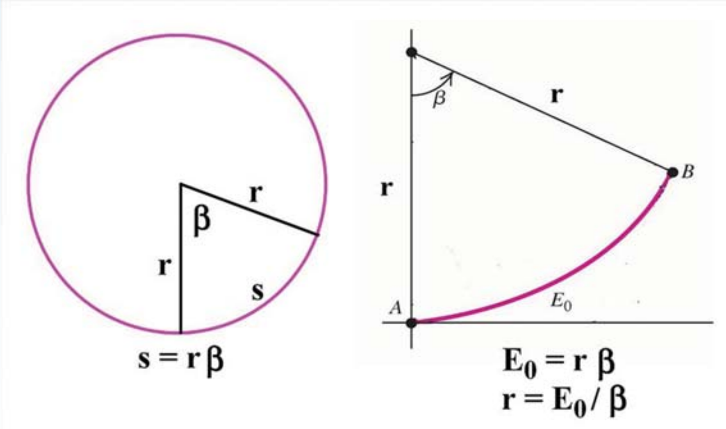


$180^o - \beta + 90^o + X + 90^o = 360^o$

$X = \beta$

Step 2

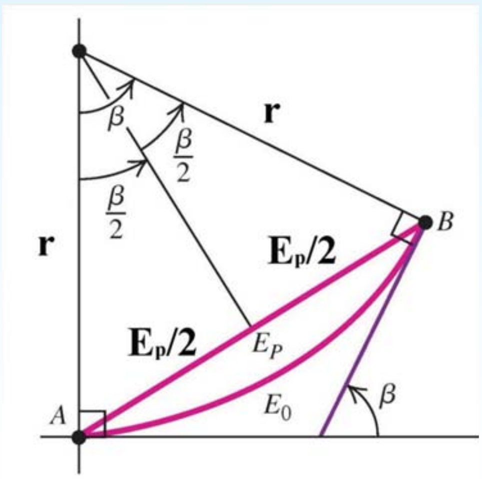
Find r !



Diffraction

Step 1 & 2

$r = E_0 / \beta$



Step 3

Triangle:

$\sin(\beta/2) = (E_p/2) / r$

$\sin(\beta/2) = (E_p/2) / (E_0/\beta)$

$$E_P = E_0 \frac{\sin(\beta/2)}{\beta/2}$$

Diffraction

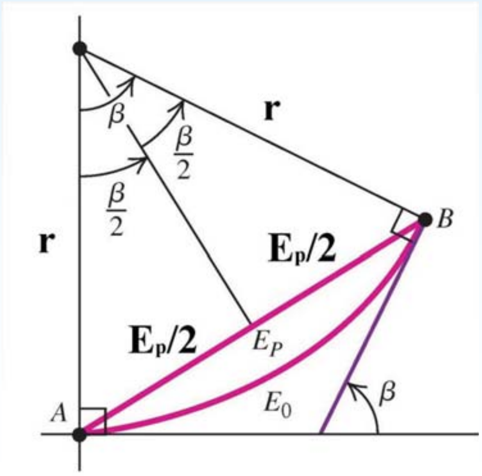
$$I \sim E_p^2$$

$$E_P = E_0 \frac{\sin(\beta/2)}{\beta/2}$$

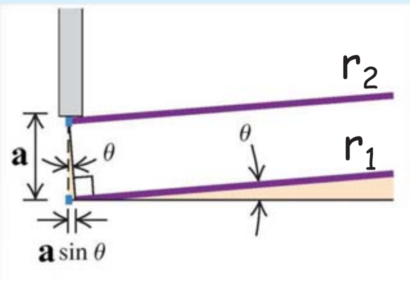
Intensity

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

But what is β ?



Diffraction



Path difference:
$r_2 - r_1 = a \sin(\theta)$

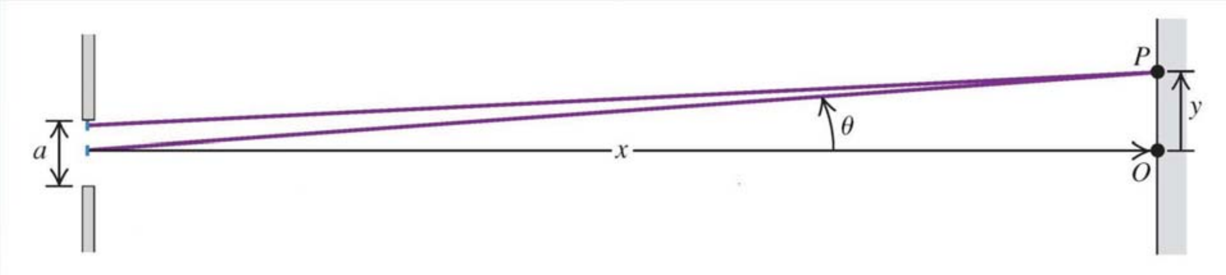
A path difference of one wavelength corresponds to a phase difference of 2π

$$\frac{\beta}{2\pi} = \frac{r_2 - r_1}{\lambda}$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

r_2-r_1 is the path difference between a ray at the top and bottom of the slit.

Diffraction



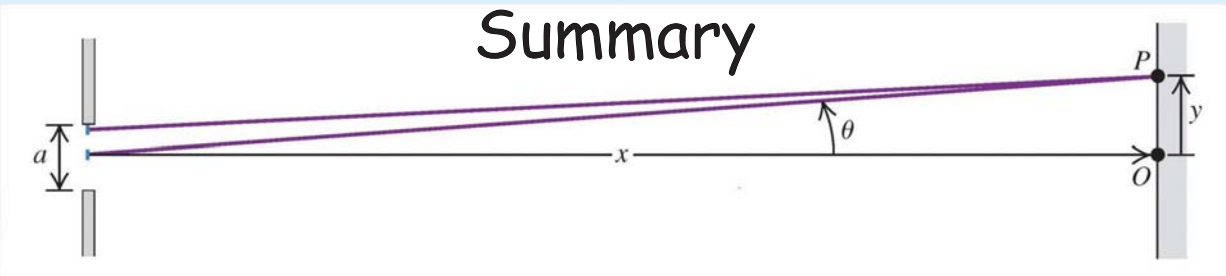
$$\tan(\theta) = y / x \approx \sin(\theta)$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta \approx \frac{2\pi a y}{\lambda x}$$

Diffraction

Summary



$$I \sim E_p^2$$

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

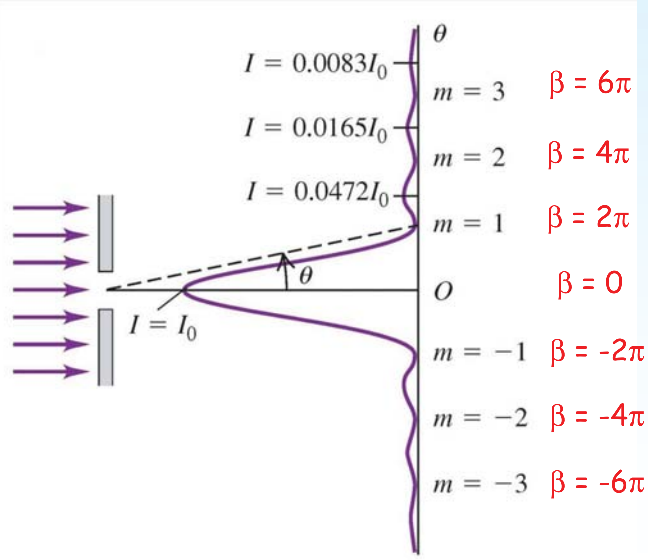
where

$$E_P = E_0 \frac{\sin(\beta/2)}{\beta/2}$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta \approx \frac{2\pi a y}{\lambda x}$$

Diffraction

$$I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$



Destructive interference:

Intensity is minimum

$$0 = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

$$0 = \sin^2(\beta/2)$$

$$0 = \sin(\beta/2)$$

$$\beta = 0, 2\pi, 4\pi, 6\pi\ldots\ldots = \pm 2\pi\, m$$

This gives again:

$$y_m = x\frac{m\lambda}{a}$$

Constructive interference:

Intensity is maximum

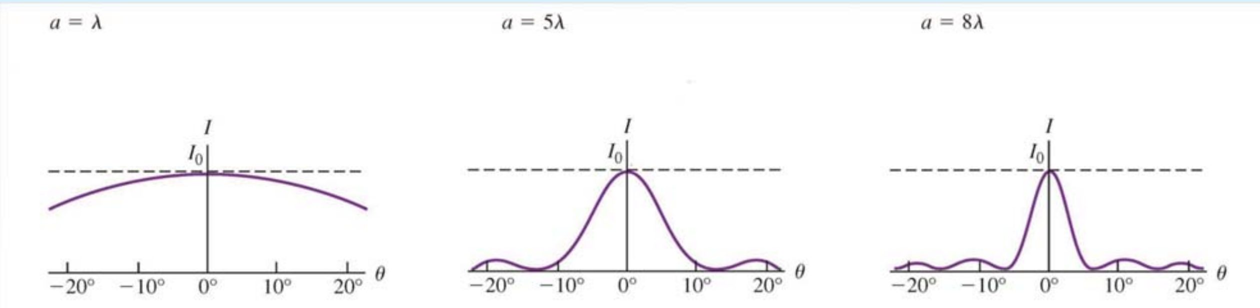
$$\frac{dI}{d\beta} = 0$$

Gives maximum (and minimum)

But the resulting equation has no analytical solution.

The peaks are close but not exactly at $\beta = 3\pi, 5\pi, 7\pi$....

Diffraction

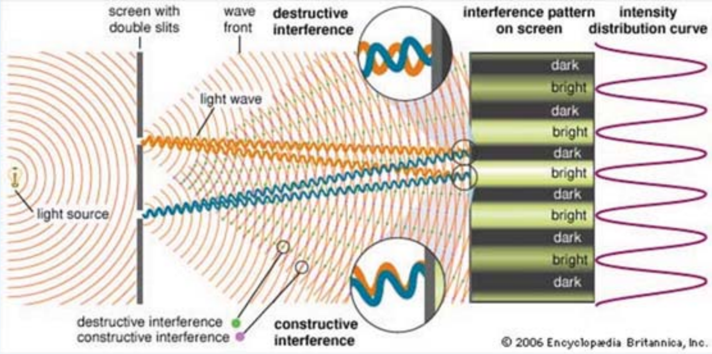


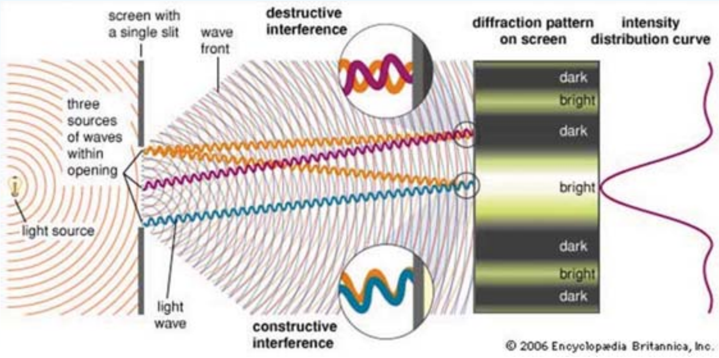
$$y_m = x\frac{m\lambda}{a}$$

If the width of the slit is equal or smaller than λ then only one broad maximum is observed.

A broader slit makes a narrower centre peak.

Diffraction





Intensity:

$$I = I_0 \cos^2 \frac{\phi}{2}$$

$$\phi = \frac{2\pi d}{\lambda} \sin\theta$$

$$\tan(\theta) = y / R \approx \sin(\theta)$$

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

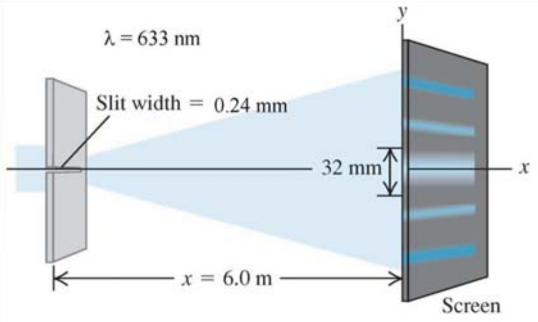
$$\tan(\theta) = y / x \approx \sin(\theta)$$

Diffraction

Problem solving

Diffraction

intensity at the center of the pattern is I_0. What is the intensity at a point on the screen 3.0 mm from the center of the pattern?



$\lambda = 633$ nm
$x = 6.00$ m
$a = 0.24$ mm
$y = 3.00$ mm

$$\tan\theta = y/x = (3.0 \times 10^{-3}\ \text{m})/(6.0\ \text{m}) = 5 \times 10^{-4} = \sin(\theta)$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta = \frac{2\pi(2.4 \times 10^{-4}\ \text{m})(5.0 \times 10^{-4})}{6.33 \times 10^{-7}\ \text{m}} = 1.20$$

$$I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 = I_0\left(\frac{\sin 0.60}{0.60}\right)^2 = 0.89 I_0$$

Diffraction

(a) The intensity at the center of a single-slit diffraction pattern is I_0. What is the intensity at a point in the pattern where there is a 66-radian phase difference between wavelets from the two edges of the slit? (b) If this point is 7.0° away from the central maximum, how many wavelengths wide is the slit?

$$I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

$\beta = 66$ rad

$$I = I_0\left[\frac{\sin(33\ \text{rad})}{33\ \text{rad}}\right]^2 = (9.2 \times 10^{-4}) I_0$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

$\theta = 7.0°$

$$\frac{a}{\lambda} = \frac{\beta}{2\pi \sin\theta} = \frac{66\ \text{rad}}{(2\pi\ \text{rad})\sin 7.0°} = 86$$

Diffraction

Two broad slits

Diffraction

In the analysis of interference from two slits it was assumed that they were very narrow. What if they are broad ?

Two narrow slits:

$$I = I_0 \cos^2 \frac{\phi}{2}$$

One broad slit:

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

Two broad slits:

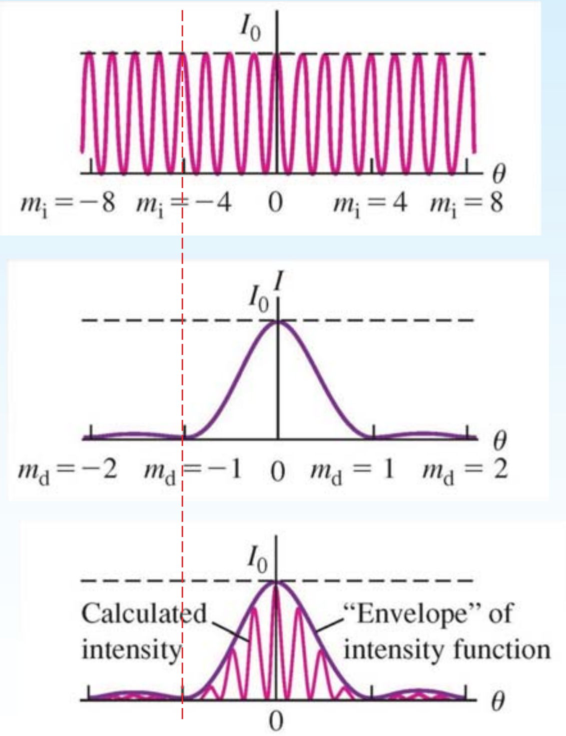
$$I = I_0 \cos^2 \frac{\phi}{2} \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

where

$$\phi = \frac{2\pi d}{\lambda} \sin\theta$$

$$\beta = \frac{2\pi a}{\lambda} \sin\theta$$

Diffraction



Two narrow slits:

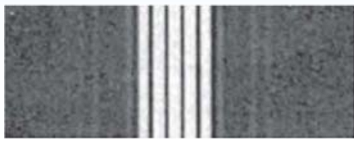
$$I = I_0 \cos^2 \frac{\phi}{2}$$

One broad slit:

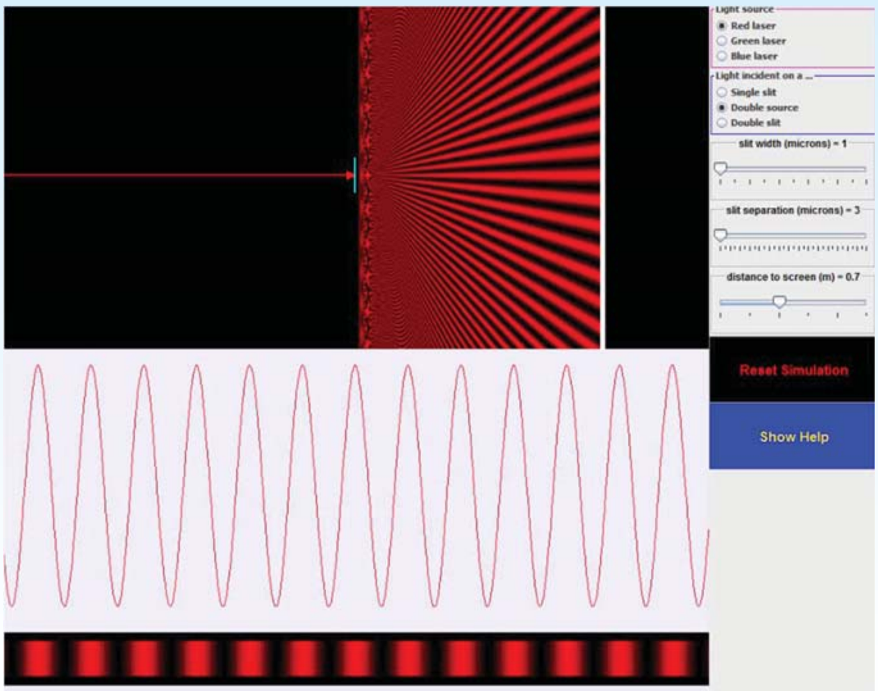
$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

Two broad slits:

$$I = I_0 \cos^2 \frac{\phi}{2} \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$



Diffraction

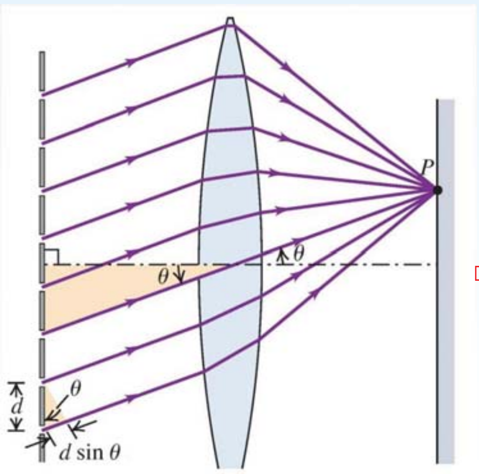


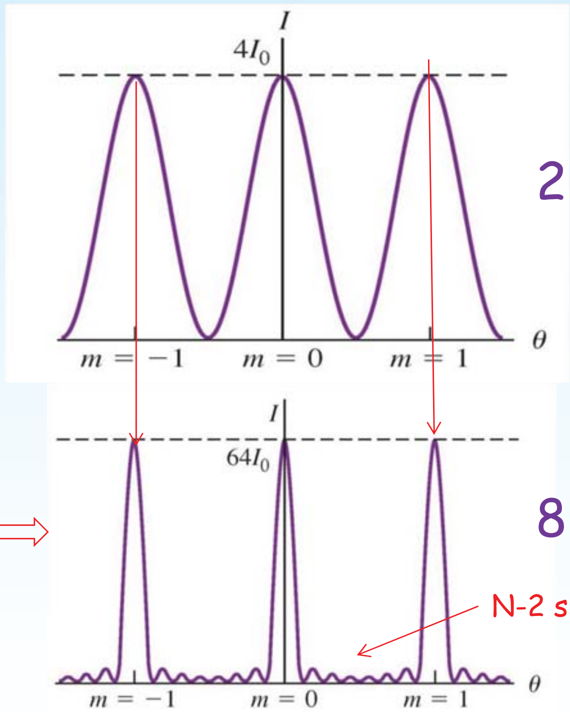
http://www.opensourcephysics.org/items/detail.cfm?ID=9988

Multiple slits

The path difference between adjacent slits that gives maximum intensity with many slits is always:

$$d \sin\theta = m\lambda \qquad (m = 0, \pm 1, \pm 2, \dots)$$





2 slits

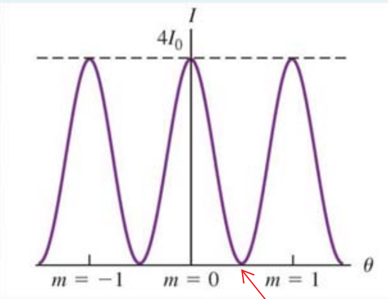
8 slits

N-2 small peaks

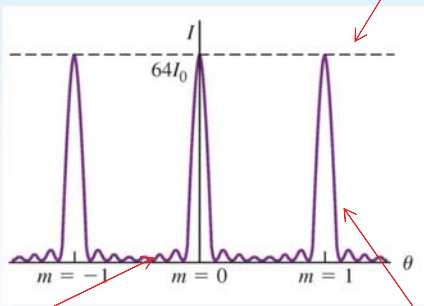
Diffraction

$I_{max} \sim N^2$

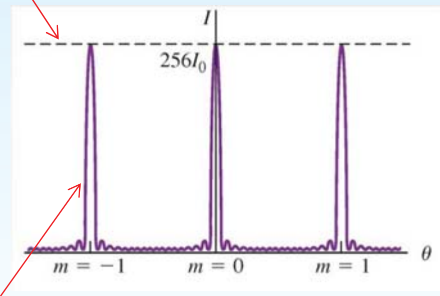
N = 2



N = 8



N = 16

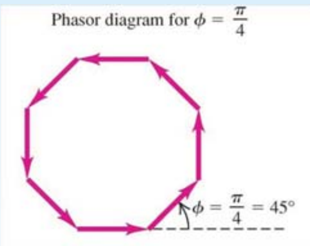


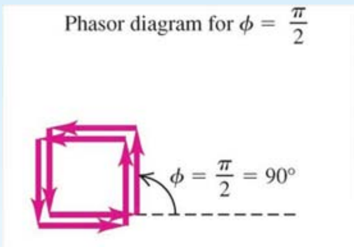
N-1 minima

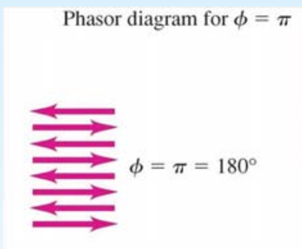
$I_{width} \sim 1 / N$

Principal maxima: $d \sin\theta = m\lambda \qquad (m = 0, \pm1, \pm2, \pm3, \ldots)$

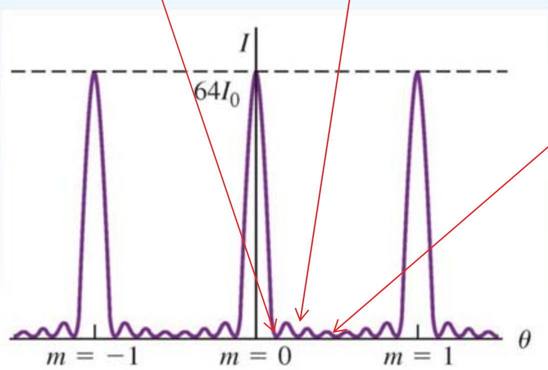
Diffraction







N = 8



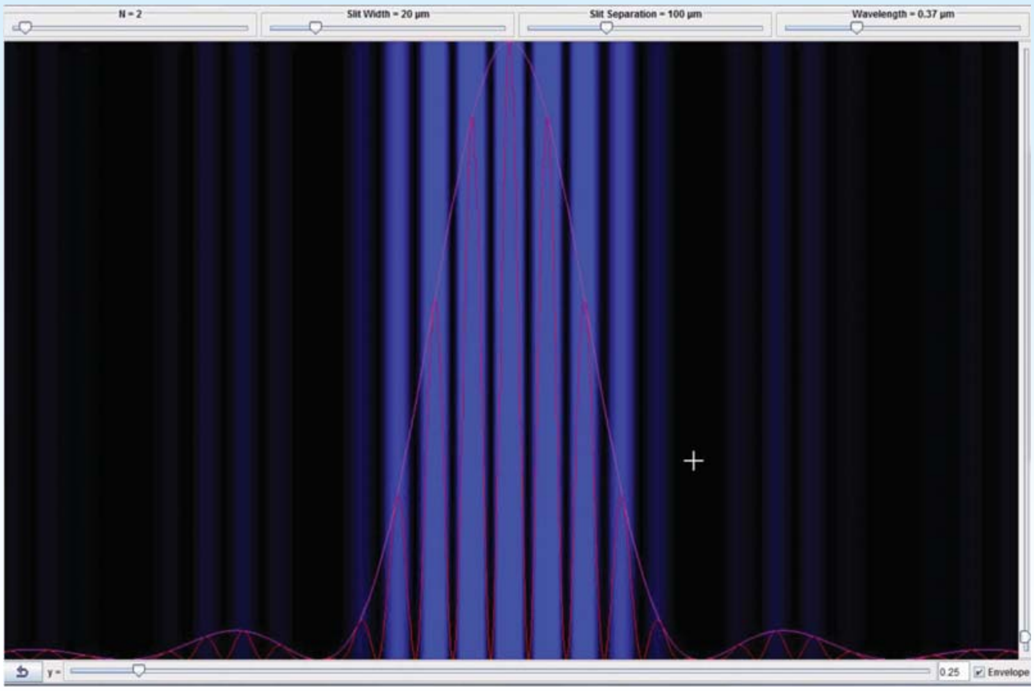
$\phi = 0$ $\phi = 2\pi$

mimima for

$\phi = k\, 2\pi / N$

where k = 1, 2......N-1

Diffraction



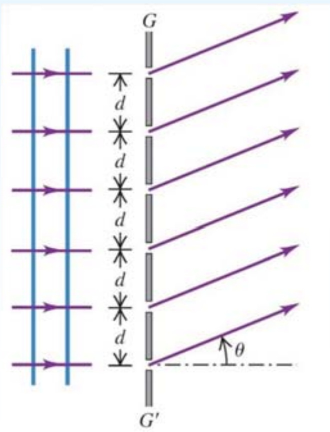
http://www.opensourcephysics.org/items/detail.cfm?ID=8331

Diffraction

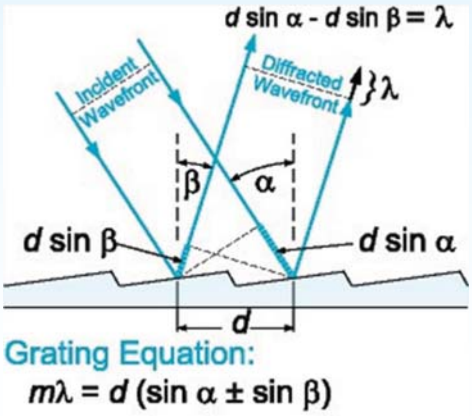
In diffraction grating one uses devices with thousands of slits or reflecting surfaces.

This gives very narrow principal maximum that can be used to determine the wavelength of light.

Transmission grating



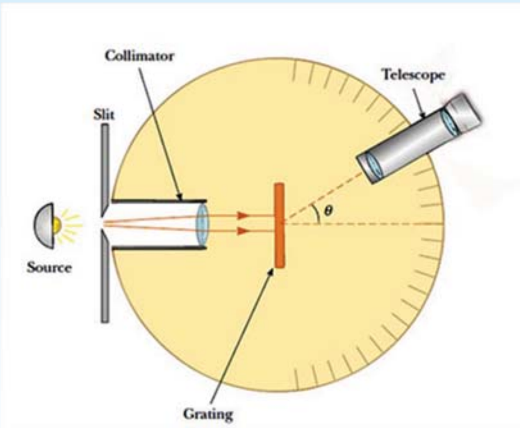
Reflection grating



Diffraction

Problem solving

Diffraction



Grating: 1000 slits per mm   1st order maximum at 24^0 What is λ ?

$$d \sin\theta = m\lambda$$

with

d = 1 mm / 1000 slits = 10^{-6} m
$\theta = 24^o$

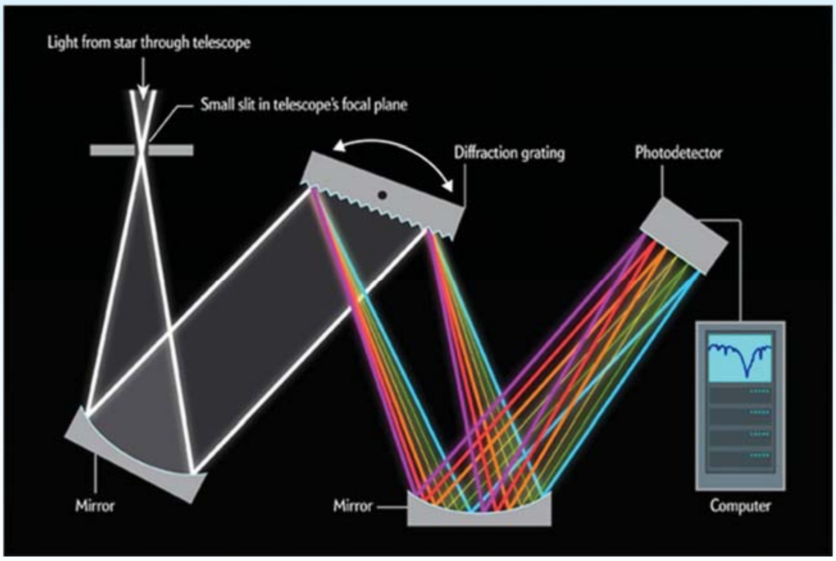
$\lambda = d \sin(\theta) = 10^{-6} \sin(24^o) = 0.407 \times 10^{-6} = 407$ nm

Diffraction

Spectrometers

Diffraction

Spectrometer for astronomy

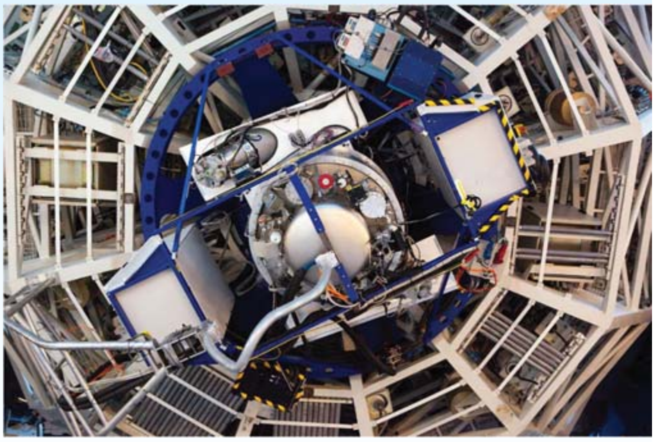


Light incident on a grating is dispursed into a spectrum. The angles of deviations of the maxima are measured to calculate the wave length.

Diffraction

The ESO Very Large Telescope (VLT) in Chile

The XSHOOTER spectrometer in the VLT

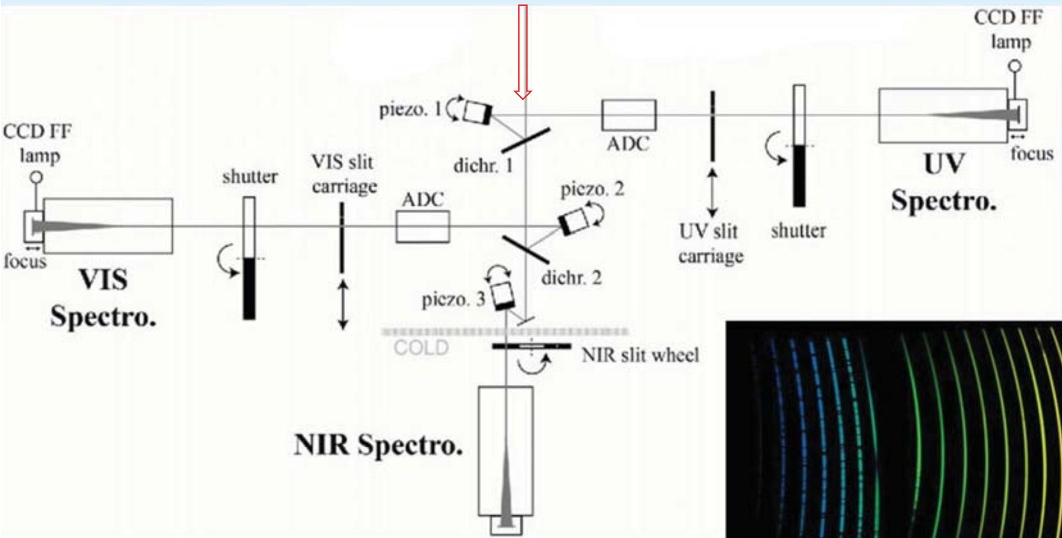


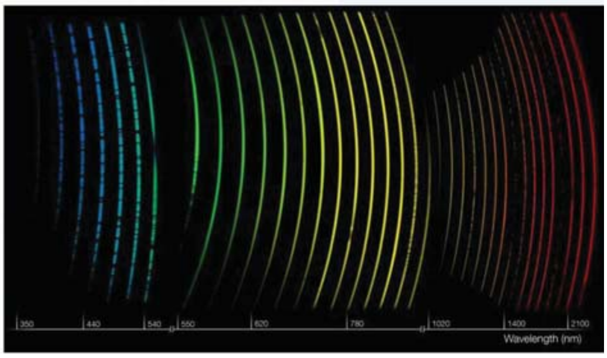
ESO: European Southern Observatory

https://www.eso.org/public/

Diffraction

Light from the VLT





Diffraction

Chromatic resolving power:

The minimum wavelength difference (Δλ) that can be distinguished by a spectrograph.

$$R = \frac{\lambda}{\Delta\lambda} \qquad \text{(chromatic resolving power)}$$

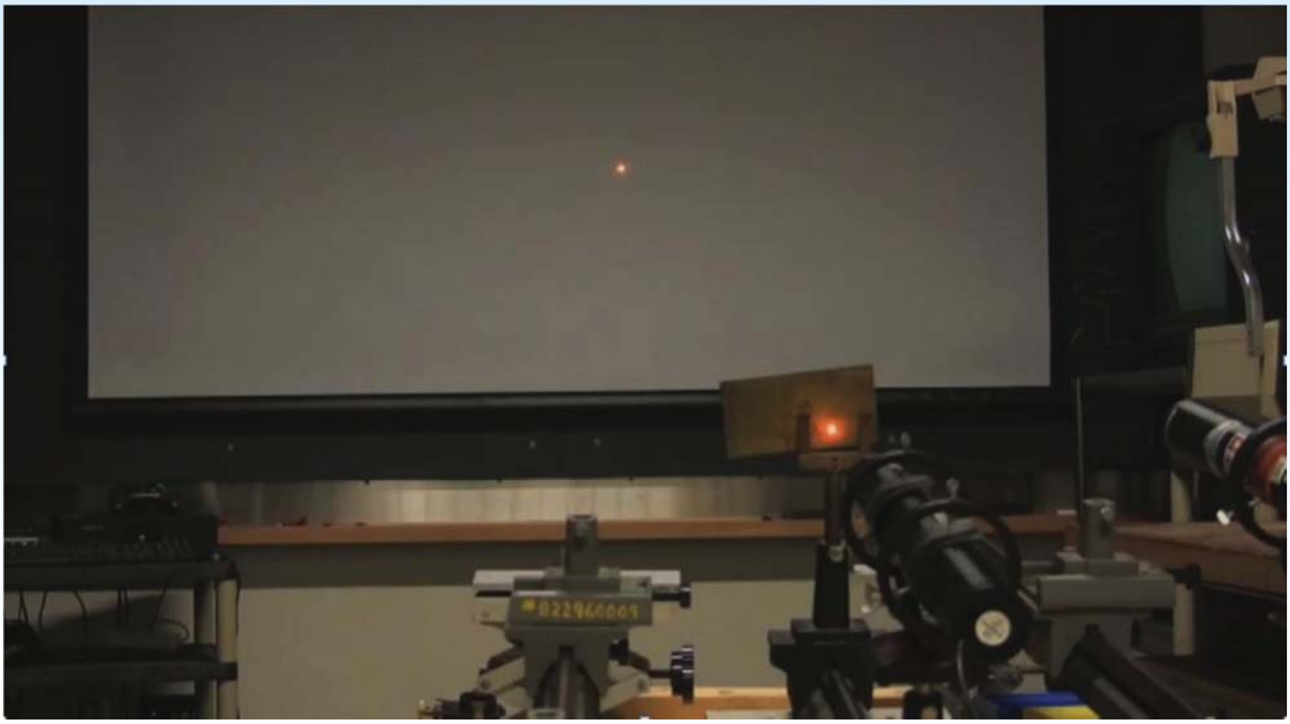
$$R = \frac{\lambda}{\Delta\lambda} = Nm$$

R is higher for many slits and higher orders !

Diffraction

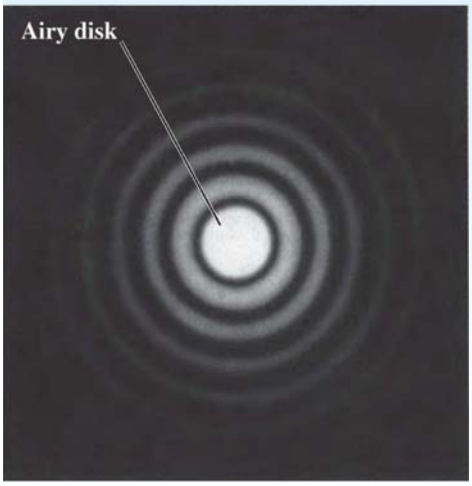
Pinhole diffraction

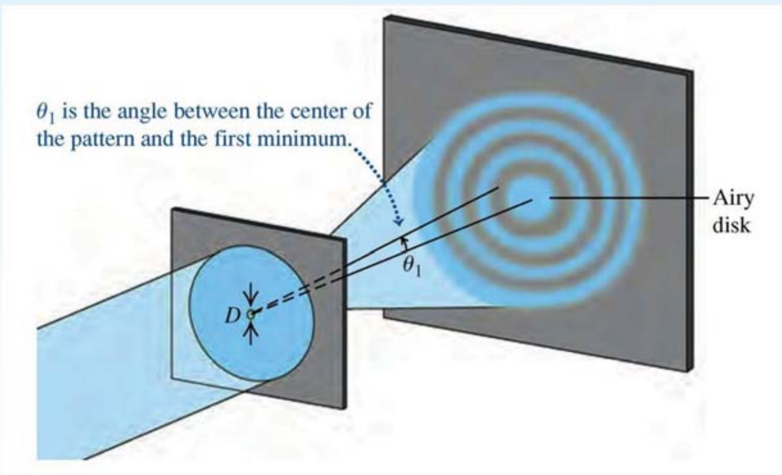
Diffraction



Diffraction

Diffraction limits the angular resolution of optical intruments.





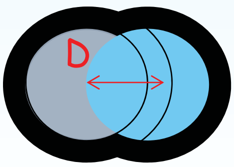
$$\sin\theta_1 = 1.22\frac{\lambda}{D} \qquad \text{(diffraction by a circular aperture)}$$

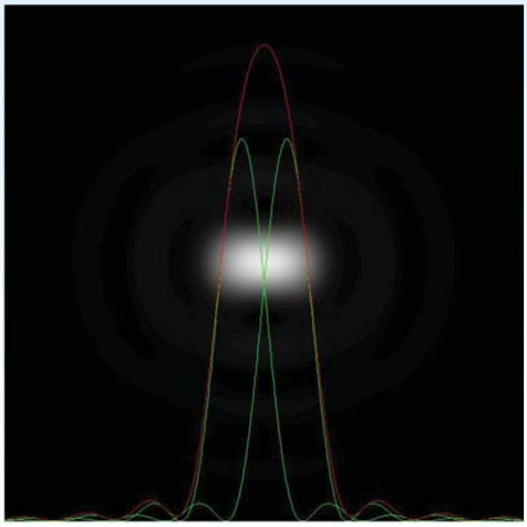
Diffraction

Rayleigh's criterion:
Two point objects can be resolved by an optical system if their angular separation is larger than θ_1 where

$$\sin\theta_1 = 1.22\frac{\lambda}{D}$$

The limit for two objects to be resolved is when the center of one diffraction pattern is in the first minimum of the other.





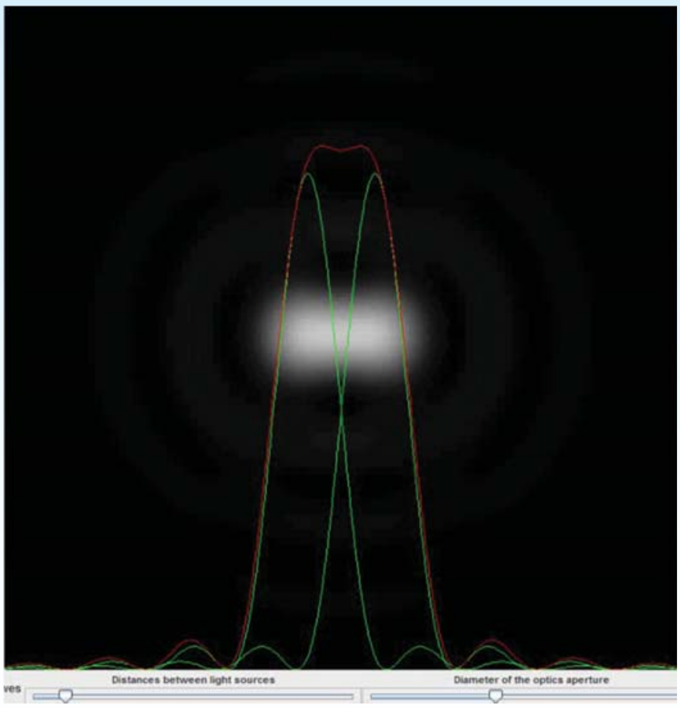
Diffraction

Resolution or resolving power:

Minimum separation of two objects that can be resolved by an instrument.

Higher resolution for larger optical aperture and shorter wavelenght.

$$\sin\theta_1 = 1.22\frac{\lambda}{D}$$



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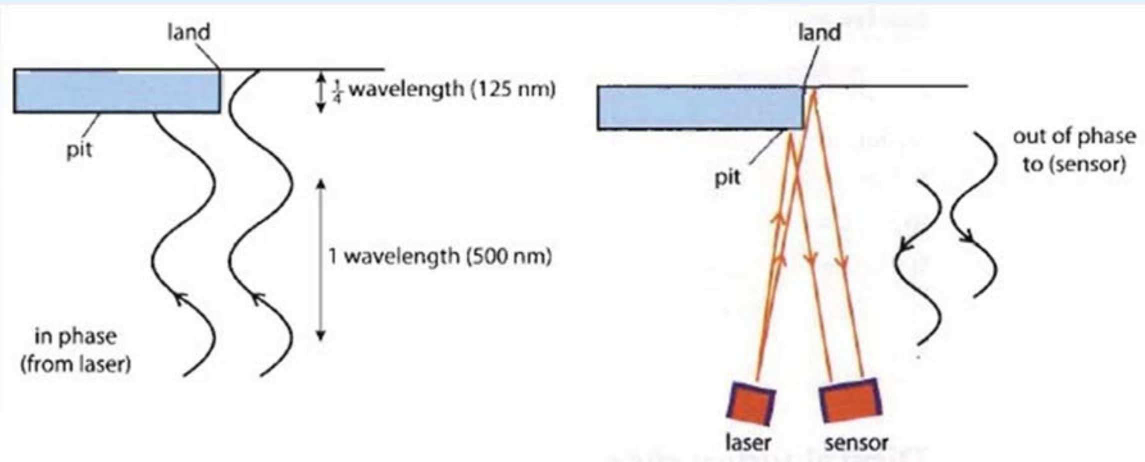
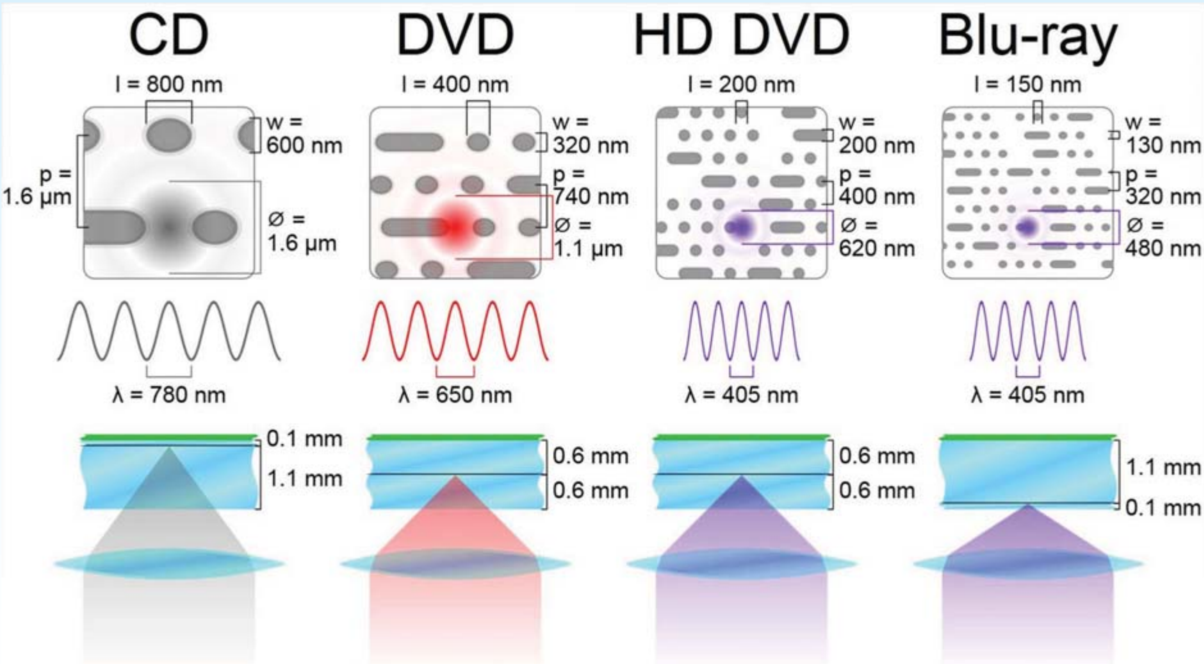
DVD players

The principle of a DVD player: Destructive interference



Diffraction



The End