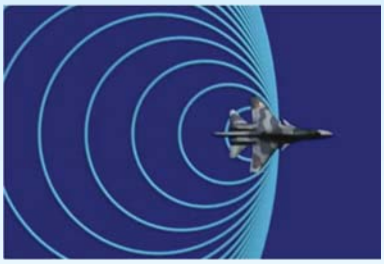
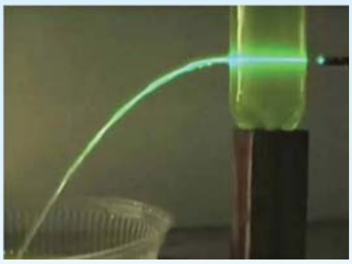
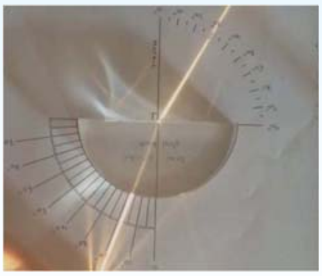
Vågrörelselära och optik



Kapitel 34 - Optik

Vågrörelselära och optik

Kurslitteratur: University Physics by Young & Friedman

Harmonisk oscillator:	Kapitel 14.1 – 14.4
Mekaniska vågor:	Kapitel 15.1 – 15.8
Ljud och hörande:	Kapitel 16.1 – 16.9
Elektromagnetiska vågor:	Kapitel 32.1 & 32.3 & 32.4
Ljusets natur:	Kapitel 33.1 – 33.4 & 33.7
Stråloptik:	Kapitel 34.1 – 34.8
Interferens:	Kapitel 35.1 – 35.5
Diffraktion:	Kapitel 36.1 - 36.5 & 36.7

Vågrörelselära och optik

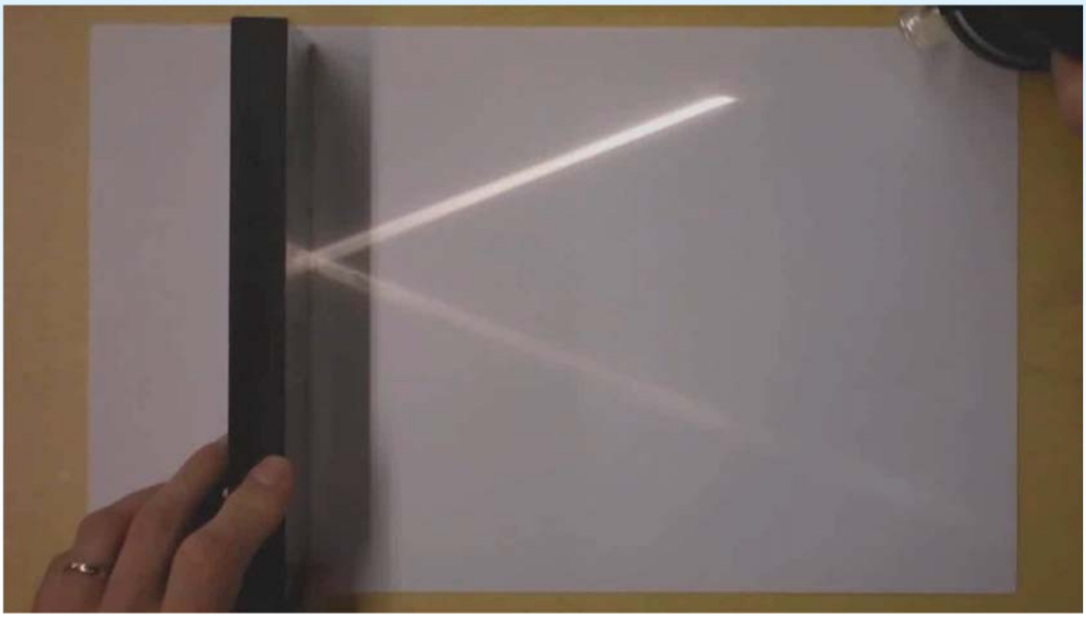
Tid	Må 02-nov	Ti 03-nov	On 04-nov	To 05-nov	Fr 06-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 14	Kvantfysik (A)	Våglära/optik (A)	Kvantfysik (A)
10-12	Intro period 2 (A) Informationssökning (A)	Kvantfysik (A)	Våglära/optik (A) kap 14+15 ÄFYA11 (L218)	Kvantfysik (A)	Kvantfysik (A) kap 15
13-15	Utvärdering (A) 12-13	Övningar Optik&Våg (L218-19)	SI gp6-10 (L219) ÄFYA11 (L218)	SI gp11-15 (L219)	Övningar Optik&Våg (L218-19)
15-17					

Tid	Må 09-nov	Ti 10-nov	On 11-nov	To 12-nov	Fr 13-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 16	Våglära/optik (A) kap 16+32	Kvantfysik (A)	Kvantfysik (A)
10-12	Våglära/optik (A) kap 15+16 ÄFYA11 (L218)	Kvantfysik (A)	Kvantfysik (A)	Våglära/optik (A) kap 32+33	Våglära/optik (A) kap 33
13-15	SI gp1-5 (L219) ÄFYA11 (L218)	Övningar Optik&Våg (L218-19)	ÄFYA11 (l218) SI gp6-10 (L219)	SI gp1-5 (L218) SI gp11-15 (L219)	Övningar Optik&Våg (L218-19)
15-17					

Tid	Må 16-nov	Ti 17-nov	On 18-nov	To 19-nov	Fr 20-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 34	Kvantfysik (A)	Våglära/optik (A) kap 35 ÄFYA11	Våglära/optik (A) kap 36
10-12	Våglära/optik (A) kap 34	Kvantfysik (A)	Våglära/optik (A) kap 34+35	Våglära/optik (A) kap 36 (L218)	Kvantfysik (A)
13-15	SI gp6-10 (L219)	Övningar Optik&Våg (L218-19)	Seminar.gen.gång (A) 12-13 SI gp1-5 (L218) 13-15 SI gp11-15 (L219) 13-15	Labbintroduktion (A) O2-O3, K1-K3	Övningar Optik&Våg (L218-19)
15-17					

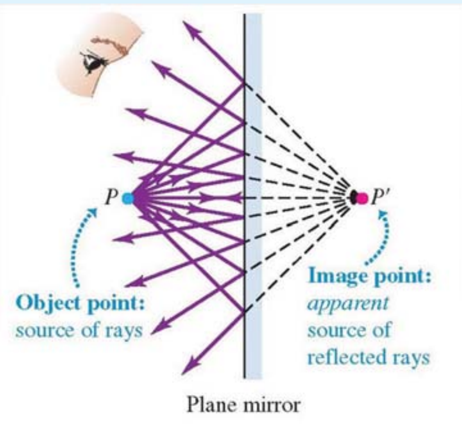
Geometrical optics

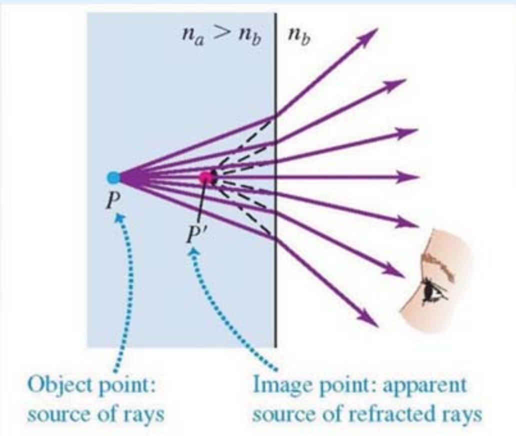
Examples of geometrical optics



Mirrors

Virtual Images: outgoing rays diverge



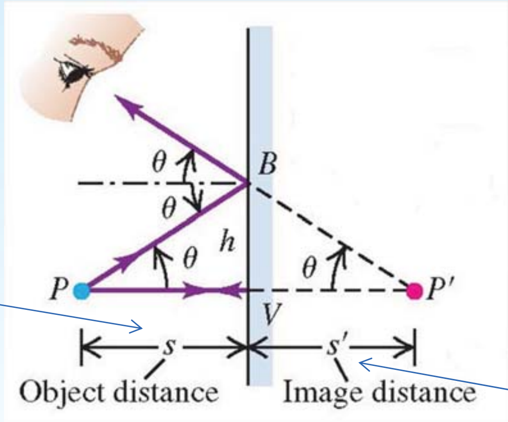


Real Images: outgoing rays converge to an image that can be shown on a screen

Geometrical optics

Point object

positive



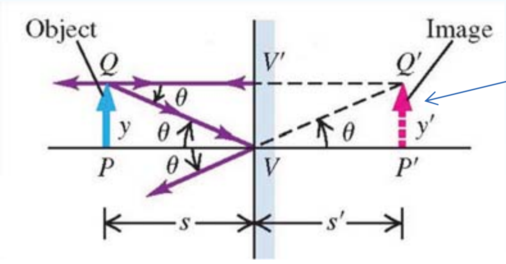
negative

Sign rules:

Object distance (s) – positive if same side as incoming light.

Image distance (s') – positive if same side as outgoing light.

Extended object

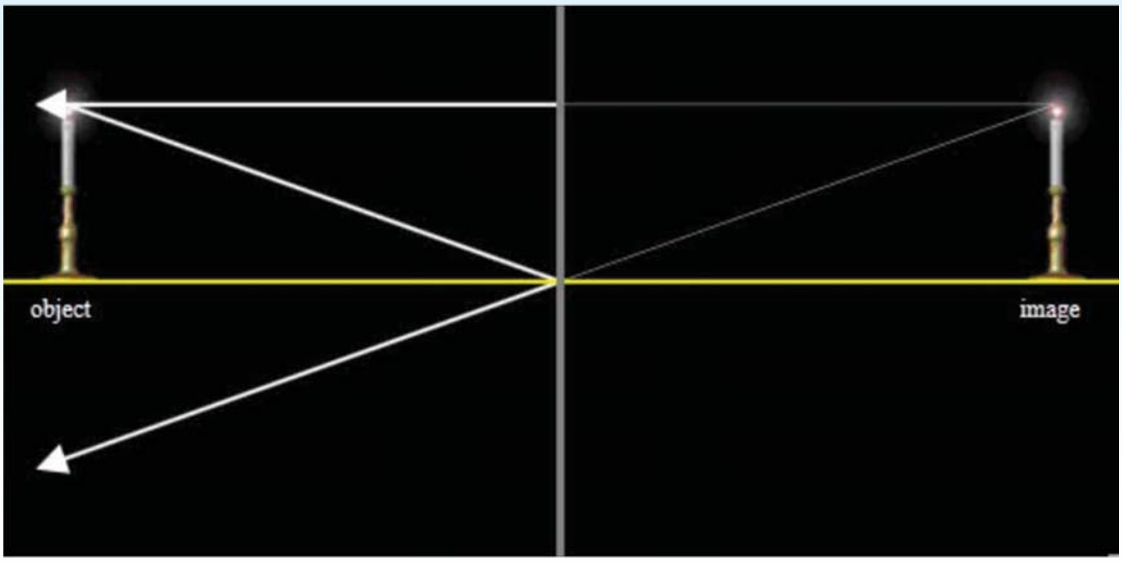


Virtual image

$$m = \frac{y'}{y} \qquad \text{(lateral magnification)}$$

Geometrical optics

Flat mirror



Geometrical optics

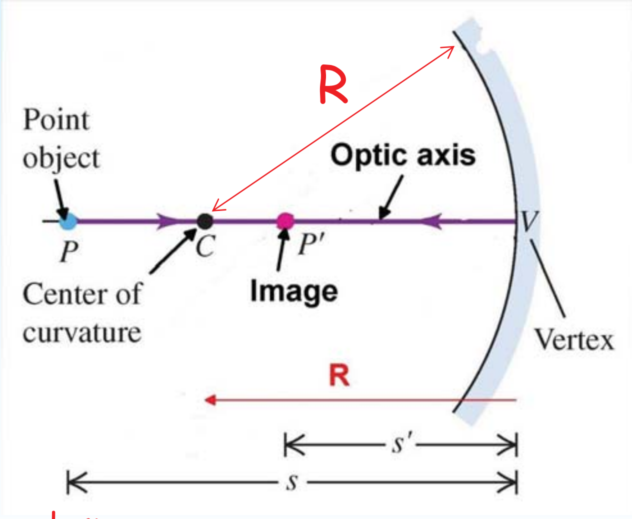
Spherical mirror

A point object on an optical axis will have the image on the optical axis.

s = distance of mirror to object

s' = distance of mirror to image

R = the mirror's radial distance

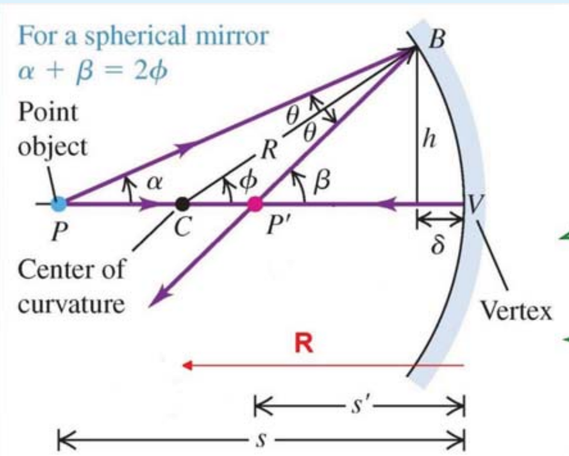


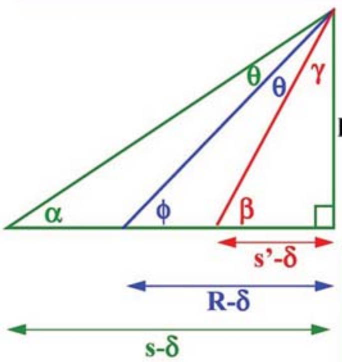
Sign rules:

Radius of curvature (R) – positive if center is on same side as outgoing light.

Geometrical optics

Trigonometry





$\beta + \gamma + 90^0 = 180^0$
$\gamma = 90^0 - \beta$

$\phi + \gamma + \theta + 90^0 = 180^0$
$\phi + 90^0 - \beta + \theta + 90^0 = 180^0$
$\theta = \beta - \phi$

$\alpha + \gamma + 2\theta + 90^0 = 180^0$
$\alpha + 90^0 - \beta + 2(\beta - \phi) + 90^0 = 180^0$
$\alpha + \beta - 2\phi = 0$

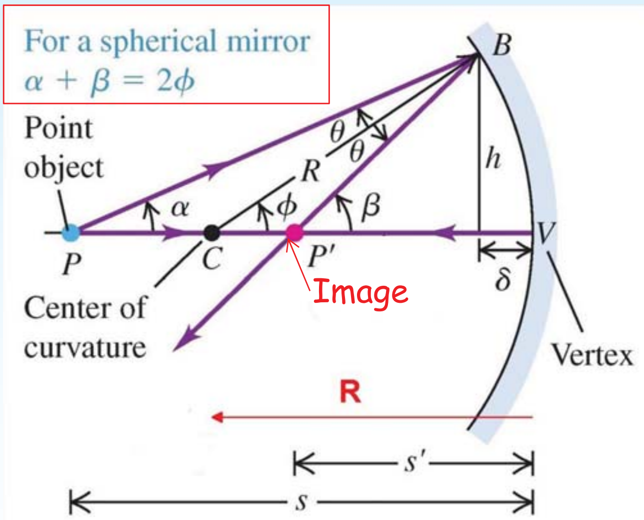
$$\boxed{\alpha + \beta = 2\phi}$$

$$\tan\alpha = \frac{h}{s - \delta} \qquad \tan\beta = \frac{h}{s' - \delta} \qquad \tan\phi = \frac{h}{R - \delta}$$

Geometrical optics

Spherical mirror

$$\tan\alpha = \frac{h}{s-\delta} \qquad \tan\beta = \frac{h}{s'-\delta} \qquad \tan\phi = \frac{h}{R-\delta}$$



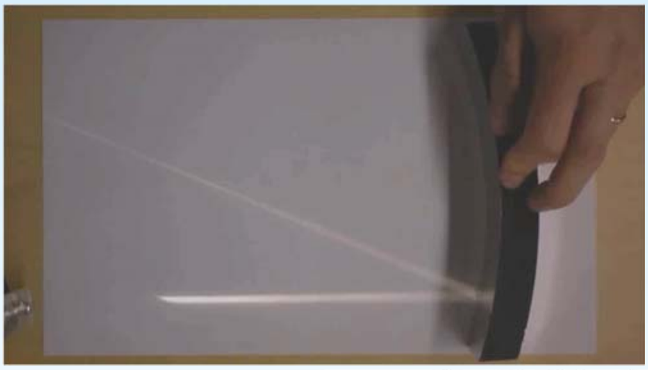
If the angles and δ are small then

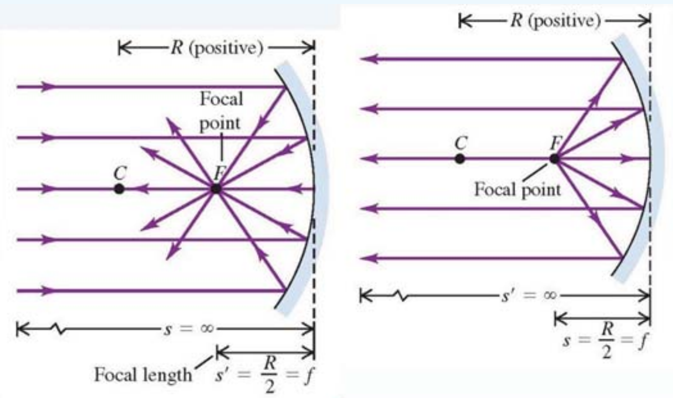
$$\alpha = \frac{h}{s} \qquad \beta = \frac{h}{s'} \qquad \phi = \frac{h}{R}$$

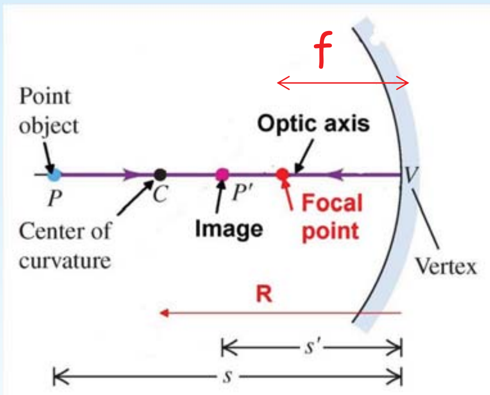
$$\alpha + \beta = 2\phi$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$ (object–image relationship, spherical mirror)

Geometrical optics







$$f = \frac{R}{2}$$

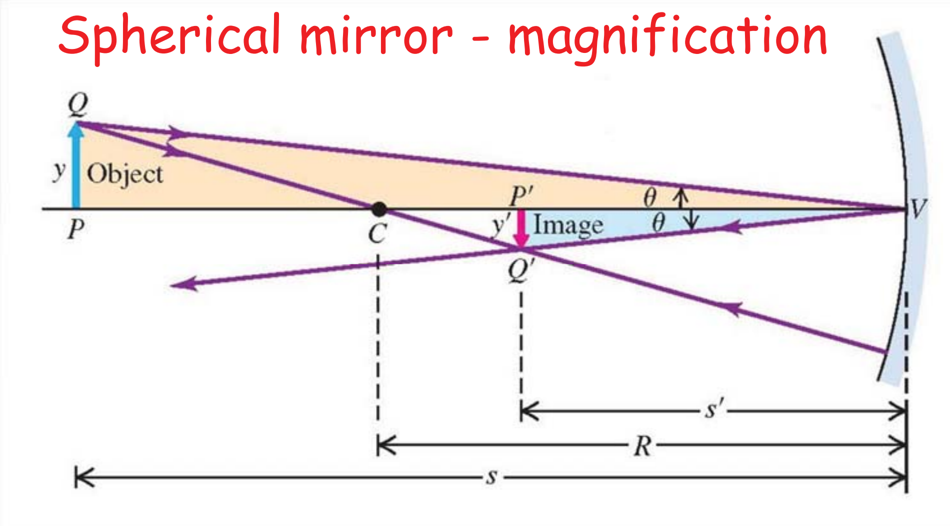
$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Spherical mirror - magnification

Definition of magnification

$$m = \frac{y'}{y}$$



tan(θ) = y/s

tan(θ) = -y′/s′

Image direction inverted

$$\frac{y}{s} = -\frac{y'}{s'} \Rightarrow m = \frac{y'}{y} = -\frac{s'}{s}$$

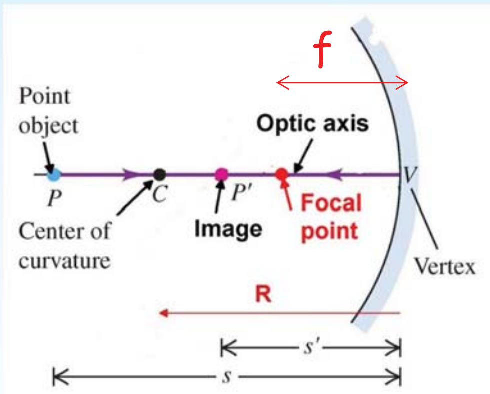
Summary spherical mirrors

Sign rules:

Object distance (s) – positive if same side as incoming light.

Image distance (s′) – positive if same side as outgoing light.

Radius of curvature (R) – positive if center is on same side as outgoing light.

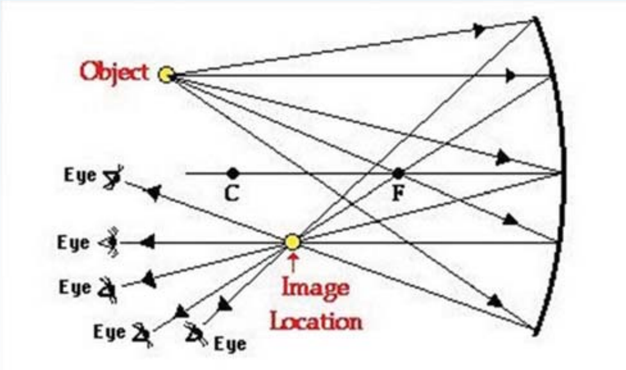


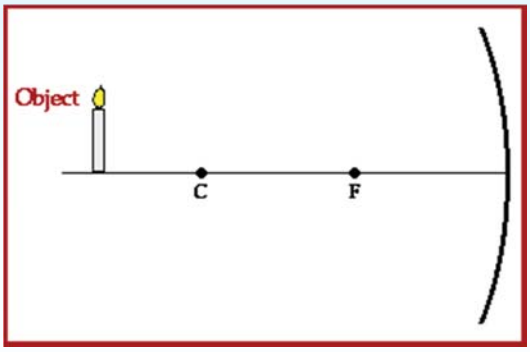
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad f = \frac{R}{2} \qquad m = \frac{y'}{y} = -\frac{s'}{s}$$

Geometrical optics

An infinite number of rays can be drawn from an object to its image.

But only two rays are needed to determine the location of the image.



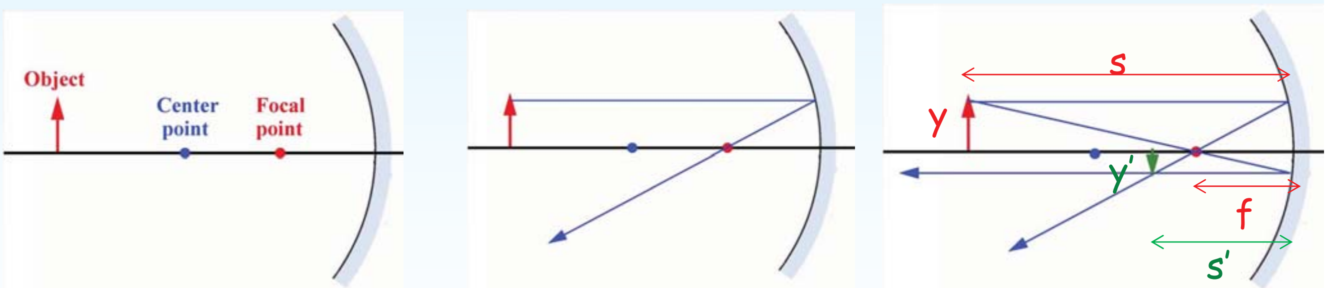


Geometrical optics

How to find the image in a concave mirror

The bottom of the object is on the optical axis and so the bottom of the image will also be on the optical axis.

The top of the image can be found with any two rays. Use for example two rays that goes through the focal point.

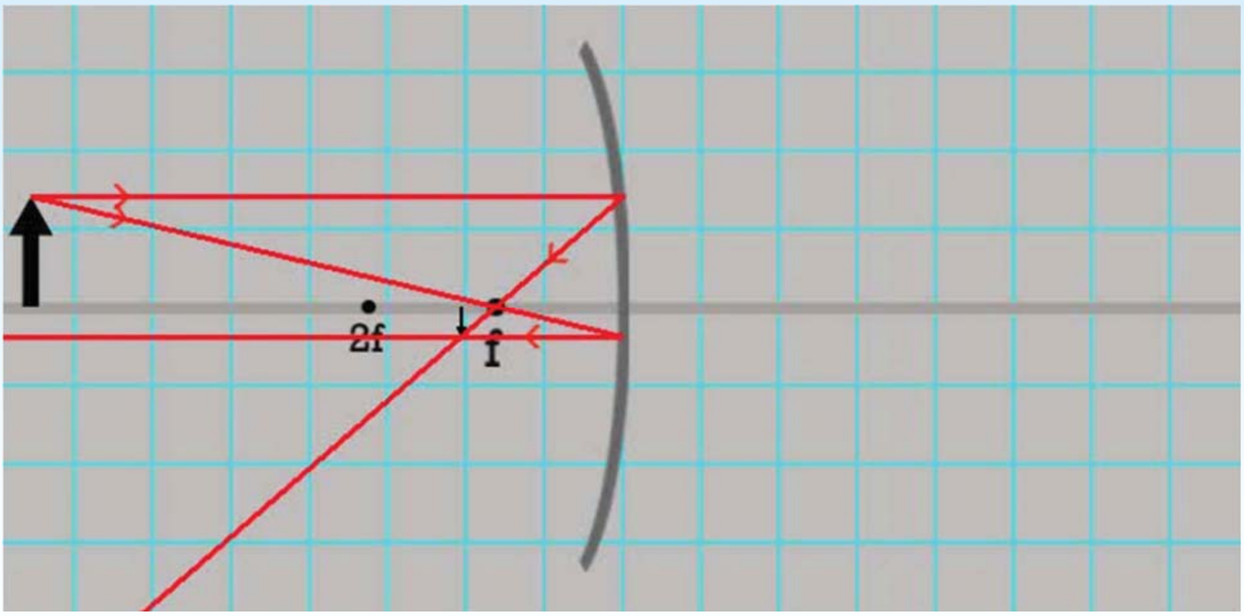


$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$f = \frac{R}{2}$$

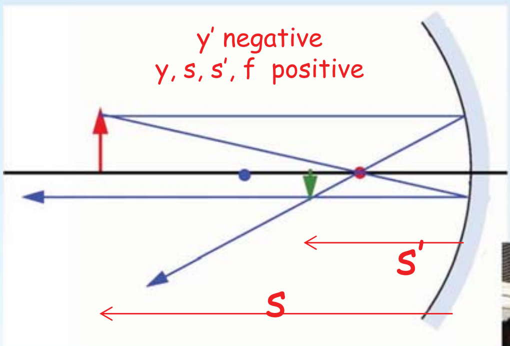
$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Geometrical optics

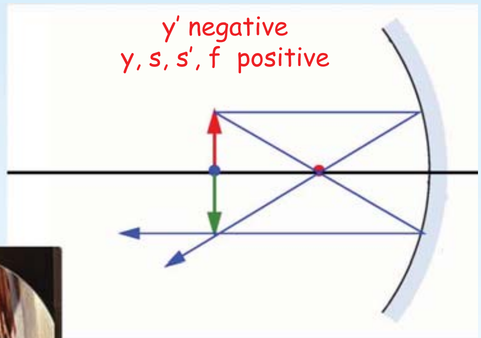


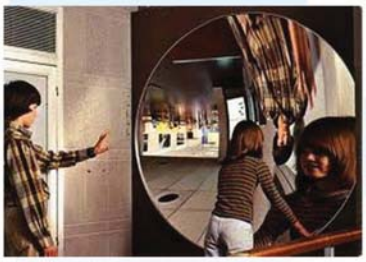
http://simbucket.com/lensesandmirrors/

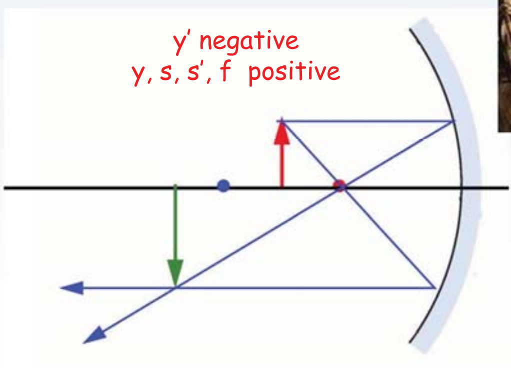
Geometrical optics

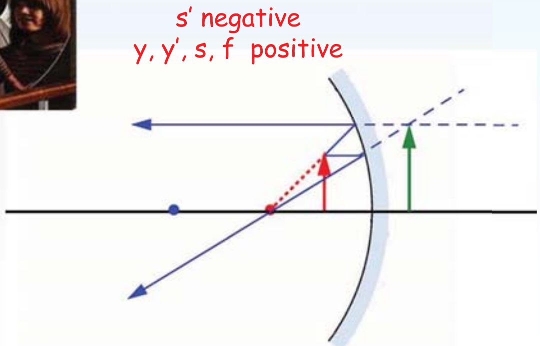


$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$









$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Problem solving

An object is put in front of a concave mirror with R = 20 cm at distances 30cm, 20cm, 10 cm and 5 cm. Where will the image be ? And what will the maginification be ?

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

$$m=\frac{y'}{y}=-\frac{s'}{s}$$

Always positive for a concave mirror

s = 30 cm

$$\frac{1}{30\text{ cm}}+\frac{1}{s'}=\frac{1}{10\text{ cm}} \qquad s'=15\text{ cm} \qquad m=-\frac{15\text{ cm}}{30\text{ cm}}=-\frac{1}{2}$$

s = 20 cm

$$\frac{1}{20\text{ cm}}+\frac{1}{s'}=\frac{1}{10\text{ cm}} \qquad s'=20\text{ cm} \qquad m=-\frac{20\text{ cm}}{20\text{ cm}}=-1$$

s = 10 cm

$$\frac{1}{10\text{ cm}}+\frac{1}{s'}=\frac{1}{10\text{ cm}} \qquad s'=\infty\ (\text{or } -\infty) \qquad m=-\frac{\infty\ \text{cm}}{10\text{ cm}}=-\infty\ (\text{or } +\infty)$$

s = 5 cm

$$\frac{1}{5\text{ cm}}+\frac{1}{s'}=\frac{1}{10\text{ cm}} \qquad s'=-10\text{ cm} \qquad m=-\frac{-10\text{ cm}}{5\text{ cm}}=+2$$

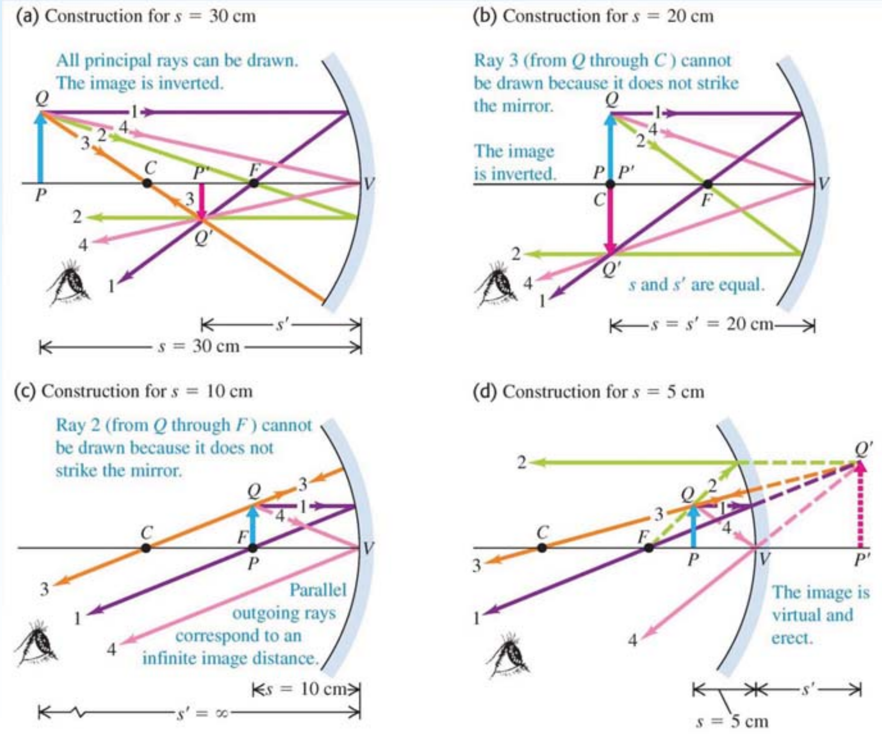
Geometrical optics

$s' = 15$ cm

$s' = 20$ cm

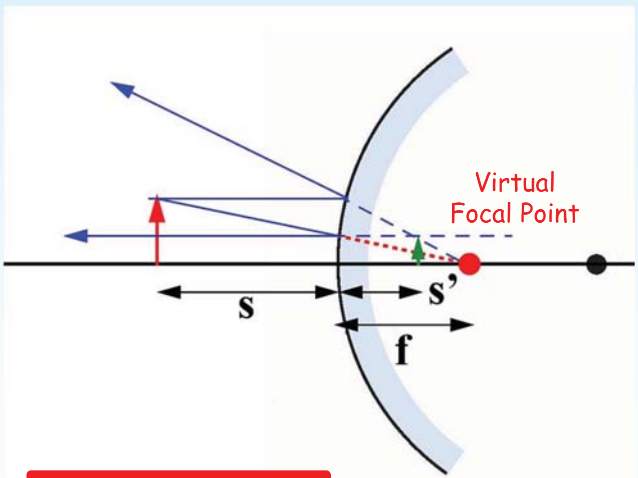
$s' = \infty$ (or $-\infty$)

$s' = -10$ cm



Geometrical optics

Convex mirrors

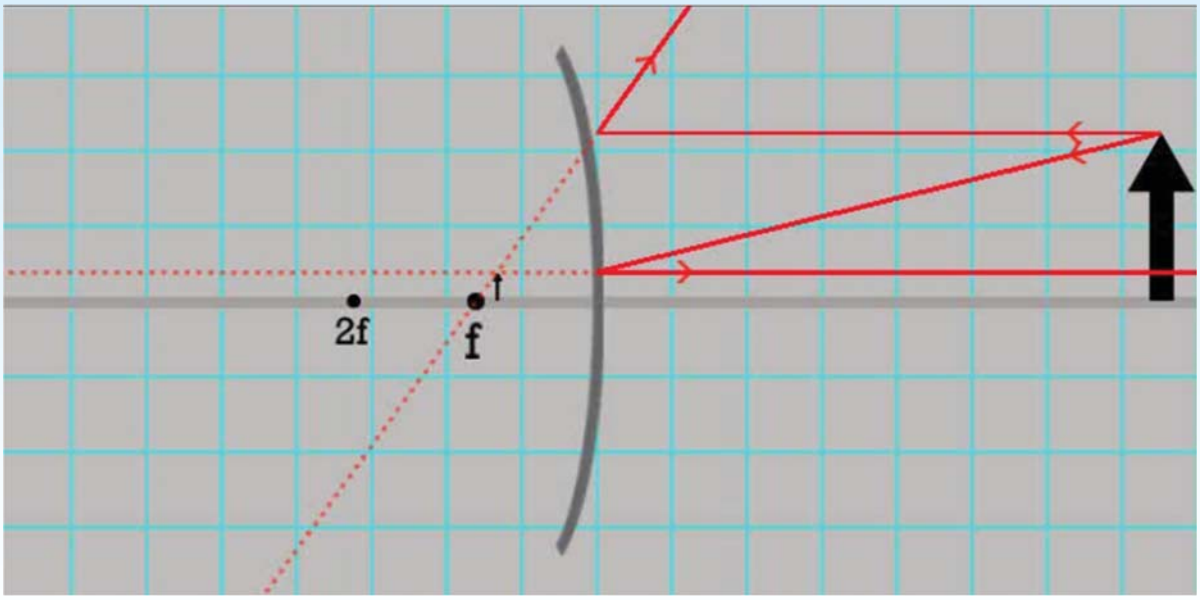


$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

s', f negative
y, y', s positive

Geometrical optics



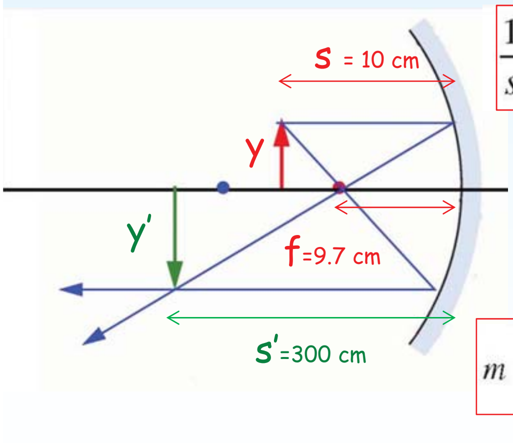
http://simbucket.com/lensesandmirrors/

Geometrical optics

Problem solving

Geometrical optics

A concave mirror forms an image, on a wall 3.00 m in front of the mirror, of a headlamp filament 10.0 cm in front of the mirror. (a) What are the radius of curvature and focal length of the mirror? (b) What is the lateral magnification? What is the image height if the object height is 5.00 mm?



$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

$$\frac{1}{10.0\text{ cm}} + \frac{1}{300\text{ cm}} = \frac{2}{R}$$

$$R = 2\left(\frac{1}{10.0\text{ cm}} + \frac{1}{300\text{ cm}}\right)^{-1} = 19.4\text{ cm}$$

$$f = \frac{R}{2}$$

$$f = R/2 = 9.7\text{ cm}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

$$m = -\frac{s'}{s} = -\frac{300\text{ cm}}{10.0\text{ cm}} = -30.0$$

The height of the image is 30 x 5 mm = 150 mm

Geometrical optics

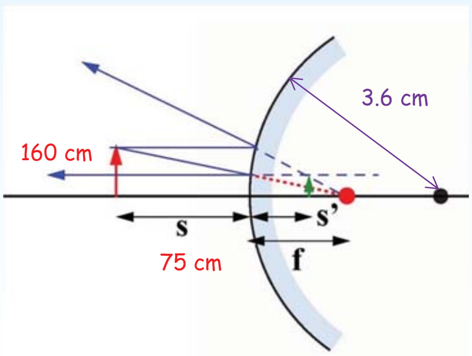
Santa checks himself for soot, using his reflection in a silvered Christmas tree ornament 0.750 m away The diameter of the ornament is 7.20 cm.

his height to be 1.6 m.

Where and how tall is the image of Santa formed by the ornament? Is it erect or inverted?

$$f = \frac{R}{2} = 7.2 / 2 / 2 = -1.80\text{ cm}$$

f is negative for a convex mirror



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

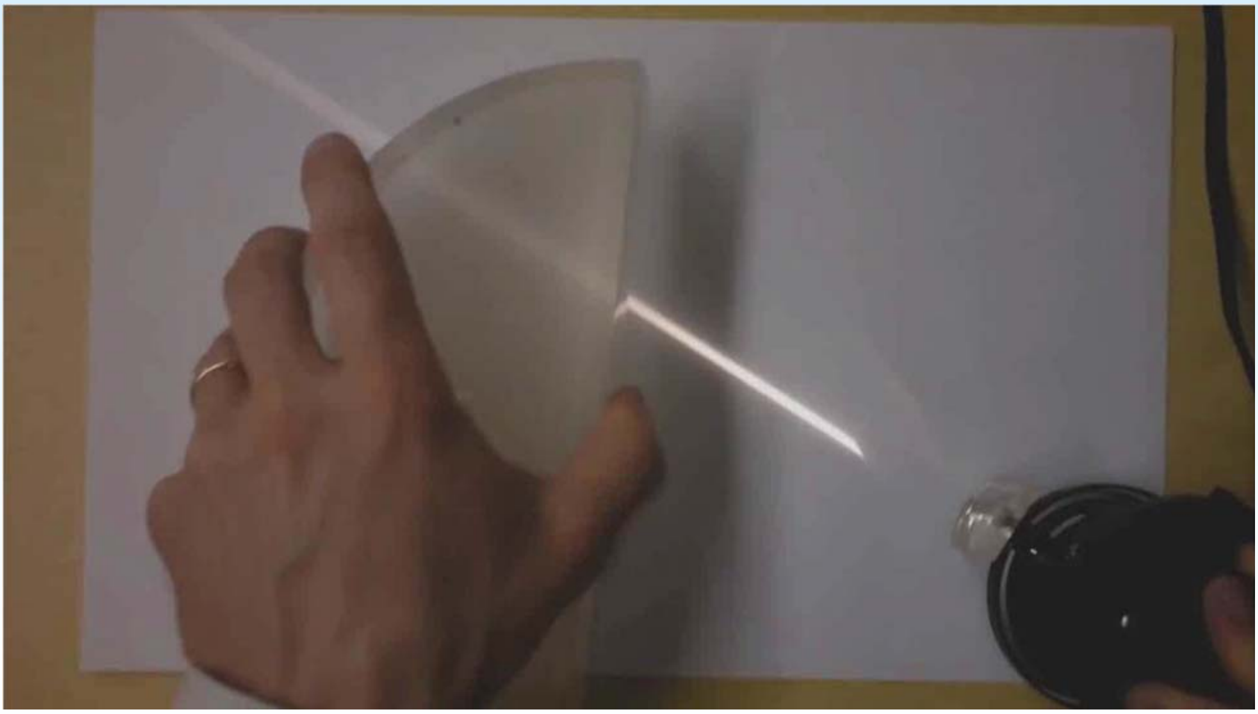
$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-1.80\text{ cm}} - \frac{1}{75.0\text{ cm}}$$

$$s' = -1.76\text{ cm}$$

$$m = \frac{y'}{y} = -\frac{s'}{s} = -\frac{-1.76\text{ cm}}{75.0\text{ cm}} = 0.0234$$

$$y' = my = (0.0234)(1.6\text{ m}) = 3.8 \times 10^{-2}\text{m} = 3.8\text{ cm}$$

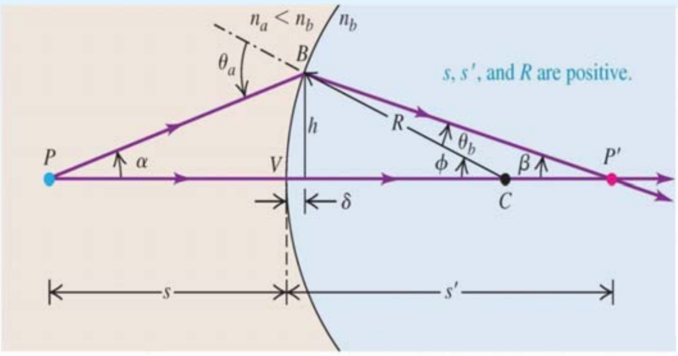
Spherical surface



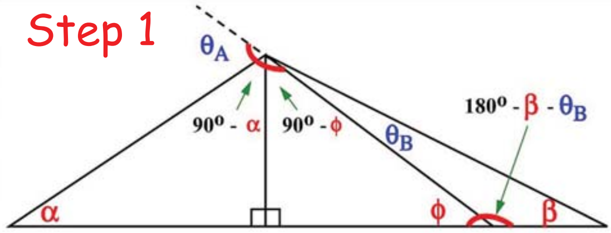
Geometrical optics

Trigonometry

to find a relationship between s' and s and R.



Step 1



$\theta_A + (90^o - \alpha) + (90^o - \phi) = 180^o \longrightarrow$

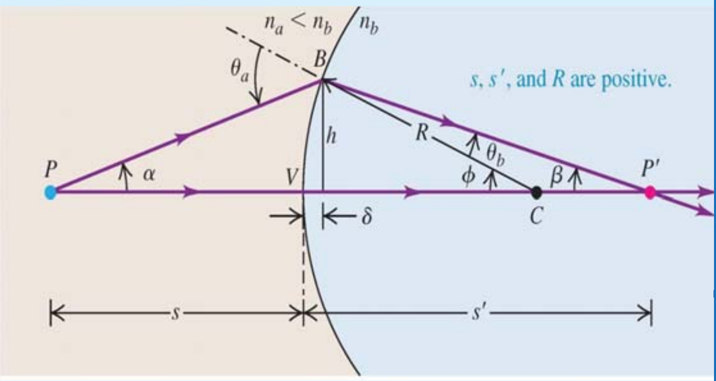
$\phi + (180^o - \beta - \theta_B) = 180^o \longrightarrow$

$\theta_A = \alpha + \phi$
$\theta_B = \phi - \beta$

Step 2

$$\tan\alpha = \frac{h}{s+\delta} \qquad \tan\beta = \frac{h}{s'-\delta} \qquad \tan\phi = \frac{h}{R-\delta}$$

Geometrical optics



Step 3

$$\tan\alpha = \frac{h}{s+\delta} \qquad \tan\beta = \frac{h}{s'-\delta} \qquad \tan\phi = \frac{h}{R-\delta}$$

If the angles and δ are small then

$$\alpha = \frac{h}{s} \qquad \beta = \frac{h}{s'} \qquad \phi = \frac{h}{R}$$

Step 4

the law of refraction $\quad n_a \sin\theta_a = n_b \sin\theta_b$

If the angles are small then

$$n_a\theta_a = n_b\theta_b$$

Combine step 1 and step 4

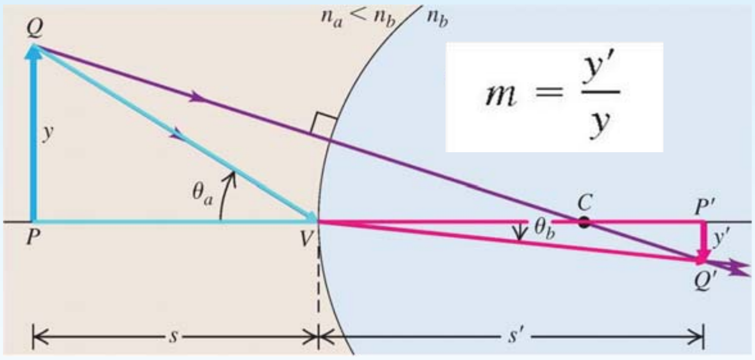
$\theta_A = \alpha + \phi$
$\theta_B = \phi - \beta$

$$n_a\alpha + n_b\beta = (n_b - n_a)\phi$$

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

Geometrical optics

Spherical surface - magnification



$$m = \frac{y'}{y}$$

Step 1 - Geometry

Image direction inverted

$$\tan\theta_a = \frac{y}{s} \qquad \tan\theta_b = \frac{-y'}{s'}$$

Small angle approximation:

$\theta_a = y/s$ $\theta_b = -y'/s'$

Step 2 – The law of refraction

$$n_a \sin\theta_a = n_b \sin\theta_b$$

Small angle approximation:

$n_a \theta_a = n_b \theta_b$

Step 3 – Combine

$$\frac{n_a y}{s} = -\frac{n_b y'}{s'}$$

$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s}$$

Geometrical optics

The small angle approximation
How good is it ?

$\sin(\theta) = \theta$
$\tan(\theta) = \theta$

sin(1º) = sin(0.0175 rad) = 0.0175
tan(1º) =tan(0.0175 rad) = 0.0175

sin(5º) = sin(0.0873 rad) = 0.0872
tan(5º) =tan(0.0873 rad) = 0.0875

sin(10º) = sin(0.175 rad) = 0.174
tan(10º) =tan(0.175 rad) = 0.176

sin(20º) = sin(0.349 rad) = 0.342
tan(20º) =tan(0.349 rad) = 0.364

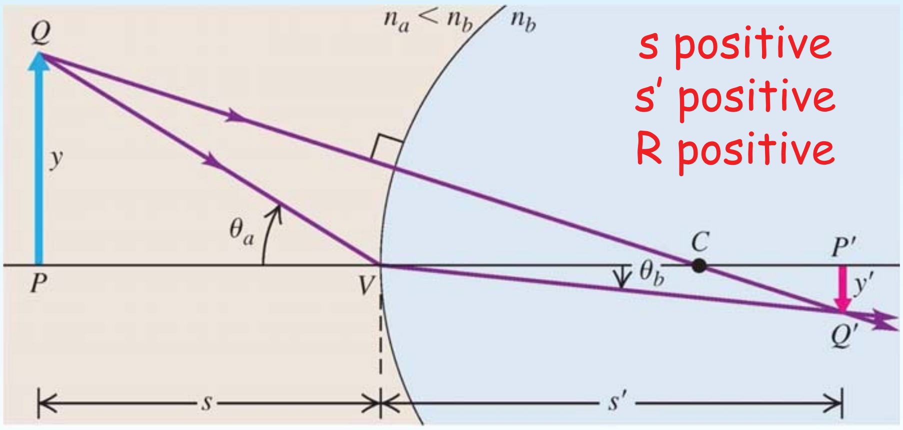
Spherical surface -Summary

Sign rules:

Object distance (s) – positive if same side as incoming light.

Image distance (s') – positive if same side as outgoing light.

Radius of curvature (R) – positive if center is on same side as outgoing light.



s positive
s' positive
R positive

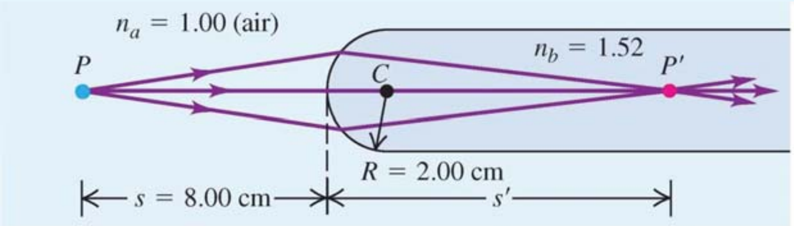
$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s}$$

Problem solving

Geometrical optics

A cylindrical glass rod has index of refraction 1.52. It is surrounded by air. One end is ground to a hemispherical surface with radius $R = 2.00$ cm. A small object is placed on the axis of the rod, 8.00 cm to the left of the vertex. Find (a) the image distance and (b) the lateral magnification.



$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

Image distance

$$\frac{1.00}{8.00\ \text{cm}} + \frac{1.52}{s'} = \frac{1.52 - 1.00}{+2.00\ \text{cm}}$$

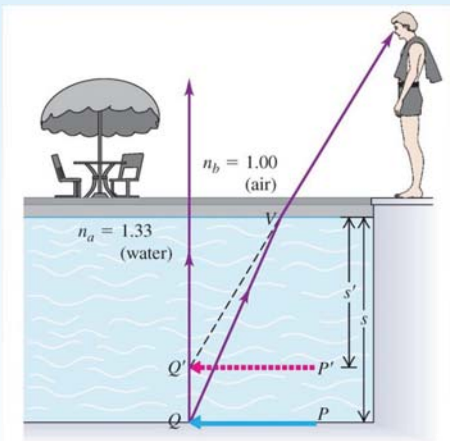
$$s' = +11.3\ \text{cm}$$

Magnification

$$m = -\frac{n_a s'}{n_b s} = -\frac{(1.00)(11.3\ \text{cm})}{(1.52)(8.00\ \text{cm})} = -0.929$$

Geometrical optics

Special case: flat surface



$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} = 0$$

$R \to \infty$

$n_a / s = -n_b / s'$

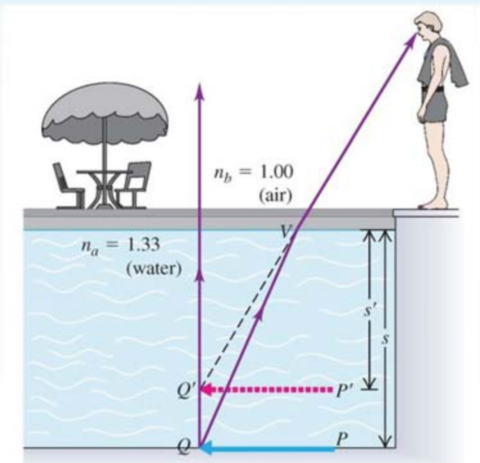
$-s'/s = n_b/n_a$

Geometrical optics

Problem solving

Geometrical optics

If you look straight down into a swimming pool where it is 2.00 m deep, how deep does it appear to be?



$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{1.33}{2.00\text{ m}} + \frac{1.00}{s'} = 0$$

$$s' = -1.50\text{ m}$$

Geometrical optics

The water in Flathead Lake is so clear that it appears very shallow. Can you believe it's actually 370 feet deep?

Image Credits: National Geographic

This is a simple illusion, but very cool nonetheless.

$n_a / s = -n_b / s'$

$-s'/s = n_b/n_a = 1.00/1.33 = 0.75$

Does it look like the depth is 0.75 x 370 feet = 278 feet = 85 m ??????

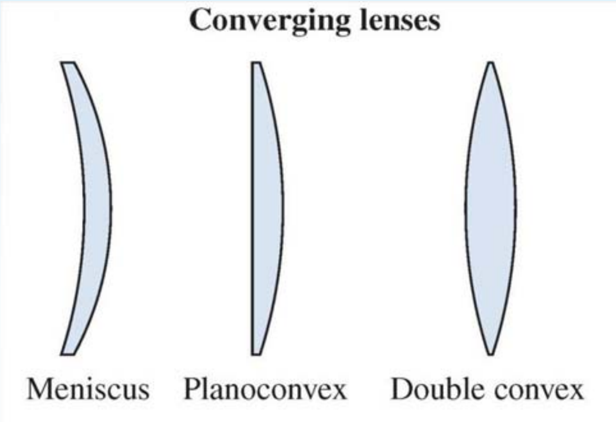
Answer: no, more like 2 m.

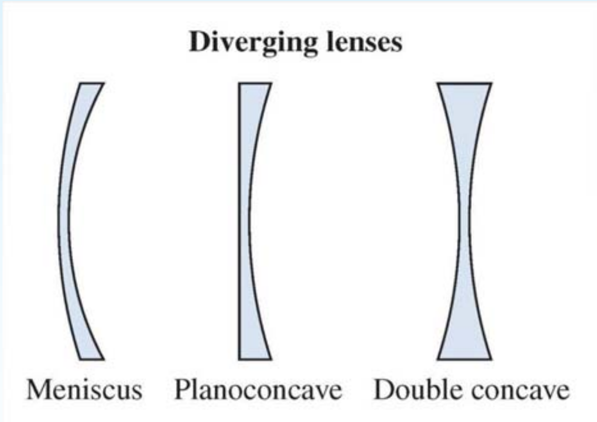
Geometrical optics

Lenses

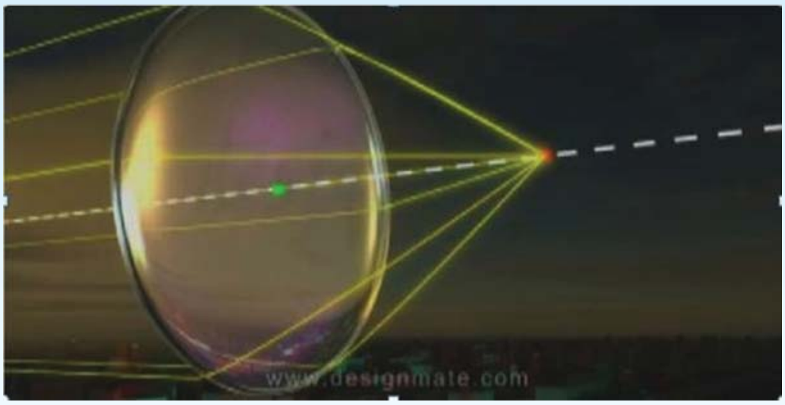
Geometrical optics

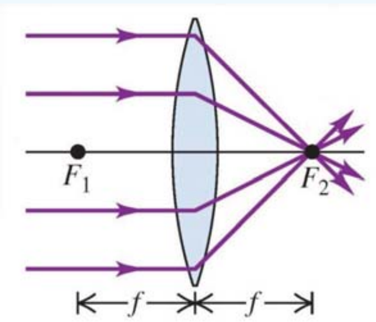
Different type of lenses

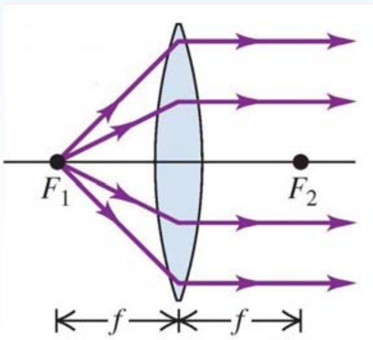




Geometrical optics

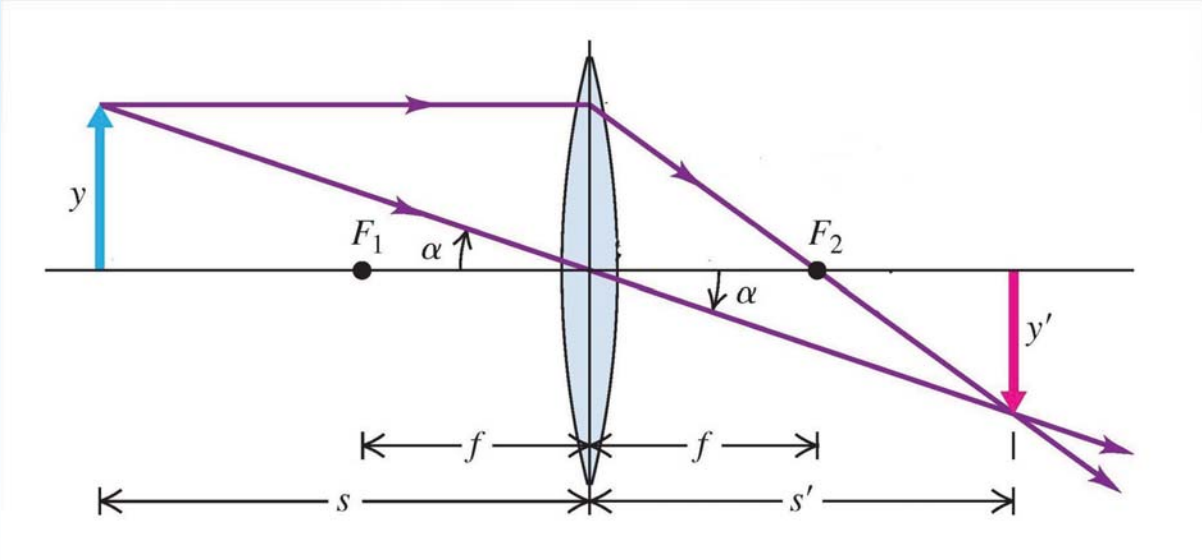






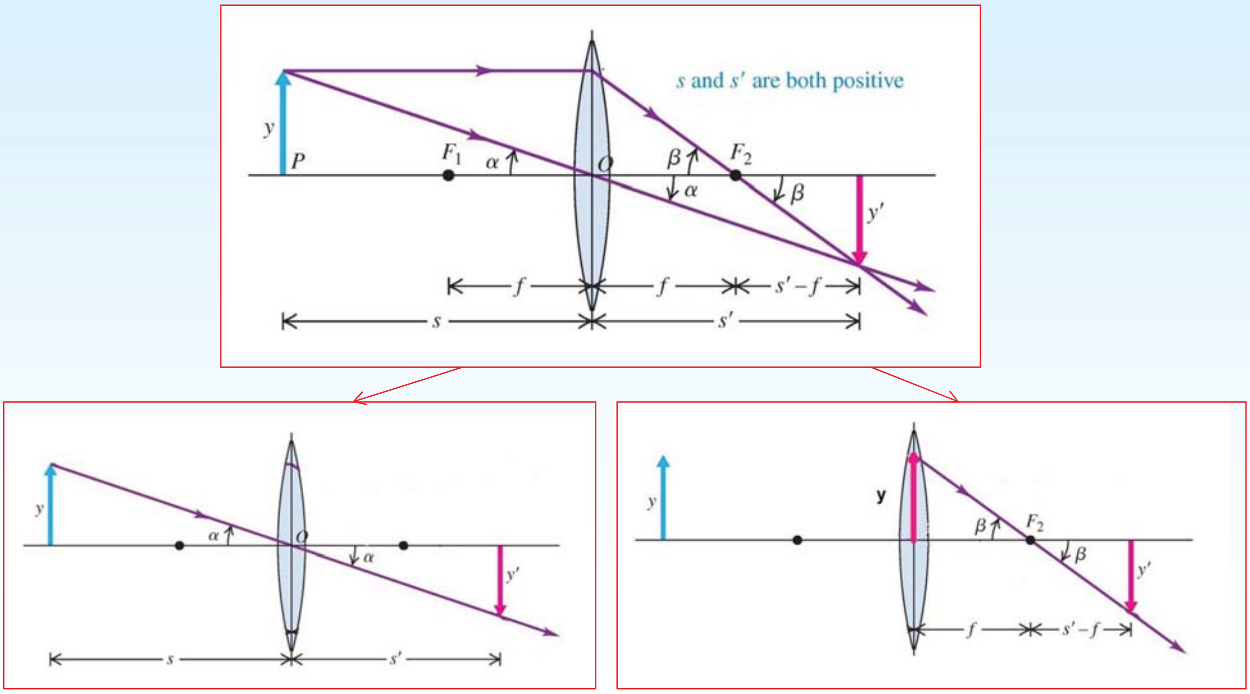
Geometrical optics

Useful rays



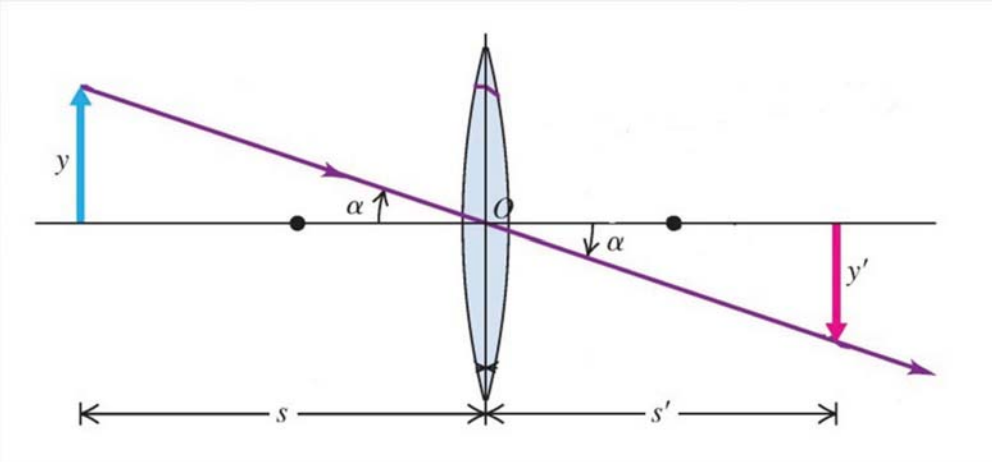
Geometrical optics

Deriving formulas for lenses



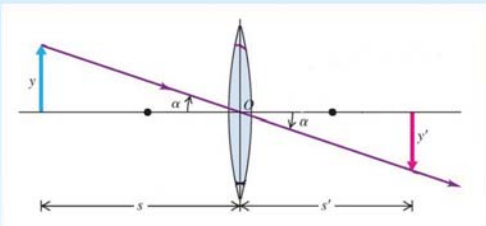
Geometrical optics

Magnification formula for lenses



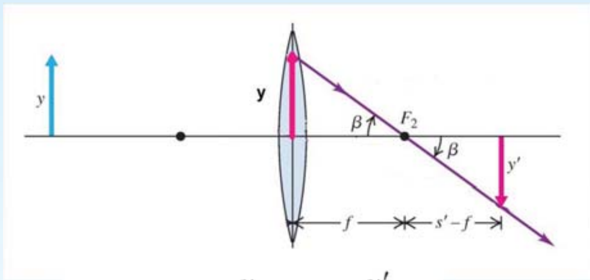
$$\tan(\alpha) = \frac{y}{s} = -\frac{y'}{s'} \Rightarrow \frac{y'}{y} = -\frac{s'}{s} \Rightarrow m = \frac{y'}{y} = -\frac{s'}{s}$$

Geometrical optics



$$\tan(\alpha) = \frac{y}{s} = -\frac{y'}{s'}$$

$$\frac{y'}{y} = -\frac{s'}{s}$$

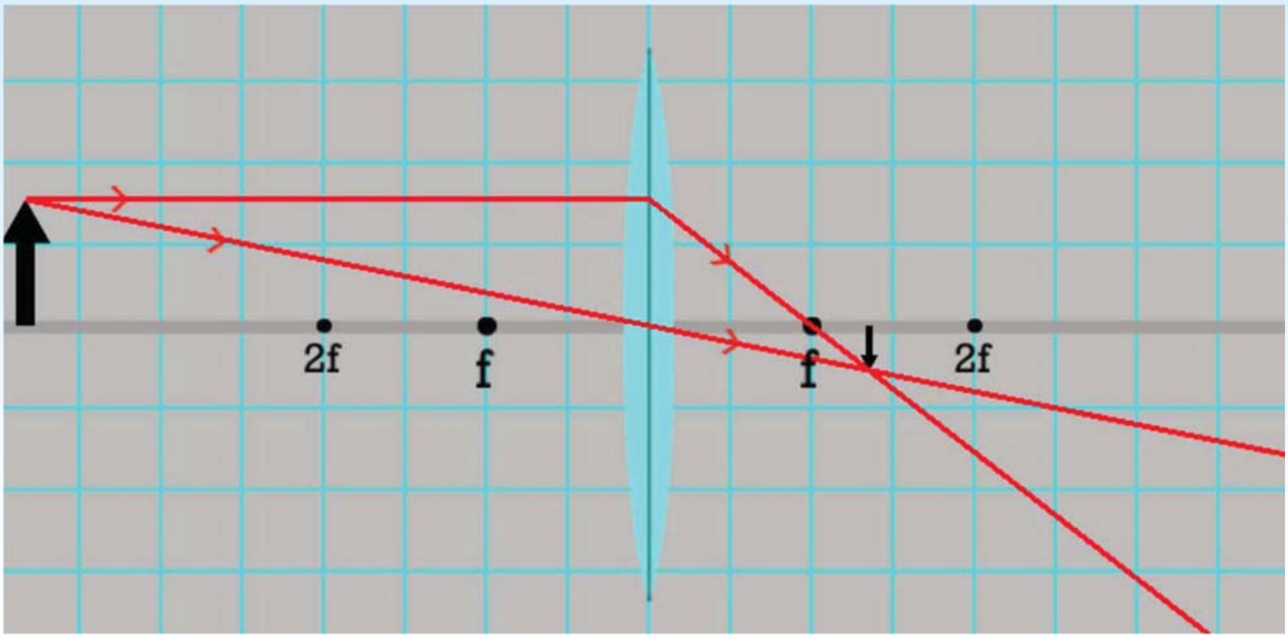


$$\tan(\beta) = \frac{y}{f} = -\frac{y'}{s' - f}$$

$$\frac{y'}{y} = -\frac{s' - f}{f}$$

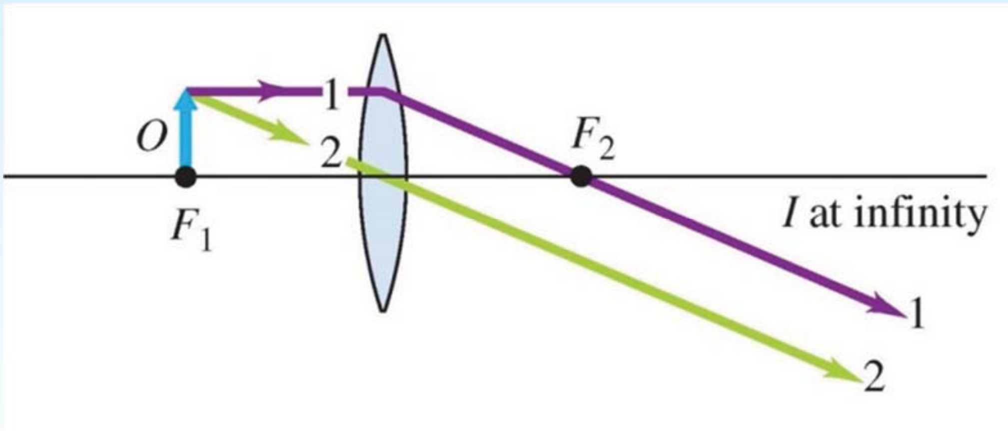
$$-\frac{s'}{s} = -\frac{s' - f}{f} \qquad -\frac{s'}{s} \Big/ s' = -\frac{s' - f}{f} \Big/ s' = -(s'-f)/s'f \Rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Geometrical optics



http://simbucket.com/lensesandmirrors/

Geometrical optics



An object placed at the focal point appear to be at infinity

Geometrical optics

Convex lenses -Summary

Sign rules:

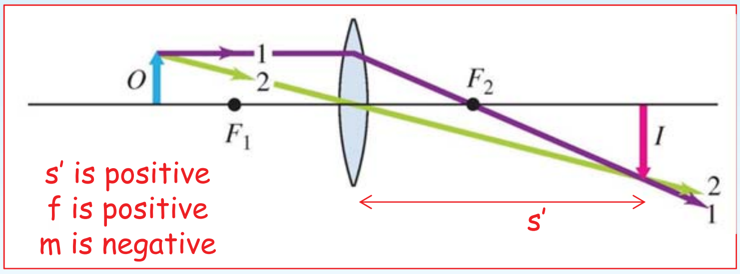
Object distance (s) – positive if same side as incoming light.

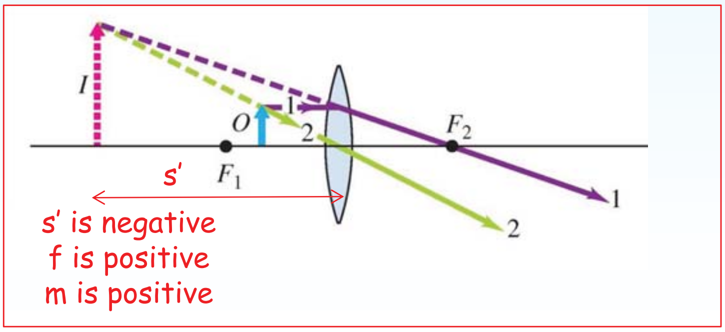
Image distance (s') – positive if same side as outgoing light.

Focal length (f) – positive for converging lenses (convex lenses)

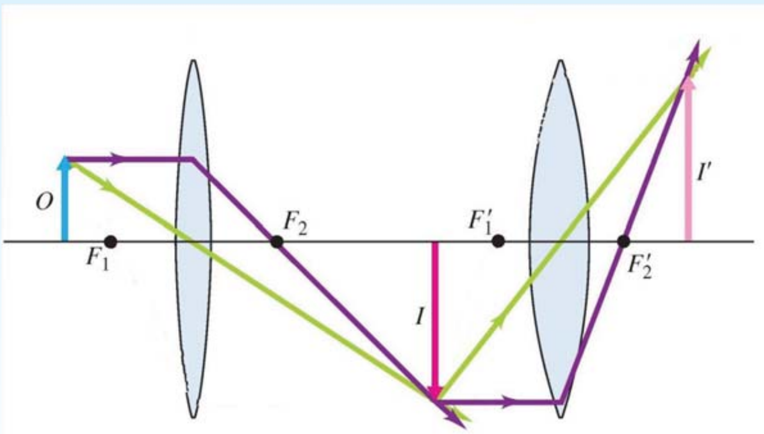
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$





Two lenses



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = -\frac{s'}{s}$$

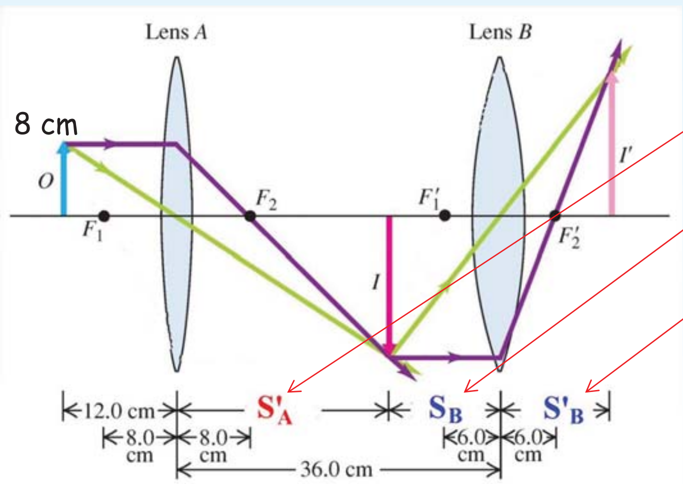
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = -\frac{s'}{s}$$

Problem solving

Converging lenses *A* and *B*, of focal lengths 8.0 cm and 6.0 cm, respectively, are placed 36.0 cm apart. Both lenses have the same optic axis. An object 8.0 cm high is placed 12.0 cm to the left of lens *A*. Find the position, size, and orientation of the image produced by the lenses in combination.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$



$$\frac{1}{12.0\text{ cm}} + \frac{1}{s'_{I,A}} = \frac{1}{8.0\text{ cm}} \qquad s'_{I,A} = +24.0\text{ cm}$$

$S_B = 36.0 - S_A' = 36 - 24 = 12$ cm

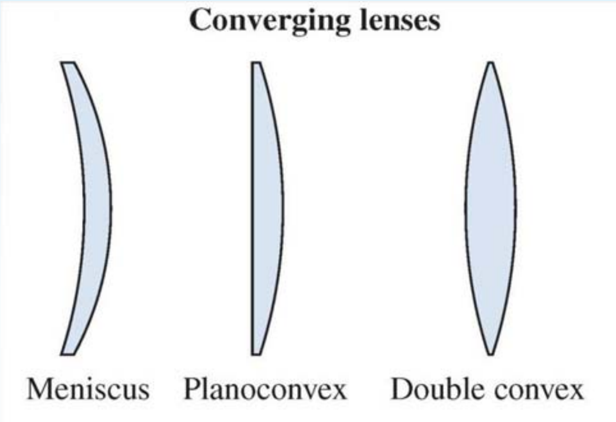
$$\frac{1}{12.0\text{ cm}} + \frac{1}{s'_{I',B}} = \frac{1}{6.0\text{ cm}} \qquad s'_{I',B} = +12.0\text{ cm}$$

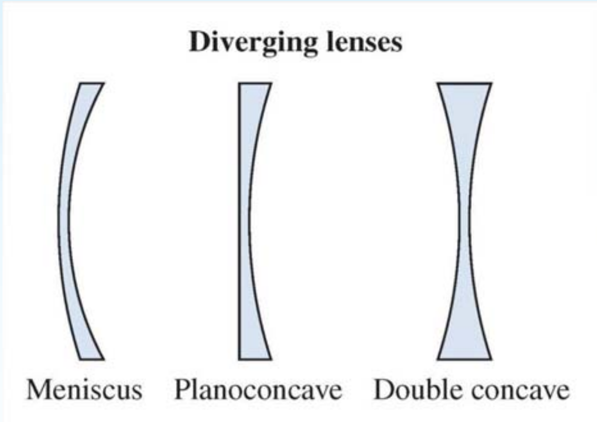
$$m_A = -(24.0\text{ cm})/(12.0\text{ cm}) = -2.00$$
$$m_B = -(12.0\text{ cm})/(12.0\text{ cm}) = -1.00$$
$$m_A m_B = (-2.00)(-1.00) = +2.00$$

I' is $(2.00)(8.0\text{ cm}) = 16$ cm

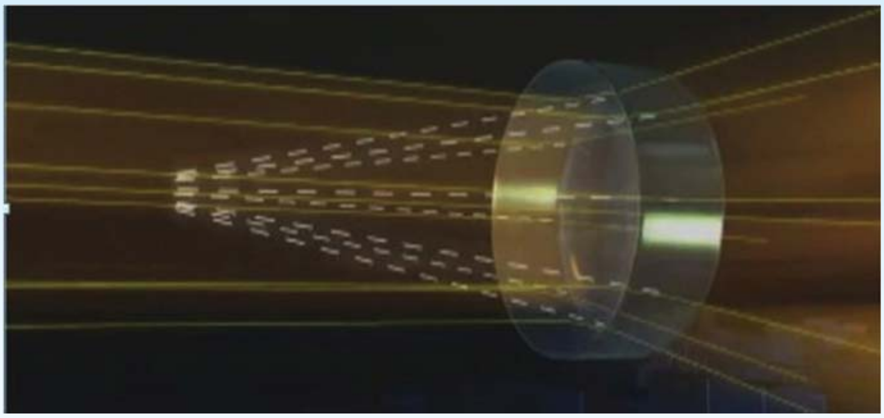
Geometrical optics

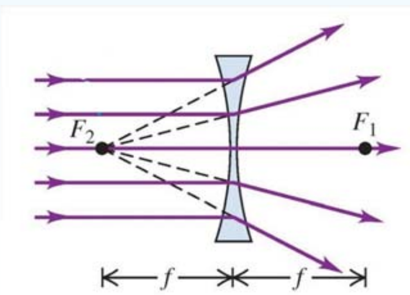
Lenses

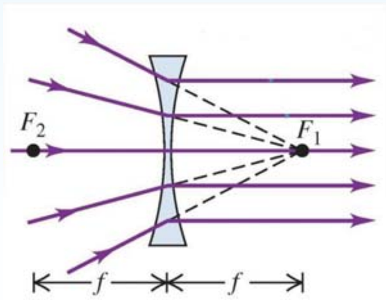




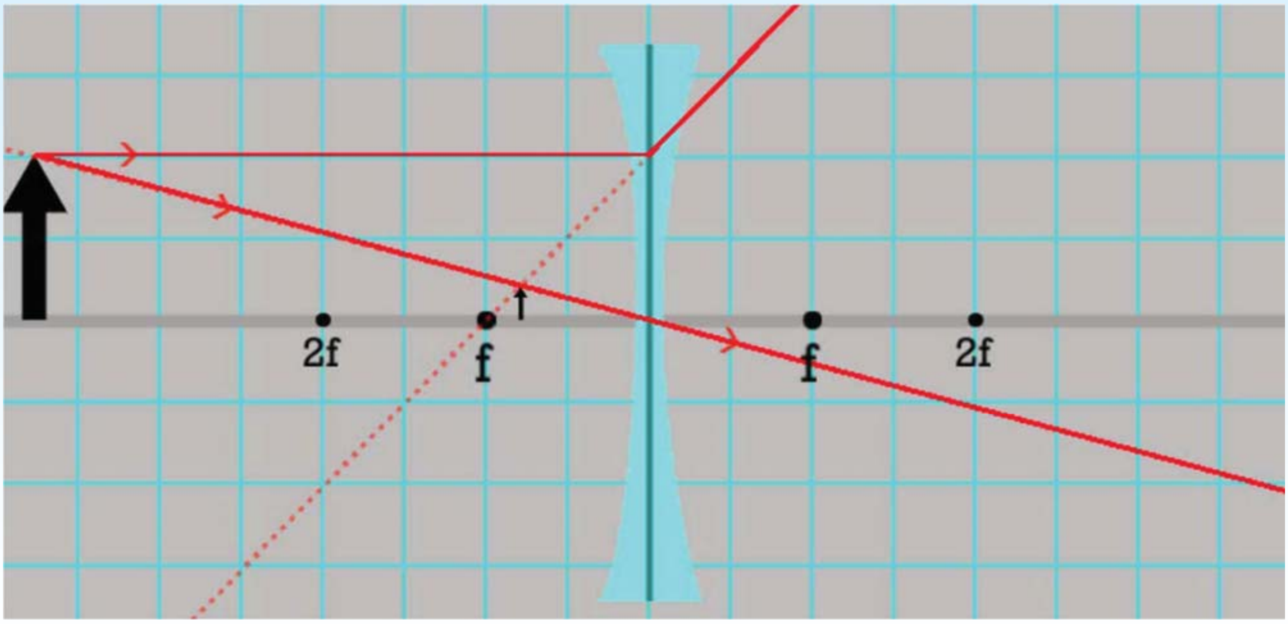
Geometrical optics







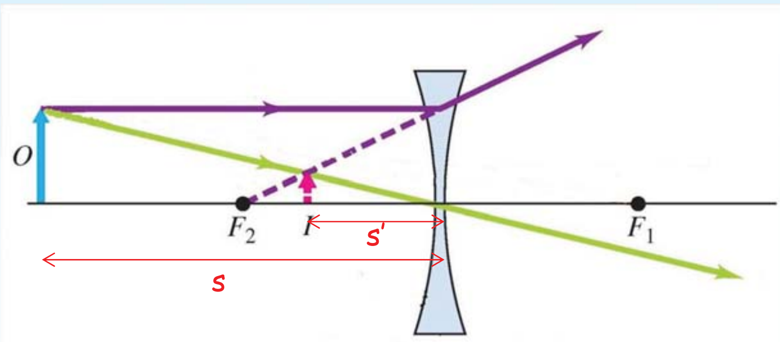
Geometrical optics



http://simbucket.com/lensesandmirrors/

Geometrical optics

Lens formula for concave lenses



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = -\frac{s'}{s}$$

f is negative for diverging lenses

s' is negative for diverging lenses

m is positive

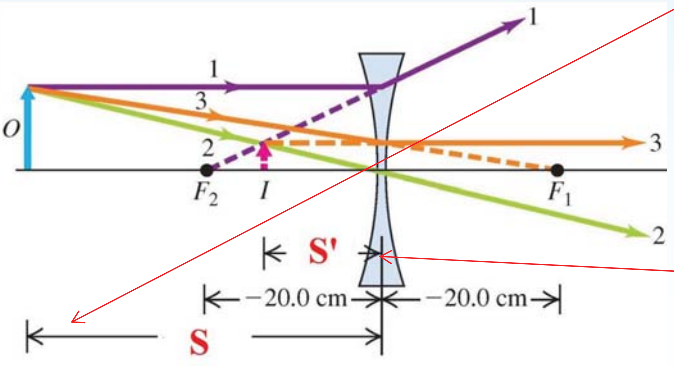
Problem solving

A beam of parallel rays spreads out after passing through a thin diverging lens, as if the rays all came from a point 20.0 cm from the center of the lens. You want to use this lens to form an erect, virtual image that is $\frac{1}{3}$ the height of the object. Where should the object be placed? Where will the image be?

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{s} + \frac{1}{-s/3} = \frac{1}{s} - \frac{3}{s} = -\frac{2}{s} = \frac{1}{f}$$

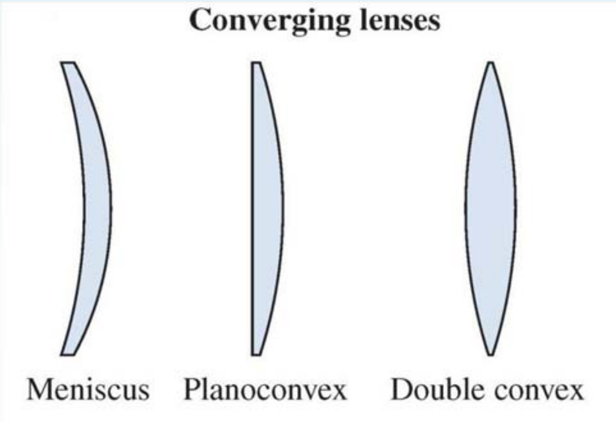
$$s = -2f = -2(-20.0\text{ cm}) = 40.0\text{ cm}$$

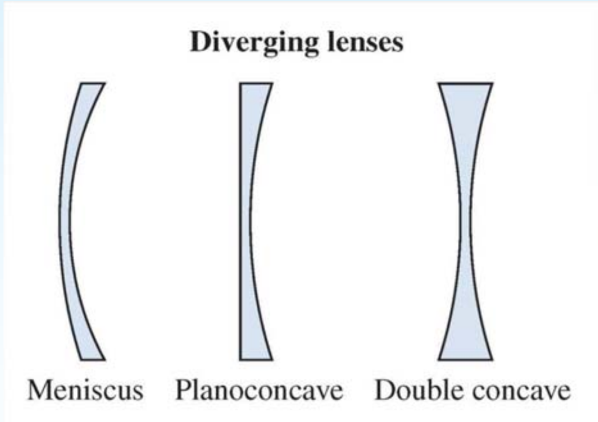


$$s' = -\frac{s}{3} = -\frac{40.0\text{ cm}}{3} = -13.3\text{ cm}$$

Geometrical optics

Lenses

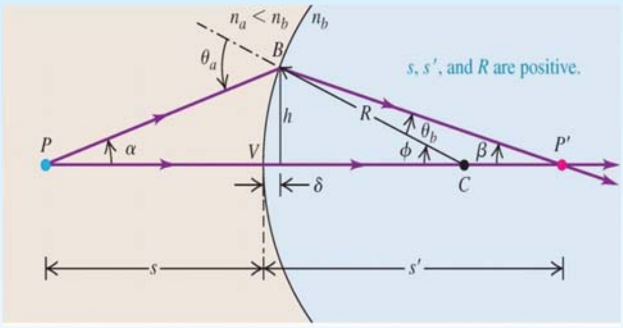




Rule:
A lens that is thicker at the center than the edges is converging (positive f)
A lens that is thinner at the center than the edges is diverging (negative f)

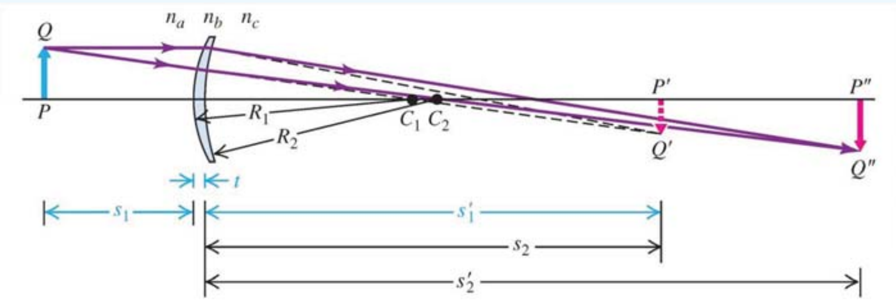
Geometrical optics

Spherical surface



$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s}$$

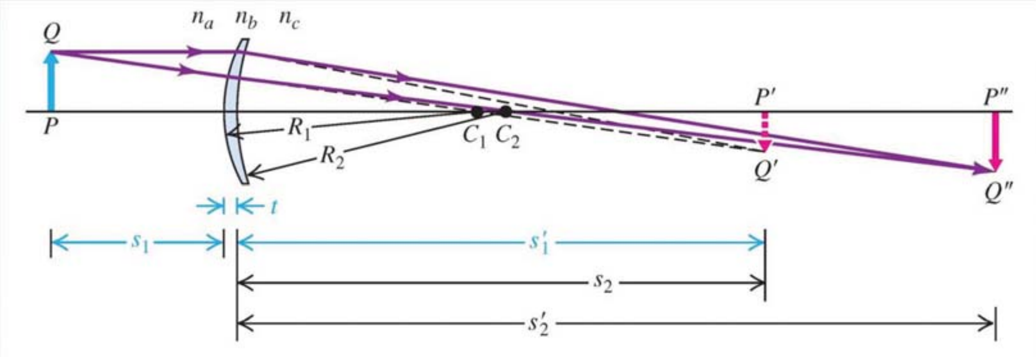


Step 1

$$\frac{n_a}{s_1} + \frac{n_b}{s'_1} = \frac{n_b - n_a}{R_1}$$

$$\frac{n_b}{s_2} + \frac{n_c}{s'_2} = \frac{n_c - n_b}{R_2}$$

Geometrical optics



Step 1

$$\frac{n_a}{s_1} + \frac{n_b}{s_1'} = \frac{n_b - n_a}{R_1}$$

$$\frac{n_b}{s_2} + \frac{n_c}{s_2'} = \frac{n_c - n_b}{R_2}$$

Step 2

$$n_a = n_c = 1$$

$$n_b = n$$

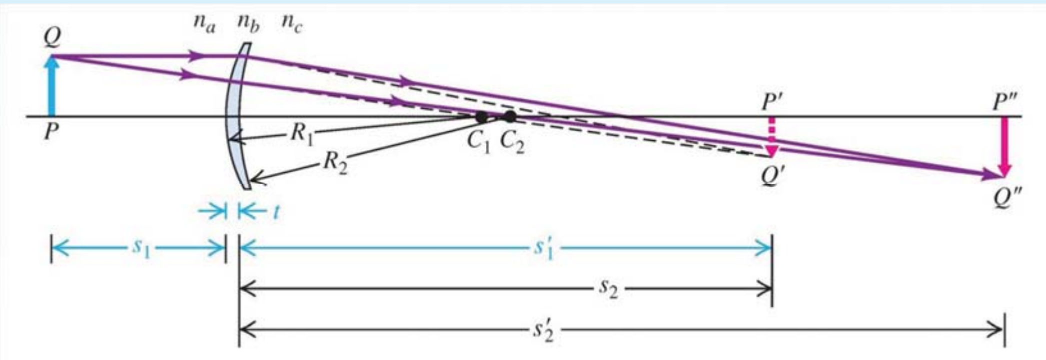
$$s_2 = -s_1'$$

⇨

$$\frac{1}{s_1} + \frac{n}{s_1'} = \frac{n-1}{R_1}$$

$$-\frac{n}{s_1'} + \frac{1}{s_2'} = \frac{1-n}{R_2}$$

Geometrical optics



Step 2

$$\frac{1}{s_1} + \frac{n}{s_1'} = \frac{n-1}{R_1}$$

$$-\frac{n}{s_1'} + \frac{1}{s_2'} = \frac{1-n}{R_2}$$

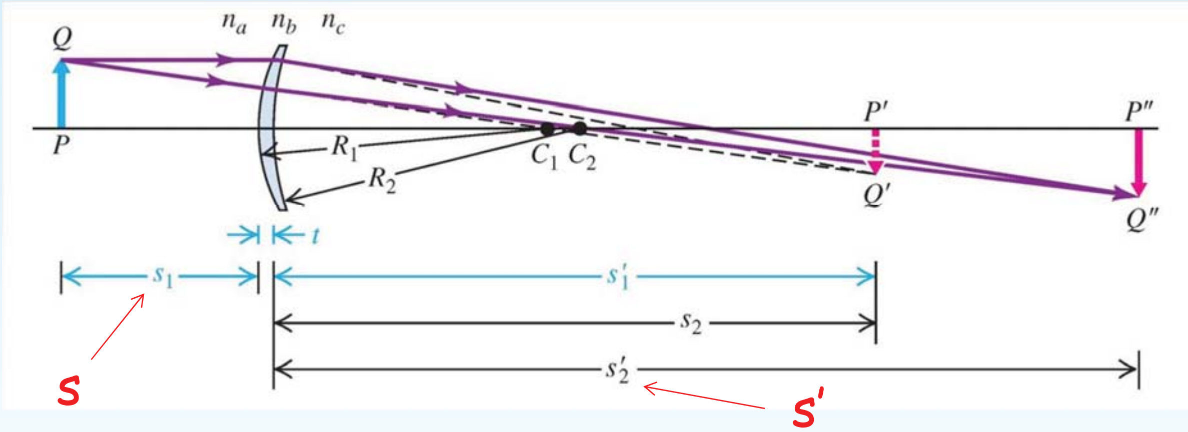
⇨

Step 3 Add the two equations

$$\frac{1}{s_1} + \frac{1}{s_2'} = \frac{n-1}{R_1} + \frac{1-n}{R_2}$$

$$\frac{1}{s_1} + \frac{1}{s_2'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Geometrical optics



Step 3

$$\frac{1}{s_1} + \frac{1}{s'_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Step 4

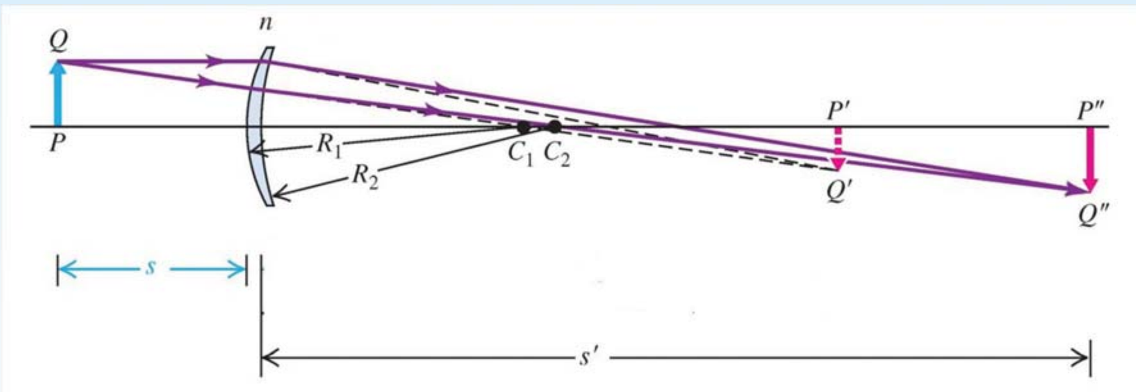
$s_1 = s$

$s_2' = s'$

⇨

$$\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Geometrical optics



Step 5 - combine new with old

$$\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

⇨

The lensmaker's equation

$$\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$

Geometrical optics

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$m = -\frac{s'}{s}$$

$$m = \frac{y'}{y}$$

Sign rule: Radius of curvature – positive if center is on same side as outgoing light.

f = positive R_1 = positive R_2 = positive s' = positive or negative

f = positive R_1 = positive R_2 = negative s' = positive or negative

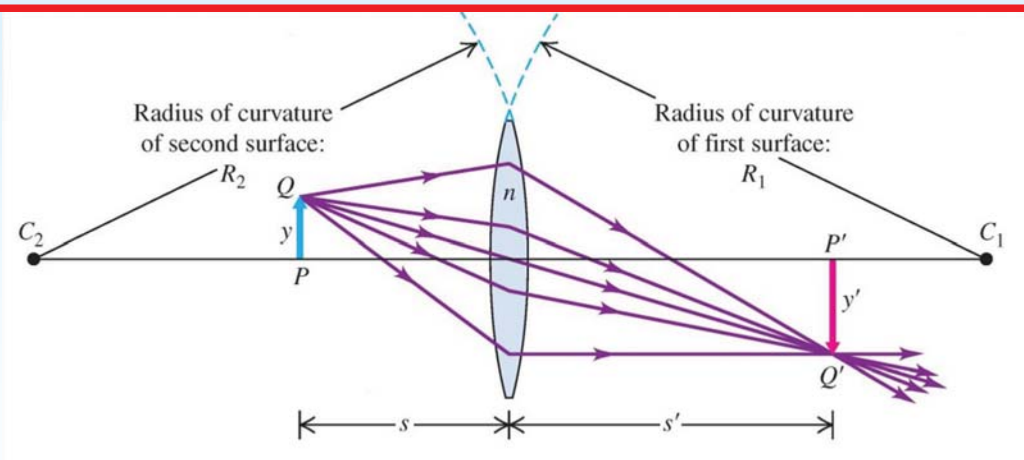
f = negative R_1 = negative R_2 = positive s' = negative

Geometrical optics

Problem solving

Geometrical optics

A double convex lens has $R_1 = R_2 = 10$ cm and $n = 1.52$
What is the focal length ?



$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (1.52 - 1)\left(\frac{1}{+10\text{ cm}} - \frac{1}{-10\text{ cm}}\right)$$

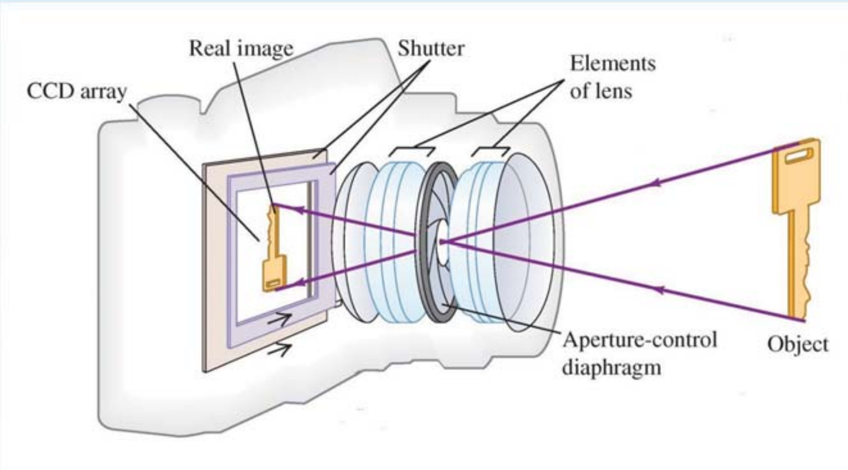
$$f = 9.6\text{ cm}$$

Geometrical optics

The camera

Geometrical optics

Geometrical optics

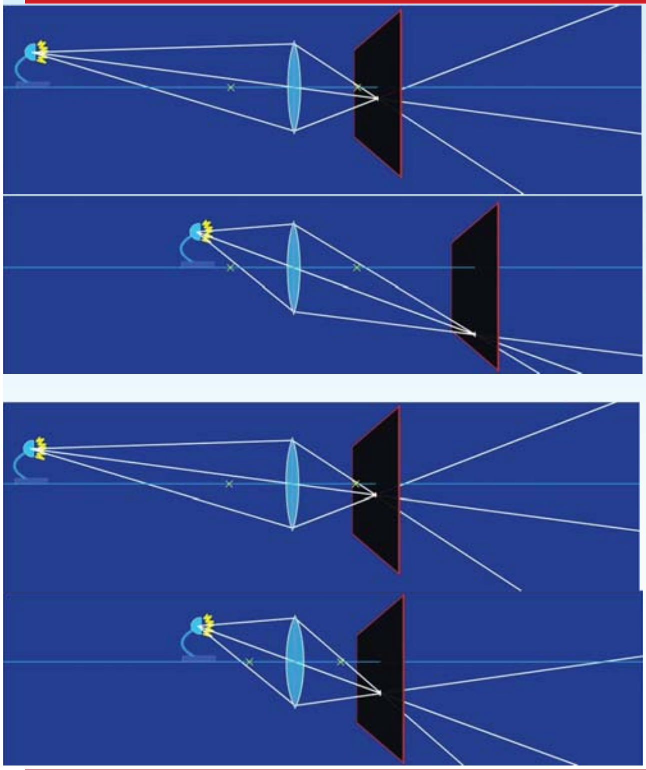


Two main issues for a camera:

1. Focussing the image on the image sensor (CCD)

2. Getting enough exposure (enough light on to the image sensor)

Geometrical optics



Focussing

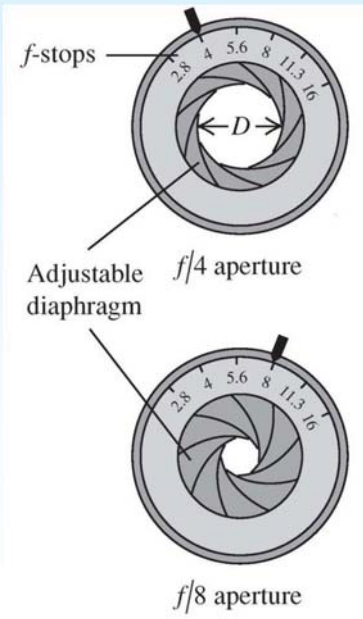
1. Changing the distance between lens and CCD.

2. Changing the focal length of the lens.

Telephoto lens: Long focal length
Wide-angle lens: Short focal length

Geometrical optics

Exposure: light energy per unit area that hits CCD



Exposure depends on shutter time and the lens aperture.

Long shutter times leads to problems if the object is moving.

The aperture is controlled by the diaphragm that can change its diameter (D).

$f_{number} = f / D$ Exposure $\sim 1 / f_{number}^2$

$f/2$ $f/2.8$ $f/4$ $f/5.6$ $f/8$ $f/11$ $f/16$

f_{number}

Geometrical optics

Camera without zoom

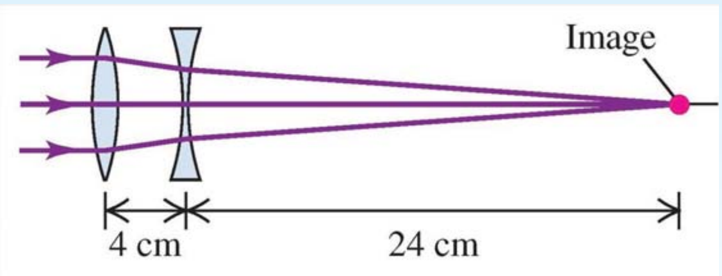
50 mm
1:1.7

focal length: f = 50 mm
f_{number} = 1.7

Aperture diameter: $D = f / f_{number} = 50/1.7 = 2.9$ mm

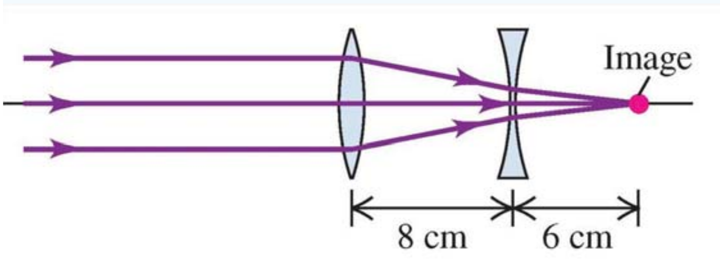
Geometrical optics

Zoom lens: A combination of several lenses



Lenses close together:

Long total focal length
Telephoto lends



Lenses further apart:

Short focal length
Wide angle lens

Geometrical optics

4.6 – 23.0 mm
1:3.2 – 6.5

18 – 135 mm
1:3.5 – 5.6

focal length: f = 4.6 – 23.0 mm
f_{number} = 3.2 – 6.5

focal length: f = 18 – 135 mm
f_{number} = 3.5 – 5.6

Geometrical optics

Problem solving

Geometrical optics

A common telephoto lens for a 35-mm camera has a focal length of 200 mm; its *f*-stops range from *f*/2.8 to *f*/22. (a) What is the corresponding range of aperture diameters? (b) What is the corresponding range of image intensities on the film?

$f_{number} = f / D$

$$D = \frac{f}{f\text{-number}} = \frac{200 \text{ mm}}{2.8} = 71 \text{ mm}$$

$$D = \frac{200 \text{ mm}}{22} = 9.1 \text{ mm}$$

Exposure $\sim 1 / f_{number}^2$

Maximum exposure = $C / 2.8^2$
Minimum exposure = $C / 22^2$

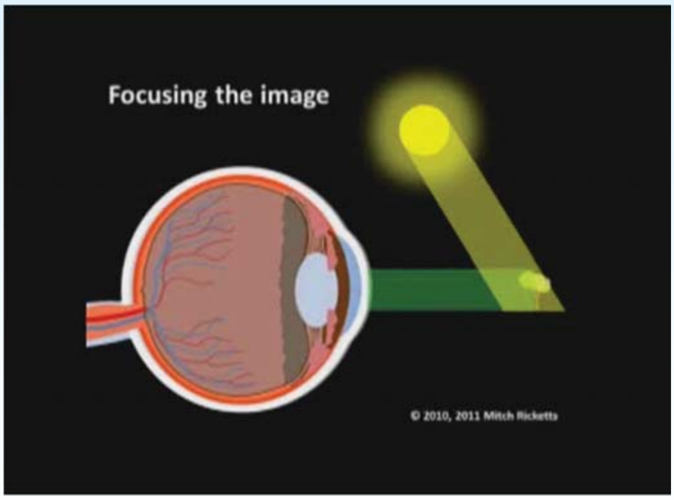
Maximum / Minimum = $22^2 / 2.8^2 = 62$

Geometrical optics

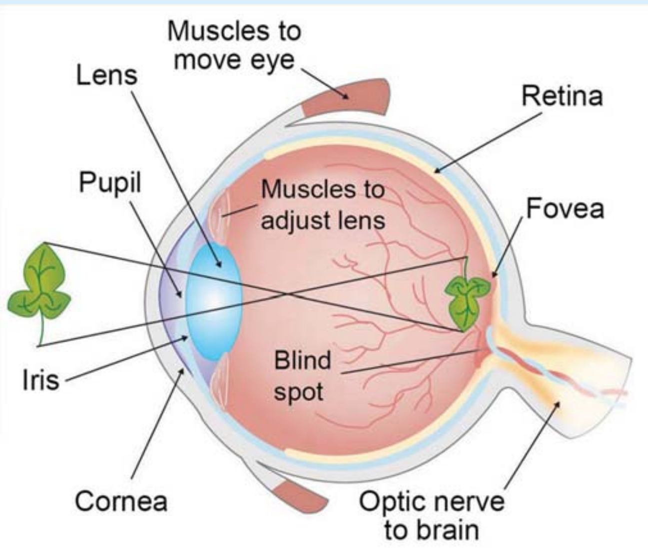
The eye

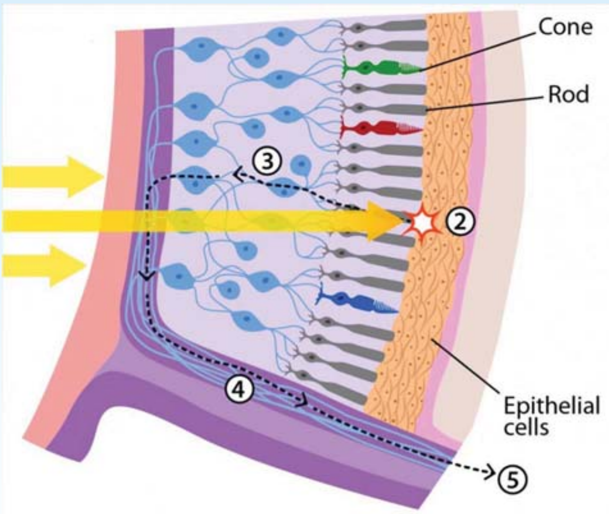
Geometrical optics

The function of the eye



Geometrical optics

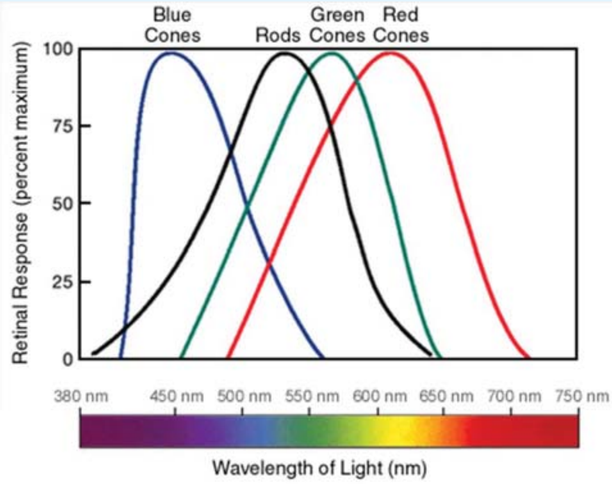




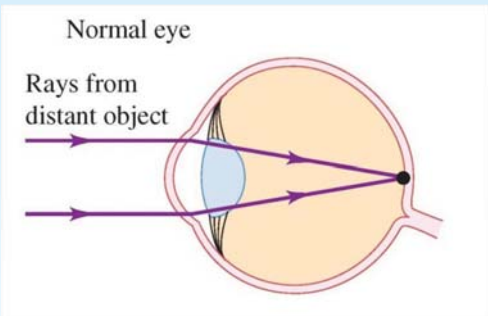
Rods: Very light sensitive. Used for night vision in black and white.
Cones: Three types (red, blue, green). Used to see colour.

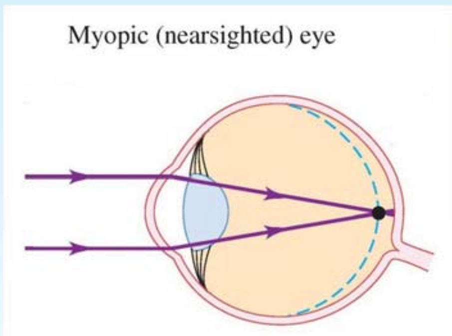
Geometrical optics

The human eye's sensitivity to different wavelengths



Geometrical optics



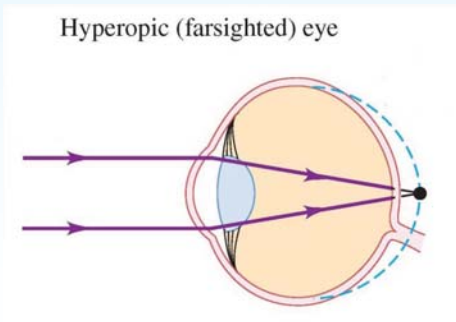


Near point: Closest distance to the eye at which people can see clear (7cm at age 10 to 40cm at age 50 for normal eye).

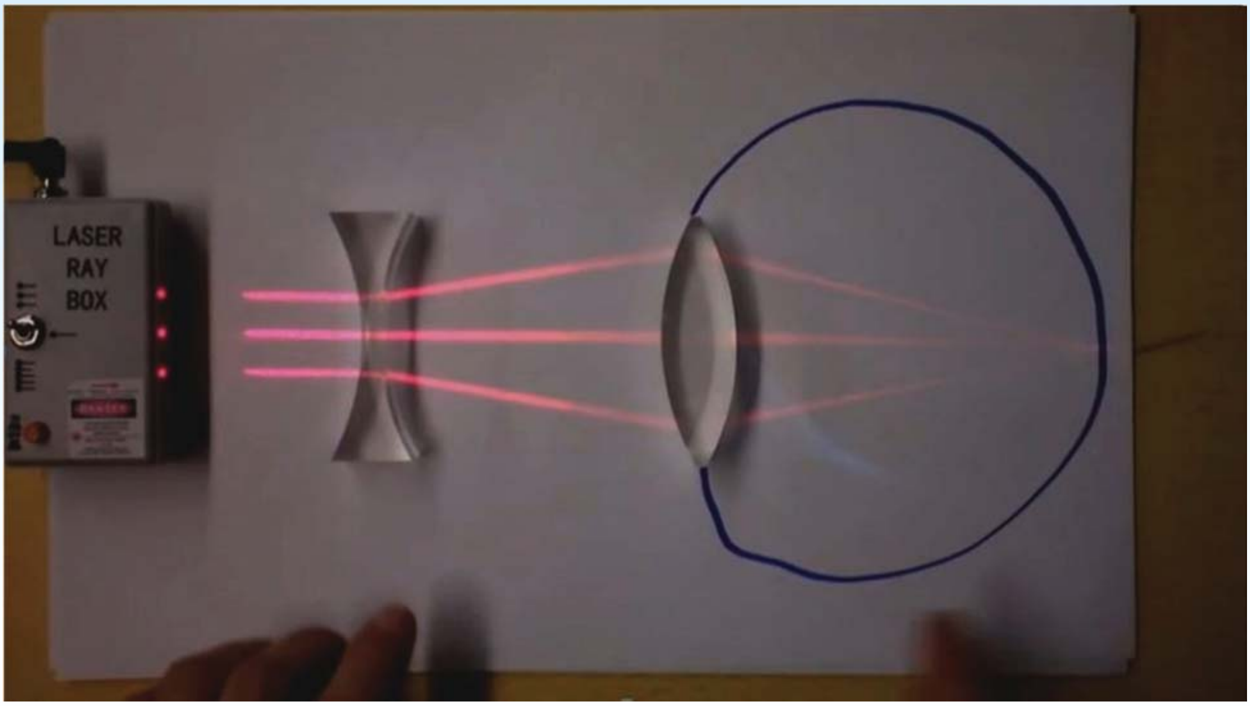
Normal reading distance: Assumed to be 25 cm when designing correction lenses.

Lenses for corrections are given in diopter.

Lens power = 1/f (unit diopter = m^{-1})



Geometrical optics

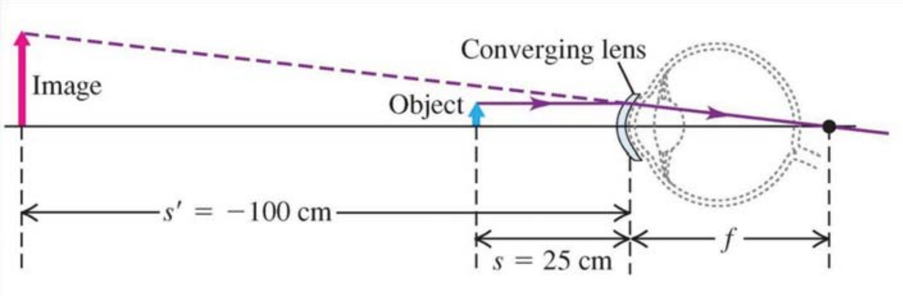


Geometrical optics

Problem solving

Geometrical optics

The near point of a certain hyperopic eye is 100 cm in front of the eye. Find the focal length and power of the contact lens that will permit the wearer to see clearly an object that is 25 cm in front of the eye.



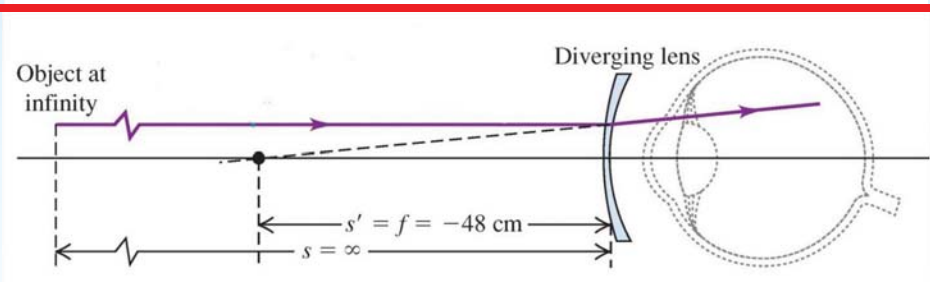
When the person puts an object at s = 25 cm from the correcting lens we want the image to end up at s′ = 100 cm because this is the nearest point the eye can see sharply.

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{+25\text{ cm}} + \frac{1}{-100\text{ cm}}$$

$$f = +33\text{ cm}$$

Geometrical optics

The far point of a certain myopic eye is 50 cm in front of the eye. Find the focal length and power of the eyeglass lens that will permit the wearer to see clearly an object at infinity. Assume that the lens is worn 2 cm in front of the eye.



The lens should move the actual far point from 50 cm to infinity.
The correcting lens should therefore have s = infinity for s′ = 50-2 = 48 cm.

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty} + \frac{1}{-48\text{ cm}}$$

$$f = -48\text{ cm}$$

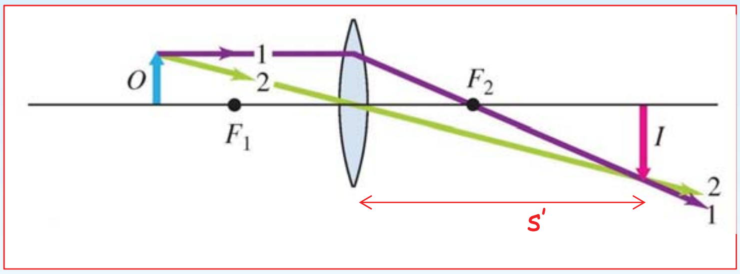
Geometrical optics

The magnifying glass

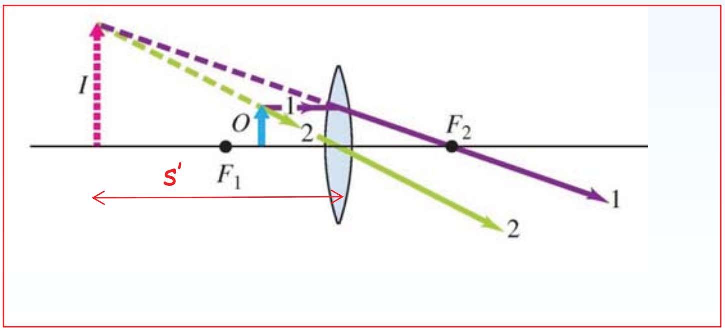
Geometrical optics

A magnifying glass is a convex lens.

If you hold a magnifying glass far away from the eye (arms lengths distance) you can see a magnified and up-side down image.

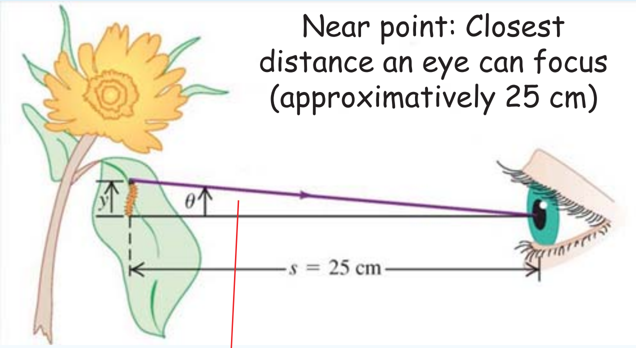


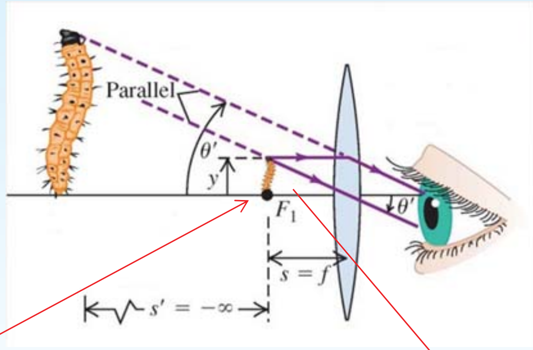
The normal use of a magnifying glass is to put the object between the focal point and the glass to get a magnified up-right image.



Geometrical optics

The magnifying glass





$$\theta = \frac{y}{25\text{ cm}}$$

When the object is at the focal point one uses angular magnification (M) instead of lateral magification (m).

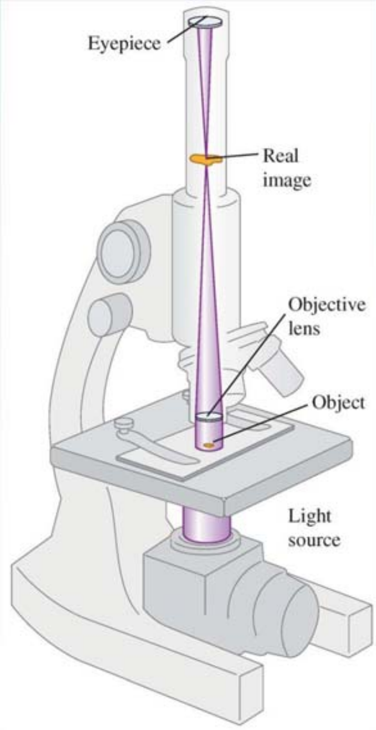
$$\theta' = \frac{y}{f}$$

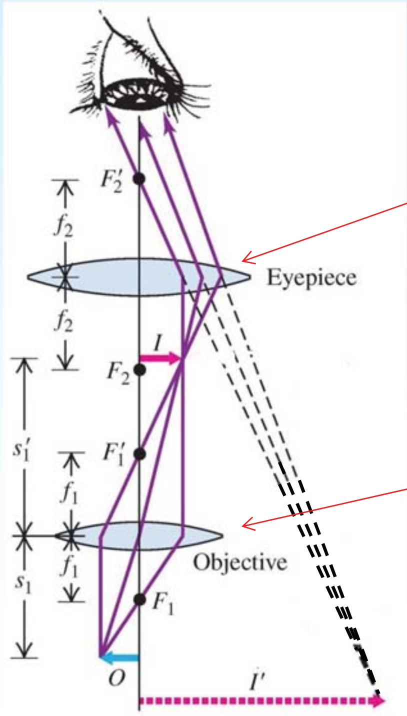
$$M = \frac{\theta'}{\theta} = \frac{y/f}{y/25\text{ cm}} = \frac{25\text{ cm}}{f} \quad \text{(angular magnification for a simple magnifier)}$$

Geometrical optics

The microscope

Geometrical optics





Magnifying glass

Creates magnified image close to the focal point of the eye piece

Geometrical optics

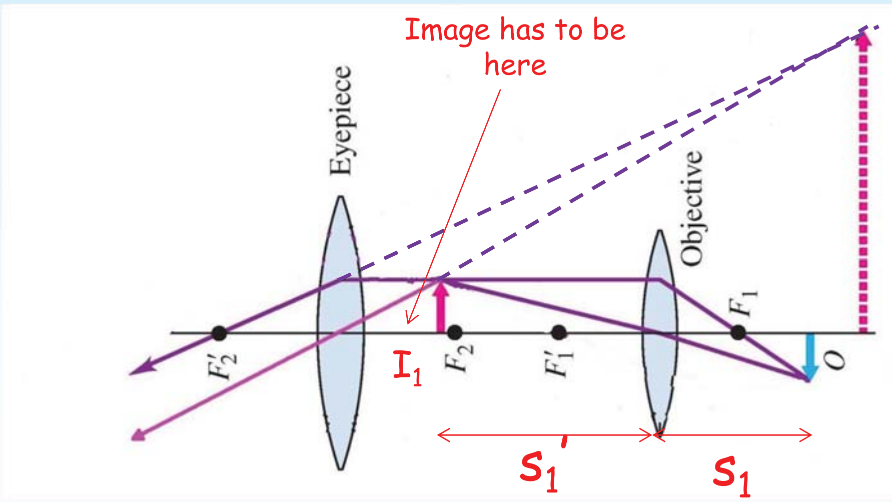


Image has to be here

I_1

s_1' s_1

Magnification first lense: $m_1 = -\frac{s_1'}{s_1}$

If one put I_1 in F_2 $M_2 = (25\text{ cm})/f_2$

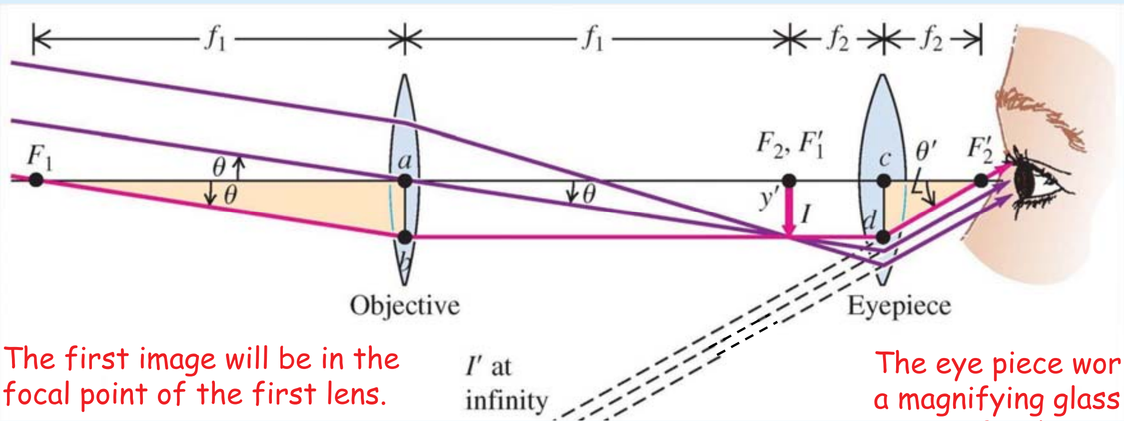
Magnifying glass

$$M = m_1 M_2 = \frac{(25\text{ cm})s_1'}{s_1 f_2}$$

Geometrical optics

The telescope

Geometrical optics



The first image will be in the focal point of the first lens.

The eye piece works as a magnifying glass with I in its focal point.

$$\tan(\theta) = \theta = \frac{-y'}{f_1}$$

$$\tan(\theta') = \theta' = \frac{y'}{f_2}$$

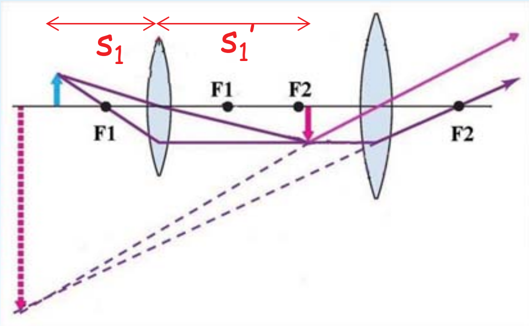
The angular magnification of a telescope is defined as the ratio of the angle of the image to that of the incoming light.

$$M = \frac{\theta'}{\theta} = -\frac{y'/f_2}{y'/f_1} = -\frac{f_1}{f_2}$$

Geometrical optics

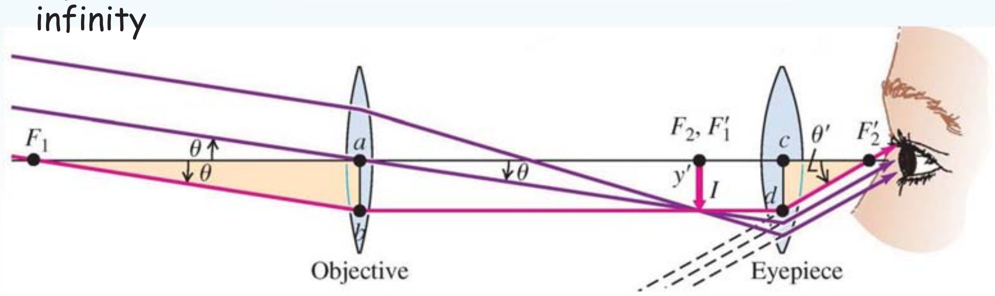
Comparing microscopes with telescopes

Object at a close distance



$$M = m_1 M_2 = \frac{(25\text{ cm})s_1'}{s_1 f_2}$$

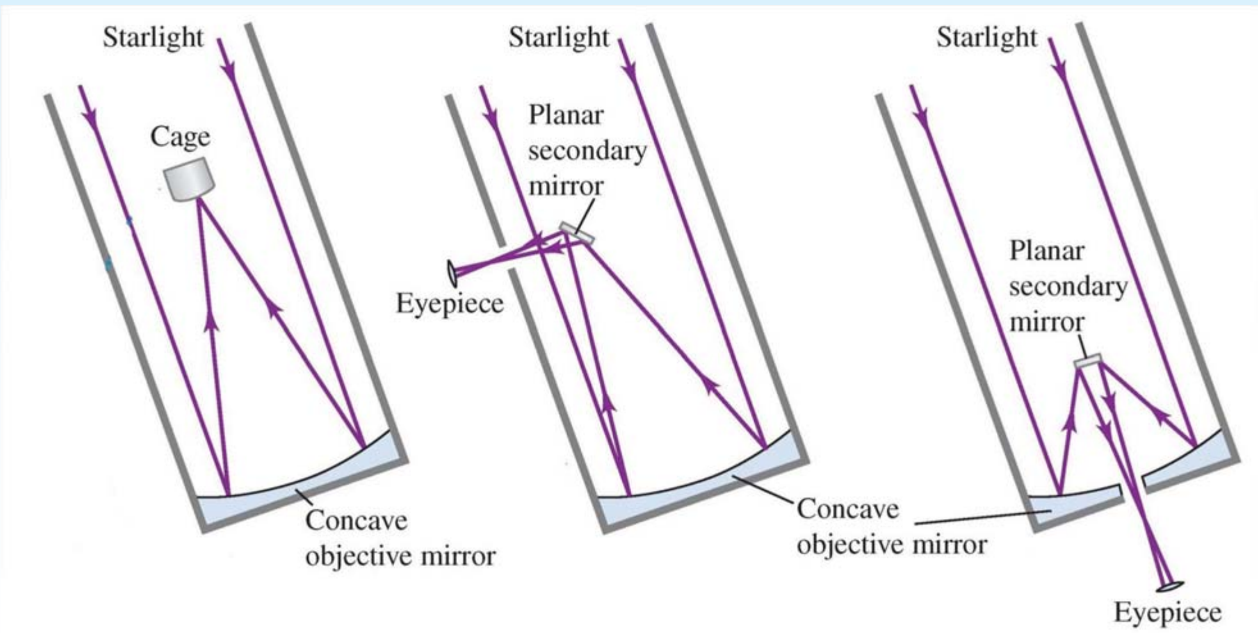
Object at infinity



Large f_1 is the challenge

$$M = -\frac{f_1}{f_2}$$

Geometrical optics



Geometrical optics