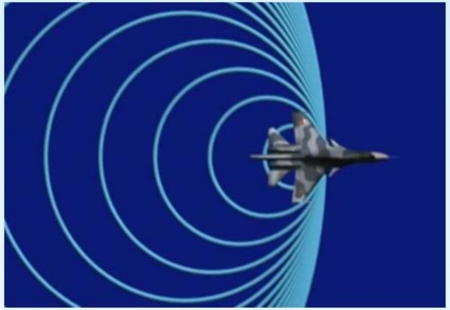
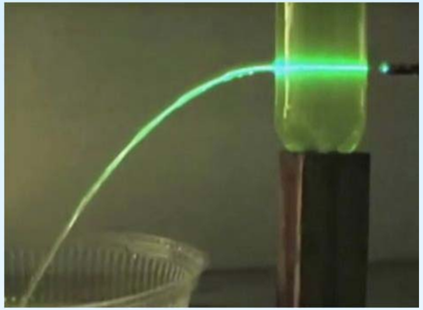
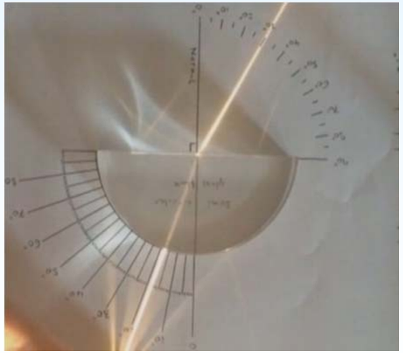
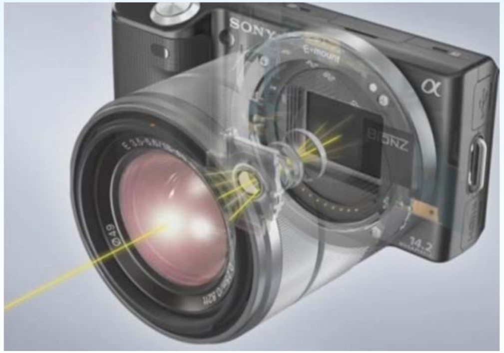
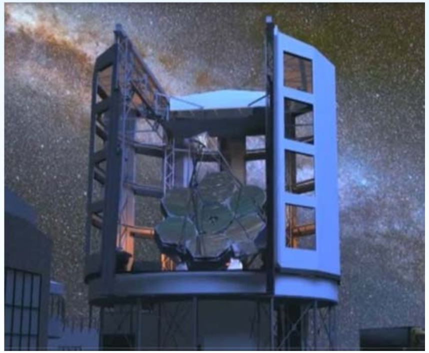
Wavemechanics and optics









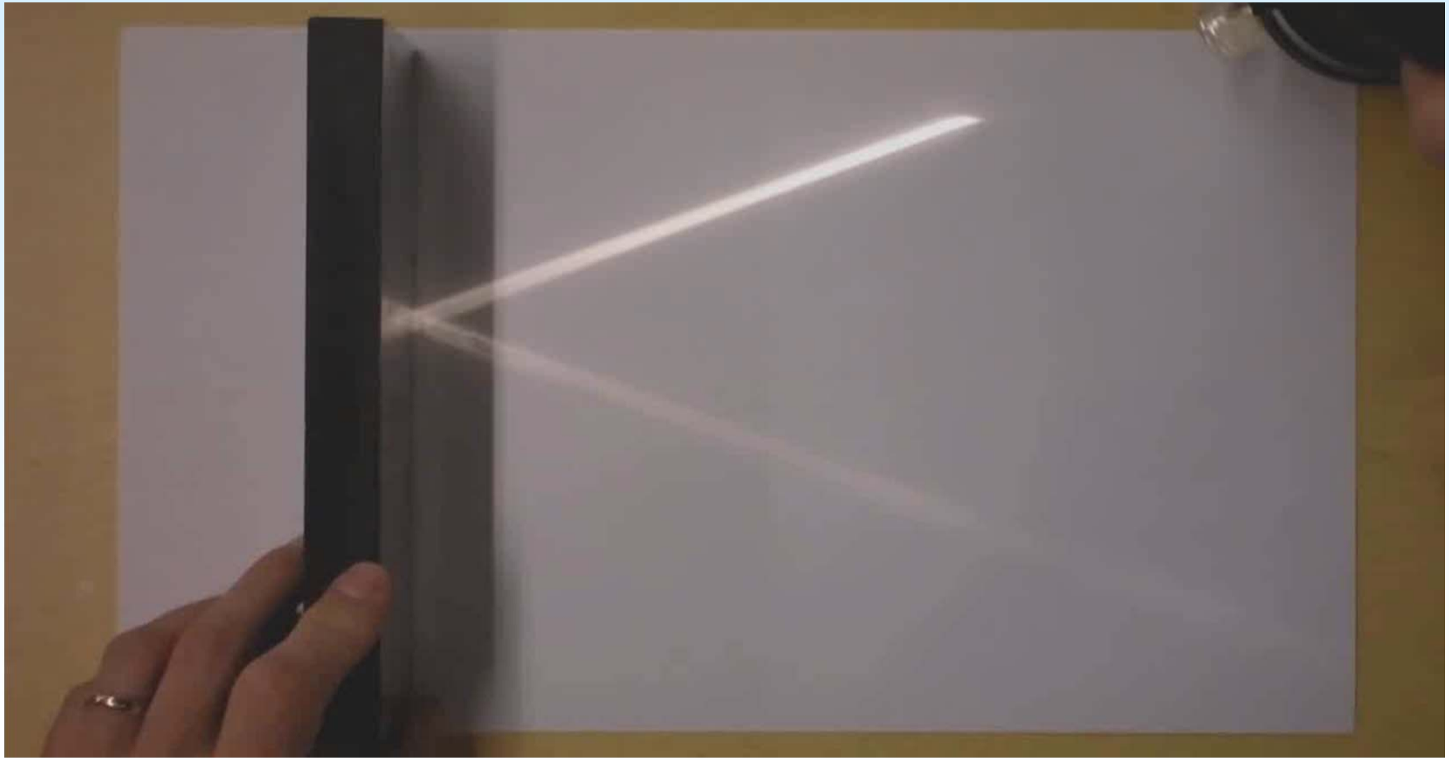


Chapter 34 – Optics

Vincent Hedberg - Lunds Universitet

Content

Geometrical optics: Introduction



https://www.youtube.com/watch?v=uQE659ICjqQ

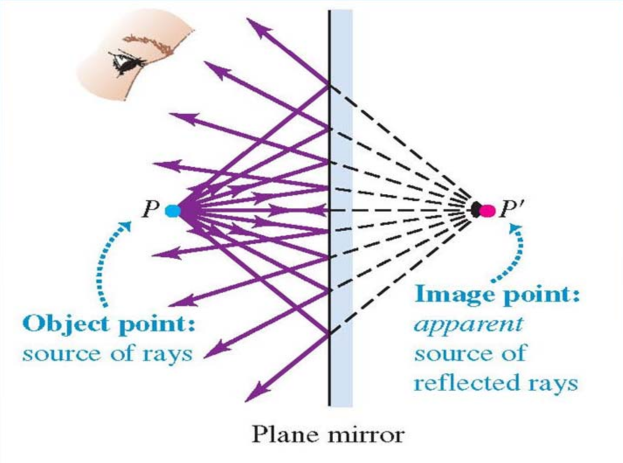
Part 1. Flat mirrors

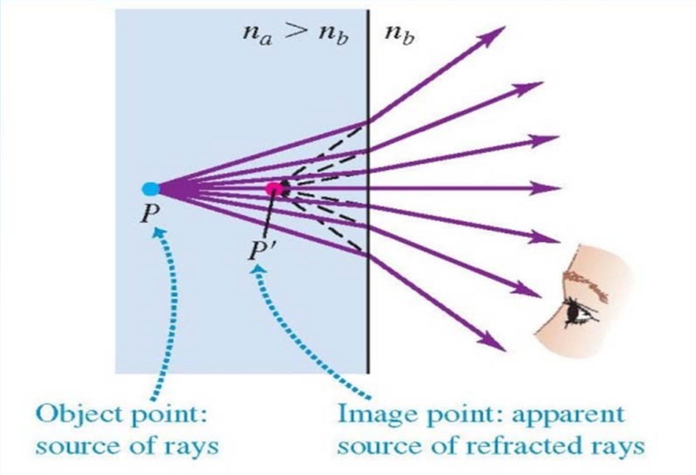
Salar de Uyuni in Bolivia is a salt flat which during the flooding season becomes the largest flat mirror on earth.

It is used for calibrating the distance measurement equipment on satellites.

Geometrical optics: Flat mirrors

Virtual Images: outgoing rays diverge

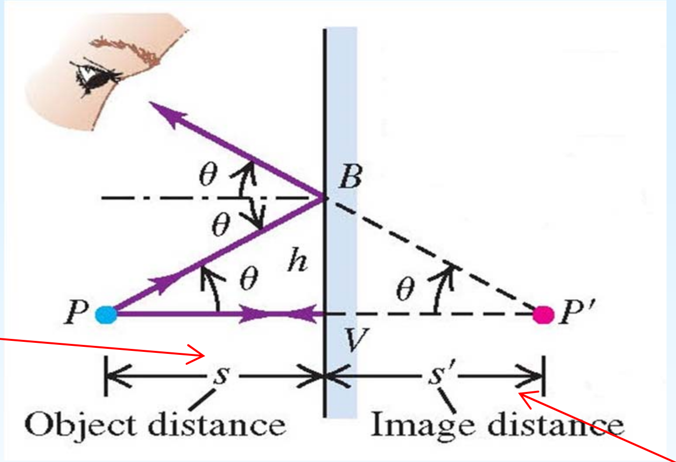




Real Images: outgoing rays converge to an image that can be shown on a screen

Geometrical optics: Flat mirrors

Point object



positive

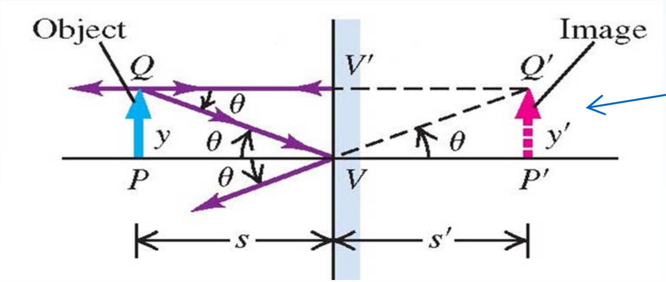
negative

Sign rules:

Object distance (s) – positive if same side as incoming light.

Image distance (s') – positive if same side as outgoing light.

Extended object

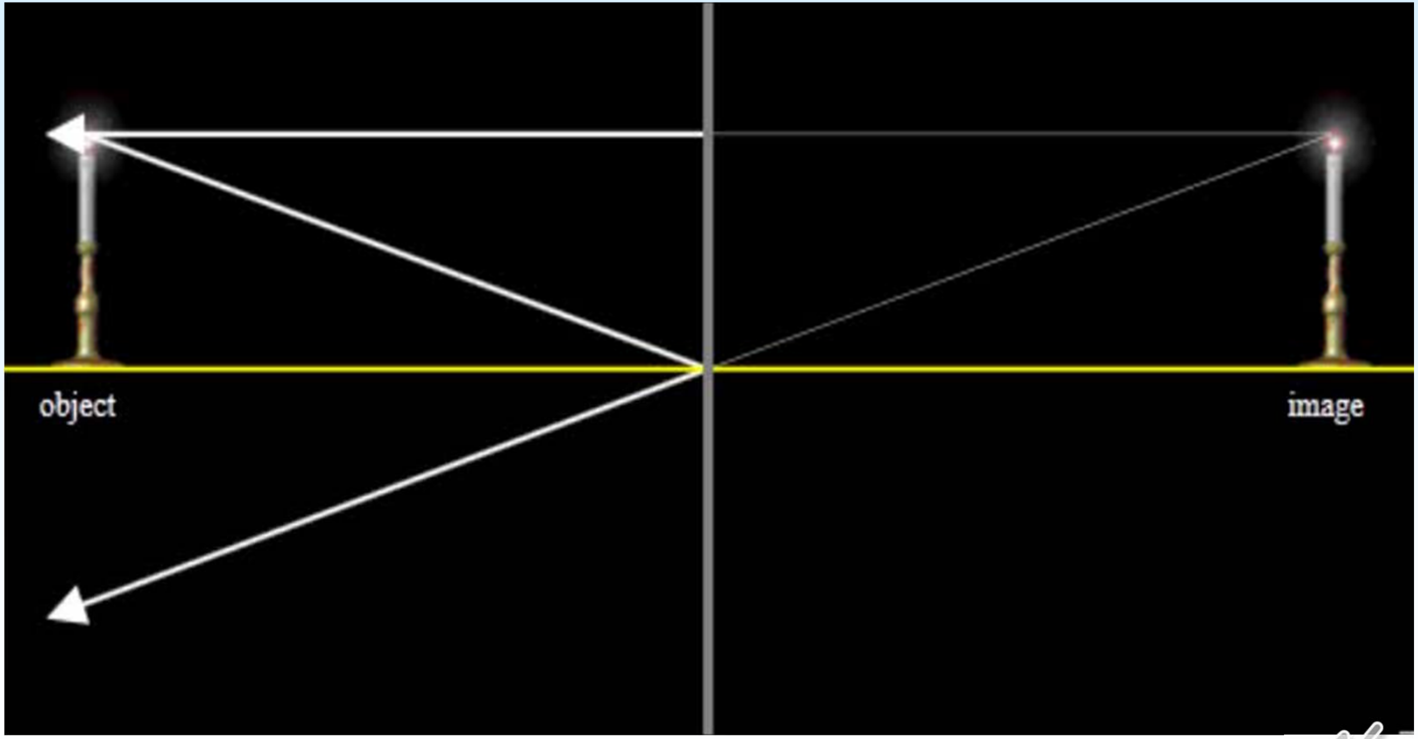


Virtual image

$$m = \frac{y'}{y} \qquad \text{(lateral magnification)}$$

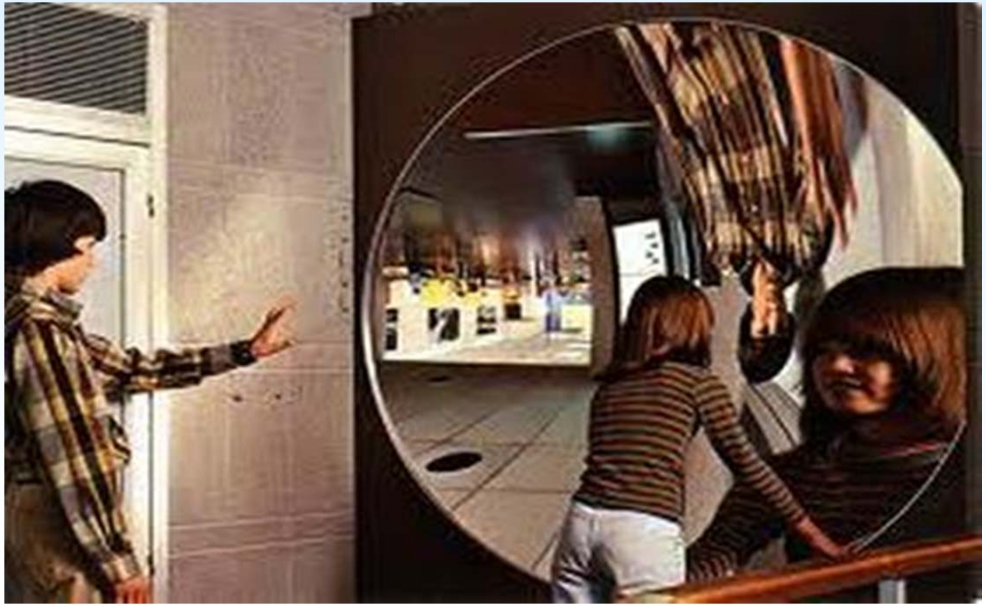
Geometrical optics: Flat mirrors

Simulation of a flat mirror:



http://www.opensourcephysics.org/osp/EJSS/3650/21.htm

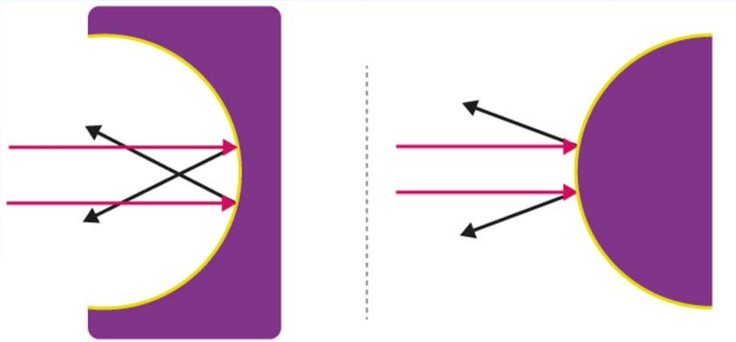
Part 2. Concave mirrors



Geometrical optics: Concave mirrors

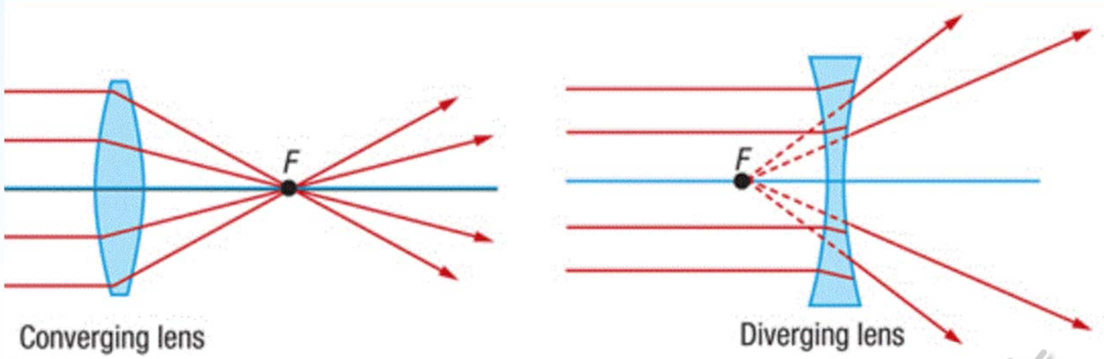
Concave means "hollowed out or rounded inward" and is easily remembered because these surfaces form a "cave".
The opposite is **convex meaning** "curved or rounded outward."

Concave & Convex mirror



Converging & Diverging

Convex & Concave lens



Geometrical optics: Concave mirrors

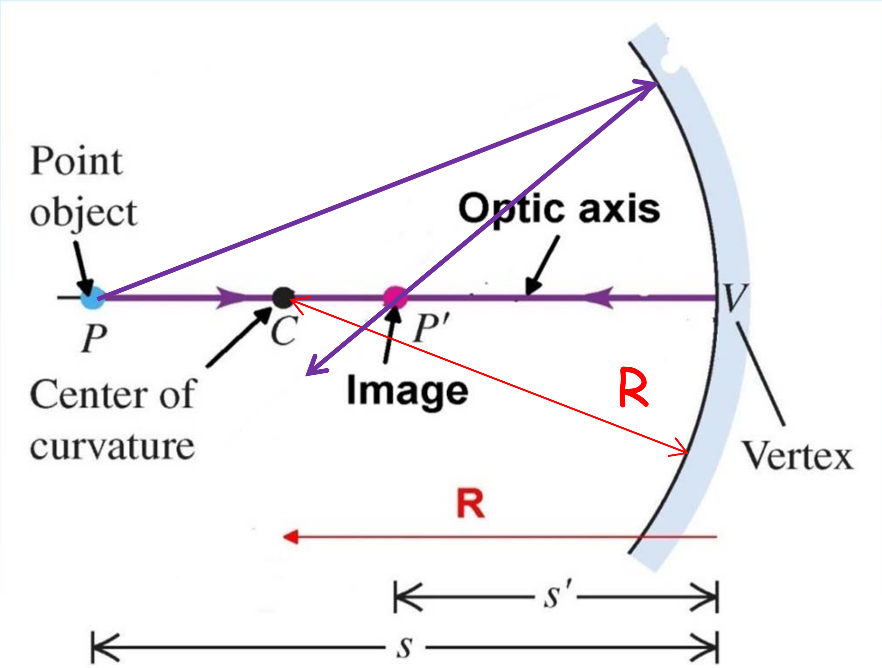
Spherical mirror

A point object on an optical axis will have the image on the optical axis.

s = distance mirror - object
s′ = distance mirror - image
R = radius of curvature

Sign rule:
R is positive if center is on same side as outgoing light.

The image can be found with two rays:



Geometrical optics: Concave mirrors

Given

A concave mirror with radius of curvature **R** that has an object at the distance **S**

Goal

Derive a formula so that one can calculate where the image ends up = **S'**

How

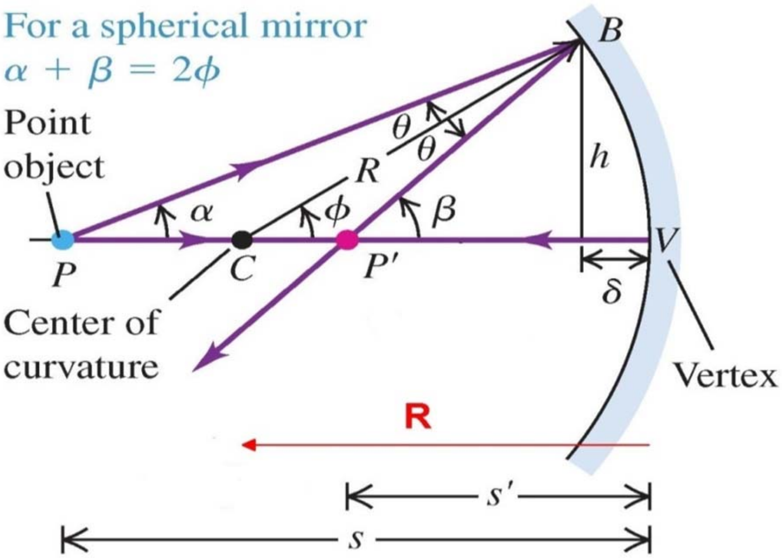
Law of reflection + Trigonometry

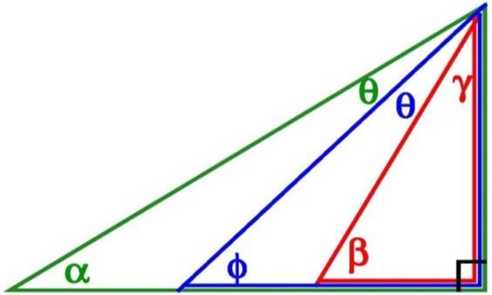
Step 1

Trigonometry

The sum of the angles in a triangle is 180 degrees

➡ Relationship between α, β and ϕ





$\beta + \gamma + 90^0 = 180^0$

$\gamma = 90^0 - \beta$

$\phi + \gamma + \theta + 90^0 = 180^0$

$\phi + 90^0 - \beta + \theta + 90^0 = 180^0$

$\theta = \beta - \phi$

$\alpha + \gamma + 2\theta + 90^0 = 180^0$

$\alpha + 90^0 - \beta + 2(\beta - \phi) + 90^0 = 180^0$

$\alpha + \beta - 2\phi = 0$

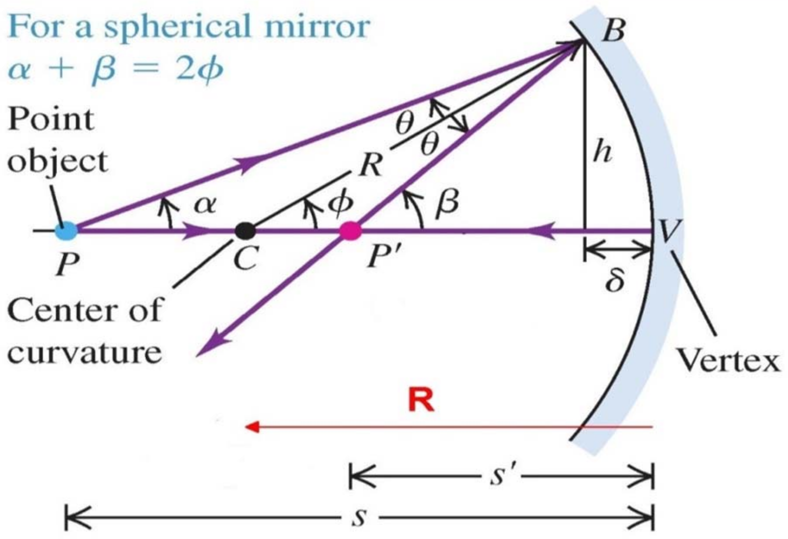
$$\alpha + \beta = 2\phi$$

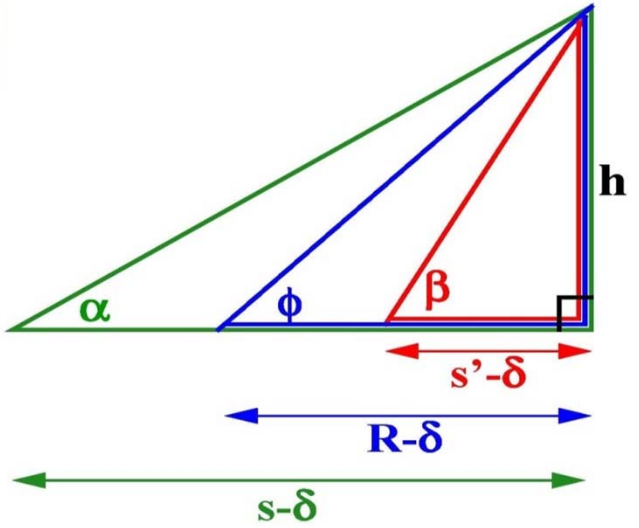
Step 2

Trigonometry

Use the tangent of the triangles

→ Relationship between α, β, ϕ and S, R, S'





$$\tan(\alpha) = \frac{h}{s-\delta}$$

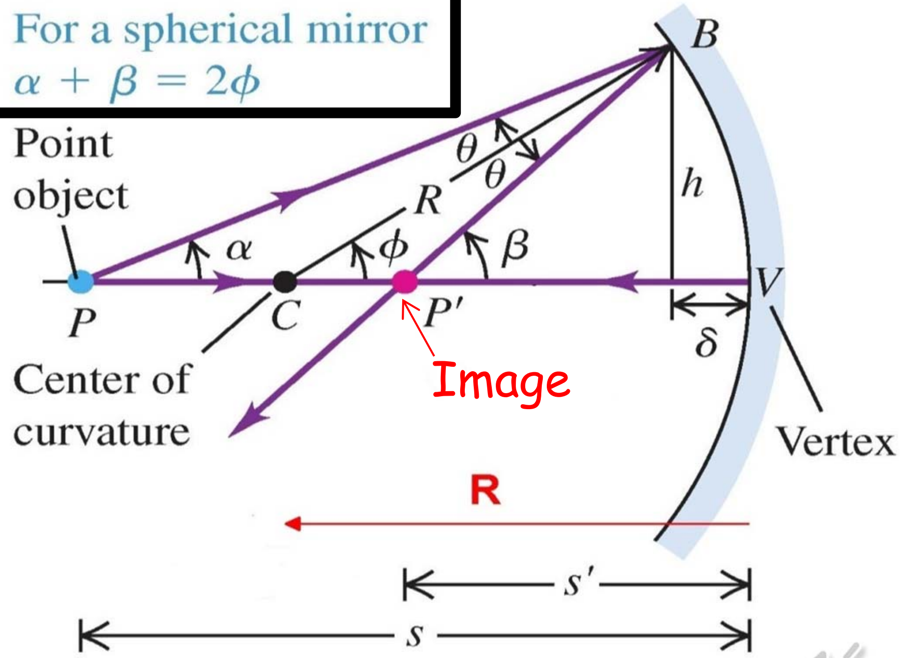
$$\tan(\phi) = \frac{h}{R-\delta}$$

$$\tan(\beta) = \frac{h}{s'-\delta}$$

Step 3 Approximation and combination of step 1 and 2

$$\tan\alpha = \frac{h}{s-\delta} \qquad \tan\beta = \frac{h}{s'-\delta} \qquad \tan\phi = \frac{h}{R-\delta}$$

For a spherical mirror
$\alpha + \beta = 2\phi$



If the angles and δ are small then

$$\alpha = \frac{h}{s} \qquad \beta = \frac{h}{s'} \qquad \phi = \frac{h}{R}$$

$$\alpha + \beta = 2\phi$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

(object–image relations

How good is the approximation for small angles ? → $\sin(\theta) = \theta$, $\tan(\theta) = \theta$

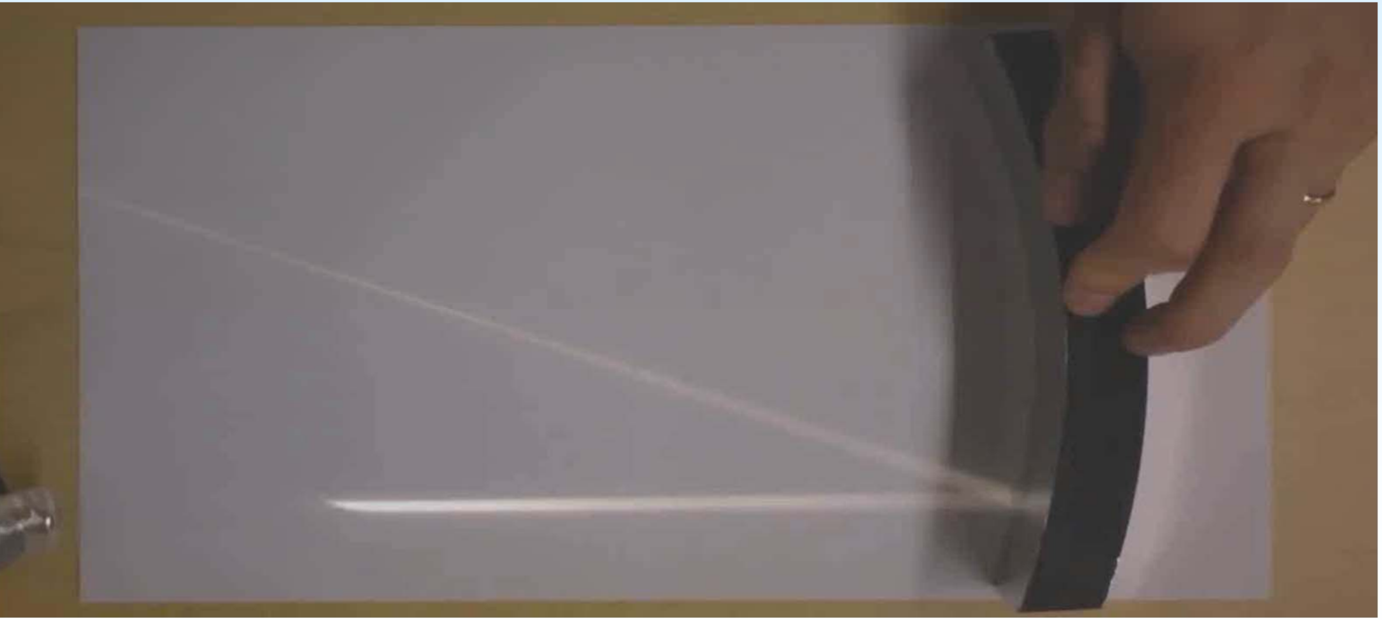
$\sin(1^\circ) = \sin(0.0175 \text{ radians}) = 0.0175$
$\tan(1^\circ) = \tan(0.0175 \text{ radians}) = 0.0175$

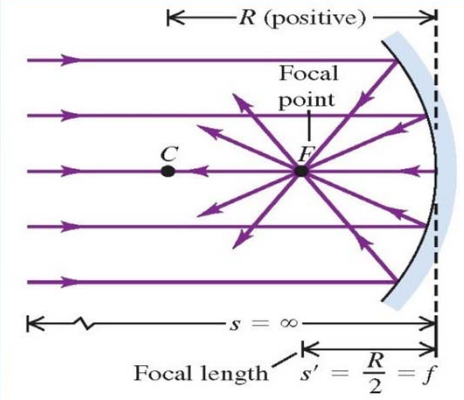
$\sin(5^\circ) = \sin(0.0873 \text{ radians}) = 0.0872$ (-0.1%)
$\tan(5^\circ) = \tan(0.0873 \text{ radians}) = 0.0875$ (+0.4%)

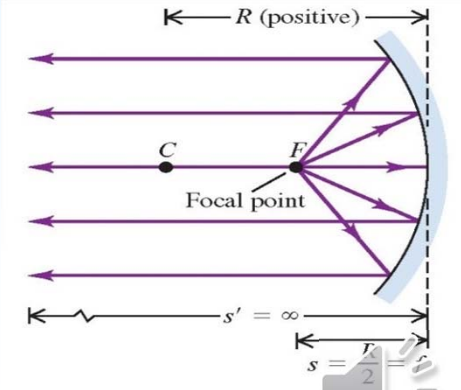
$\sin(10^\circ) = \sin(0.175 \text{ radians}) = 0.174$ (-0.5%)
$\tan(10^\circ) = \tan(0.175 \text{ radians}) = 0.176$ (+1.5%)

$\sin(20^\circ) = \sin(0.349 \text{ radians}) = 0.342$ (-2.1%)
$\tan(20^\circ) = \tan(0.349 \text{ radians}) = 0.364$ (+6.0%)

Focal point




R (positive)
Focal
point
C
F
s = ∞
Focal length
s′ = R/2 = f

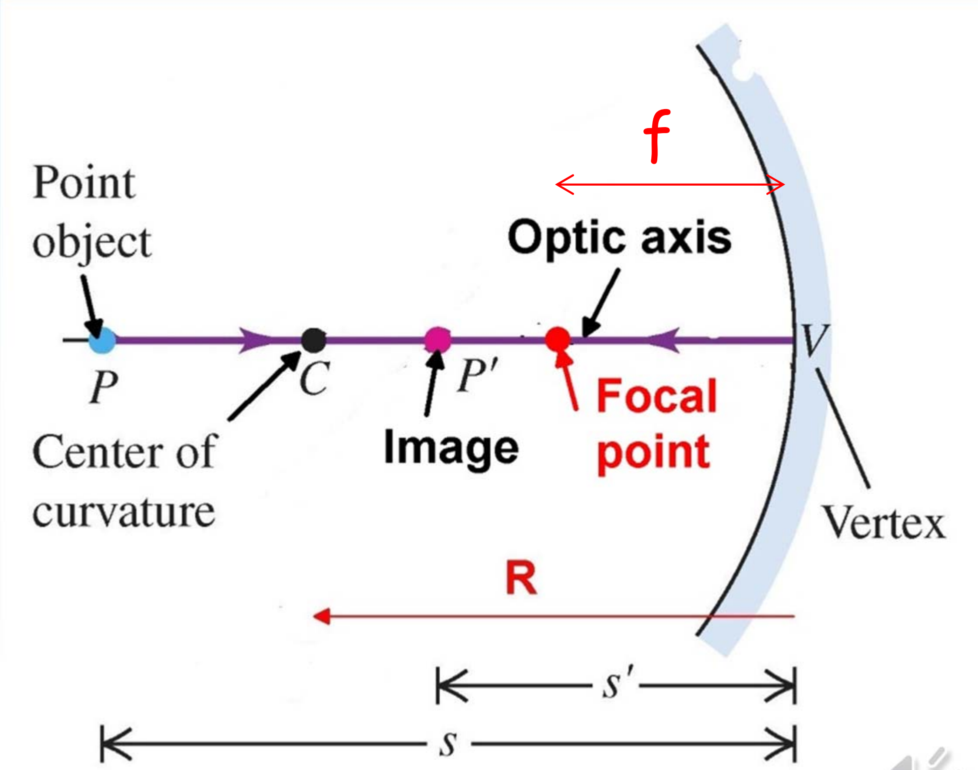

R (positive)
C
F
Focal point
s′ = ∞

Focal distance

$$f = \frac{R}{2}$$

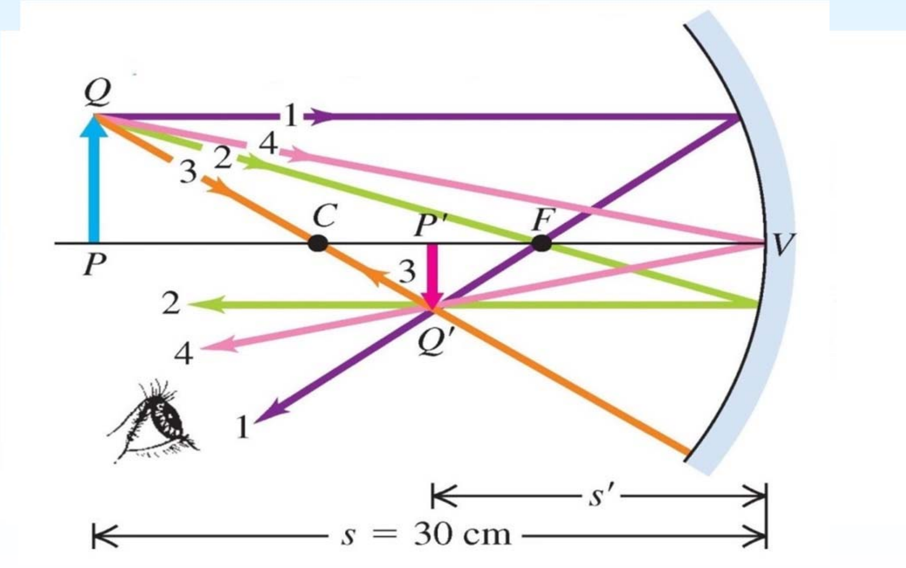
$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

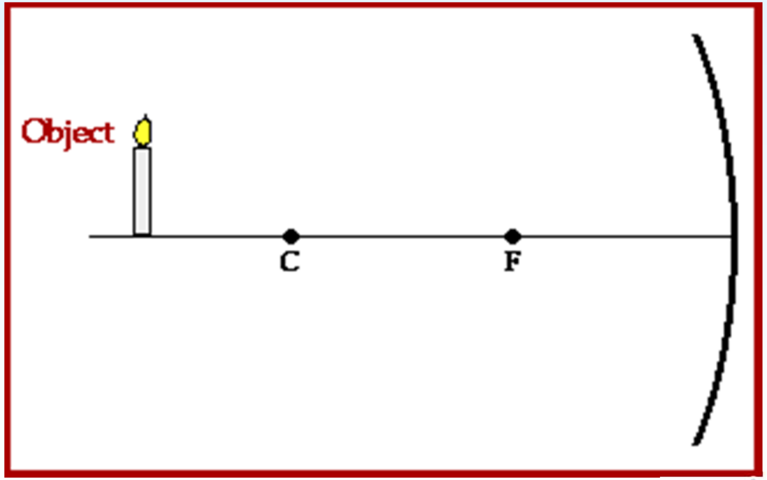


Geometrical optics: Concave mirrors

An infinite number of rays can be drawn from an object to its image.



But only two rays are needed to determine the location of the image.

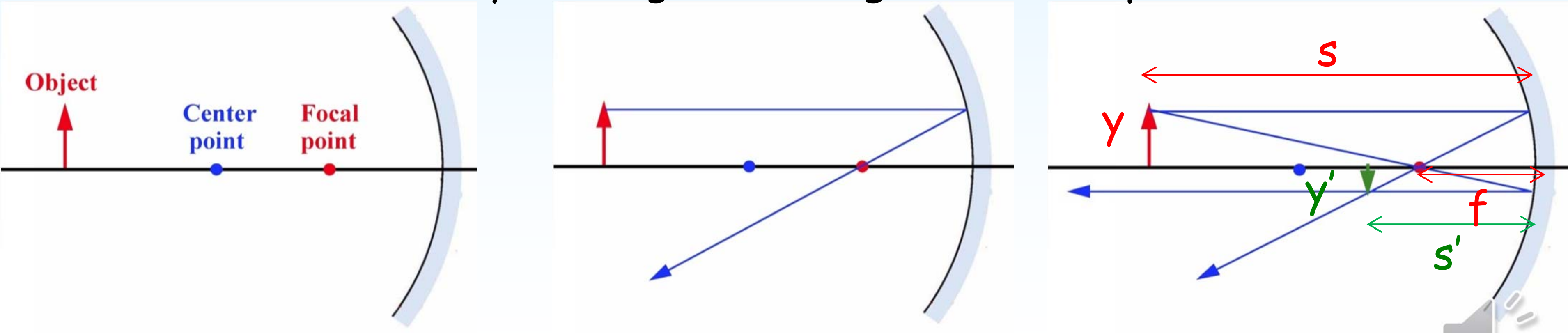


Geometrical optics: Concave mirrors

How to find the image in a concave mirror

The bottom of the object is on the optical axis and so the bottom of the image will also be on the optical axis.

The top of the image can be found with any two rays. Use for example two rays that goes through the focal point.



Geometrical optics: Concave mirrors

Given

A concave mirror with radius of curvature **R** that has an object at a distance **S** and an image at a distance **S'**

Goal

Derive a formula so that the magnification m can be calculated

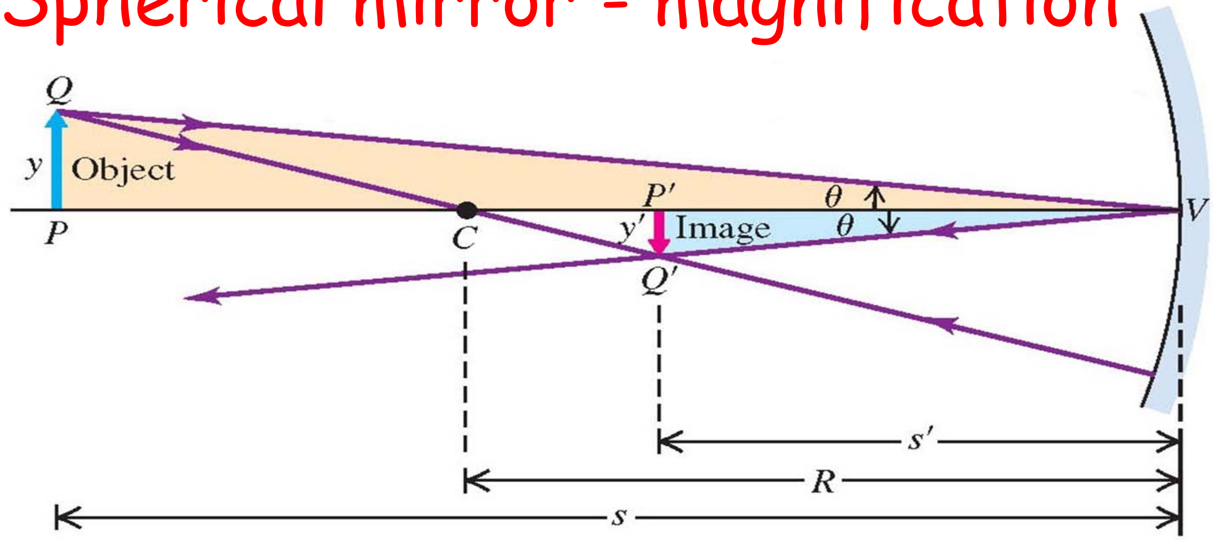
How

The law of reflection + Trigonometry

Spherical mirror - magnification

Definition of magnification

$$m = \frac{y'}{y}$$



$\tan(\theta) = y/s$

$\tan(\theta) = -y'/s'$

Image direction inverted

$$\frac{y}{s} = -\frac{y'}{s'}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Summary spherical mirrors

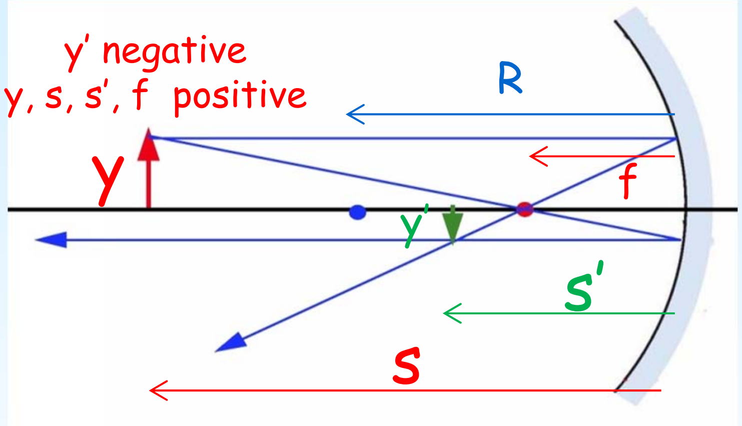
Sign rules:

Positive object distance (s) =
Object is on the side of the incoming light.

Positive image distance (s') =
Image and outgoing light on the same side.

Positive radius of curvature (R) =
Center is on the side of outgoing light.

Positive magnification (m) =
Direction of object and image is the same.

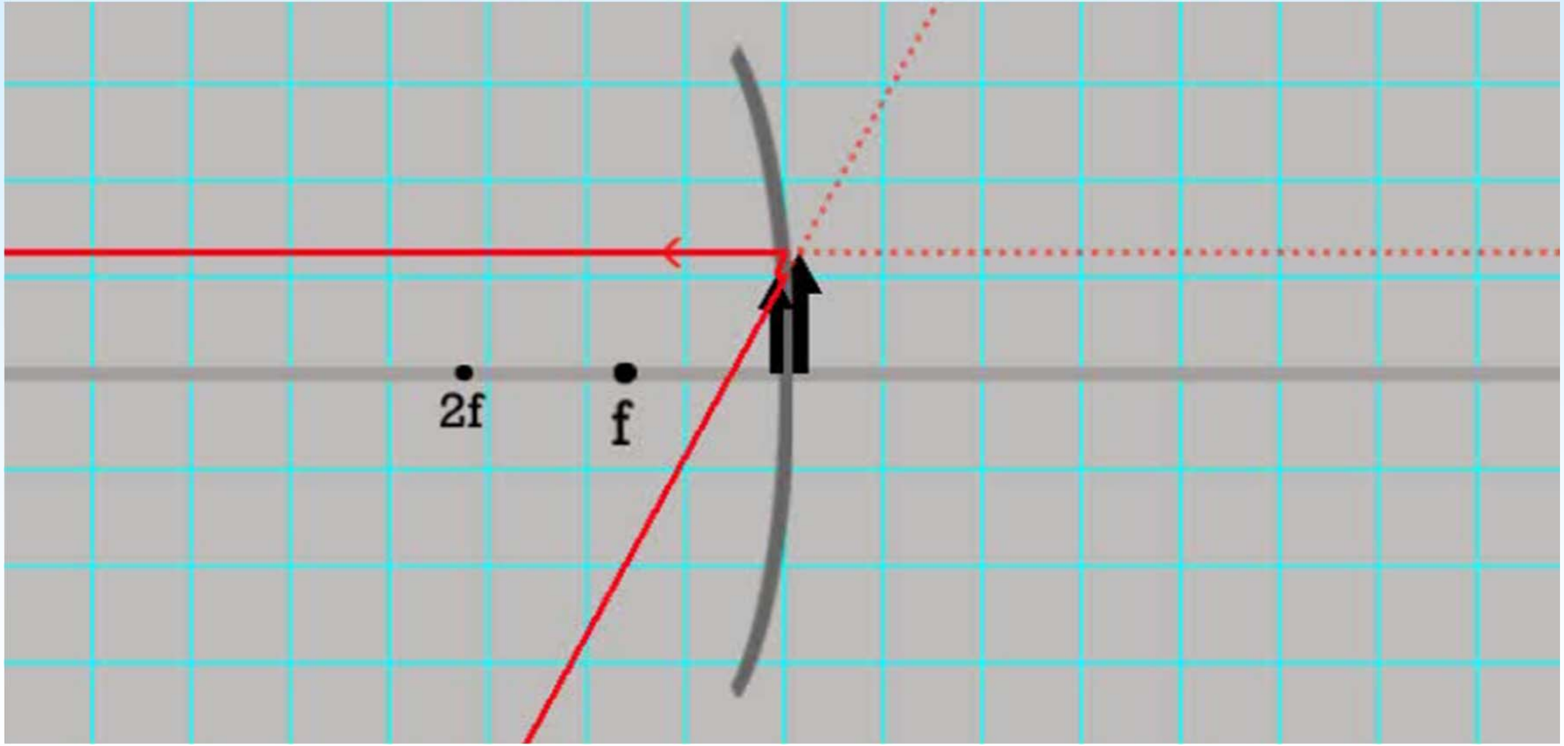


$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$f = \frac{R}{2}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Geometrical optics: Concave mirrors

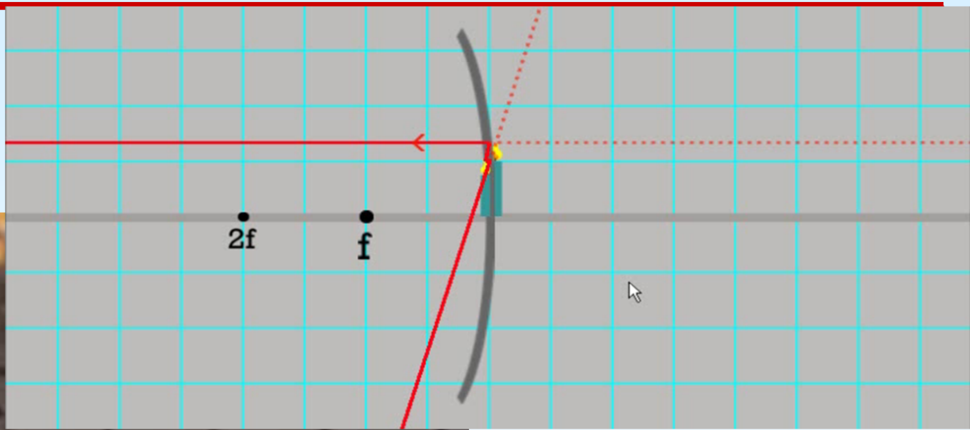


http://simbucket.com/lensesandmirrors/

Geometrical optics: Concave mirrors

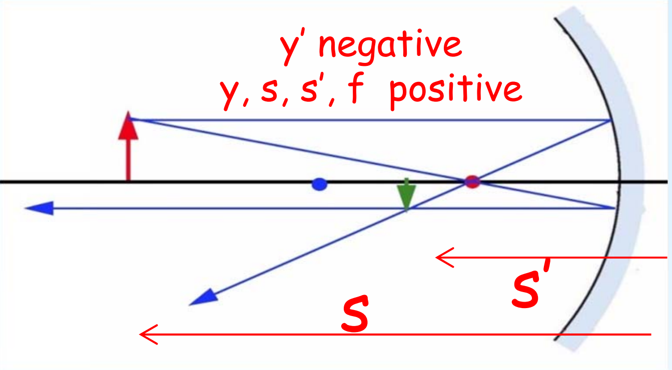
https://www.youtube.com/watch?v=7zv-4Zh-9R4

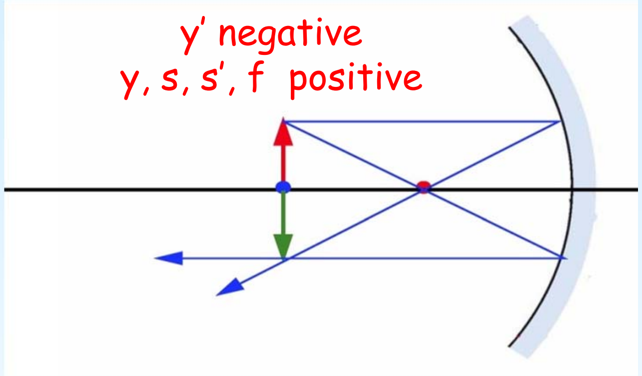
Geometrical optics: Concave mirrors



Concave mirrors can produce real images

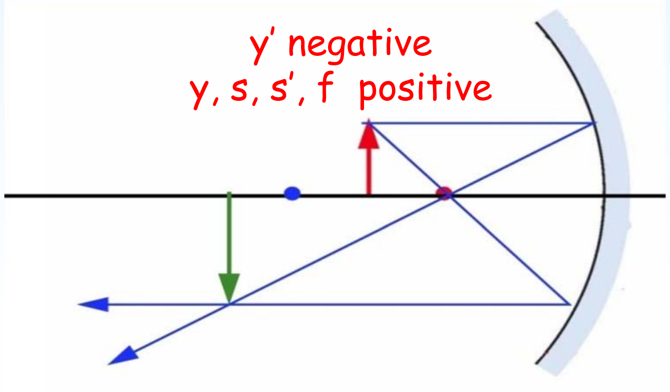
Geometrical optics: Concave mirrors

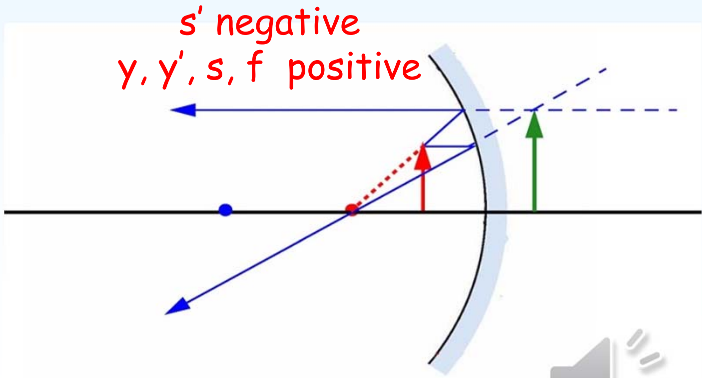




$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$





Part 3. Problems

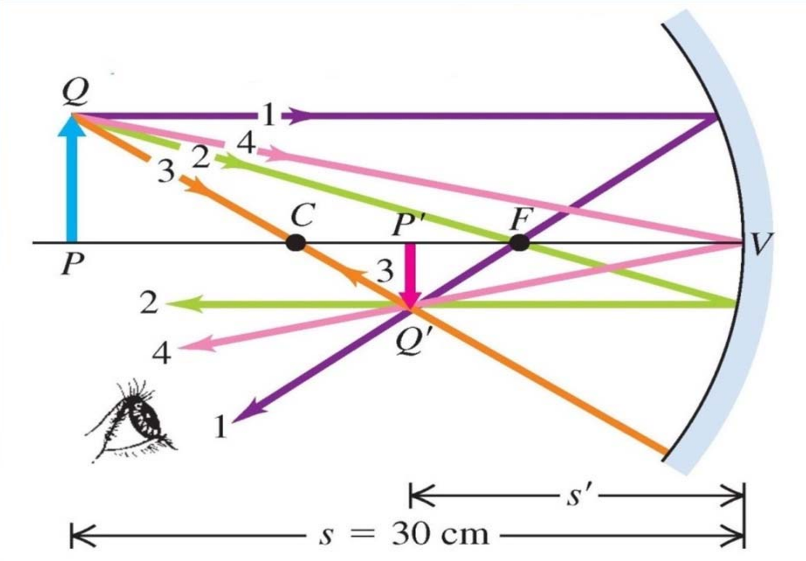
$$\lim_{x \to 8} \frac{1}{x-8} = \infty$$

$$\lim_{x \to 5} \frac{1}{x-5} =$$

Geometrical optics: Problems

An object is put 30 cm in front of a concave mirror with R = 20 cm.

Where will the image be ? And what will the magnification be ?



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Always positive for a concave mirror

f = R/2 = 10 cm and s = 30 cm

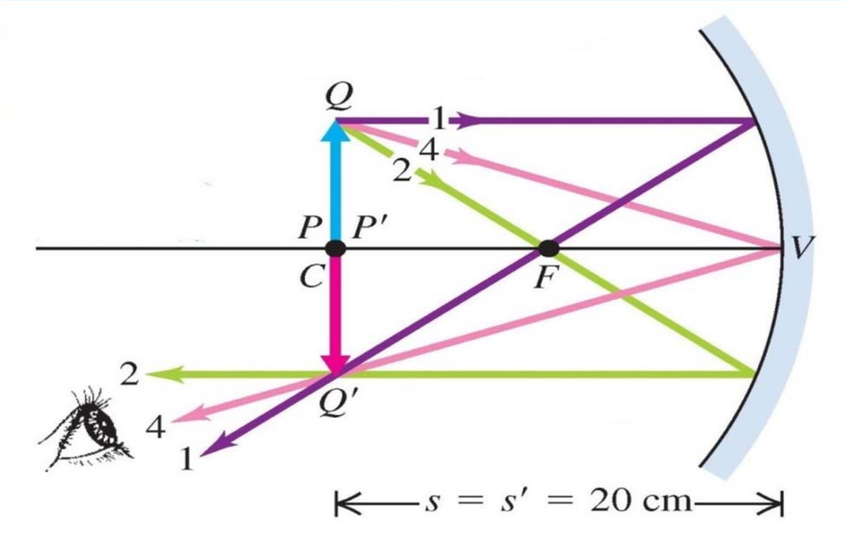
$$\frac{1}{30\text{ cm}} + \frac{1}{s'} = \frac{1}{10\text{ cm}} \Rightarrow s' = 15\text{ cm}$$

$$m = -\frac{15\text{ cm}}{30\text{ cm}} = -\frac{1}{2}$$

Geometrical optics: Problems

An object is put 20 cm in front of a concave mirror with R = 20 cm.

Where will the image be ? And what will the magnification be ?



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Always positive for a concave mirror

f = R/2 = 10 cm and s = 20 cm

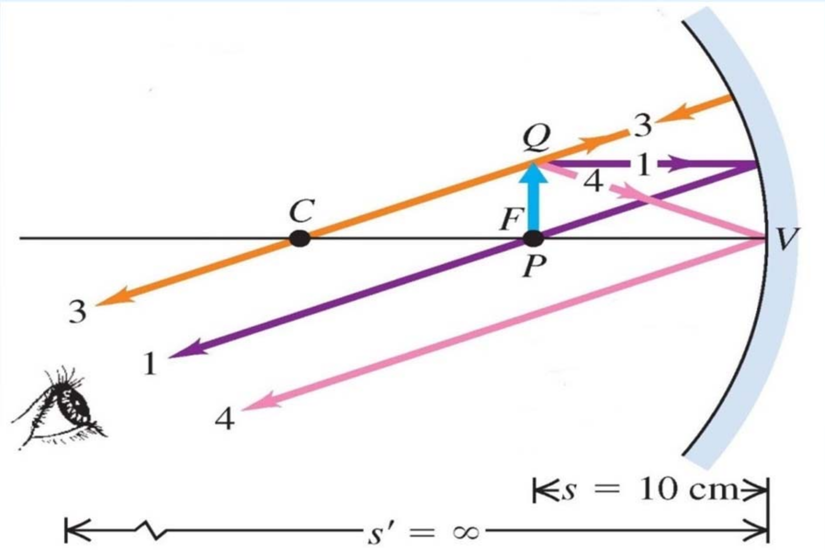
$$\frac{1}{20\text{ cm}} + \frac{1}{s'} = \frac{1}{10\text{ cm}} \Rightarrow s' = 20\text{ cm}$$

$$m = -\frac{20\text{ cm}}{20\text{ cm}} = -1$$

Geometrical optics: Problems

An object is put 10 cm in front of a concave mirror with R = 20 cm.

Where will the image be ? And what will the magnification be ?



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Always positive for a concave mirror

f = R/2 = 10 cm and s = 10 cm

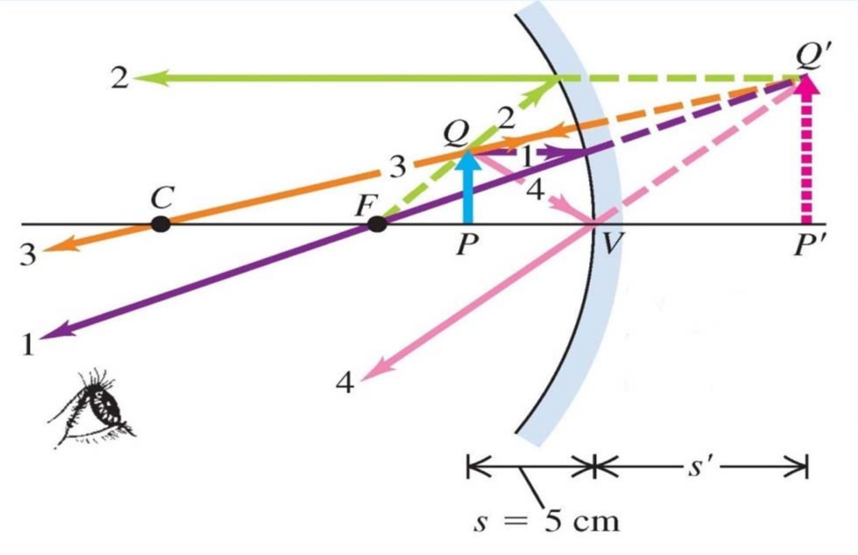
$$\frac{1}{10\text{ cm}} + \frac{1}{s'} = \frac{1}{10\text{ cm}} \Rightarrow s' = \infty \;(\text{or } -\infty)$$

$$m = -\frac{\infty \text{ cm}}{10\text{ cm}} = -\infty \;(\text{or } +\infty)$$

Geometrical optics: Problems

An object is put 5 cm in front of a concave mirror with R = 20 cm.

Where will the image be ? And what will the magnification be ?



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Always positive for a concave mirror

f = R/2 = 10 cm and s = 5 cm

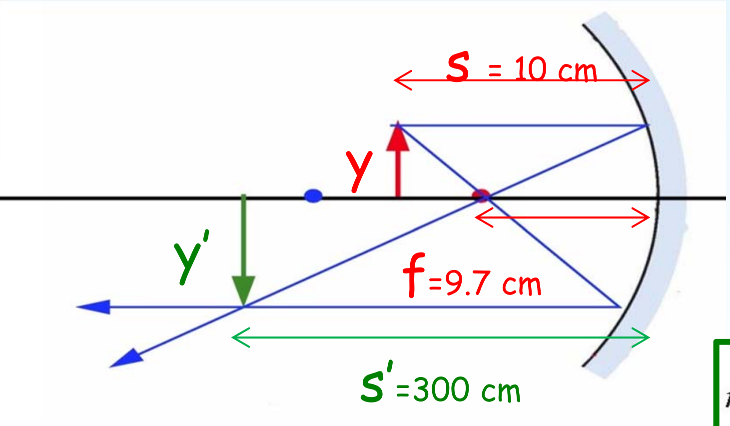
$$\frac{1}{5\text{ cm}} + \frac{1}{s'} = \frac{1}{10\text{ cm}} \Rightarrow s' = -10\text{ cm}$$

$$m = -\frac{-10\text{ cm}}{5\text{ cm}} = +2$$

A 5 mm large object is placed 10.0 cm in front of a concave mirror and produces an image on a wall that is 3.00 m away.

What is the radius of curvature and the distance to the focal point ?

What is the magnification and the size of the image ?



$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

$$\frac{1}{10.0 \text{ cm}} + \frac{1}{300 \text{ cm}} = \frac{2}{R}$$

$$R = 2\left(\frac{1}{10.0 \text{ cm}} + \frac{1}{300 \text{ cm}}\right)^{-1} = 19.4 \text{ cm}$$

$$f = \frac{R}{2}$$

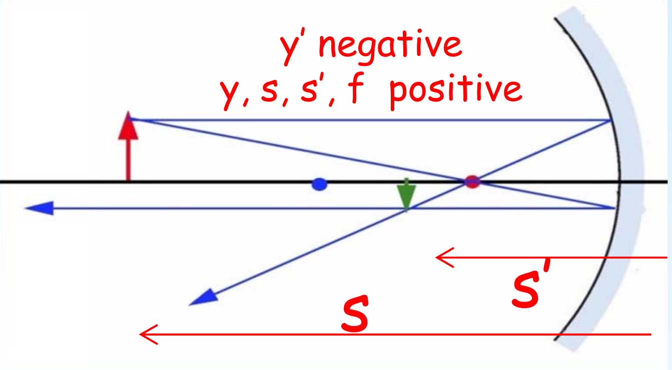
$$f = R/2 = 9.7 \text{ cm}$$

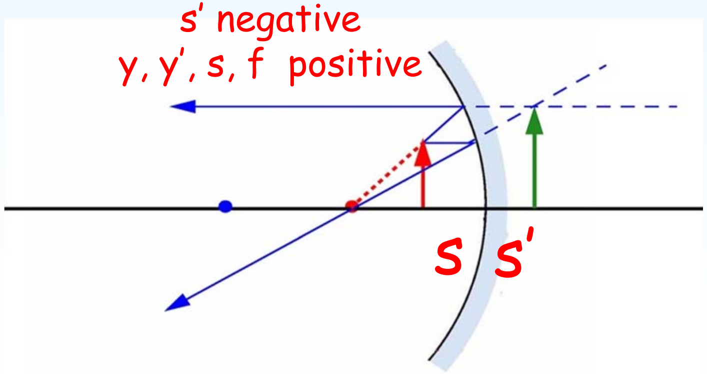
$$m = \frac{y'}{y} = -\frac{s'}{s}$$

$$m = -\frac{s'}{s} = -\frac{300 \text{ cm}}{10.0 \text{ cm}} = -30.0$$

The height of the image: 30 x 5 mm = 150 mm

Summary: Concave mirrors





$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Sign rules for mirrors:

Positive object distance (s) = Object is on the side of the incoming light.

Positive image distance (s′) = Image and outgoing light on the same side.

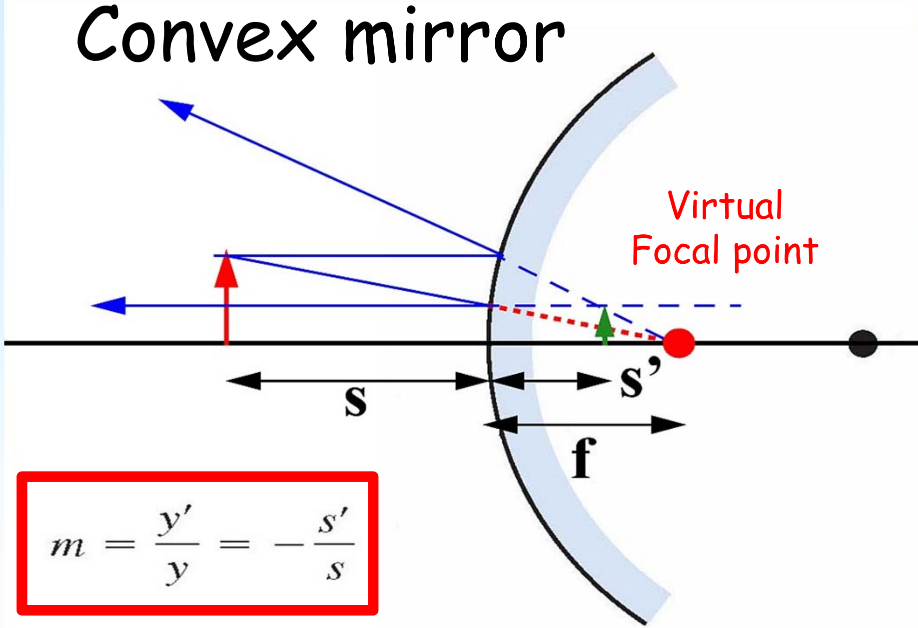
Positive radius (R) = Center is on the side of outgoing light.

Positive magnification (m) = Direction of object and image is the same.

Part 4. Convex mirrors

Geometrical optics: Convex mirrors

https://www.youtube.com/watch?v=J6LQM6re_1s

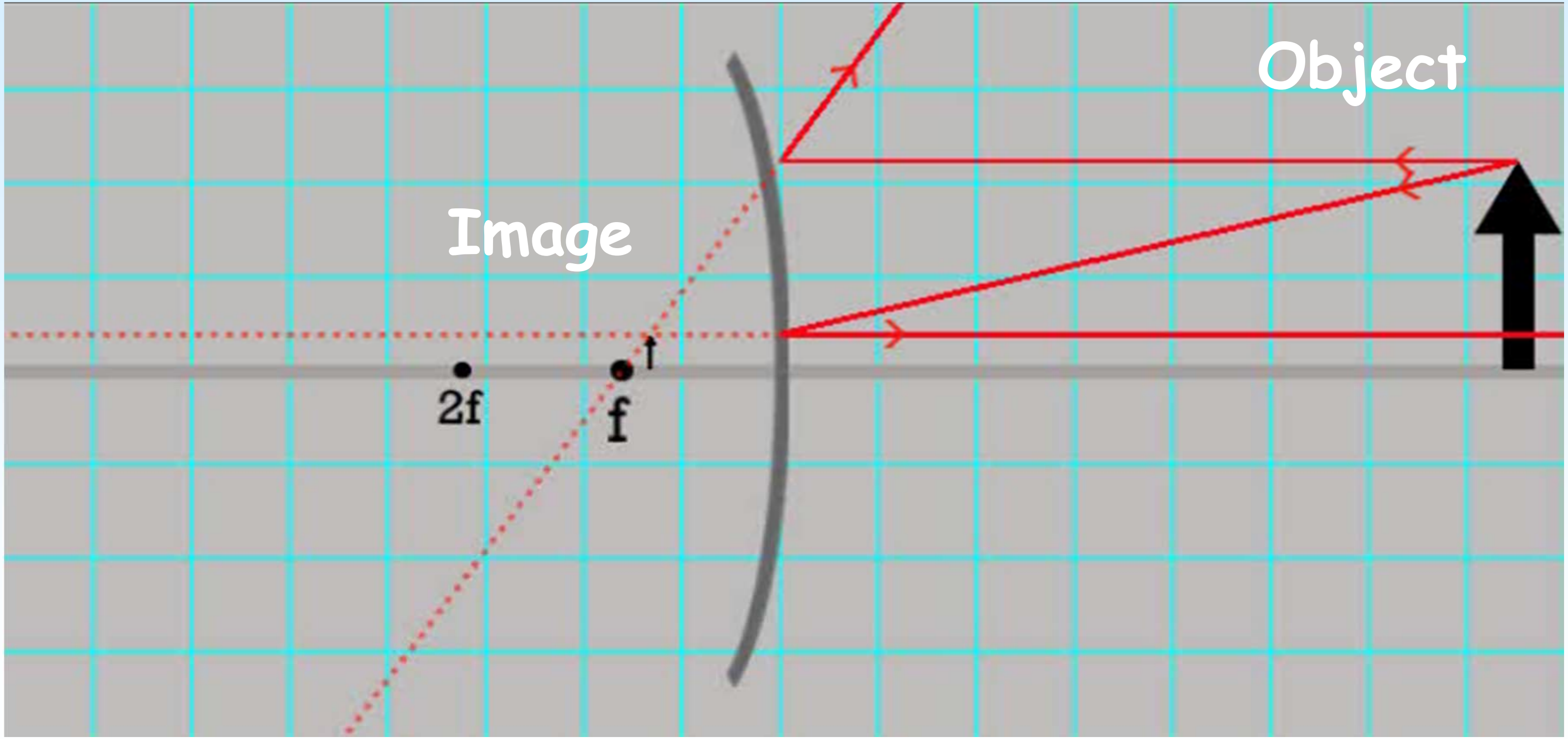


$$m = \frac{y'}{y} = -\frac{s'}{s}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

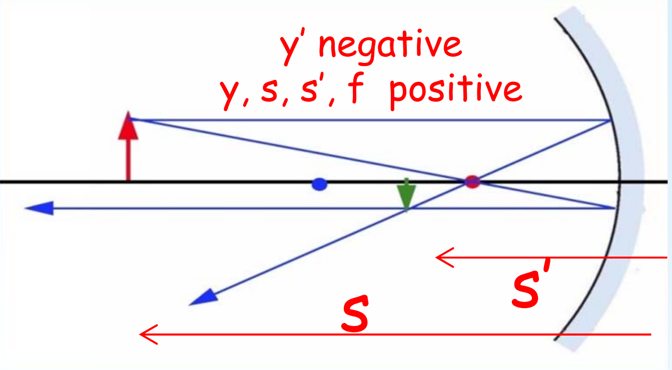
s', f negative
y, y', s positive

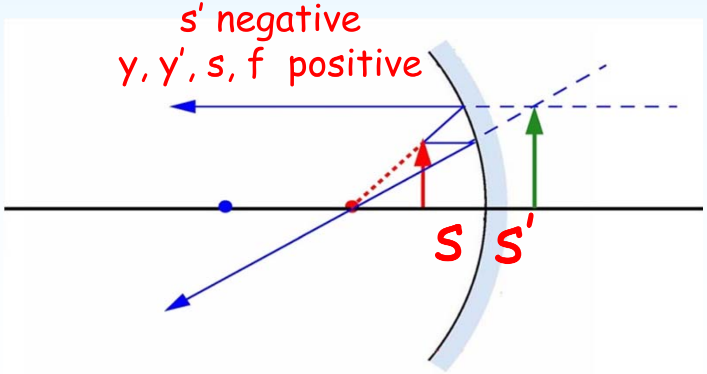
Geometrical optics: Convex mirrors



http://simbucket.com/lensesandmirrors/

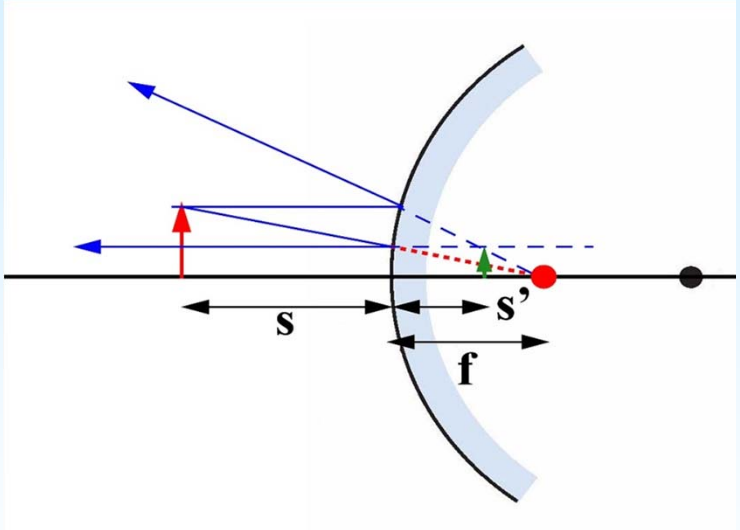
Summary: All spherical mirrors





$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$



s′, f negative
y, y′, s positive

Part 5. Problems

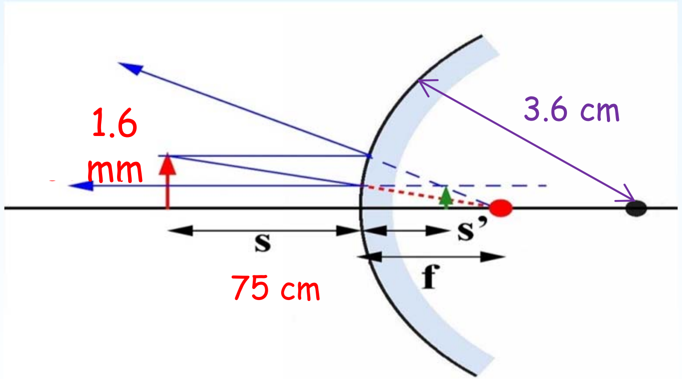
$$\lim_{x \to 8} \frac{1}{x-8} = \infty$$

$$\lim_{x \to 5} \frac{1}{x-5} = 5$$

Geometrical optics: Problems

Santa who is 1.60 m high is reflected in a Christmas tree ornament with a diameter of 7.20 cm at a distance of 0.750 m.
A 1.6 mm mosquito sits on his nose.

Where does the image of the mosquito end up and how big is it?



$$f = \frac{R}{2} = 7.2 / 2 / 2 = -1.80 \text{ cm}$$

f is negative for a convex mirror

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

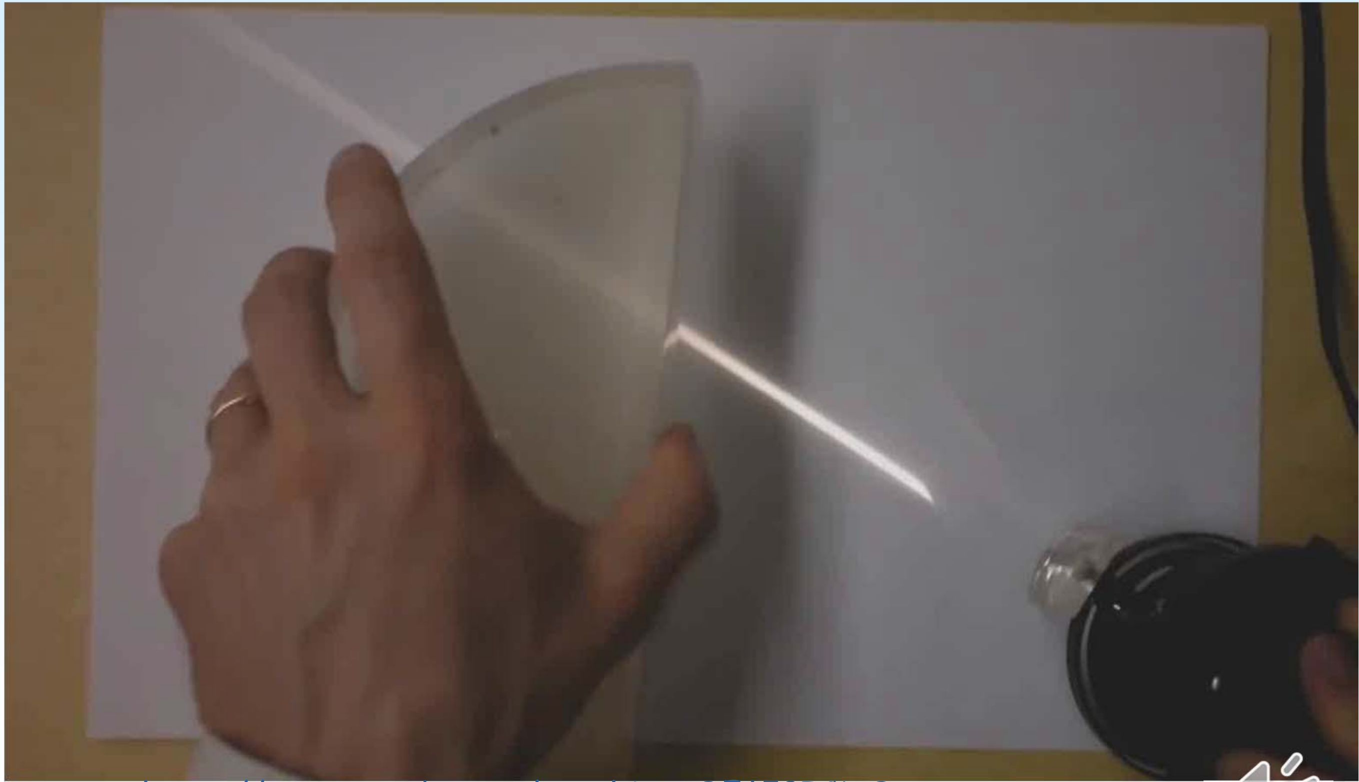
$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-1.80 \text{ cm}} - \frac{1}{75.0 \text{ cm}}$$

$$s' = -1.76 \text{ cm}$$

$$m = \frac{y'}{y} = -\frac{s'}{s} = -\frac{-1.76 \text{ cm}}{75.0 \text{ cm}} = 0.0234$$

$$y' = my = 0.0234 \times 1.6 \text{ mm} = 3.8 \times 10^{-2} \text{ mm}$$

Part 6. Spherical surface



https://www.youtube.com/watch?v=uQE659ICjqQ

Geometrical optics: Spherical surface

Given

A spherical surface with radius of curvature **R** that has an object at a distance **S**

Goal

Derive a formula so that one can calculate where the image ends up = **S'**

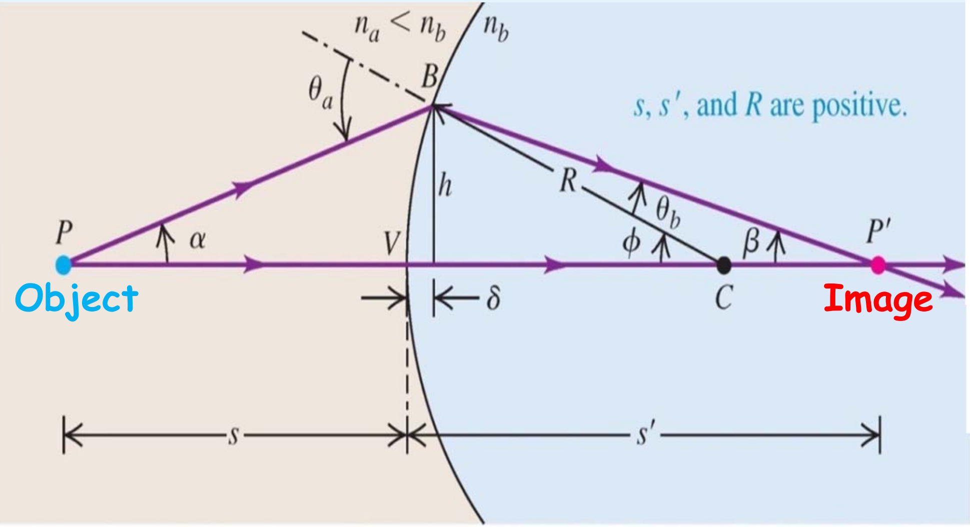
How

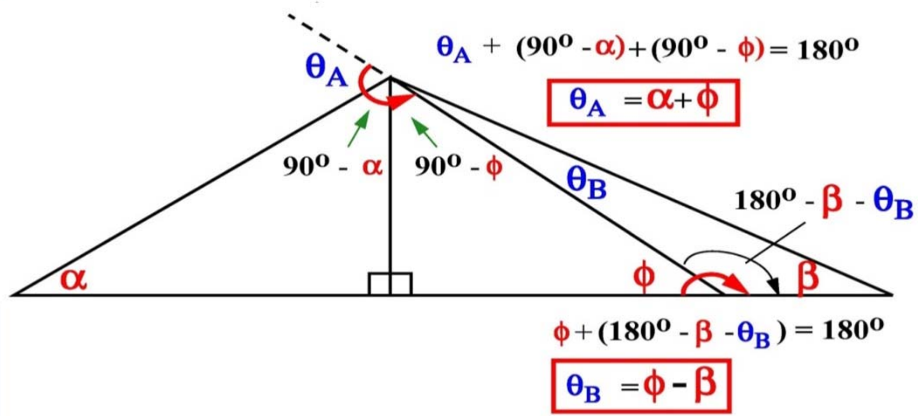
Law of refraction + Trigonometry

Step 1 Trigonometry

The sum of the angles covering a straight line is 180 degrees.

➡ **Relationship between θ and α, β, ϕ**





$$\theta_A = \alpha + \phi$$
$$\theta_B = \phi - \beta$$

Step 2

The law of refraction

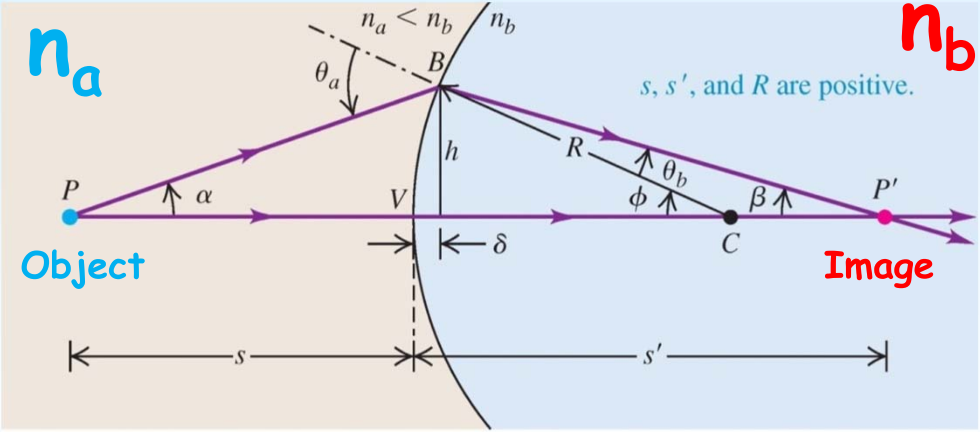
➡ Relationship between α, β, ϕ and n_a, n_b

The law of refraction

$$n_a \sin\theta_a = n_b \sin\theta_b$$

Small angle approximation:

$$n_a\theta_a = n_b\theta_b$$



$$\theta_A = \alpha + \phi$$
$$\theta_B = \phi - \beta$$

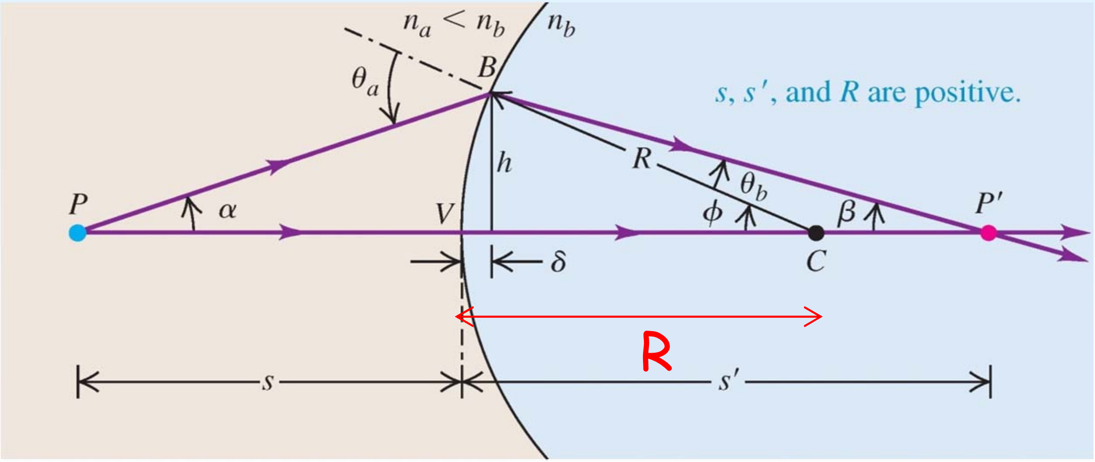
$$n_a\alpha + n_b\beta = (n_b - n_a)\phi$$

Step 3

Trigonometry

Apply tangens on angles

➡ **Relationship between α, β, ϕ and S, R, S'**



If small angles and small δ :

$$\alpha = \frac{h}{s} \qquad \beta = \frac{h}{s'} \qquad \phi = \frac{h}{R}$$

$$\tan\alpha = \frac{h}{s+\delta} \qquad \tan\beta = \frac{h}{s'-\delta} \qquad \tan\phi = \frac{h}{R-\delta}$$

Step 4

Combine step 2 with step 3

Step 3: $\alpha = \frac{h}{s} \qquad \beta = \frac{h}{s'} \qquad \phi = \frac{h}{R}$

Step 2: $n_a\alpha + n_b\beta = (n_b - n_a)\phi$

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

Geometrical optics: Spherical surface

Given

A spherical surface with radius of curvature **R** that has an object at a distance **S** and an image at a distance **S'**

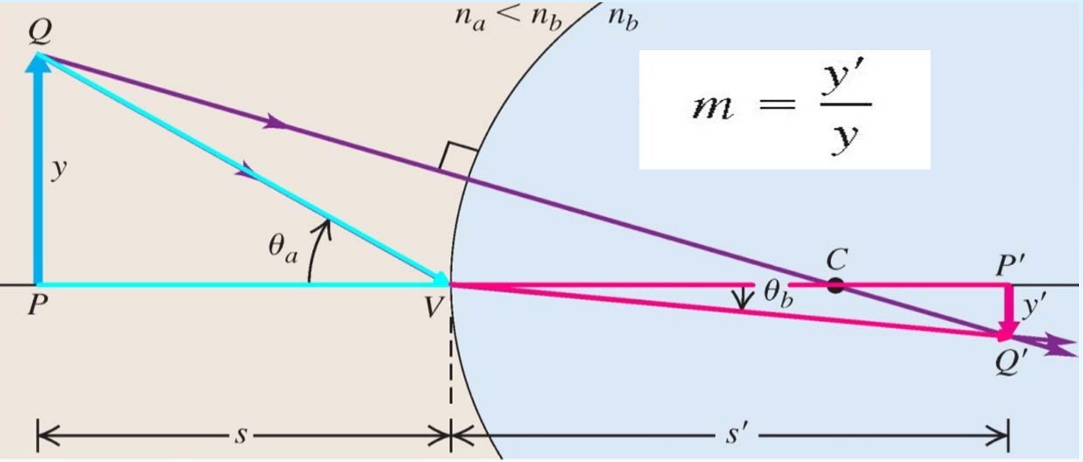
Goal

Derive a formula so that the magnification m can be calculated

How

The law of refraction + Trigonometry

Geometrical optics: Spherical surface



$$m = \frac{y'}{y}$$

Step 1 - Geometry

Image is inverted

$$\tan\theta_a = \frac{y}{s} \qquad \tan\theta_b = \frac{-y'}{s'}$$

If small angles:

$\theta_a = y/s$ $\qquad$ $\theta_b = -y'/s'$

Step 2 – Law of refraction

$$n_a \sin\theta_a = n_b \sin\theta_b$$

If small angles:

$$n_a\, \theta_a = n_b\, \theta_b$$

Combine step 1 and 2

$$\frac{n_a y}{s} = -\frac{n_b y'}{s'}$$

$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s}$$

Sign rules:

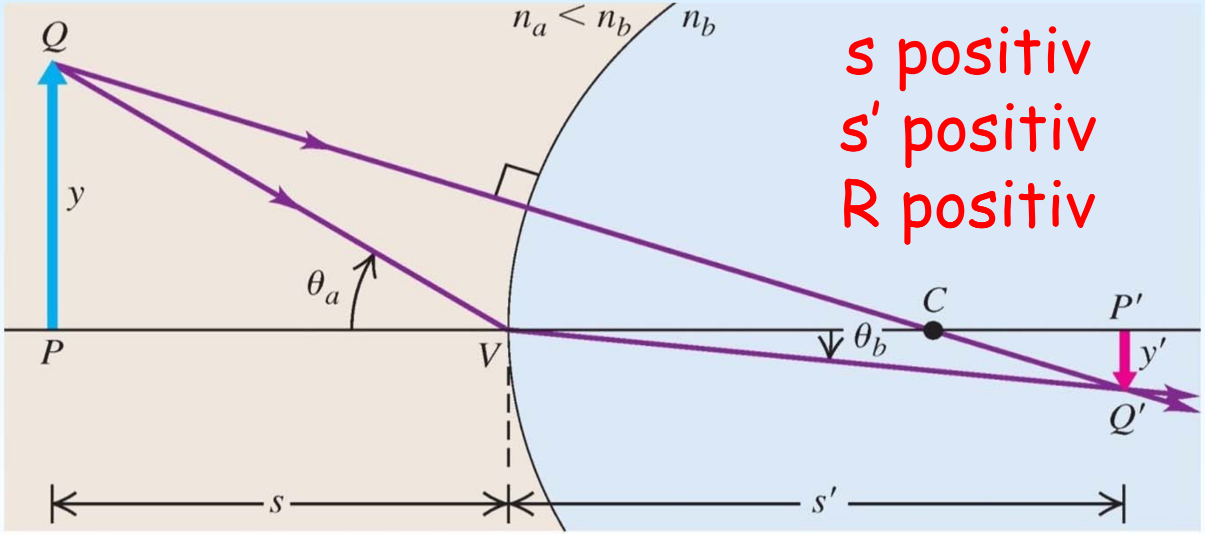
Positive object distance (s) = Object is on the side of the incoming light.

Positive image distance (s') = Image and outgoing light on the same side.

Positive radius (R) = Center is on the side of outgoing light.

Positive magnification (m) = Direction of object and image is the same.

Summary - spherical surface



s positiv

s' positiv

R positiv

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s}$$

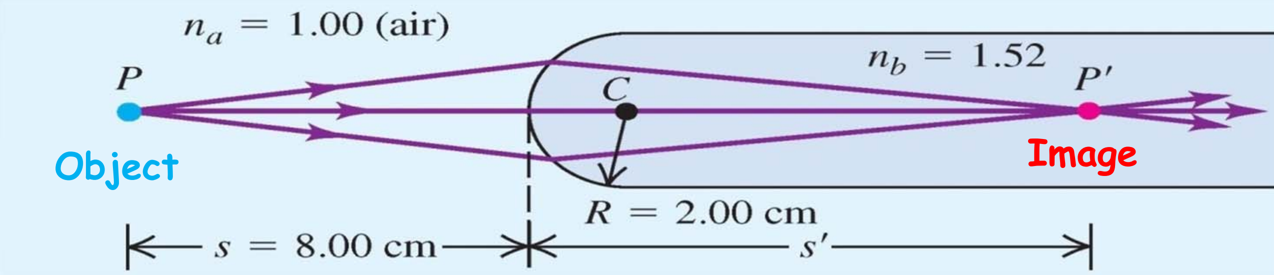
Part 7. Problems

$$\lim_{x \to 8} \frac{1}{x-8} = \infty$$

$$\lim_{x \to 5} \frac{1}{x-5} = 5$$

Geometrical optics: Problems

Where does the image end up and what will be the magnification ?



$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

Image location:

$$\frac{1.00}{8.00\ \text{cm}} + \frac{1.52}{s'} = \frac{1.52 - 1.00}{+2.00\ \text{cm}}$$

$$s' = +11.3\ \text{cm}$$

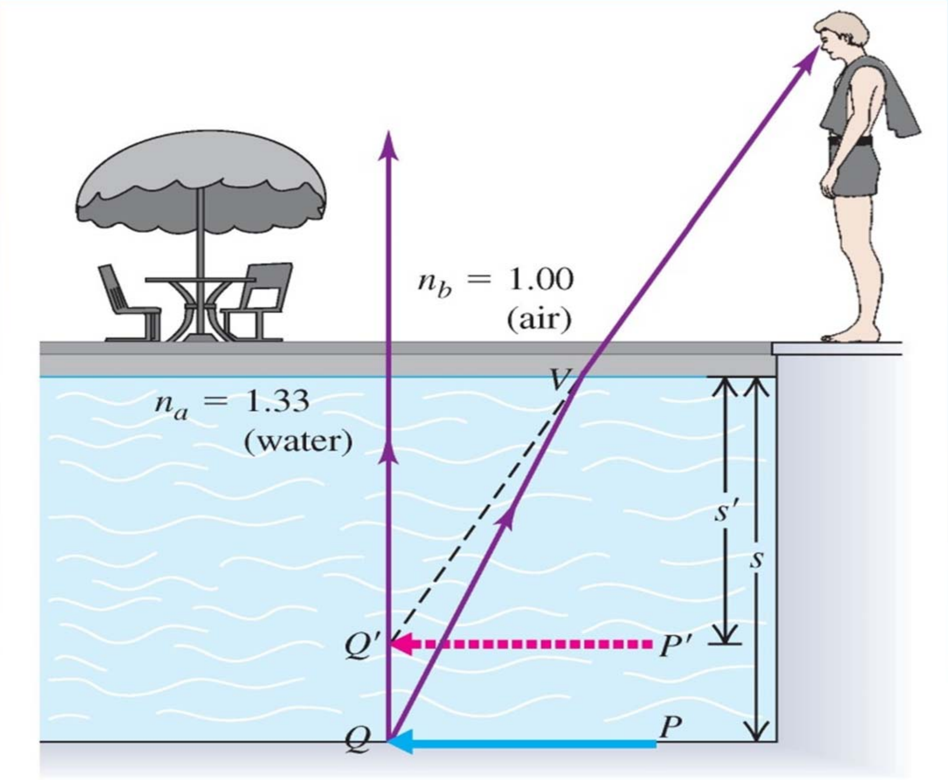
$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s}$$

Magnification:

$$m = -\frac{n_a s'}{n_b s} = -\frac{(1.00)(11.3\ \text{cm})}{(1.52)(8.00\ \text{cm})} = -0.929$$

Part 8. Flat surface

https://www.youtube.com/watch?v=7aU8sX8cFNs



Special case: Flat surface

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} = 0$$

$R \to \infty$

$n_a/s = -n_b/s'$

$-s'/s = n_b/n_a$

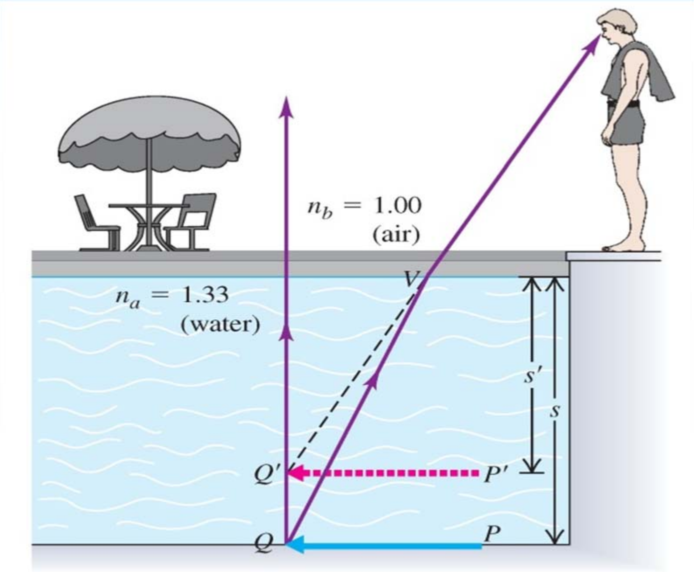
Part 9. Problems

$$\lim_{x \to 8} \frac{1}{x-8} = \infty$$

$$\lim_{x \to 5} \frac{1}{x-5} =$$

A swimming pool is 2 m deep. A person looks straight down at the bottom.

How deep does the swimming pool appear to be ?



$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{1.33}{2.00\text{ m}} + \frac{1.00}{s'} = 0$$

$$s' = -1.50\text{ m}$$

Geometrical optics: Problems

The water in Flathead Lake is so clear that it appears very shallow. Can you believe it's actually 370 feet deep?

Image Credits: National Geographic

This is a simple illusion, but very cool nonetheless.

$$n_a / s = -n_b / s'$$

$$-s'/s = n_b/n_a = 1.00/1.33 = 0.75$$

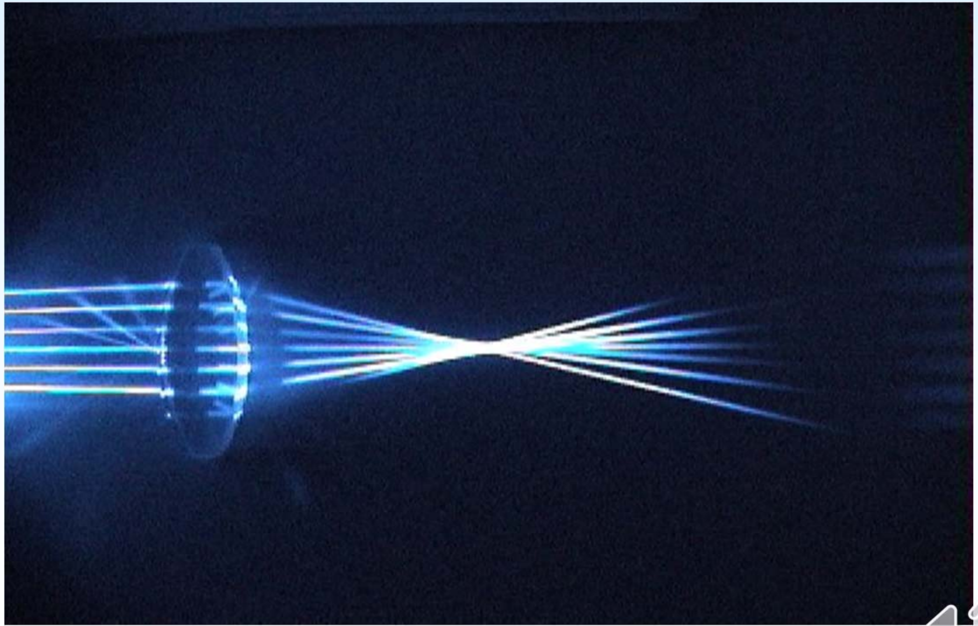
That is, the refraction of the light makes the lake look a factor of 0.75 shallower.

0.75×370 feet = 278 feet = 85 m

According to the article, the lake should look like it is 85 m deep.

This is obviously not true!
The lake is only a few meters deep here.

Part 10. Convex lenses

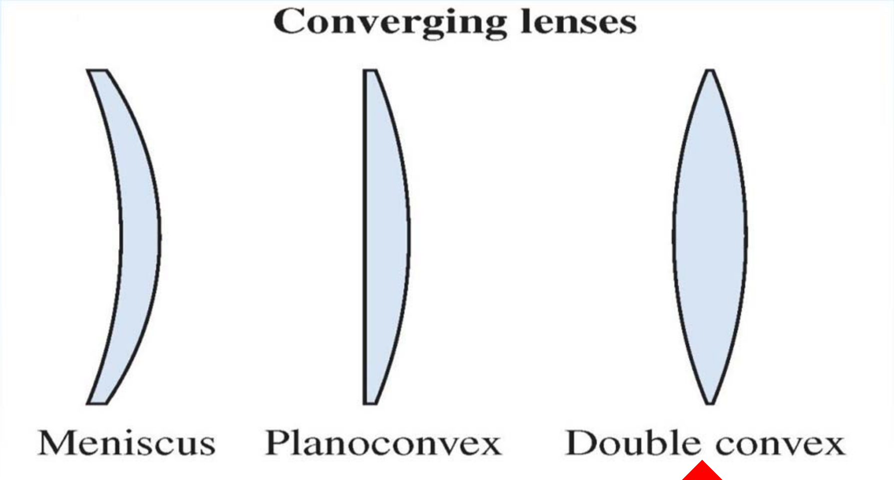


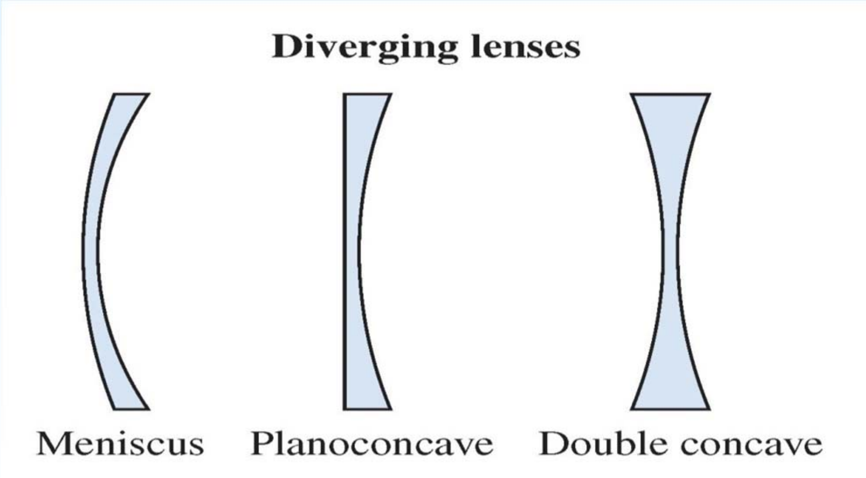
Geometrical optics: Convex lenses

Different types of lenses

A lens thicker in the middle than in the edges is convergent.

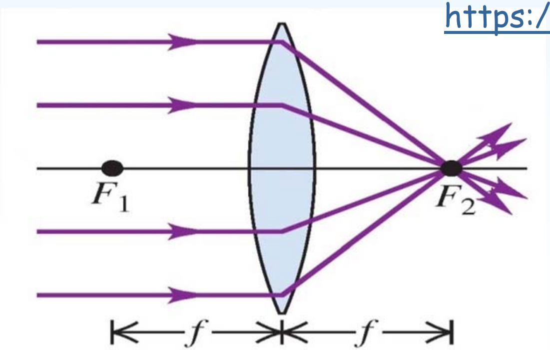
A lens thinner in the middle than in the edges is divergent.

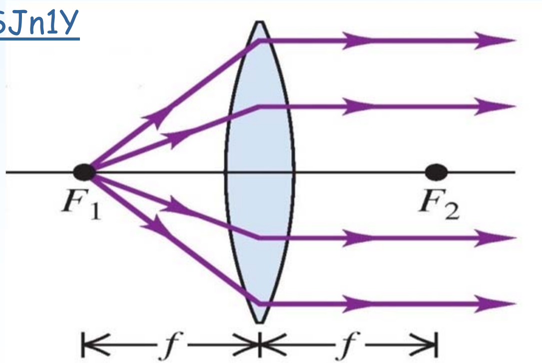




Geometrical optics: Convex lenses

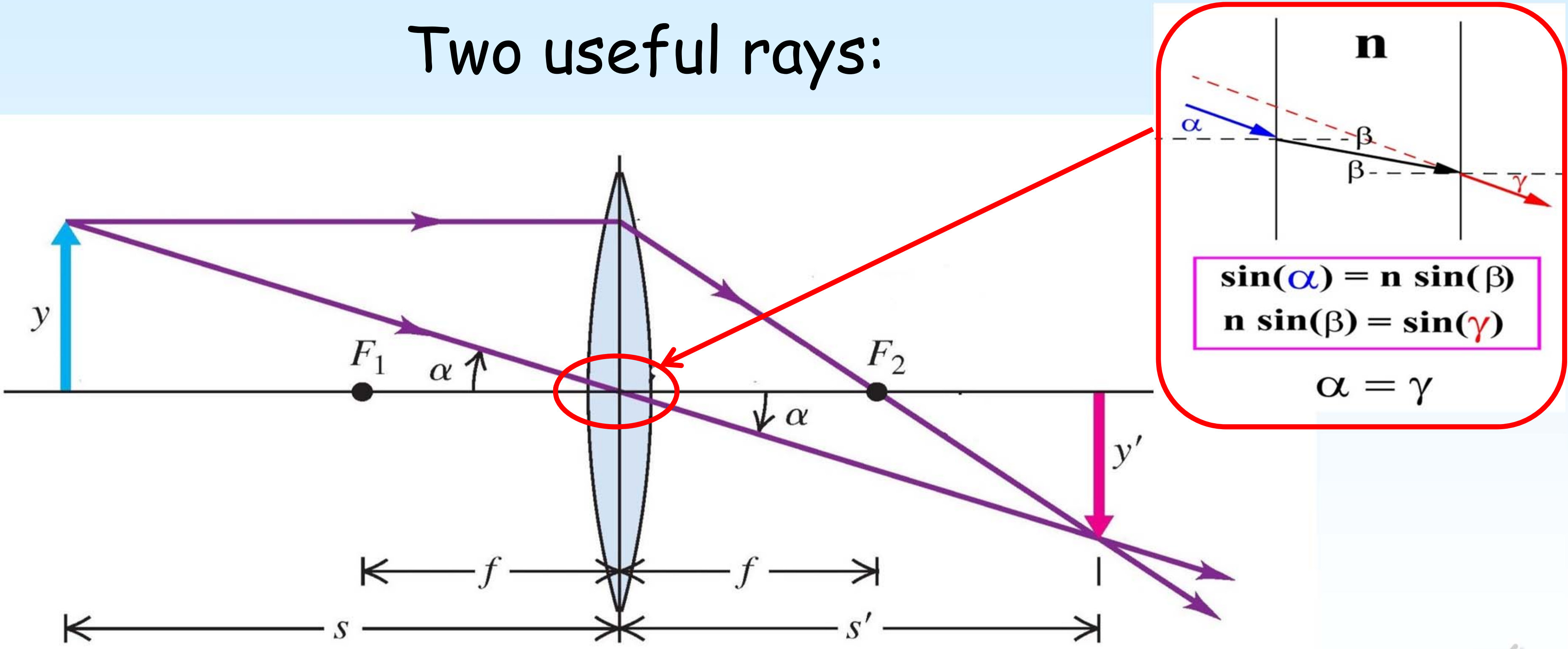
https://www.youtube.com/watch?v=4zuB_dSJn1Y





Geometrical optics: Convex lenses

Two useful rays:



Geometrical optics: Convex lenses

Given

A lens with a focal length f is having an object at a distance = S

Goal

1. Derive a formula for the magnification = m

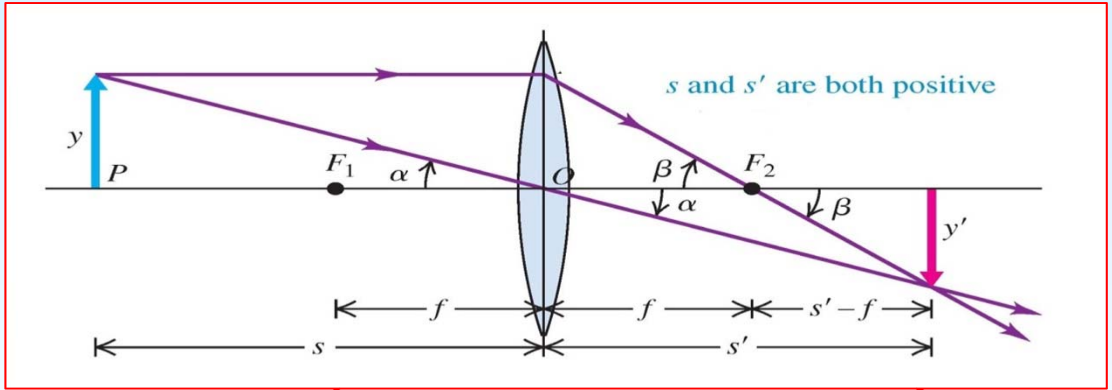
2. Derive a formula so that one can calculate where the image ends up = S '

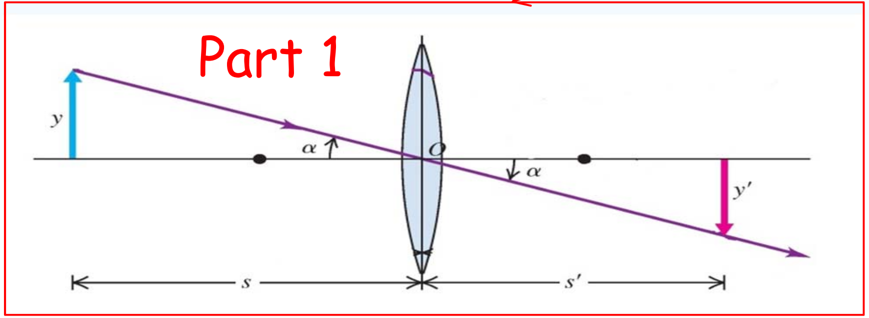
How

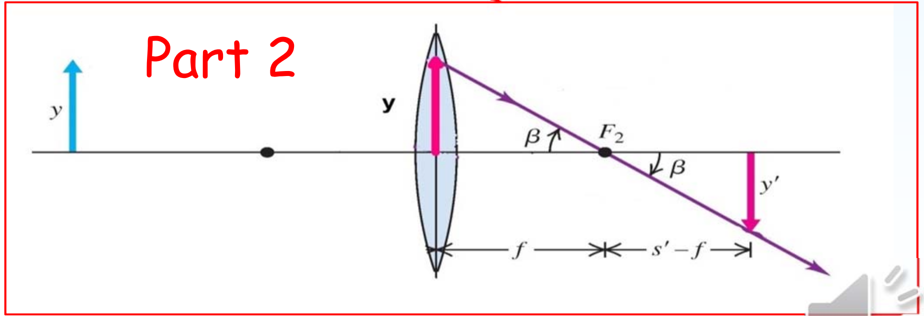
Trigonometry

Geometrical optics: Convex lenses

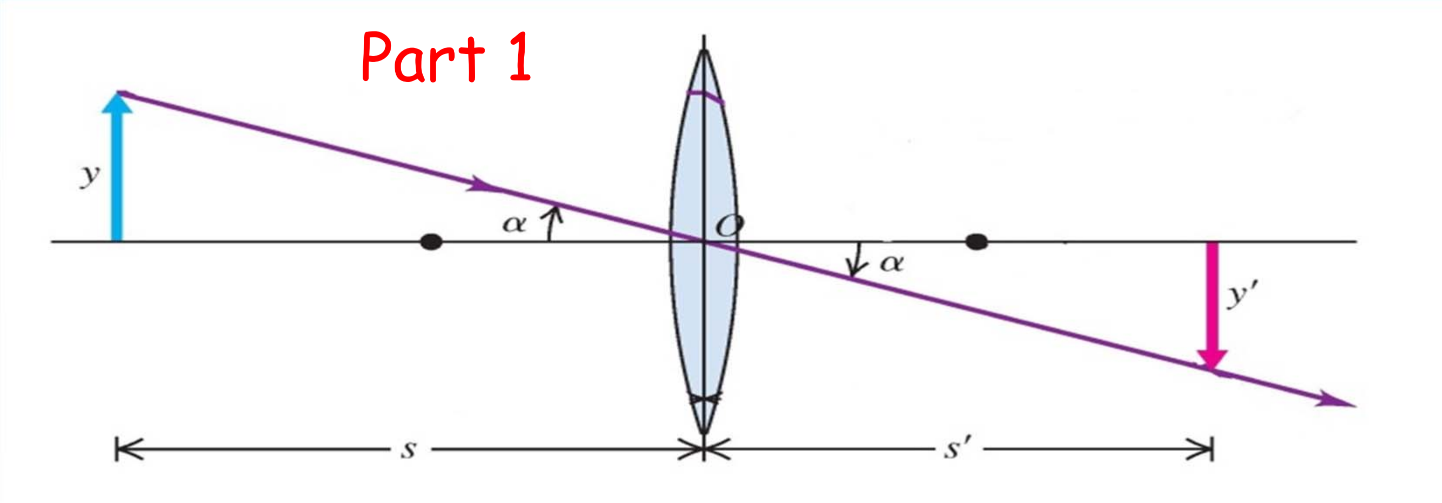
Starting point for deriving both lens formulas:







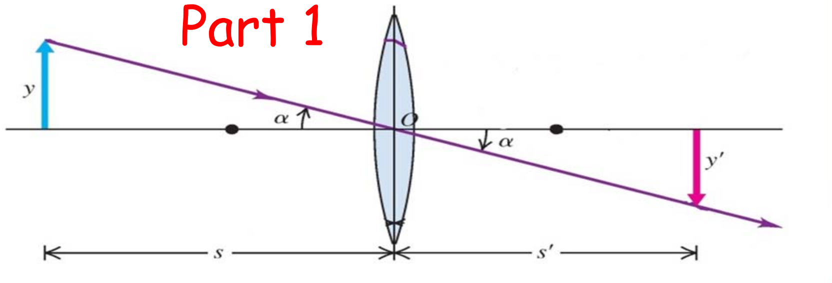
The magnification formula for lenses:



$$\tan(\alpha) = \frac{y}{s} = -\frac{y'}{s'} \Rightarrow \frac{y'}{y} = -\frac{s'}{s} \Rightarrow m = \frac{y'}{y} = -\frac{s'}{s}$$

Geometrical optics: Convex lenses

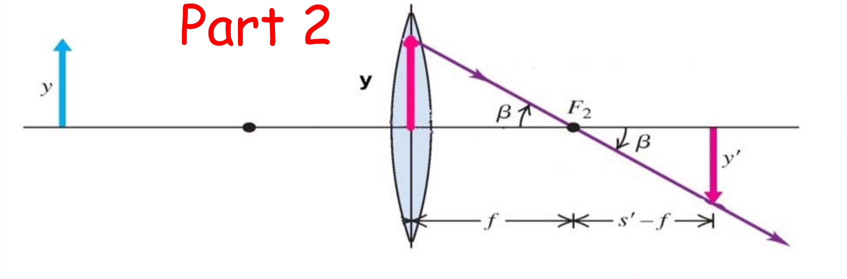
Part 1



$$\tan(\alpha) = \frac{y}{s} = -\frac{y'}{s'}$$

$$\frac{y'}{y} = -\frac{s'}{s}$$

Part 2



$$\tan(\beta) = \frac{y}{f} = -\frac{y'}{s'-f}$$

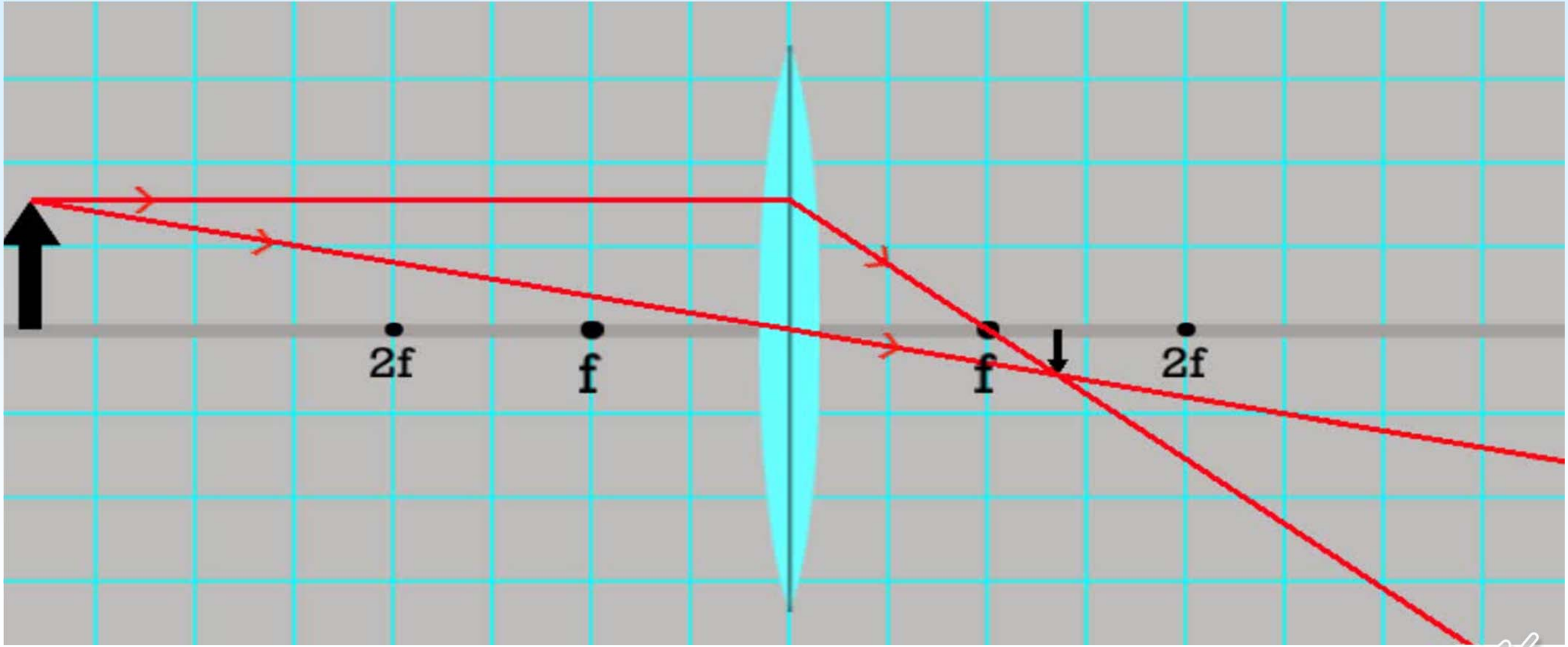
$$\frac{y'}{y} = -\frac{s'-f}{f}$$

$$-\frac{s'}{s} = -\frac{s'-f}{f} \Rightarrow -\frac{s'}{s} = -\frac{s'-f}{f}$$

s' s'

$$\Rightarrow -\frac{1}{s} = -\frac{s'-f}{fs'} = -\frac{1}{f} + \frac{1}{s'} \Rightarrow \boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

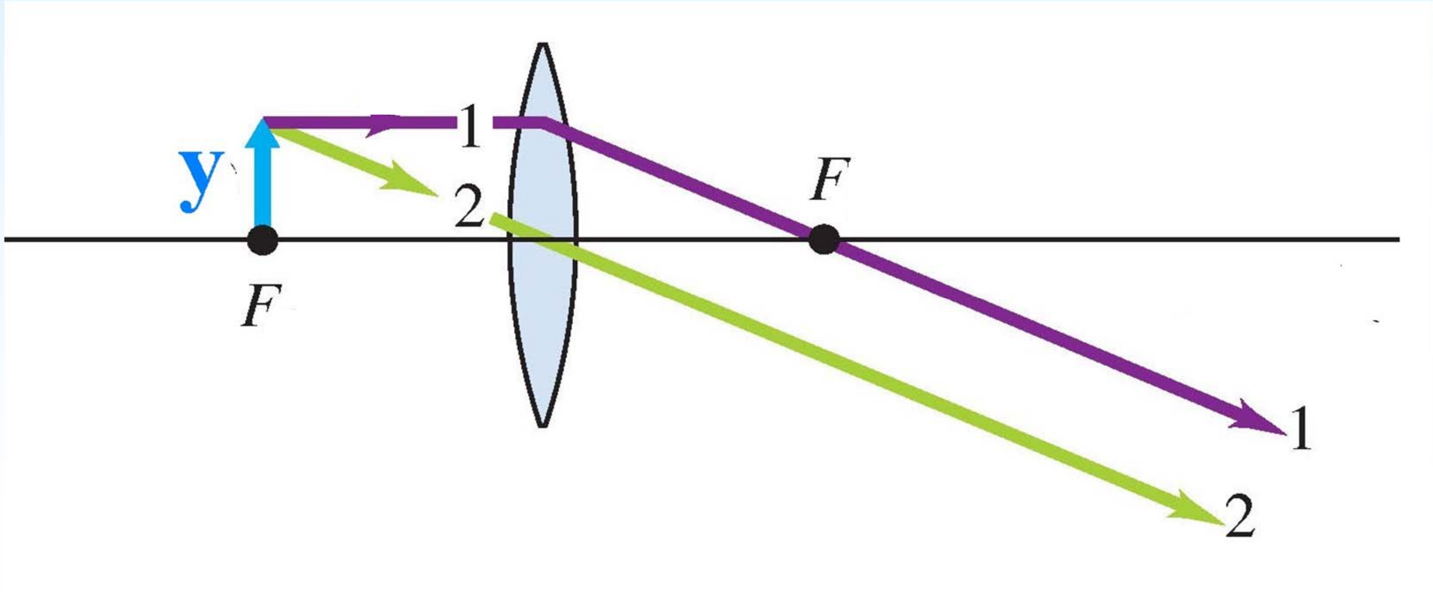
Geometrical optics: Convex lenses



http://simbucket.com/lensesandmirrors/

Geometrical optics: Convex lenses

An object placed at the focal point seems to be infinitely far away



<u>Sign rules:</u>

Positive object distance (s)
Object and incoming light is on the same side.

Positive image distance (s')
Image and outgoing light is on the same side

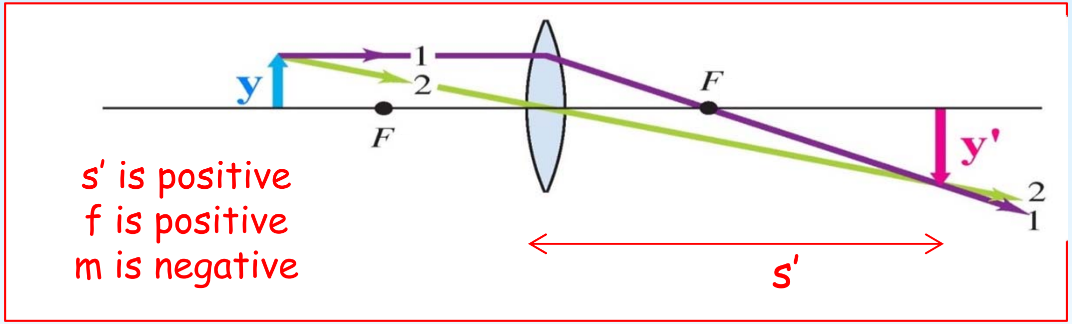
Positive focal length (f)
Converging (convex) lenses

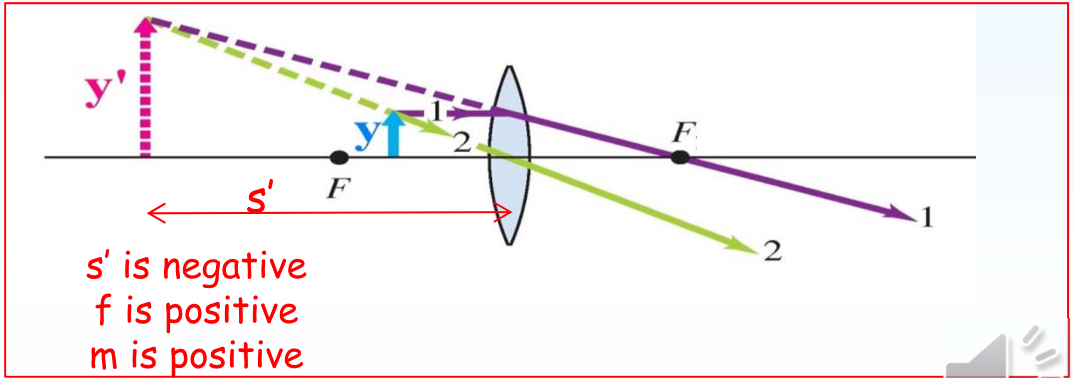
Positive magnification (m)
Same direction of object and image.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Summary of convex lenses





Geometrical optics: Convex lenses

Gauss' formula

Newton's formula

Formula collection

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

$$f = \frac{s\,s'}{s + s'}$$

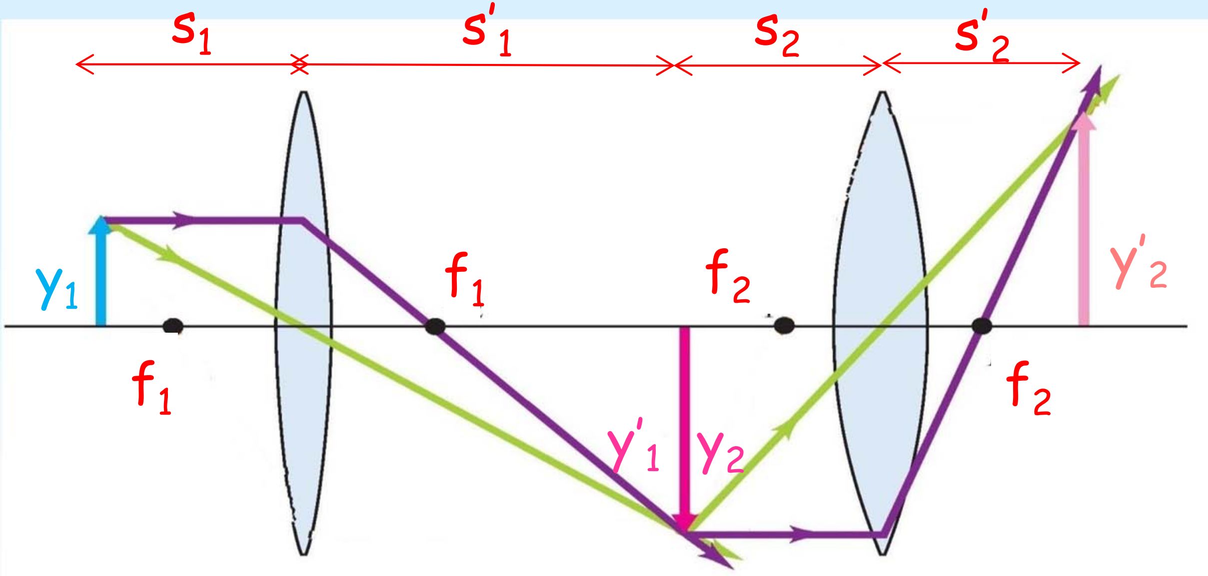
$$s = \frac{s'f}{s' - f}$$

$$s' = \frac{sf}{s - f}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

$$m = -\frac{f}{s - f}$$

Geometrical optics: Convex lenses



$$\frac{1}{f_1} = \frac{1}{s_1} + \frac{1}{s'_1}$$

$$m_1 = -\frac{s'_1}{s_1}$$

$$\frac{1}{f_2} = \frac{1}{s_2} + \frac{1}{s'_2}$$

$$m_2 = -\frac{s'_2}{s_2}$$

How to combine the magnification of two lenses:

$$\Rightarrow \quad m = m_1 m_2 = \frac{s'_1 s'_2}{s_1 s_2}$$

Part 11. Problems

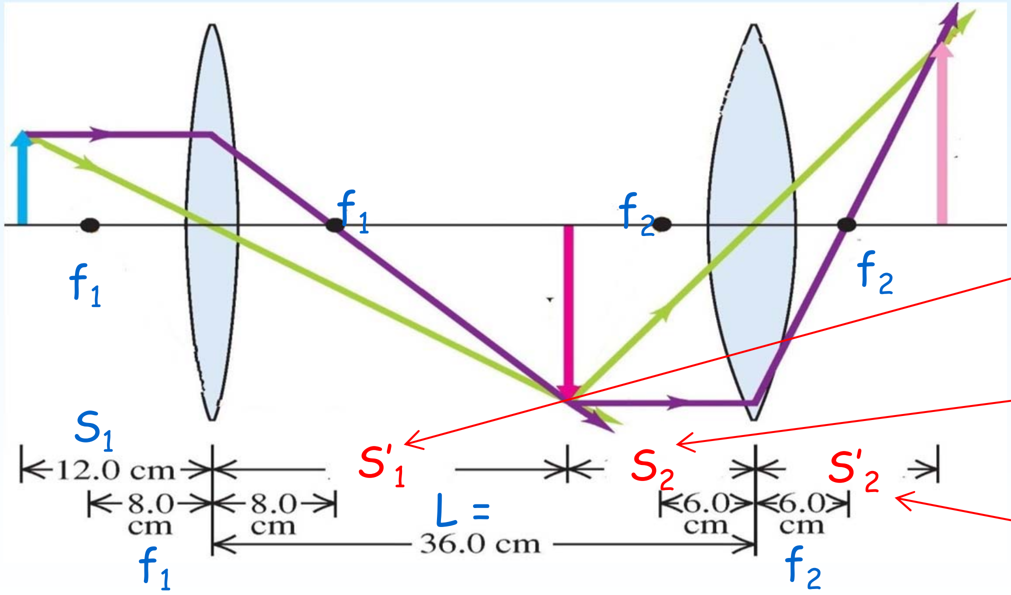
$$\lim_{x \to 8} \frac{1}{x-8} = \infty$$

$$\lim_{x \to 5} \frac{1}{x-5} = 5$$

Geometrical optics: Problems

Two lenses with f_1 = 8.0 cm and f_2 = 6.0 cm are placed 36.0 cm apart. An object is placed 12.0 cm in front of the first lens.

Where will the image end up ?



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

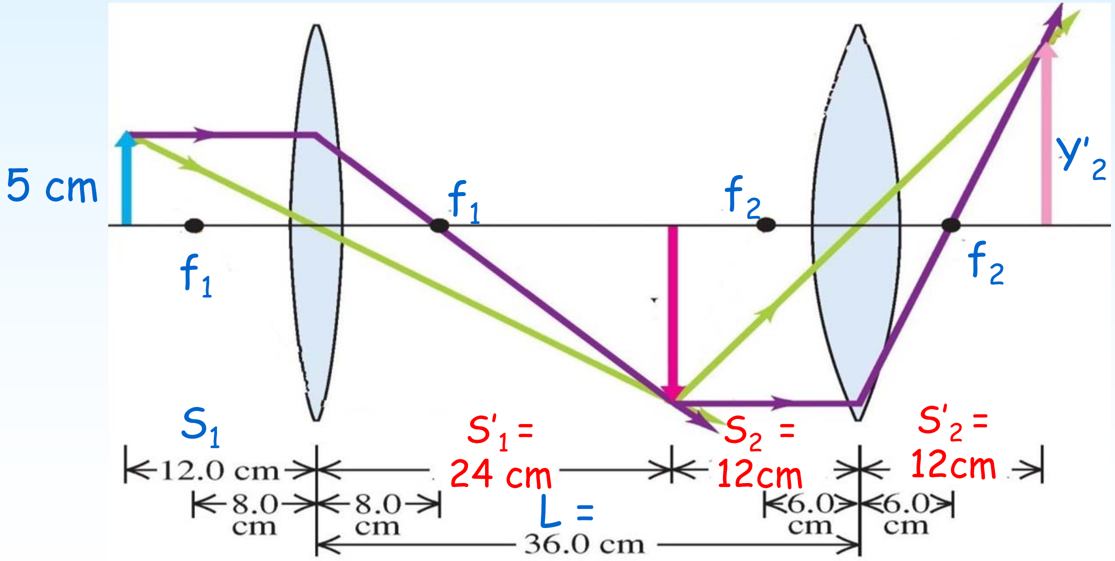
$$\frac{1}{12.0\text{ cm}} + \frac{1}{S'_1} = \frac{1}{8.0\text{ cm}} \qquad S'_1 = +24.0\text{ cm}$$

$S_2 = L - S_1' = 36 - 24 = 12$ cm

$$\frac{1}{12.0\text{ cm}} + \frac{1}{S'_2} = \frac{1}{6.0\text{ cm}} \qquad S'_2 = +12.0\text{ cm}$$

Geometrical optics: Problems

Two lenses with f_1 = 8.0 cm and f_2 = 6.0 cm are placed 36.0 cm apart.
An object that is 5.0 cm high is placed 12.0 cm in front of the first lens.
What will be the height Y'_2 of the image ?

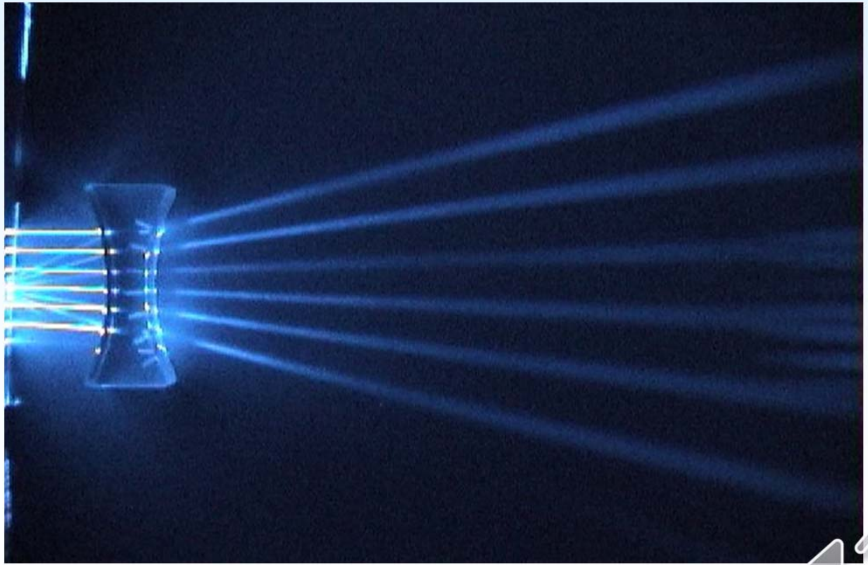


$$m = m_1 m_2 = \frac{s'_1 s'_2}{s_1 s_2}$$

$$m = m_1 m_2 = \frac{24 \cdot 12}{12 \cdot 12} = +2.0$$

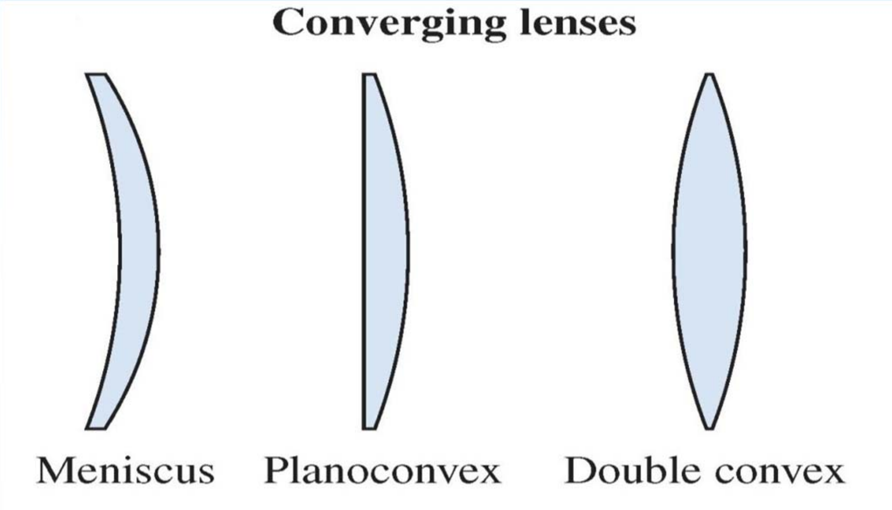
Y'_2 = 5.0 x 2.0 = 10 cm

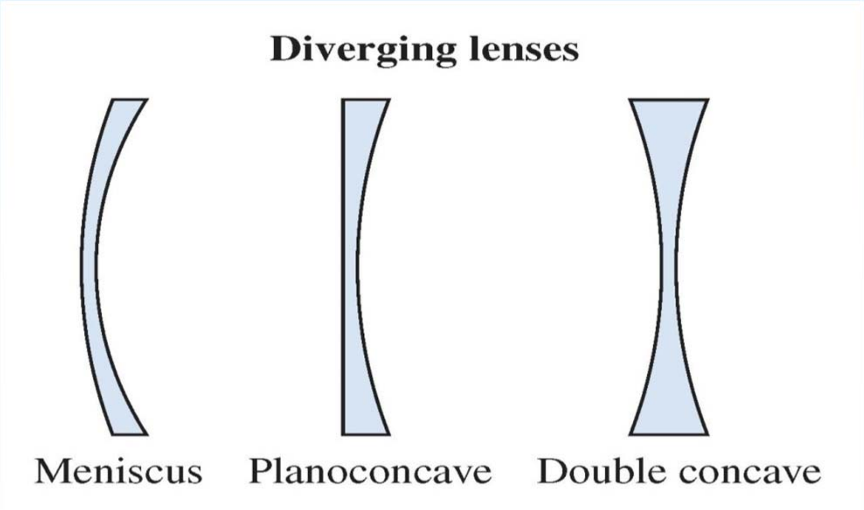
Part 12. Concave lenses



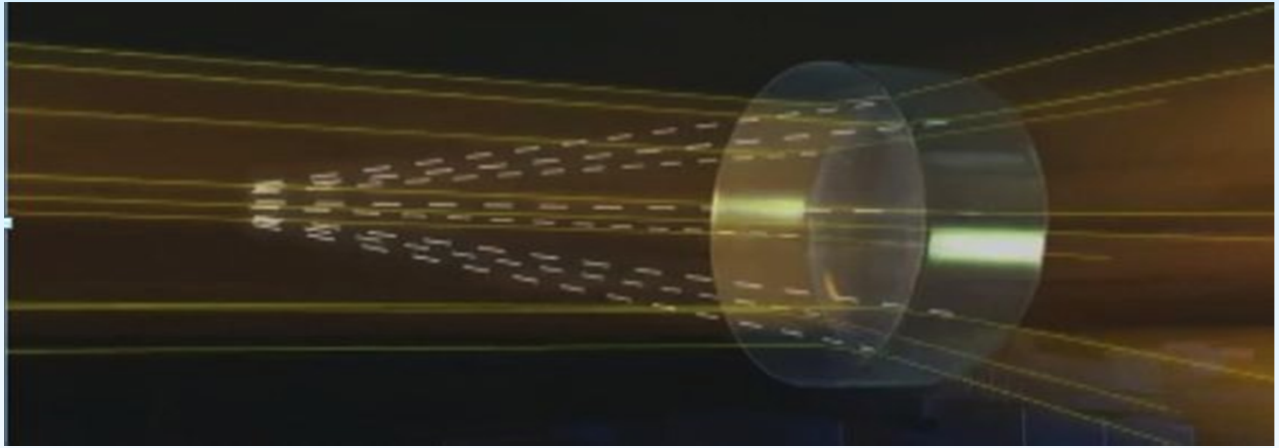
Geometrical optics: Concave lenses

Different types of lenses

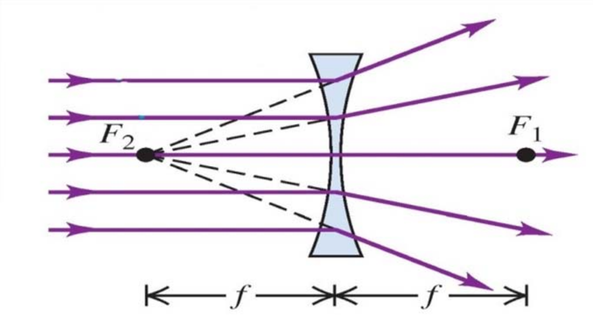


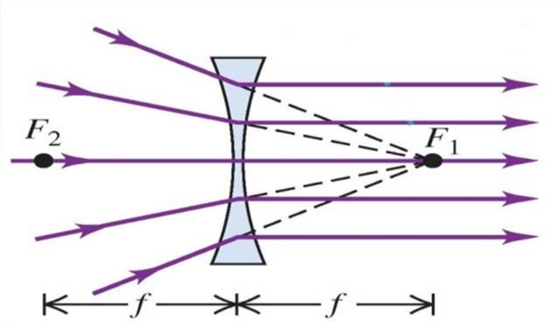


Geometrical optics: Concave lenses

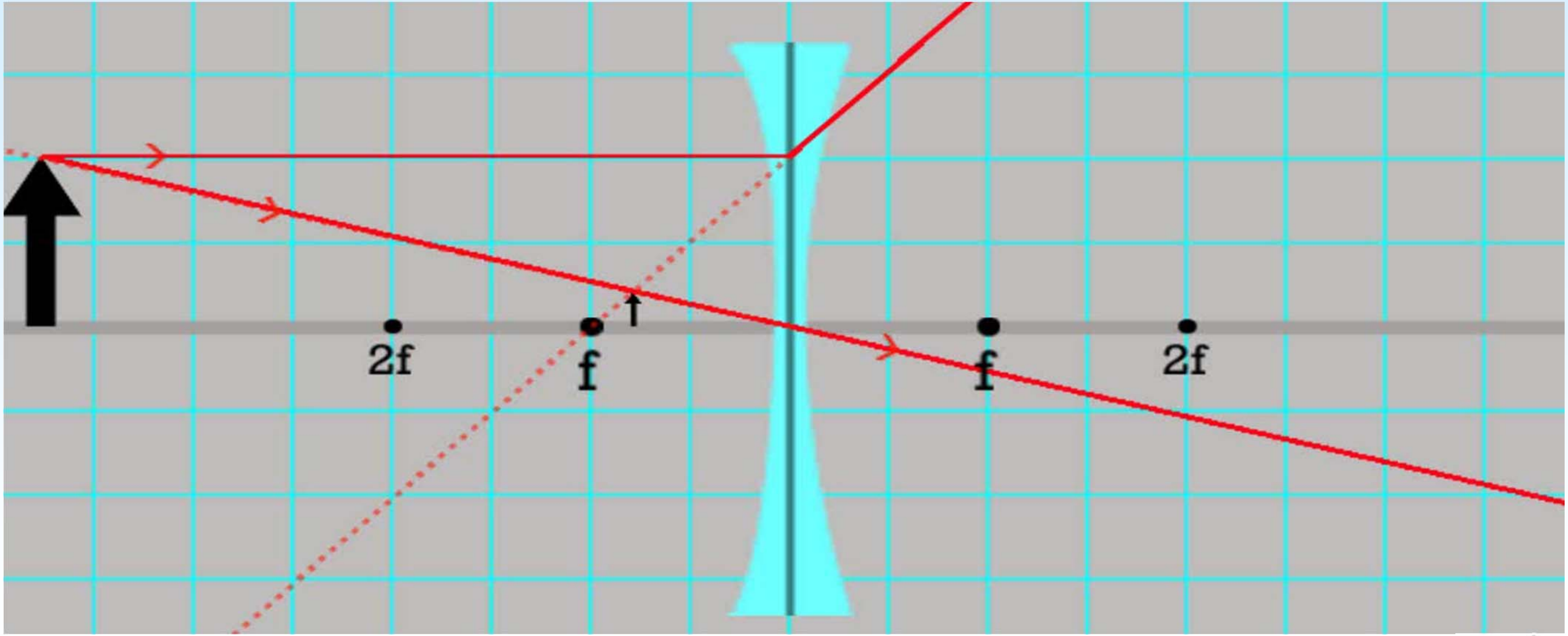


https://www.youtube.com/watch?v=4zuB_dSJn1Y





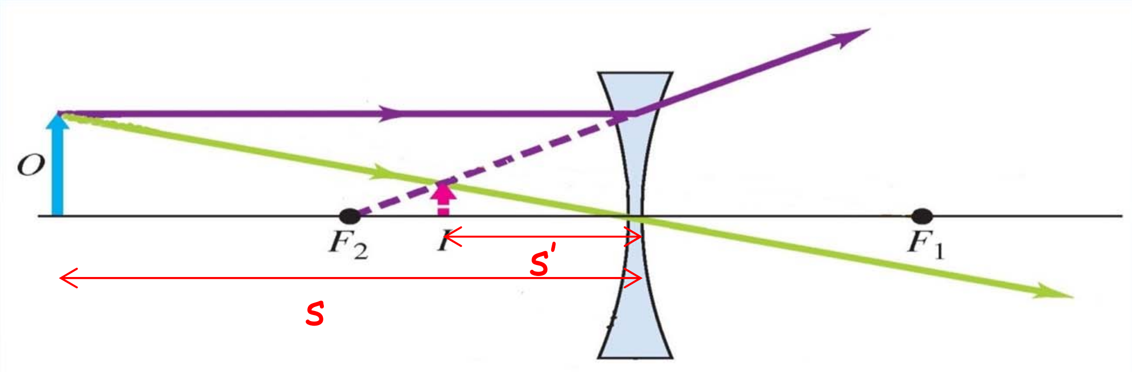
Geometrical optics: Concave lenses



http://simbucket.com/lensesandmirrors/

Geometrical optics: Concave lenses

The lens formula for concave lenses



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = -\frac{s'}{s}$$

f is negative for diverging lenses

s' is negative for diveregent lenses

m is always positive

Part 13. Problems

$$\lim_{x \to 8} \frac{1}{x-8} = \infty$$

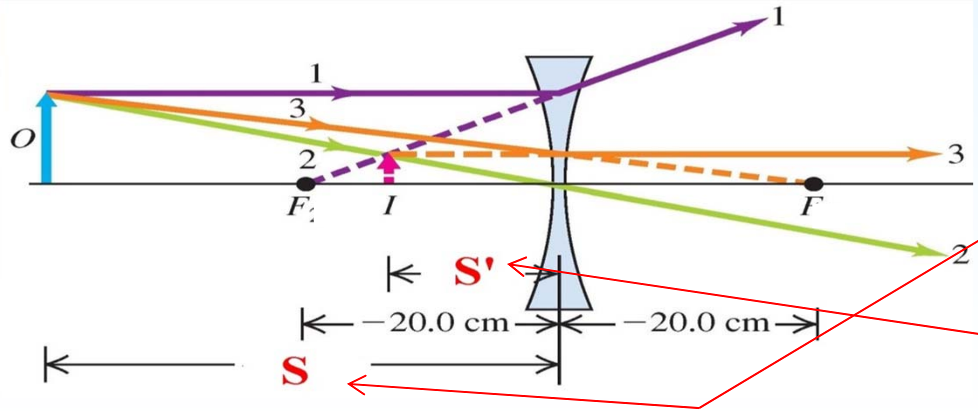
$$\lim_{x \to 5} \frac{1}{x-5} = 5$$

Geometrical optics: Problems

A divergent lens has a focal length of 20.0 cm. The magnification is 1/3.

What is the position of the object and image?

$$m = \frac{y'}{y} = -\frac{s'}{s} \Rightarrow m = -\frac{s'}{s} = \frac{1}{3} \Rightarrow s' = -\frac{s}{3} \Rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$



f = - 20.0 cm

$$\frac{1}{s} + \frac{1}{-s/3} = \frac{1}{s} - \frac{3}{s} = -\frac{2}{s} = \frac{1}{f}$$

$$s = -2f = -2(-20.0\text{ cm}) = 40.0\text{ cm}$$

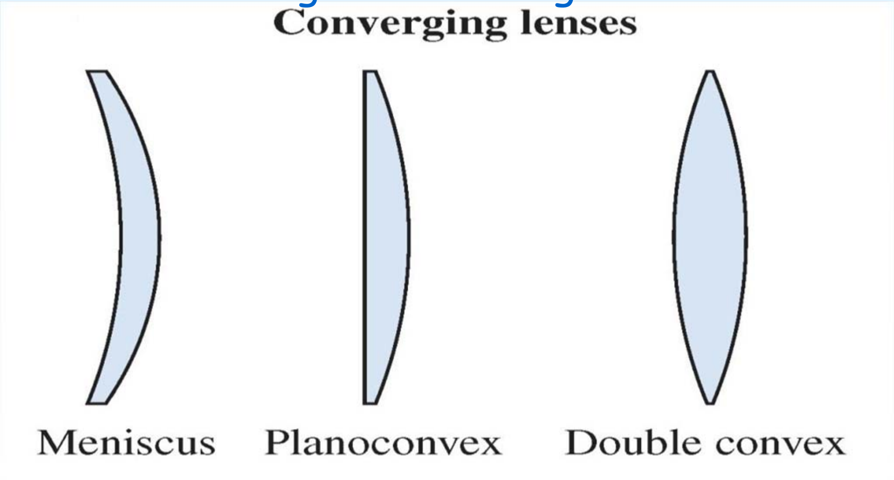
$$s' = -\frac{s}{3} = -\frac{40.0\text{ cm}}{3} = -13.3\text{ cm}$$

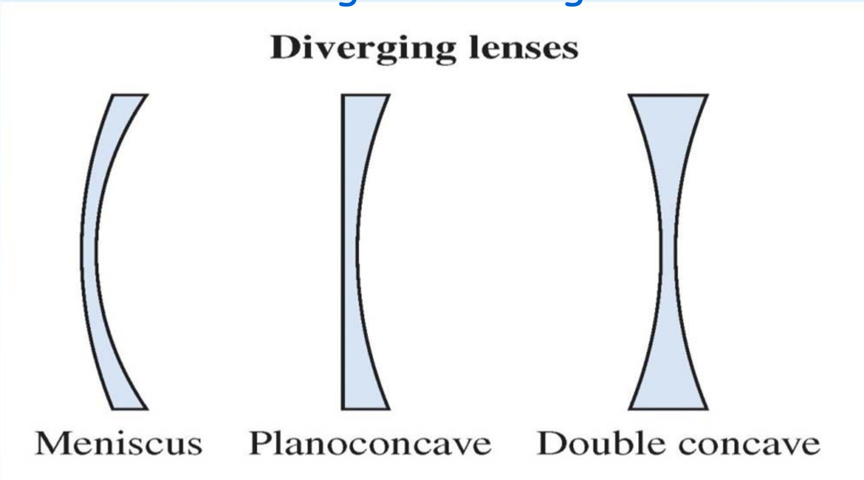
Part 14. The lensmaker's equation

Different types of lenses

A lens thicker in the middle than in the edges is convergent.

A lens thinner in the middle than in the edges is divergent.





Optics: The lensmaker's equation

Given

A lens with refractive index **n** and radiuses R_1 and R_2 and which has an object at a distance **S**

Goal

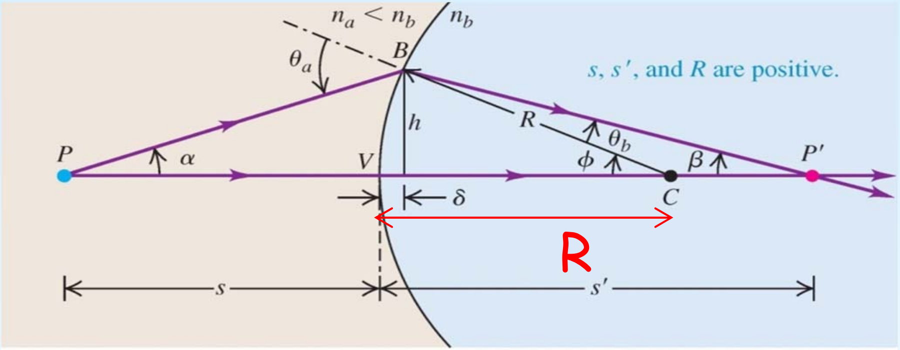
Derive the lensmaker's formula so that one can calculate where the image ends up = **S'**

How

Use the formula for the refraction in a spherical surface

Optics: The lensmaker's equation

Spherical surface



$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

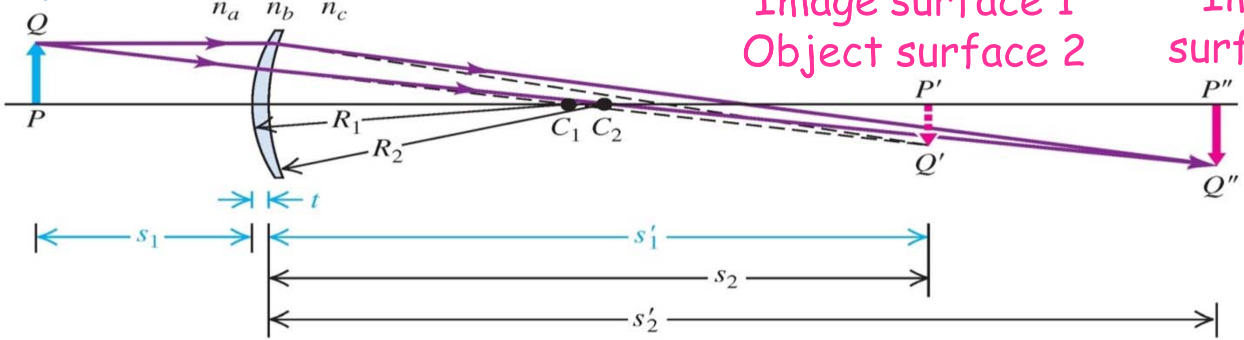
$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s}$$

Object surface 1

Image surface 1
Object surface 2

Image surface 2

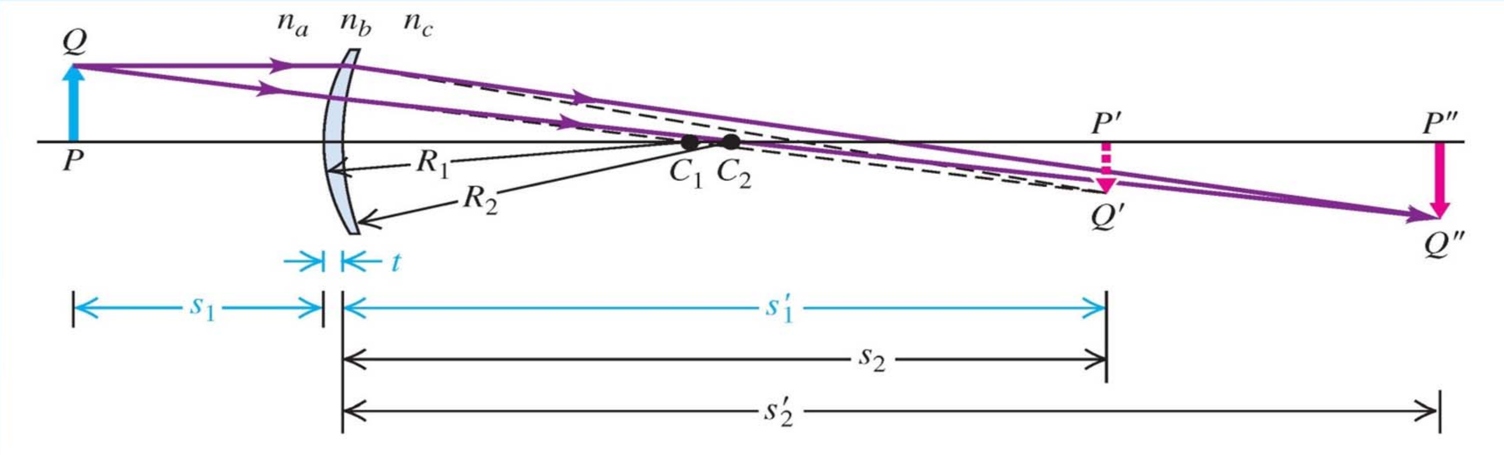
Step 1



$$\frac{n_a}{s_1} + \frac{n_b}{s'_1} = \frac{n_b - n_a}{R_1}$$

$$\frac{n_b}{s_2} + \frac{n_c}{s'_2} = \frac{n_c - n_b}{R_2}$$

Optics: The lensmaker's equation



Step 1

$$\frac{n_a}{s_1} + \frac{n_b}{s_1'} = \frac{n_b - n_a}{R_1}$$

$$\frac{n_b}{s_2} + \frac{n_c}{s_2'} = \frac{n_c - n_b}{R_2}$$

Step 2

$$n_a = n_c = 1$$

$$n_b = n$$

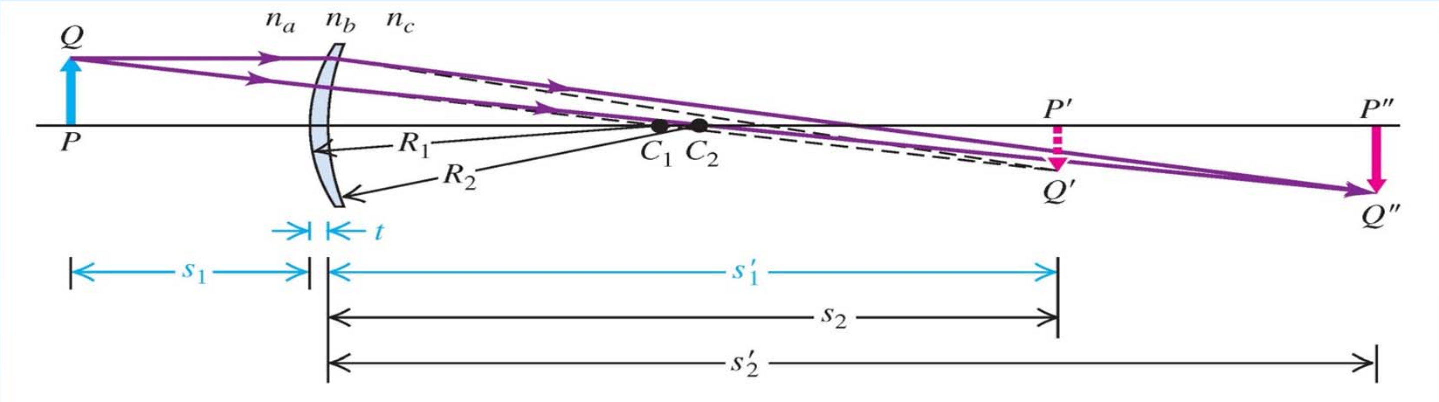
$$s_2 = -s_1'$$

⇨

$$\frac{1}{s_1} + \frac{n}{s_1'} = \frac{n-1}{R_1}$$

$$-\frac{n}{s_1'} + \frac{1}{s_2'} = \frac{1-n}{R_2}$$

Optics: The lensmaker's equation



Step 2

$$\frac{1}{s_1} + \frac{n}{s'_1} = \frac{n-1}{R_1}$$

$$-\frac{n}{s'_1} + \frac{1}{s'_2} = \frac{1-n}{R_2}$$

Step 3

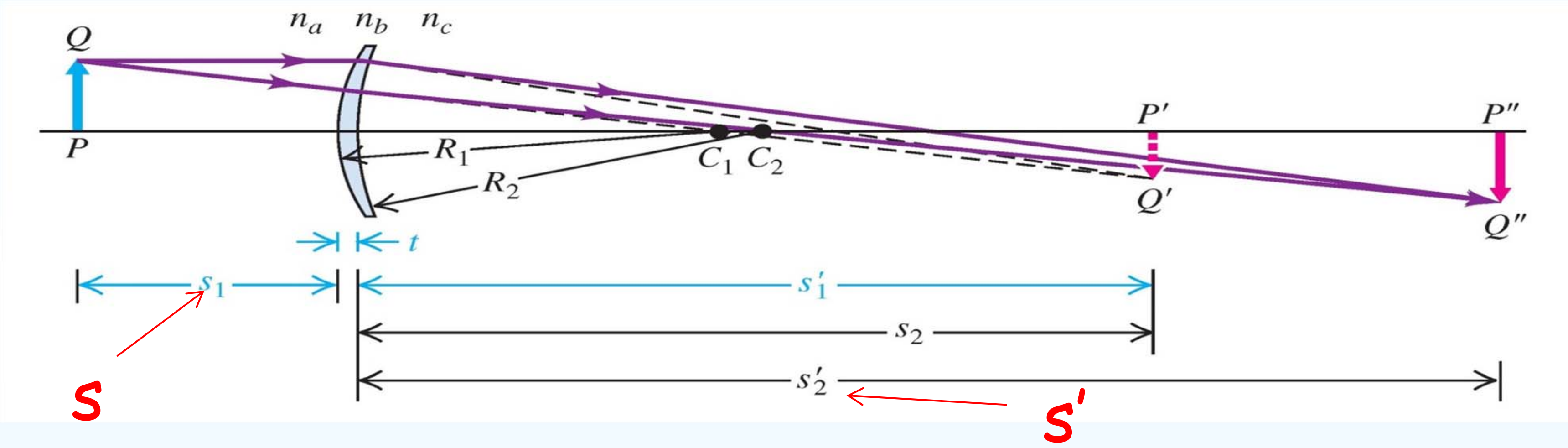
Add the two equations:

$$\frac{1}{s_1} + \frac{1}{s'_2} = \frac{n-1}{R_1} + \frac{1-n}{R_2}$$

Simplify:

$$\frac{1}{s_1} + \frac{1}{s'_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Optics: The lensmaker's equation



Step 3

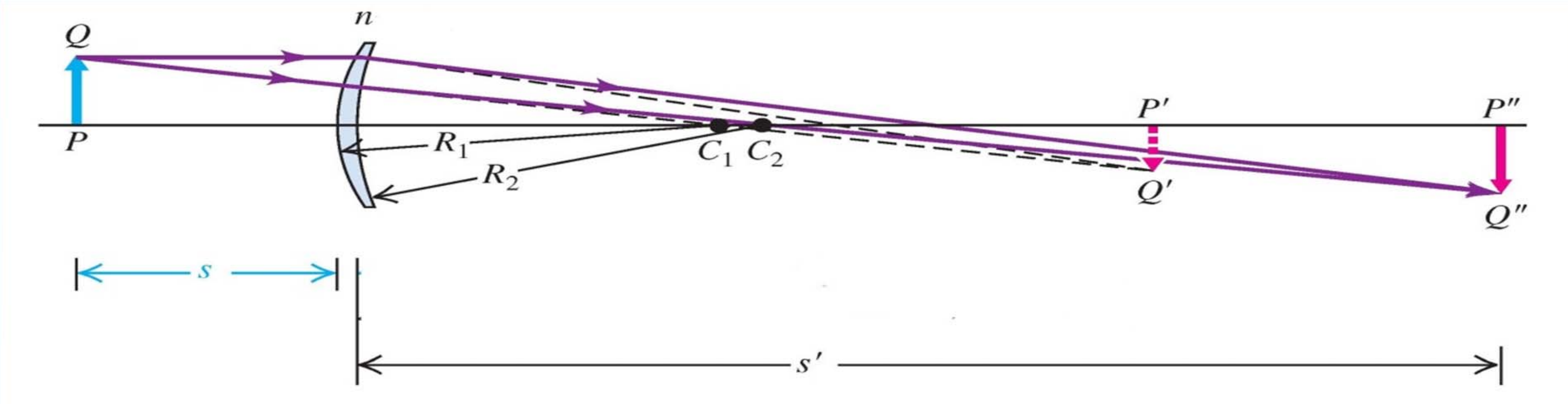
$$\frac{1}{s_1} + \frac{1}{s'_2} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Step 4

$s_1 = s$

$s_2' = s'$

$$\Rightarrow \quad \frac{1}{s} + \frac{1}{s'} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



Step 5

Combine new with old formula

$$\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

The lensmaker's equation

$$\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$

Optics: The lensmaker's equation

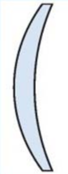
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$m = -\frac{s'}{s}$$

$$m = \frac{y'}{y}$$

Sign rule for the radius (R) says it is positive if center is on same side as outgoing light.



f = positive R_1 = positive R_2 = positive s′ = positive or negative

f = positive R_1 = positive R_2 = negative s′ = positive or negative

f = negative R_1 = negative R_2 = positive s′ = negative

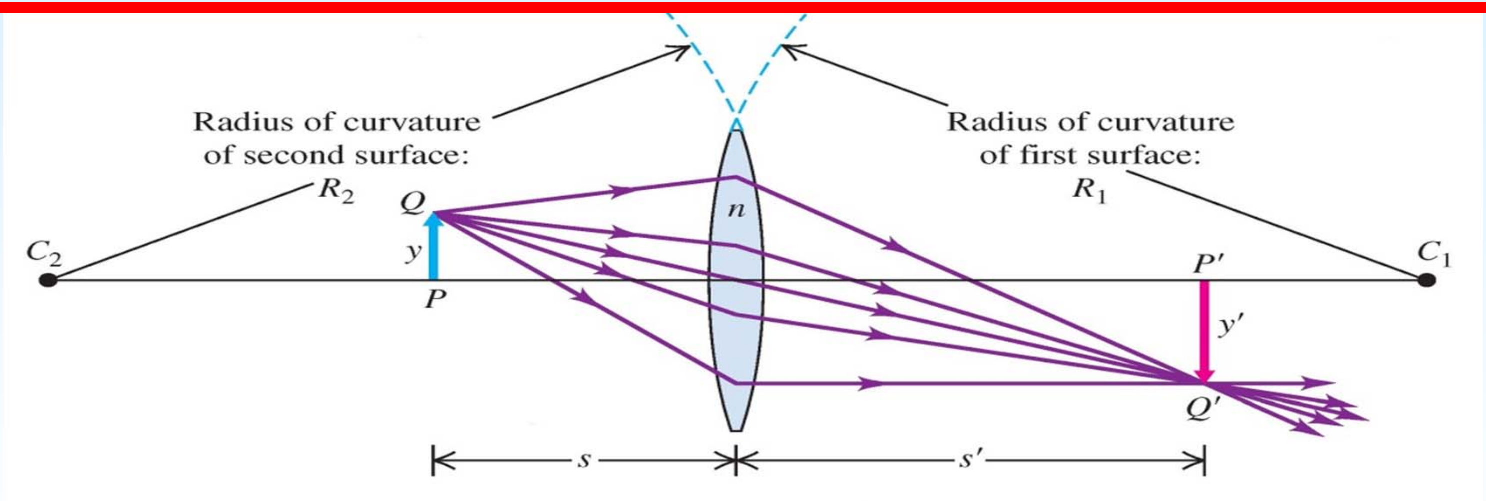
Part 15. Problems

$$\lim_{x \to 8} \frac{1}{x-8} = \infty$$

$$\lim_{x \to 5} \frac{1}{x-5} = 5$$

A double convex lens has $R_1 = R_2 = 10$ cm and $n = 1.52$

What is the focal length ?



$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

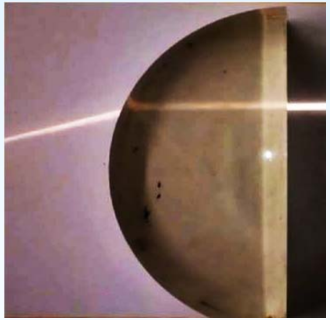
$$\frac{1}{f} = (1.52 - 1)\left(\frac{1}{+10\text{ cm}} - \frac{1}{-10\text{ cm}}\right)$$

$$f = 9.6\text{ cm}$$

Part 16. Summary

Concave mirror

Convex mirror



Spherical surface

Convex lens

Concave lens

Equations

Concave mirror / Convex mirror

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

$$m=\frac{y'}{y}=-\frac{s'}{s}$$

$$f=\frac{R}{2}$$

Spherical surface

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$$

$$m=\frac{y'}{y}=-\frac{n_a s'}{n_b s}$$

Convex lens / Concave lens

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

$$m=\frac{y'}{y}=-\frac{s'}{s}$$

$$\frac{1}{s}+\frac{1}{s'}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Geometrical optics: Summary

Sign rules for mirrors:

Positive object distance (s) = Object is on the side of the incoming light.

Positive image distance (s') = Image and outgoing light on the same side.

Positive radius (R) = Center is on the side of outgoing light.

Positive magnification (m) = Direction of object and image is the same.

Sign rules for lenses:

Positive object distance (s) Object and incoming light is on the same side.

Positive image distance (s') Image and outgoing light is on the same side.

Positive focal length (f) Converging (convex) lenses.

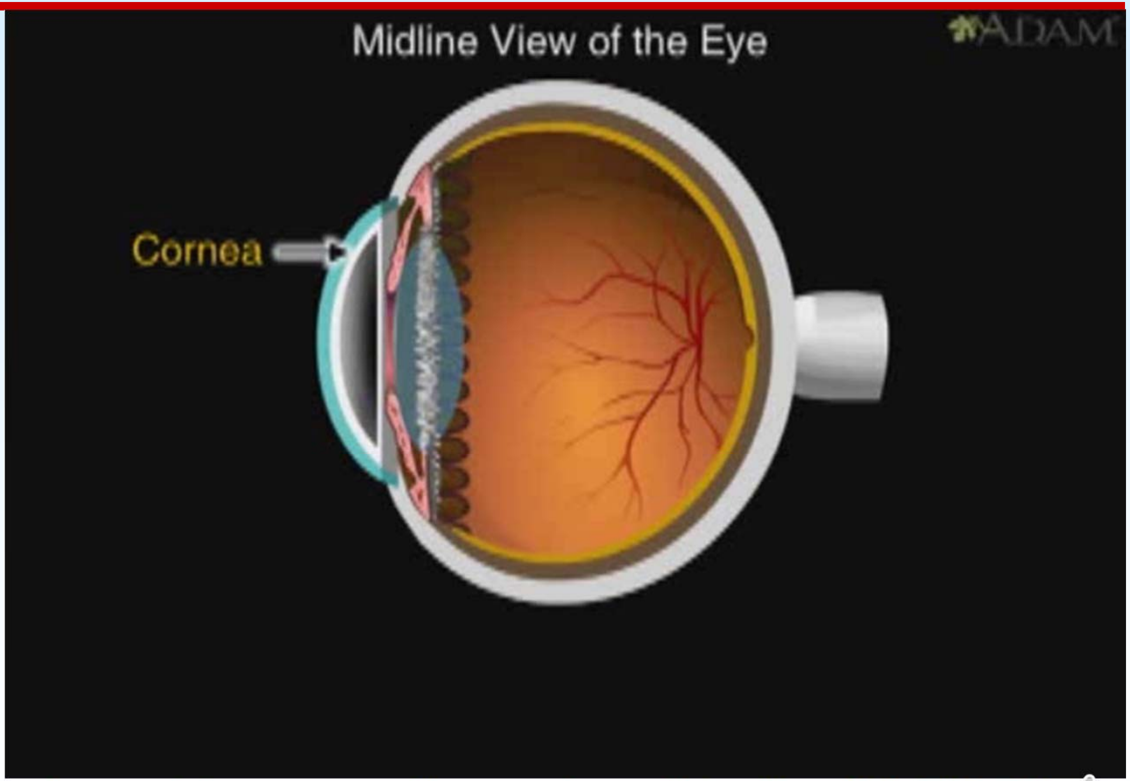
Positive magnification (m) Same direction of object and image.

Part 17. The Eye

In 1936, 9% of Swedish recruits were nearsighted.
In 2009, 38% of Swedish recruits were nearsighted.

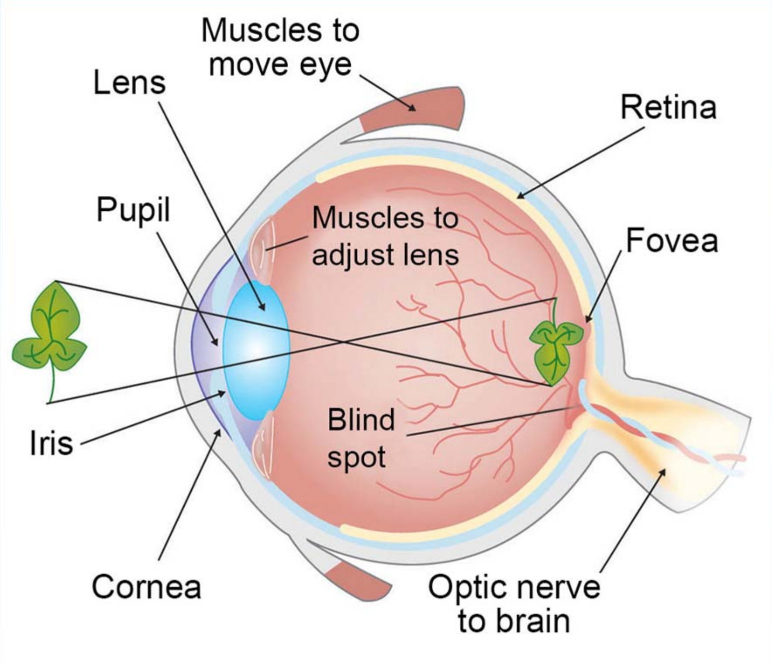
The reason: Time spent outdoors (exposure to daylight).

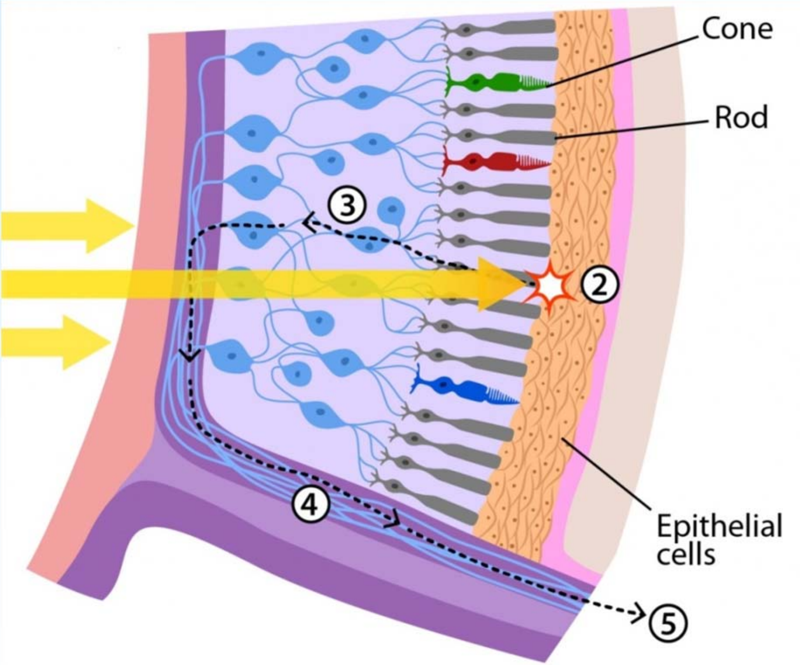
The function of the eye



https://www.youtube.com/watch?v=YcedXDN6a88

Geometrical optics: The eye

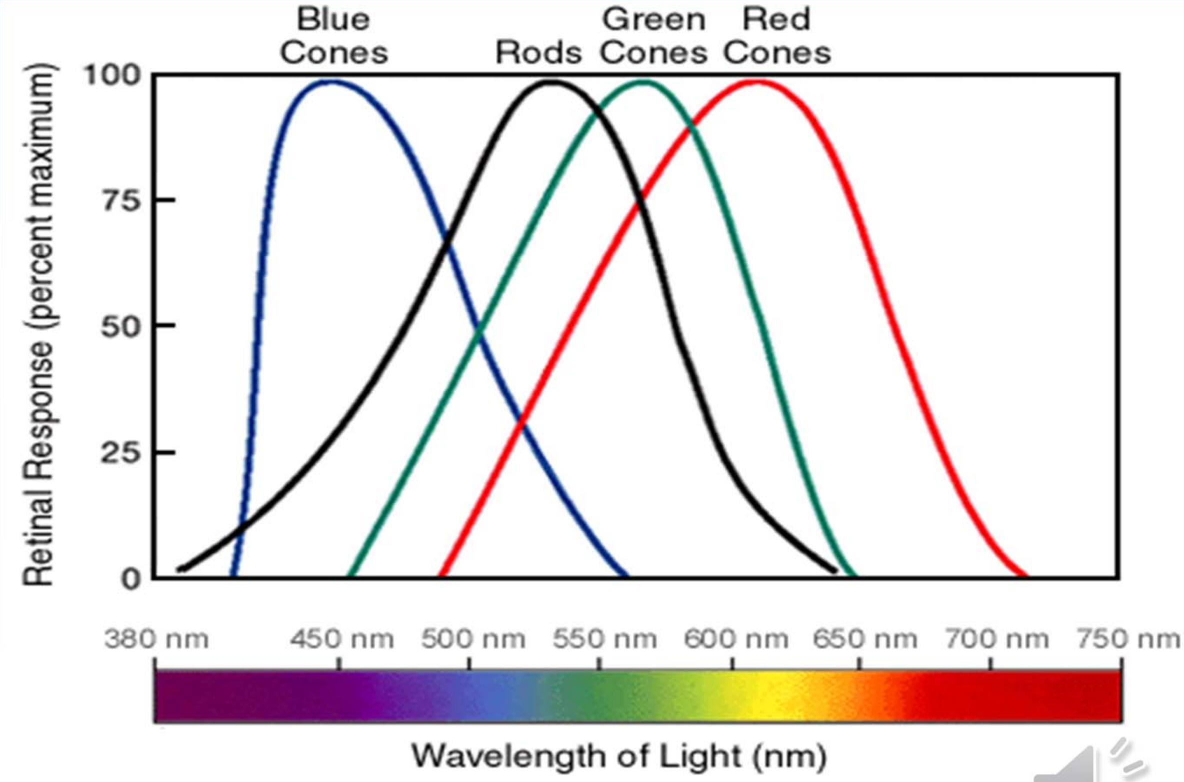




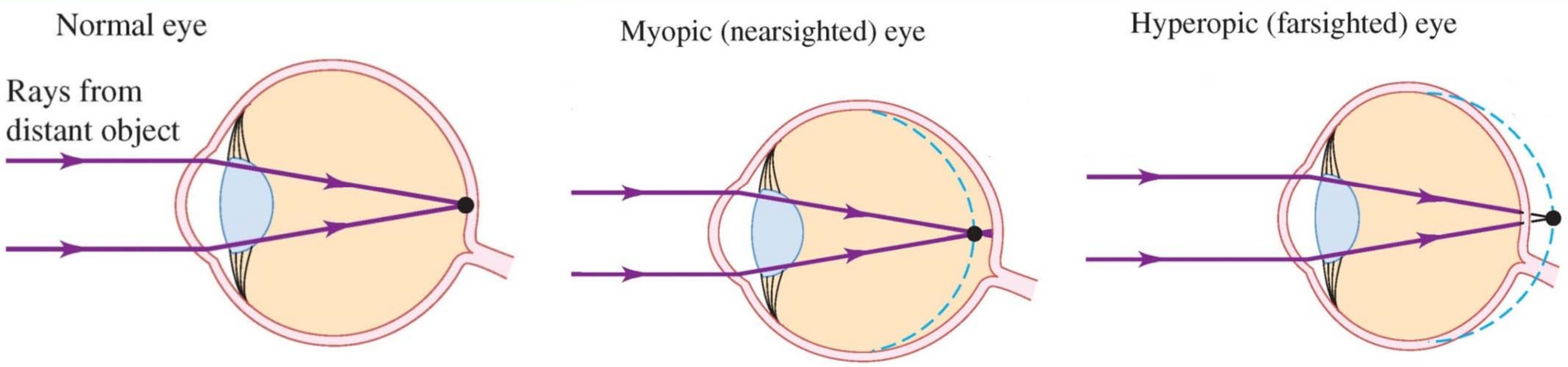
Rods: Very light sensitive. Used for night vision in black and white.
Cones: Three types (red, blue, green). Used to see colour.

Geometrical optics: The eye

The human eye's sensitivity to different wavelengths

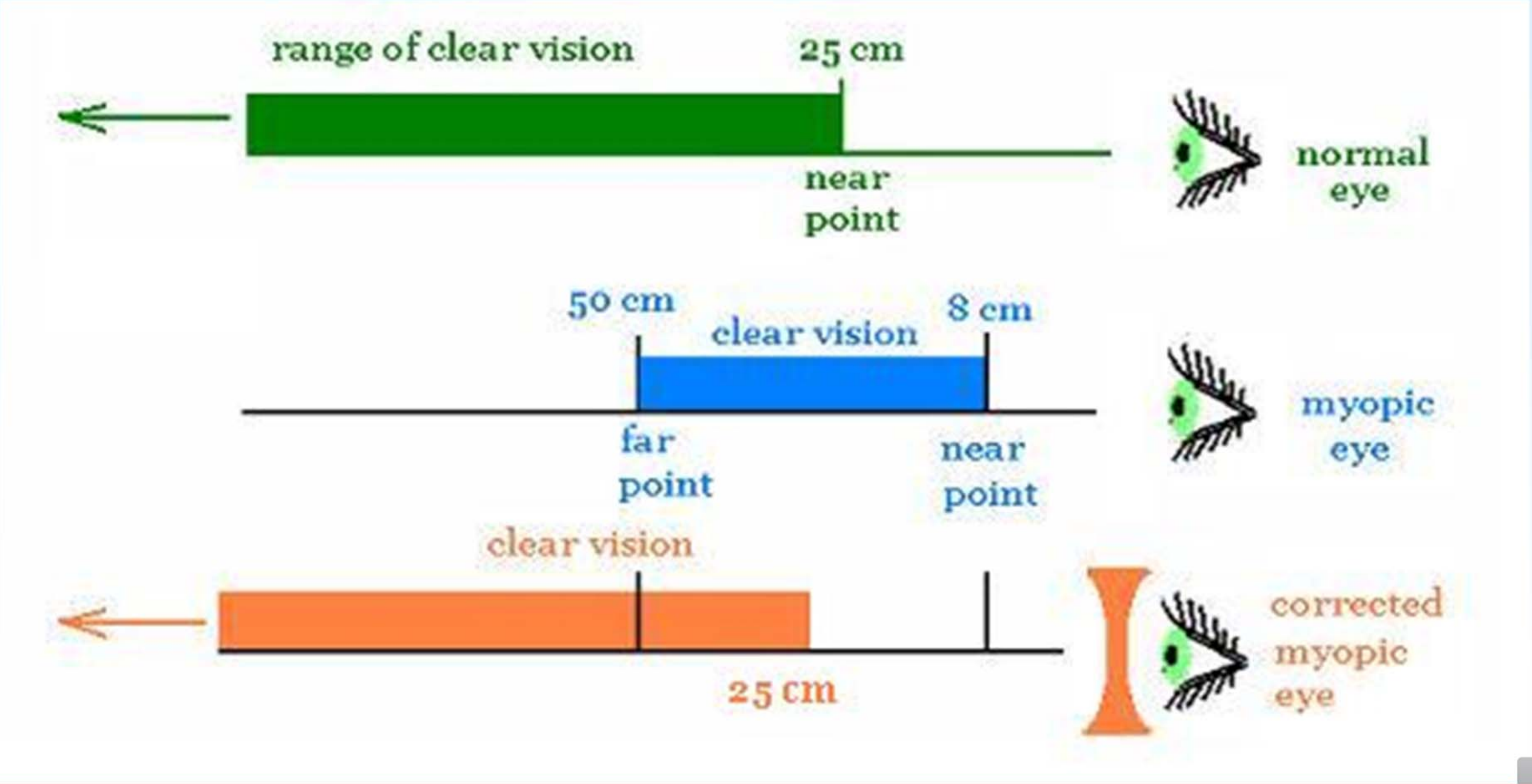


Geometrical optics: The eye

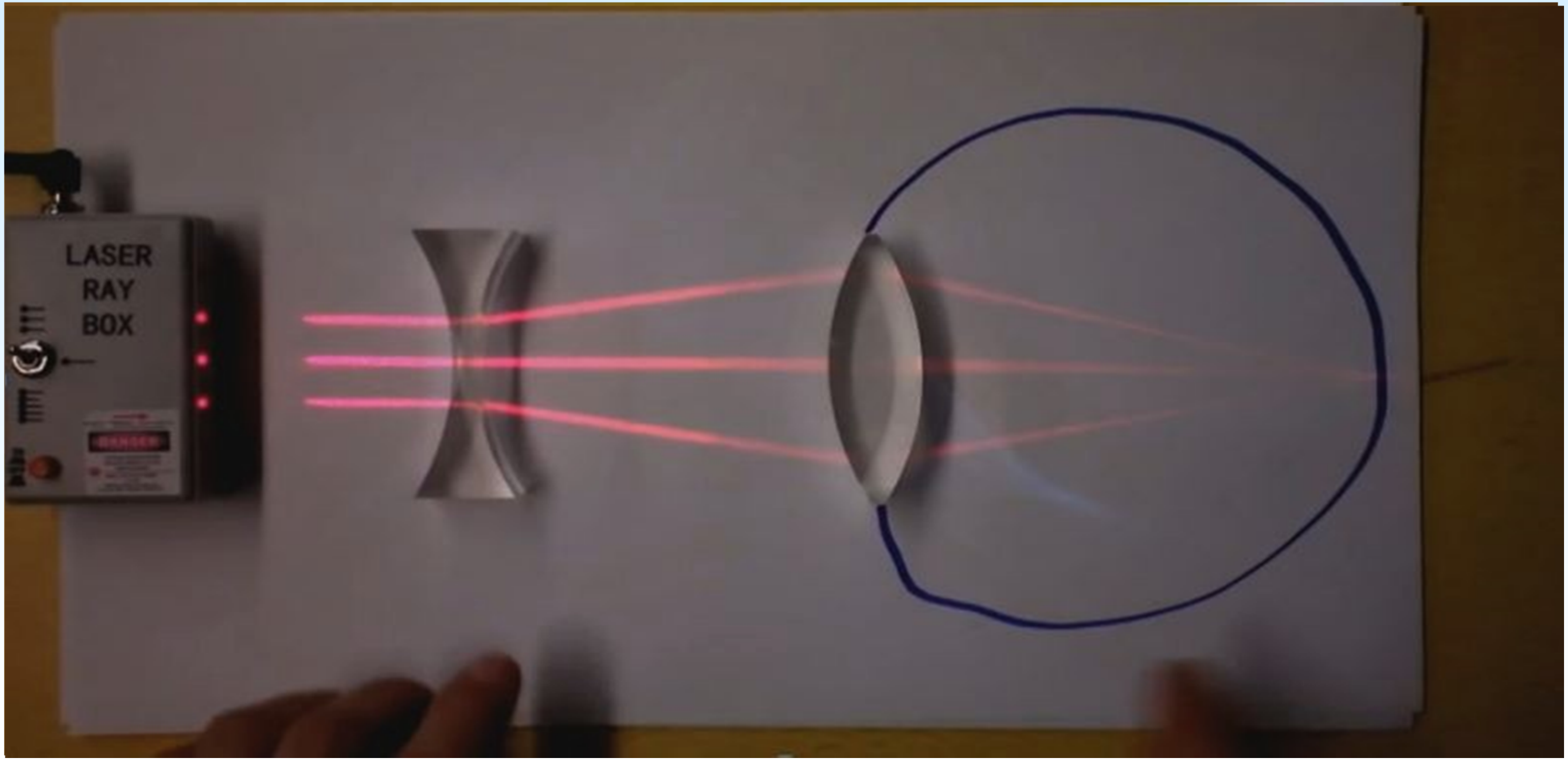


- Near point: Closest distance to the eye at which people can see clear (7cm at age 10 to 40cm at age 50 for normal eye).
- Normal reading distance: Assumed to be 25 cm when designing correction lenses.
- The far point: The longest distance to the eye at which people can see clearly.
- Lens power = 1/f (unit diopter = m^{-1}) is the quantity used for correction lenses.

Geometrical optics: The eye

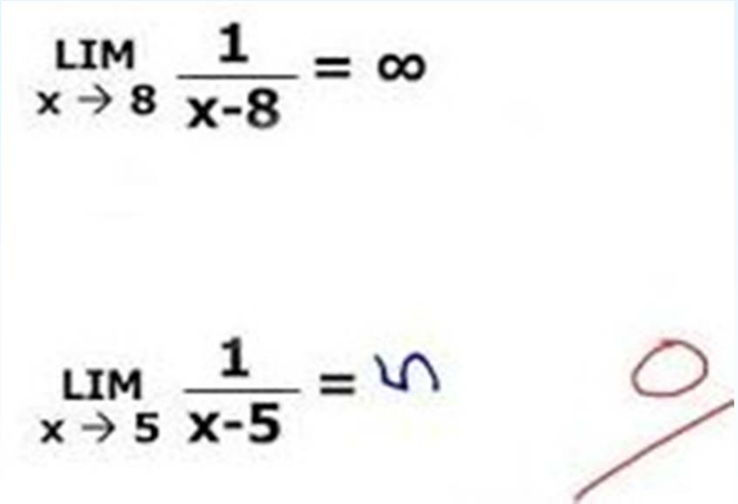


Geometrical optics: The eye



https://www.youtube.com/watch?v=VDehC_Txa1U

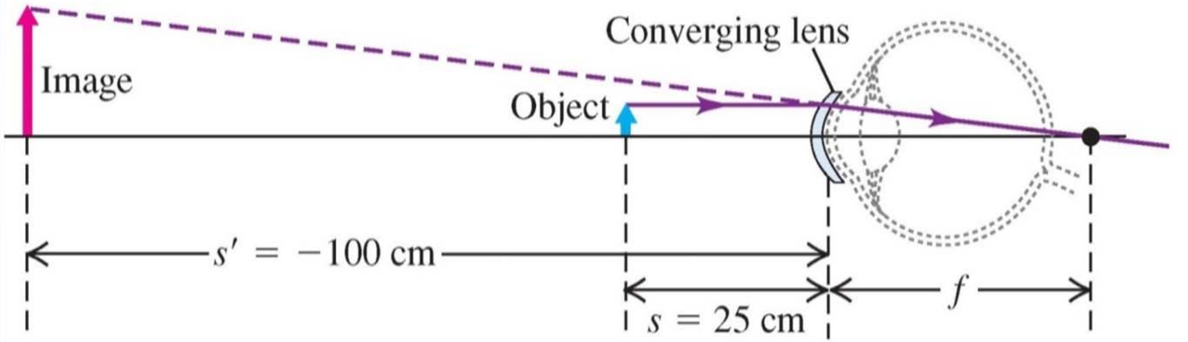
Part 18. Problems



Geometrical optics: Problems

The near point of a farsighted (hyperopic) eye is at 100 cm.

What lens power is needed to move the near point to 25 cm ?



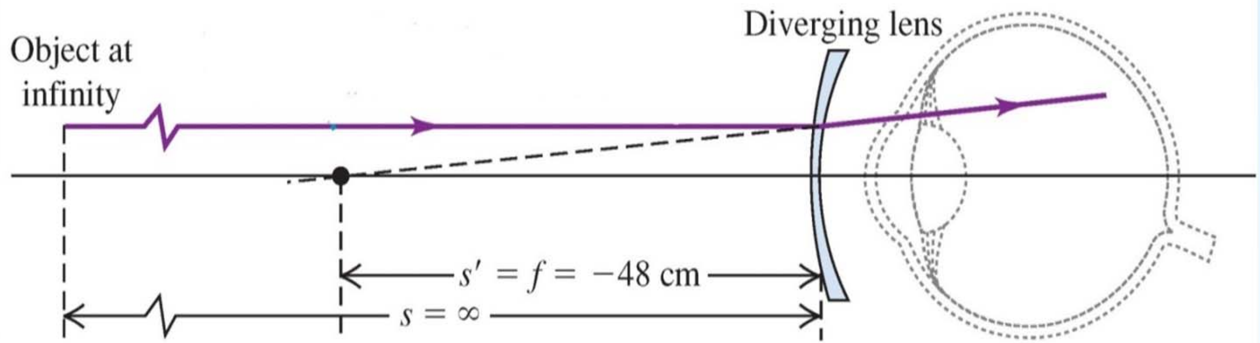
When the person puts an object at s = 25 cm from the correcting lens we want the image to end up at s′ = 100 cm because this is the nearest point the eye can see sharply.

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{+25\ \text{cm}} + \frac{1}{-100\ \text{cm}}$$

$$f = +33\ \text{cm}$$

Lens power = 1/f =
= 1/0.33 m^{-1} = 3 diopter

A nearsighted (myopic) eye has the far point at a distance of 50 cm. What lens power is needed to correct the eye if the lens is 2 cm in front of the eye ?



The lens should move the far point from 50 cm to infinity.

The correcting lens should therefore have s = infinity for s' = -50+2 = -48 cm.

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty} + \frac{1}{-48\ \text{cm}}$$

$$f = -48\ \text{cm}$$

Lens power = 1/f =
= $-1/0.48\ m^{-1}$ = -2.1 diopter

Part 19. The magnifying glass

The magnifying glass was invented by the Franciscan friar and scholar Roger Bacon in Oxford, UK. The first mention of its use was in 1268. He adapted its use as primitive spectacles, allowing scholars with failing eyesight to continue their work.

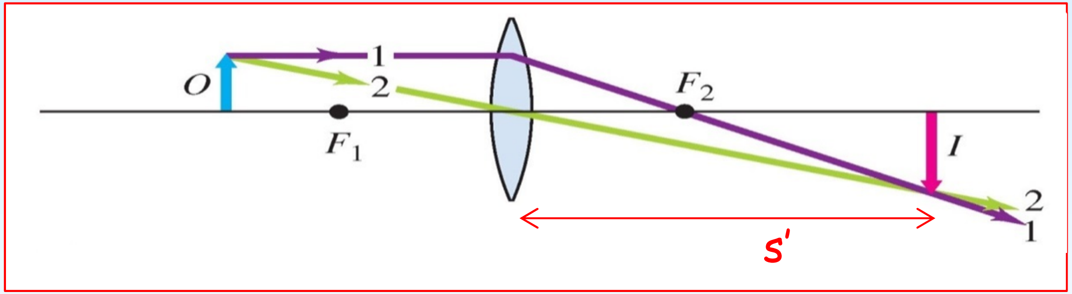
Optics: The magnifying glass

https://www.youtube.com/watch?v=CIXemjuLMGg

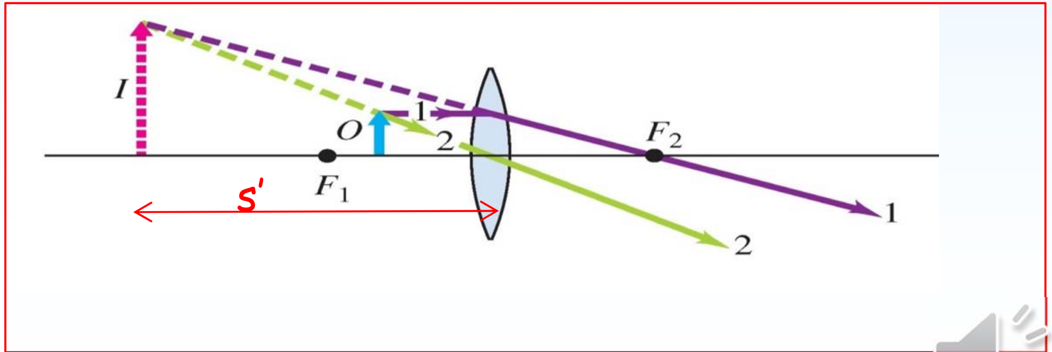
Optics: The magnifying glass

A magnifying glass is a convex lens.

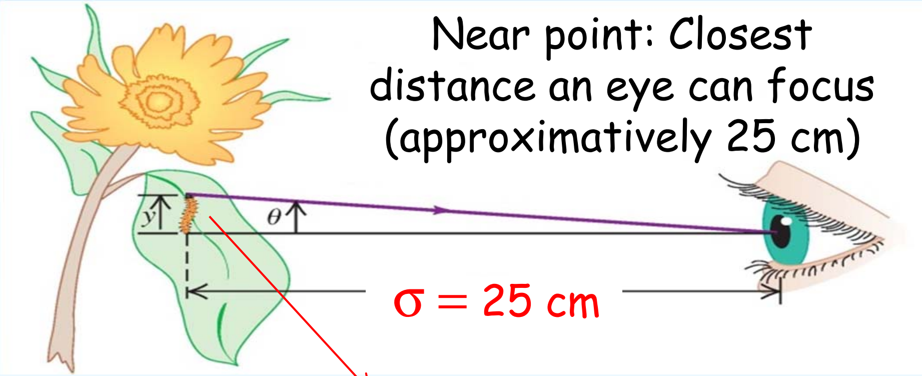
If you hold a magnifying glass far away from the eye (arms lengths distance) you can see a magnified and up-side down image.

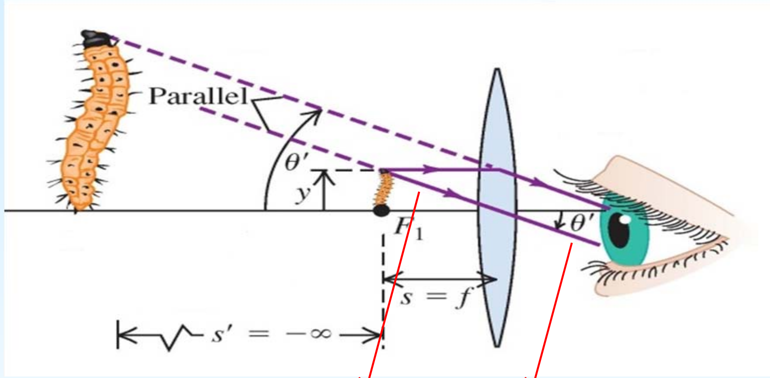


The normal use of a magnifying glass is to put the object between the focal point and the glass to get a magnified up-right image.



Optics: The magnifying glass





Angle without magnifying glass

$$\tan(\theta) \approx \theta = \frac{y}{\sigma} \approx \frac{y}{25\ cm}$$

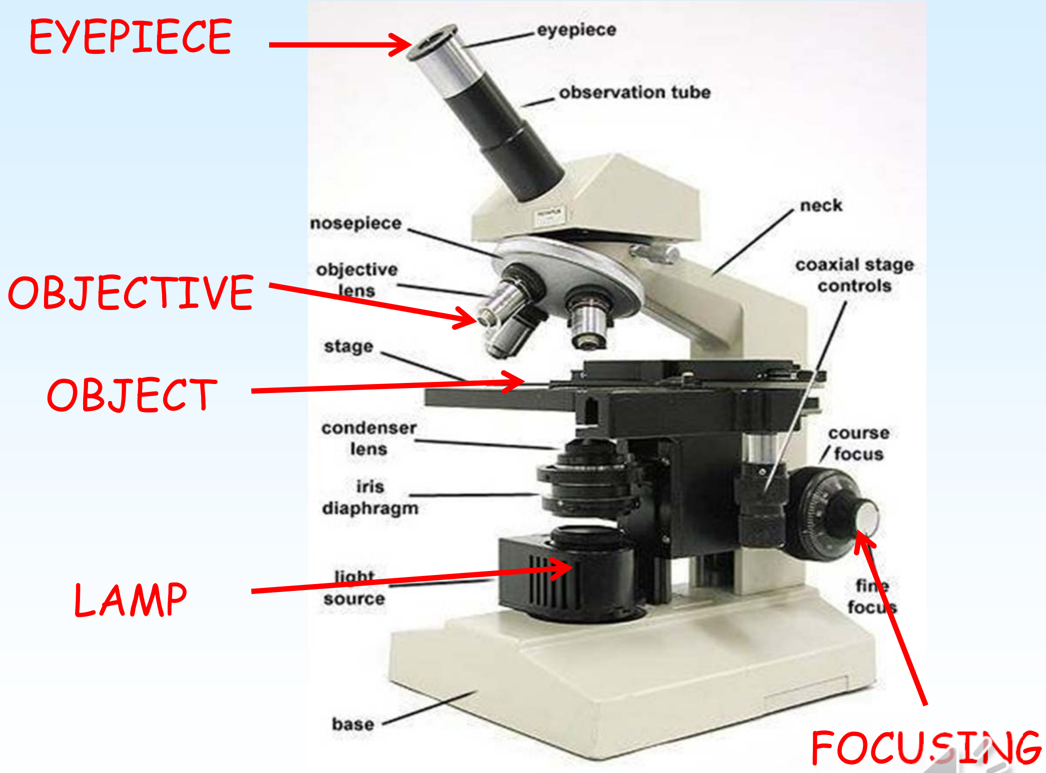
Angle with magnifying glass

$$\tan(\theta') \approx \theta' = \frac{y}{f}$$

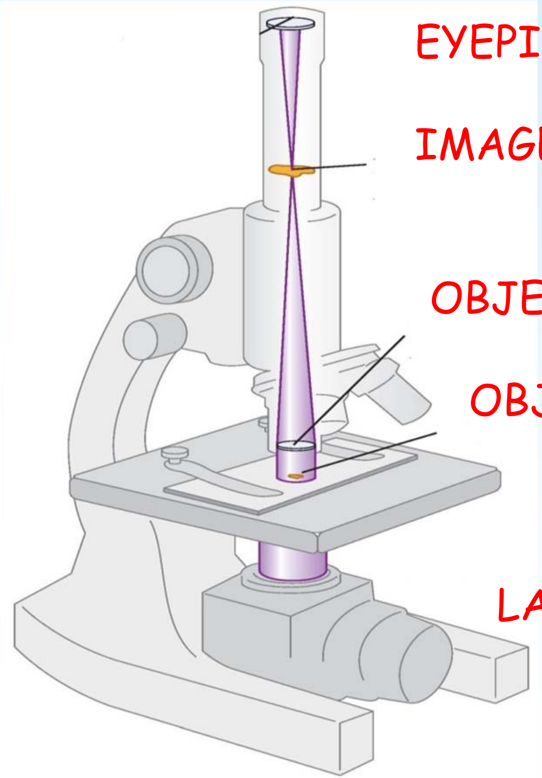
When the object is at the focal point one uses angular magnification (M) instead of lateral magnification (m).

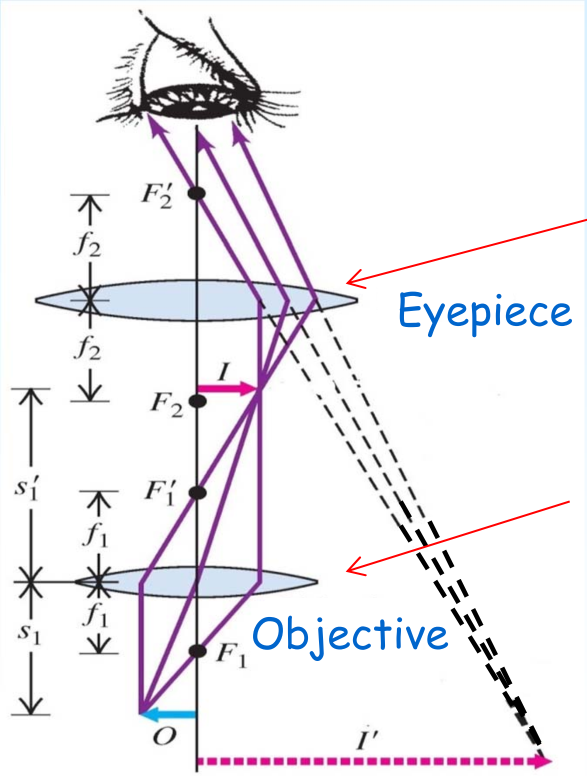
$$\mathrm{M} = \frac{\theta'}{\theta} = \frac{y/f}{y/\sigma} = \frac{\sigma}{f} = \frac{25\ cm}{f}$$

Part 20. The Microscope



Geometrical optics: The microscope

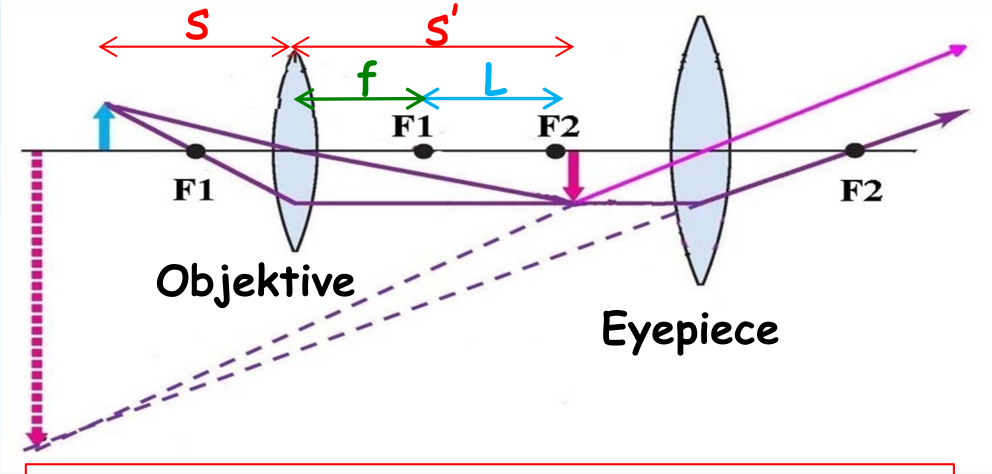




Magnifying glass
(f is a couple of cm)

Creates magnified image close to the focal point of the eye piece
(f < 1 cm)

Geometrical optics: The microscope



EYEPIECE

Angular magnification of magnifying glass:

$$M = \frac{\sigma}{f} \qquad where\ \sigma = 25\ cm$$

OBJEKTIVE

$$s' \approx f + L$$

$$m = -\frac{s'}{s} = -\frac{s' - f}{f} \approx -\frac{f + L - f}{f} = -\frac{L}{f}$$

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \Rightarrow s = \frac{s'f}{s' - f}$$

MICROSCOPE

Magnification:

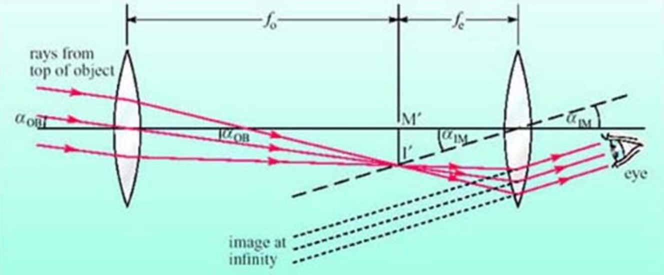
$$M = m_1 M_2 = -\frac{s'_1}{s_1}\frac{\sigma}{f_2} = -\frac{L\sigma}{f_1 f_2}$$

σ is the nearpoint distance which is typically 25 cm

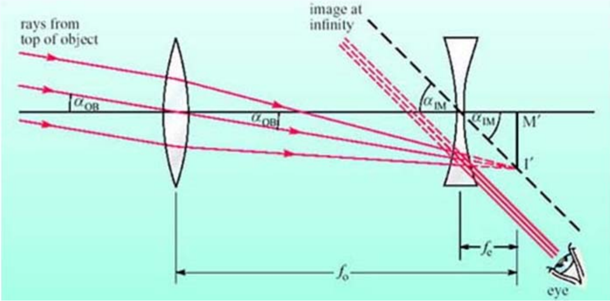
Part 21. The Telescope

Different types of telescopes

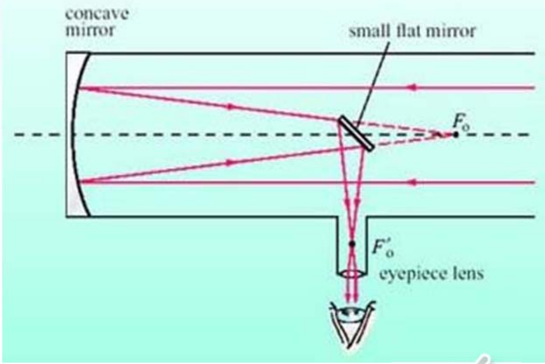
Keplerian Refracting Telescope



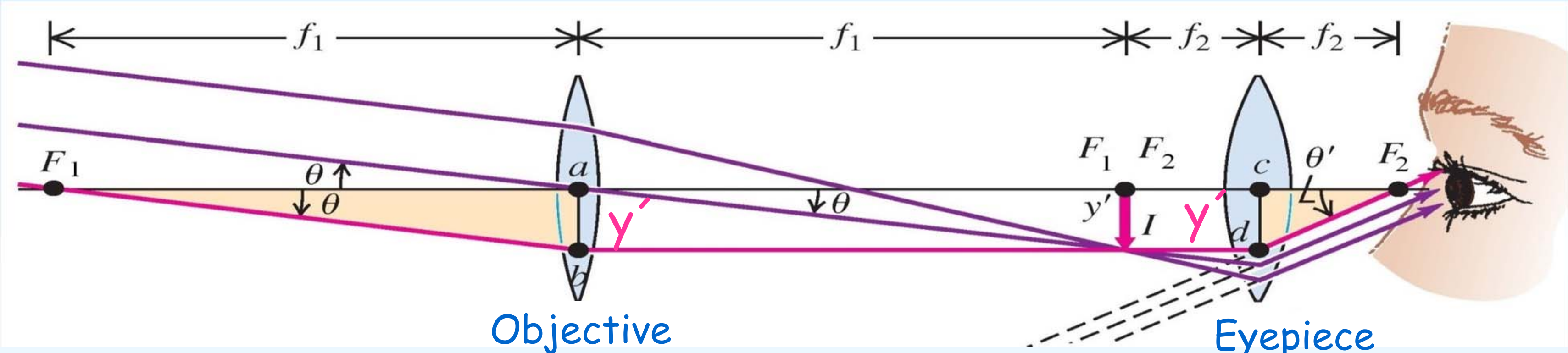
Galilean Refracting Telescope



Newtonian Reflecting Telescope



Geometrical optics: The Telescope



The object is infinitely far away so the image will be at the focal point of the lens.

$$\tan(\theta) = \theta = \frac{-y'}{f_1}$$

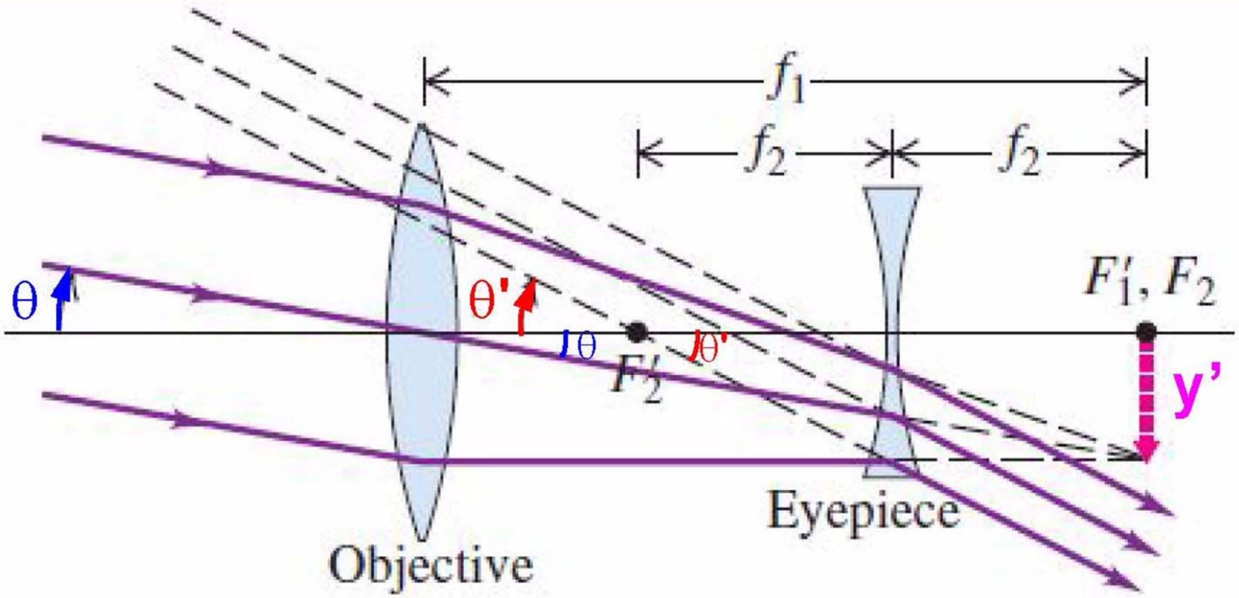
The eye piece works as a magnifying glass with the image y′ in its focal point.

$$\tan(\theta') = \theta' = \frac{y'}{f_2}$$

The angular magnification of a telescope is defined as the ratio of the angle of the image to that of the incoming light.

$$M = \frac{\theta'}{\theta} = -\frac{y'/f_2}{y'/f_1} = -\frac{f_1}{f_2}$$

Geometrical optics: The Telescope



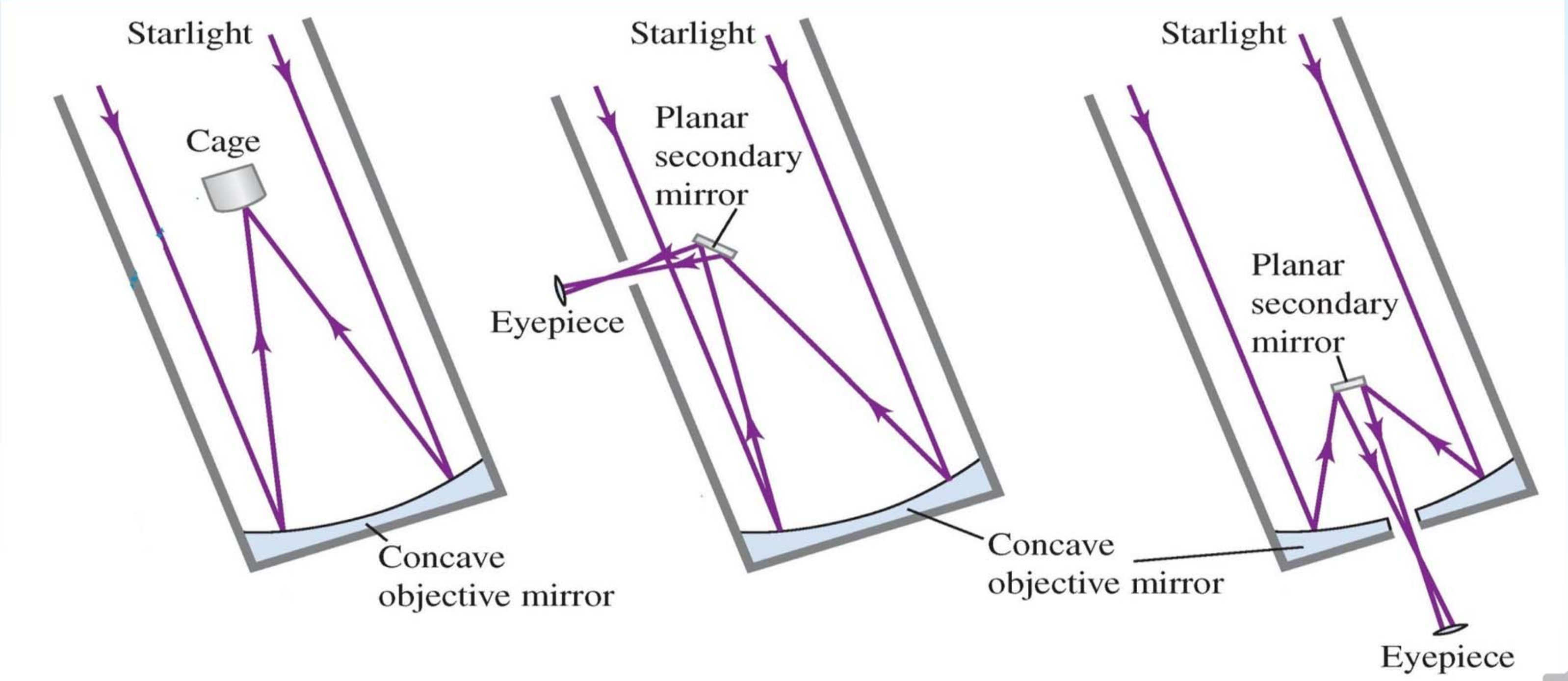
The Galilean telescope

The angular magnification

$$M = \frac{\theta'}{\theta} = \frac{\frac{-y'}{f_2}}{\frac{-y'}{f_1}} = \frac{f_1}{f_2}$$

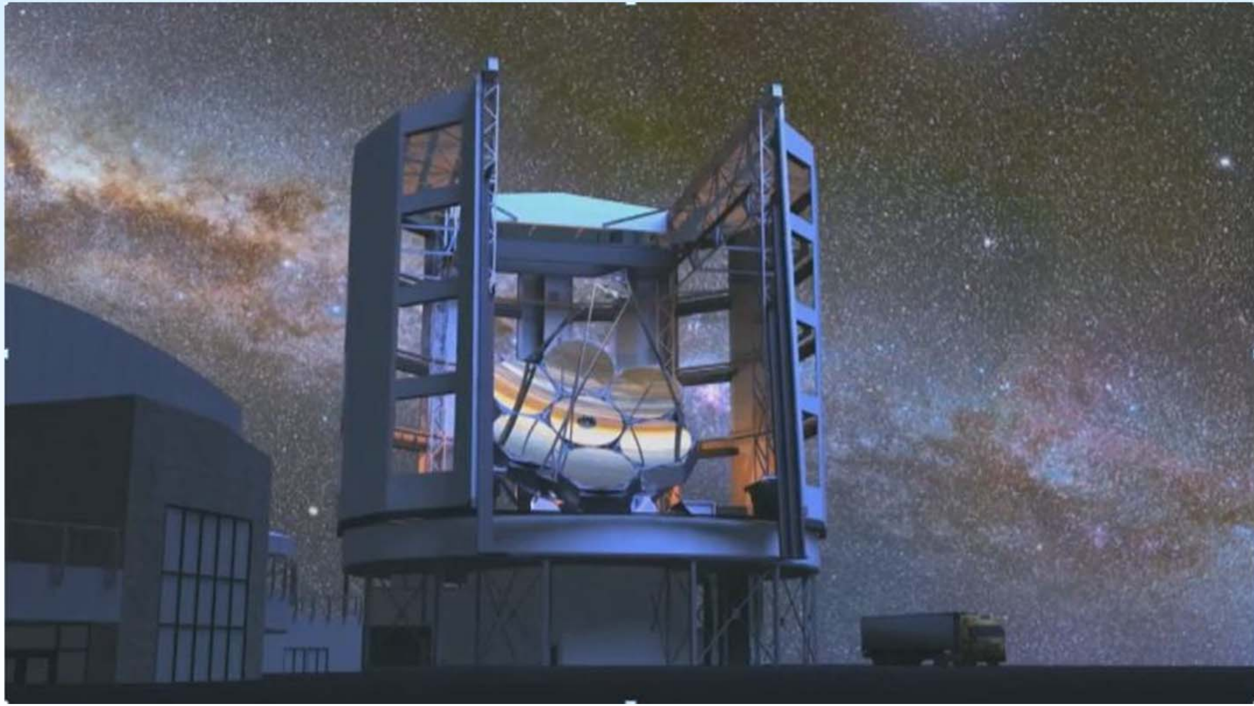
$$\tan(\theta) = \theta = \frac{-y'}{f_1} \qquad \tan(\theta') = \theta' = \frac{-y'}{f_2}$$

Different types of mirror telescopes



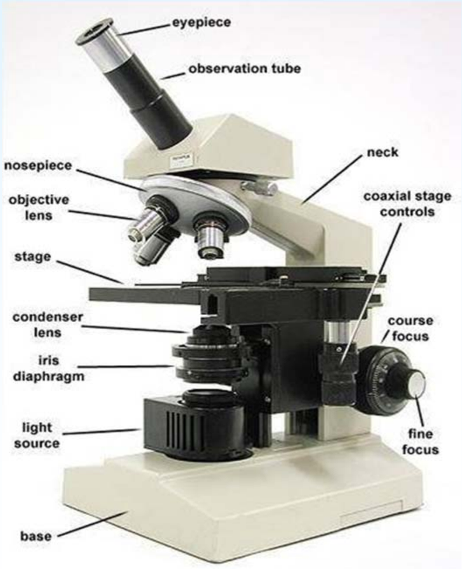
Geometrical optics: The Telescope

The Giant Magellan Telescope

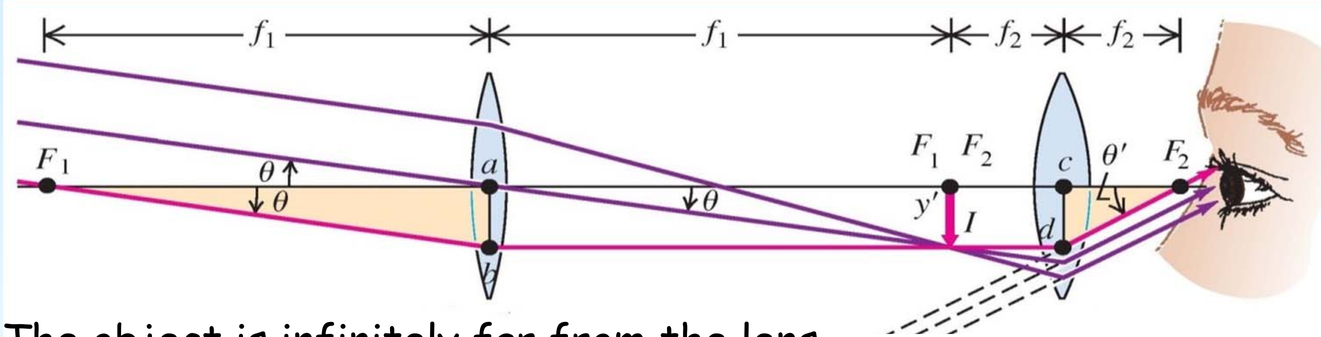


https://www.youtube.com/watch?v=7bzD8VEKMKQ

Part 22. Summary microscope and telescope



Summary: Microscope & Telescope

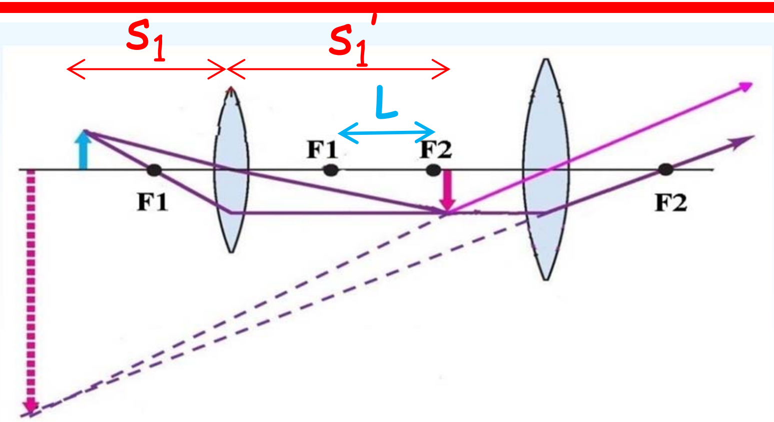


The object is infinitely far from the lens

Telescope

$$M = -\frac{f_1}{f_2}$$

Large f_1 & Small f_2



The object is close to the lens

Mikroscope

$$M = m_1 M_2 = -\frac{s'_1}{s_1}\frac{\sigma}{f_2} = -\frac{L\sigma}{f_1 f_2}$$

σ is the near point (typically 25 cm)

Small f_1 & Small f_2