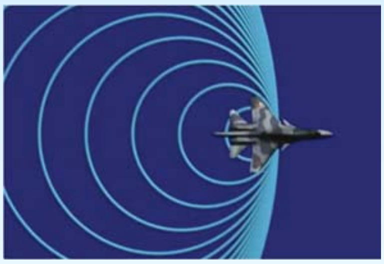
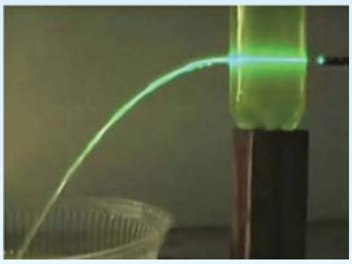
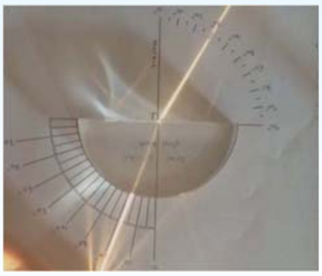
Vågrörelselära och optik

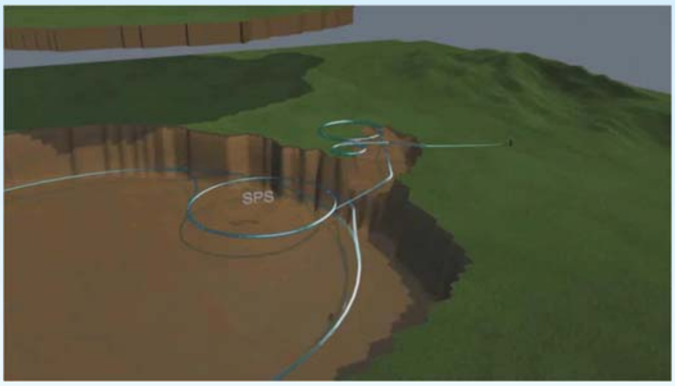






Kapitel 14 – Harmonisk oscillator

Vågrörelselära och optik



Vågrörelselära och optik

Kurslitteratur: University Physics by Young & Friedman (13th edition)

Harmonisk oscillator:	Kapitel 14.1 – 14.4
Mekaniska vågor:	Kapitel 15.1 – 15.8
Ljud och hörande:	Kapitel 16.1 – 16.9
Elektromagnetiska vågor:	Kapitel 32.1 & 32.3 & 32.4
Ljusets natur:	Kapitel 33.1 – 33.4 & 33.7
Stråloptik:	Kapitel 34.1 - 34.8
Interferens:	Kapitel 35.1 - 35.5
Diffraktion:	Kapitel 36.1 - 36.5 & 36.7

Vågrörelselära och optik

Tid	Må 02-nov	Ti 03-nov	On 04-nov	To 05-nov	Fr 06-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 14	Kvantfysik (A)	Våglära/optik (A)	Kvantfysik (A)
10-12	Intro period 2 (A) Informationssökning (A)	Kvantfysik (A)	Våglära/optik (A) kap 14+15 ÄFYA11 (L218)	Kvantfysik (A)	Kvantfysik (A) kap 15
13-15	Utvärdering (A) 12-13	Övningar Optik&Våg (L218-19)	SI gp6-10 (L219) ÄFYA11 (L218)	SI gp11-15 (L219)	Övningar Optik&Våg (L218-19)
15-17					

Tid	Må 09-nov	Ti 10-nov	On 11-nov	To 12-nov	Fr 13-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 16	Våglära/optik (A) kap 16+32	Kvantfysik (A)	Kvantfysik (A)
10-12	Våglära/optik kap 15+16 ÄFYA11 (L218)	Kvantfysik (A)	Kvantfysik (A)	Våglära/optik (A) kap 32+33	Våglära/optik (A) kap 33
13-15	SI gp1-5 (L219) ÄFYA11 (L218)	Övningar Optik&Våg (L218-19)	ÄFYA11 (L218) SI gp6-10 (L219)	SI gp1-5 (L218) SI gp11-15 (L219)	Övningar Optik&Våg (L218-19)
15-17					

Tid	Må 16-nov	Ti 17-nov	On 18-nov	To 19-nov	Fr 20-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 34	Kvantfysik (A)	Våglära/optik (A) kap 35 ÄFYA11 (L218)	Våglära/optik (A) kap 36
10-12	Våglära/optik (A) kap 34	Kvantfysik (A)	Våglära/optik (A) kap 34+35	Våglära/optik (A) kap 36	Kvantfysik (A)
13-15	SI gp6-10 (L219)	Övningar Optik&Våg (L218-19)	Seminar.gen.gång (A) 12-13 SI gp1-5 (L218) 13-15 SI gp11-15 (L219) 13-15	Labbintroduktion (A) O2-O3, K1-K3	Övningar Optik&Våg (L218-19)
15-17					

Introduction

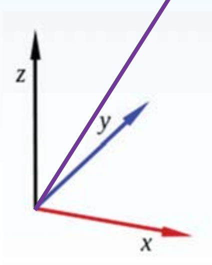
Theoretical model:

Velocity =
Distance / Time

Introduction

Theoretical model:
Position = $\bar{r}(x,y,z,t)$

$\bar{r}(x,y,z,t)$



Velocity =
the derivative of $\bar{r}$
with respect to time

$$\mathbf{r}(t) = \int \mathbf{v}(t)\, dt$$

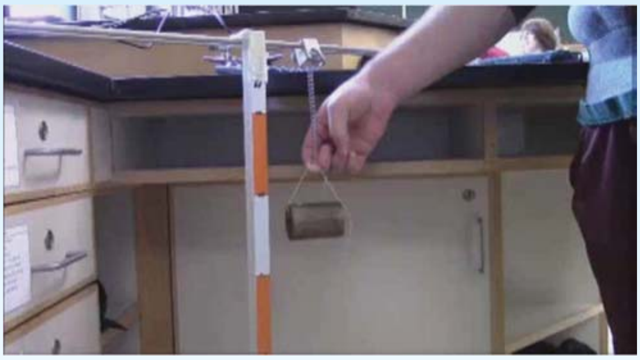
$$\mathbf{v}(t) = \frac{d\mathbf{r}}{dt} = \int \mathbf{a}(t)\, dt$$

$$\mathbf{a}(t) = \frac{d\mathbf{v}}{dt}$$

Harmonic oscillation

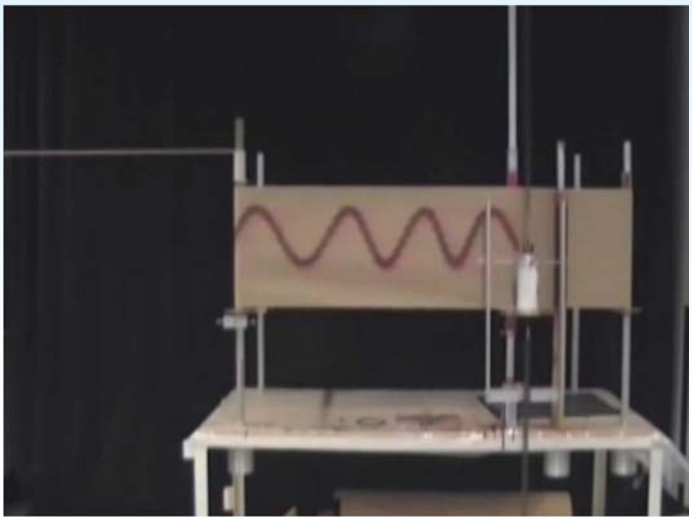
What is harmonic oscillation and how can we describe it mathematically ?

Harmonic oscillation: Examples



Harmonic oscillation: Experiment

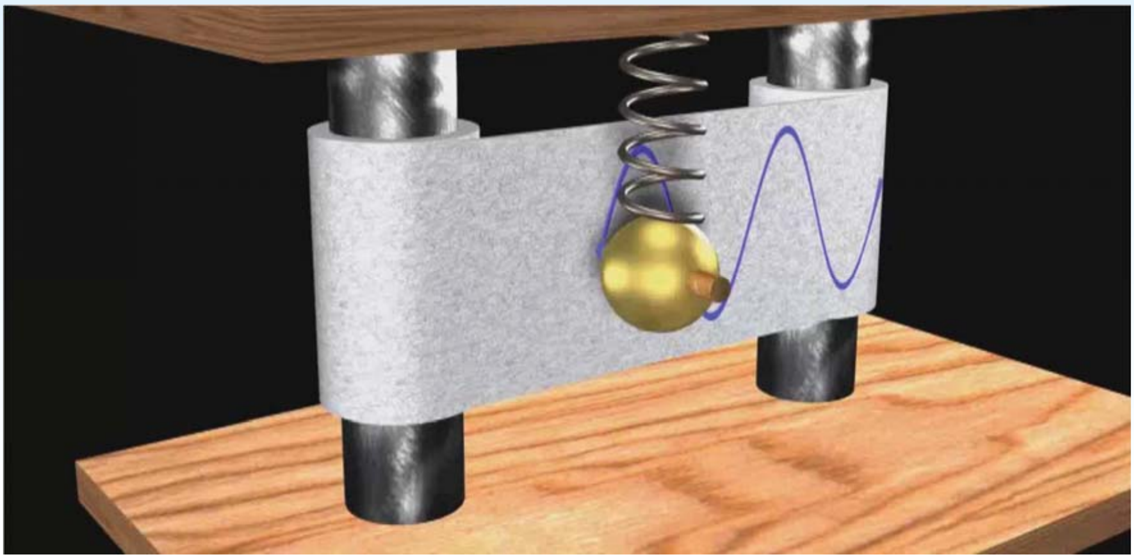
Experiment to find a mathematical description of harmonic oscillation



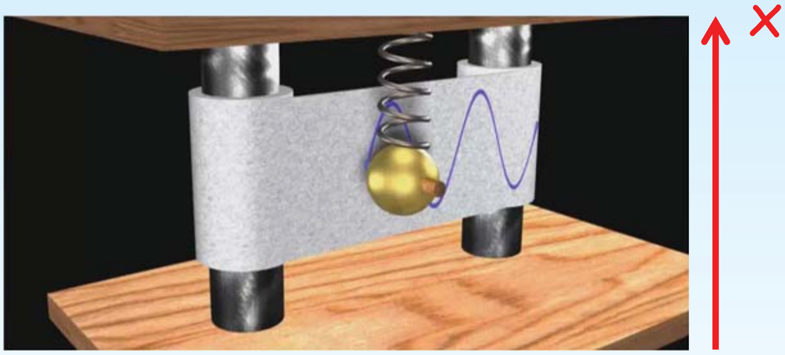
Harmonic oscillation: Experiment

Conclusion: Harmonic oscillation can be described by the function: $x = a \sin(bt + c)$

x



Harmonic oscillation: Experiment

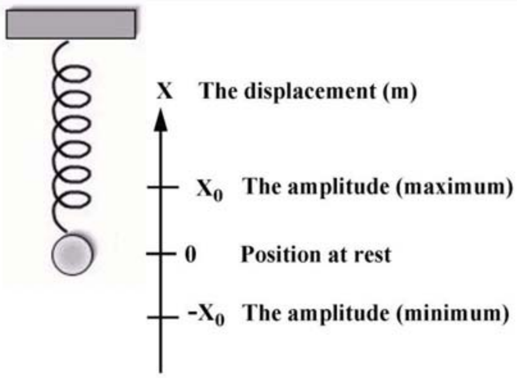


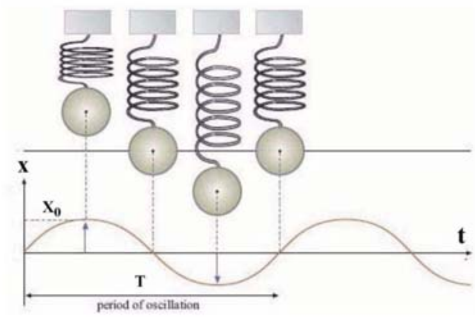
Period: The time it takes for the weight to go up and down

Frequency: The number of periods per second.

Amplitude: The maxium movement.

Harmonic oscillation: Notation





x	The displacement (m)
x_0	The amplitude (m)
t	Time (s)
T	Period (s)
f	Frequency (Hz) $= 1 / T$
ω	Angular Frequency (Hz) $= 2\pi / T = 2 \pi f$

x	Förflyttning(m)
x_0	Amplitud (m)
t	Tid (s)
T	Period (s)
f	Frekvens (Hz) $= 1 / T$
ω	Vinkelfrekvens (Hz) $= 2\pi / T = 2 \pi f$

Harmonic oscillation: velocity & acceleration

We now have a mathematical description of the displacement.

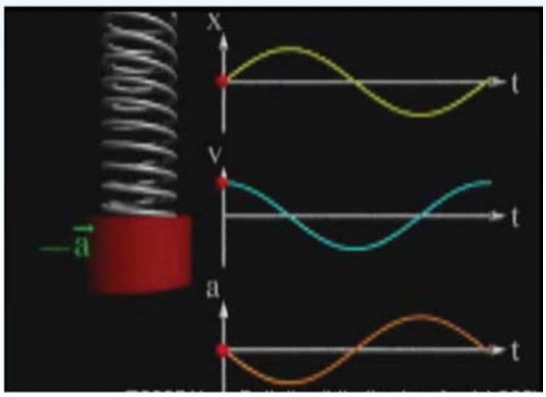
What is the velocity and acceleration ?

$$v(t) = \frac{dx}{dt}$$

$$a(t) = \frac{dv}{dt}$$

Harmonic oscillation: velocity & acceleration

Displacement:	$x = x_0 \sin(\omega t)$ →	$x_{max} = x_0$
Velocity: $v = \frac{dx}{dt}$	$v = \omega\ x_0 \cos(\omega t)$ →	$v_{max} = \omega x_0$
Acceleration: $a = \frac{dv}{dt}$	$a = -\omega^2 x_0 \sin(\omega t)$ →	$a_{max} = \omega^2 x_0$



Problem solving

Harmonic oscillation: Problem

Example 14.1 **Period, frequency, and angular frequency**

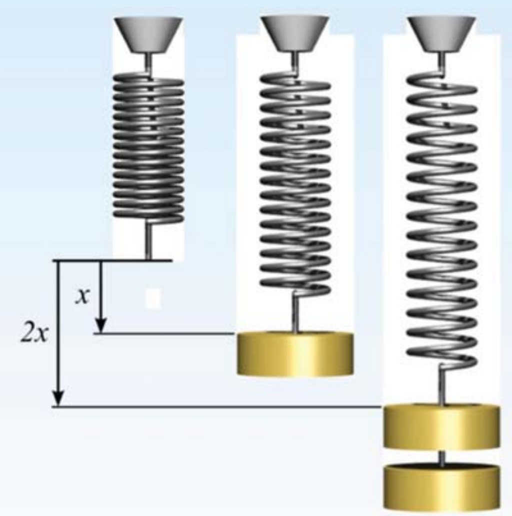
An ultrasonic transducer used for medical diagnosis oscillates at 6.7 MHz = 6.7×10^6 Hz. How long does each oscillation take, and what is the angular frequency?

$$T = \frac{1}{f} = \frac{1}{6.7 \times 10^6\ \text{Hz}} = 1.5 \times 10^{-7}\ \text{s} = 0.15\ \mu\text{s}$$

$$\begin{aligned}\omega &= 2\pi f = 2\pi(6.7 \times 10^6\ \text{Hz}) \\ &= (2\pi\ \text{rad/cycle})(6.7 \times 10^6\ \text{cycle/s}) \\ &= 4.2 \times 10^7\ \text{rad/s}\end{aligned}$$

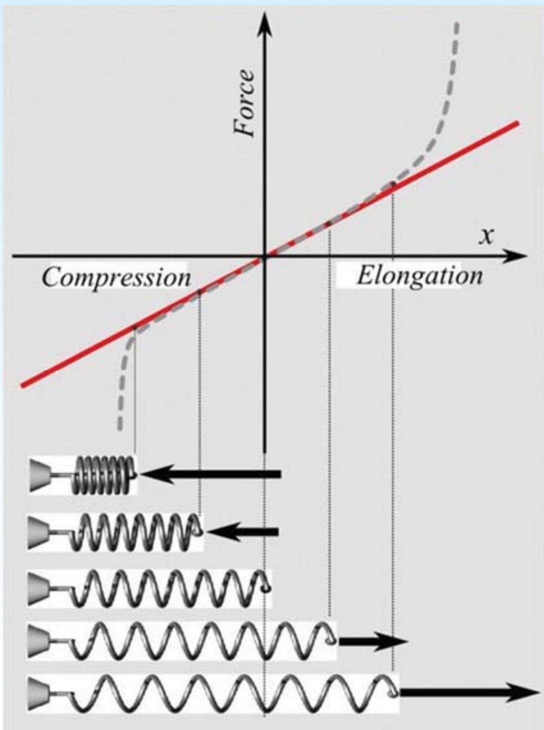
Properties of a spring

Hookes law & Forces



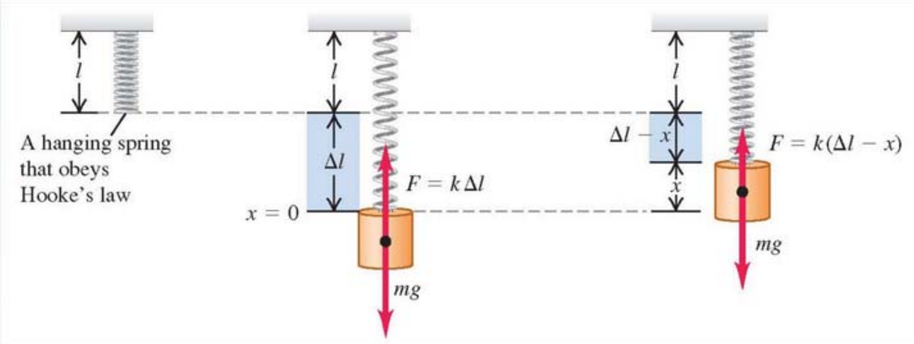
Hooke's law for a spring

$$F = -kX$$

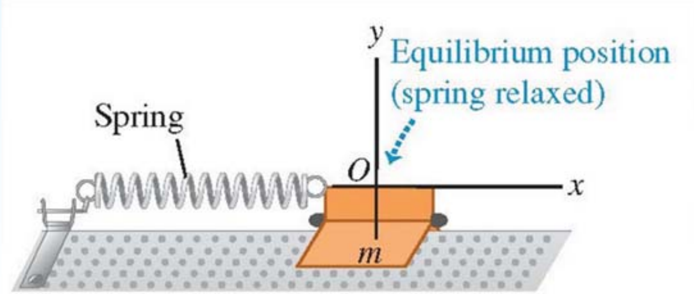


Harmonic oscillation: The spring

Gravity will stretch the spring to a new eqilibrium position.



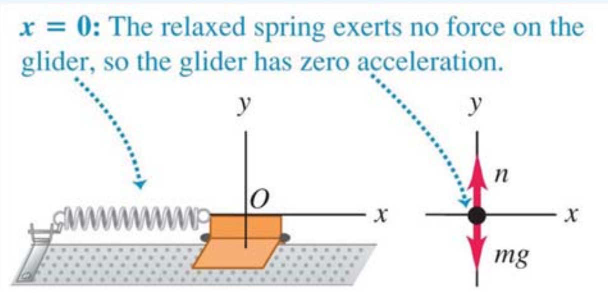
This is not the case when the spring is horizonthal.

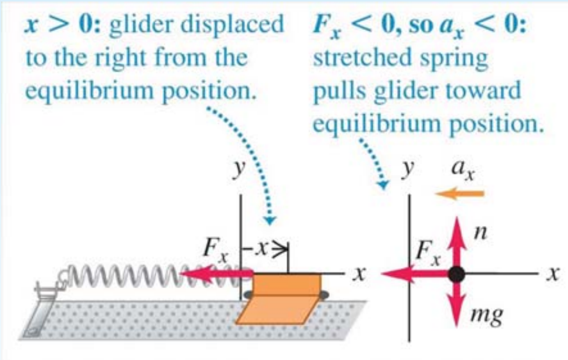


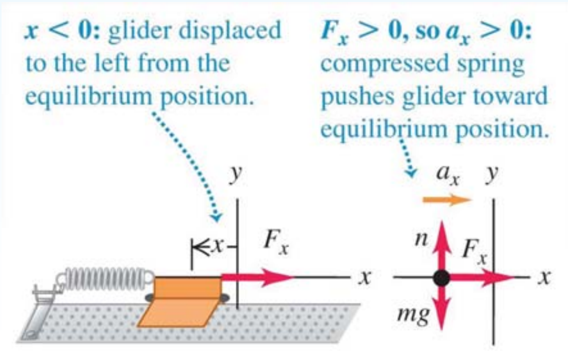
However, the oscillations will be the same.

Harmonic oscillation: Forces

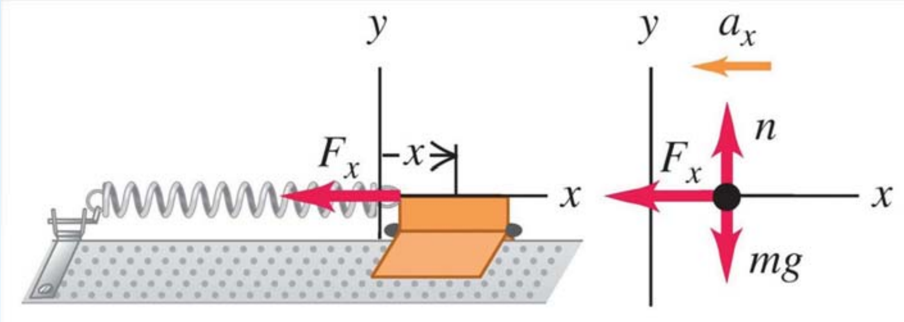
Forces on a mass connected to a horisonthal spring







Harmonic oscillation: Forces



$F_x = -kx$ (restoring force exerted by an ideal spring)

F = m a (Newton's second law)

$$a_x = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$ (simple harmonic motion)

Harmonic oscillation: Forces

Old formulas:

$x = A\cos(\omega t+\phi)$

$v = -\omega A\sin(\omega t+\phi)$

$a = -\omega^2 A\cos(\omega t+\phi)$ ⇒ $a_x = -\omega^2 x$

New formula: $$a_x = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$ (simple harmonic motion)

Combine: $-\omega^2 = -k/m$

$$\omega = \sqrt{\frac{k}{m}}$$

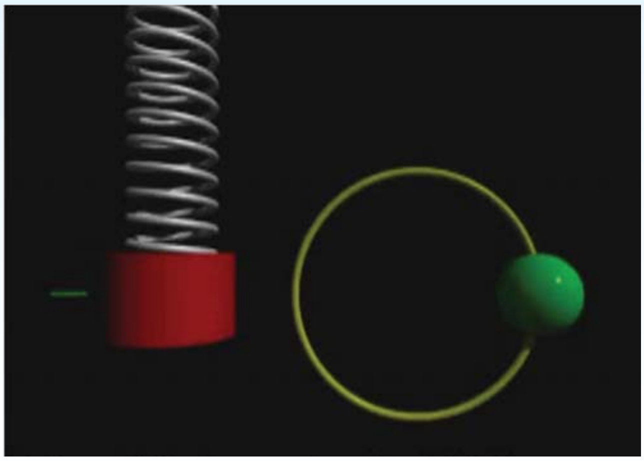
The frequency depends on the spring constant and the mass

Harmonic oscillation: Circular motion

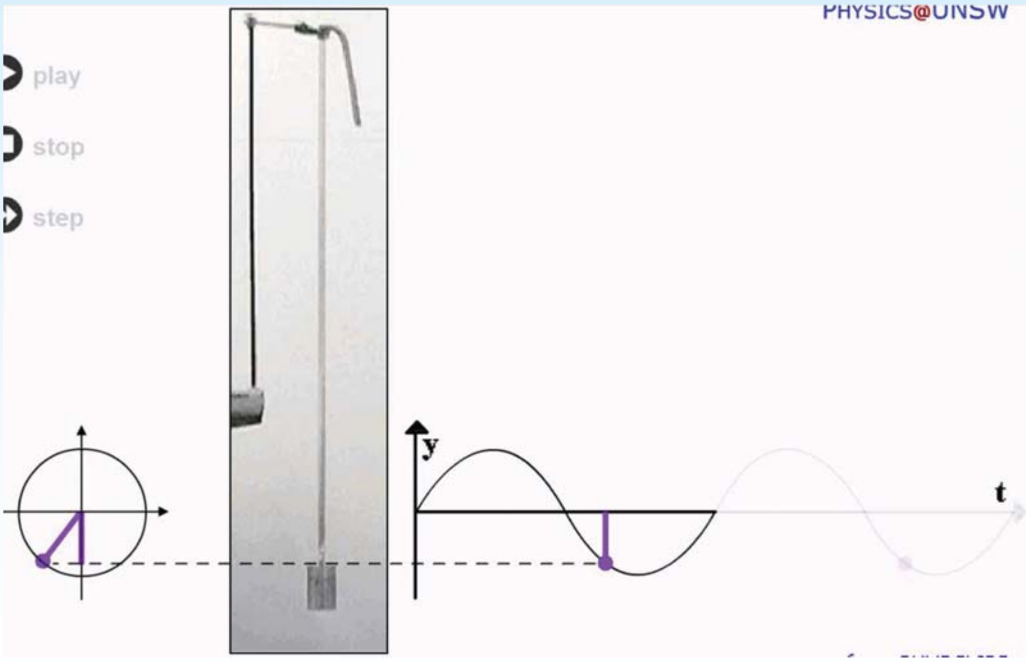
Circular motion can be used to describe harmonic oscillation

Harmonic oscillation: Circular motion

Since harmonic oscillation is described by a sinus function it can also be compared to a circular motion.

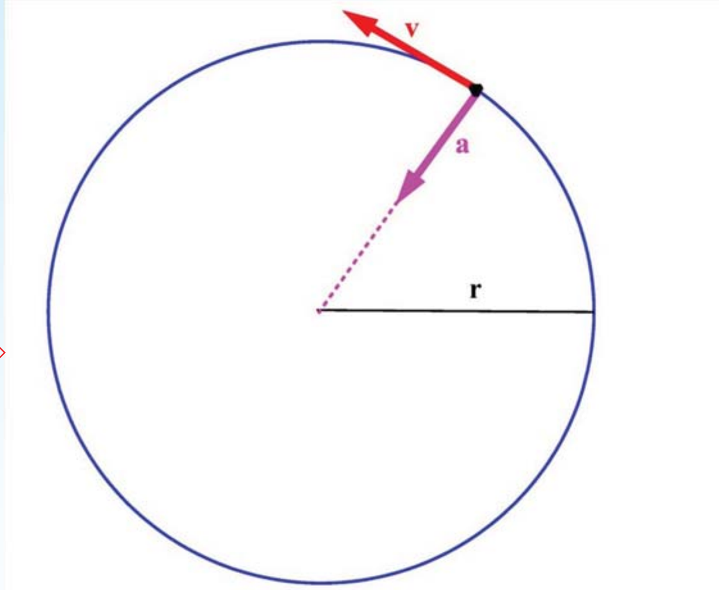


Harmonic oscillation: Circular motion



Harmonic oscillation: Circular motion

Basic description of circular motion with constant speed |v|

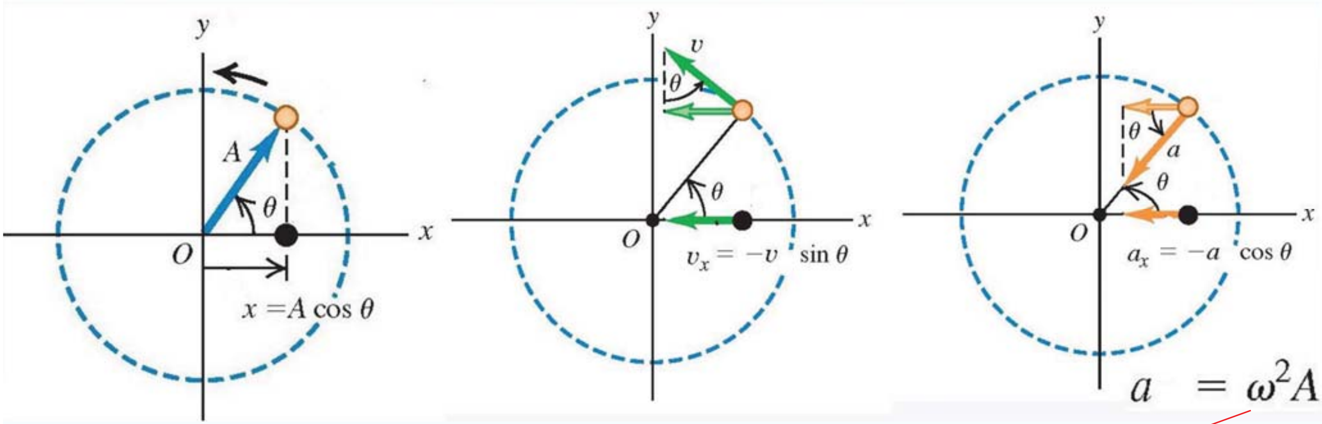


$$v = \text{velocity} = \frac{\text{distance}}{\text{time}} = \frac{\text{circumference}}{\text{time period}} = \frac{2\pi r}{T} = \omega r$$

$$a = \text{acceleration} = v^2/r = \omega^2 r$$

Harmonic oscillation: Circular motion

What is x, v and a in the x-direction ?



$$x = A\cos\theta$$

radius

$$v_x = -v \sin\theta$$

$$a_x = -a \cos\theta$$

$$a_x = -\omega^2 A\cos\theta$$

Harmonic oscillation: Circular motion

Combine

the acceleration from the discussion about forces

with

the acceleration in circular motion.

Harmonic oscillation: Frequency

Forces

$F = m\,a$

$F = -k\,x$

$a_x = -k\,x / m$

Circular Motion

$x = A\cos\theta$

$a_x = -\omega^2 A\cos\theta$

$a_x = -\omega^2\,x$

$$\omega = \sqrt{\frac{k}{m}}$$

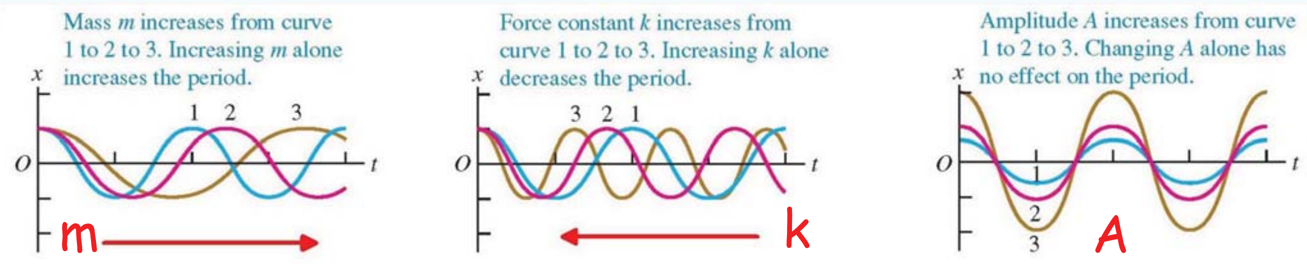
Simple harmonic motion requires a restoring force that is proportional to the displacement.

Harmonic oscillation: Frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{(simple harmonic motion)}$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \quad \text{(simple harmonic motion)}$$

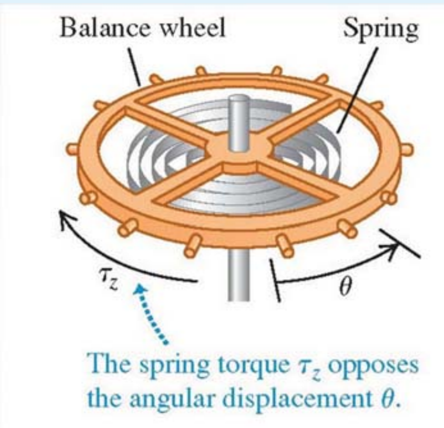
Note: f and T depends only on k and m but not on the amplitude !



Angular simple harmonic oscillation

Harmonic oscillation: Angular motion

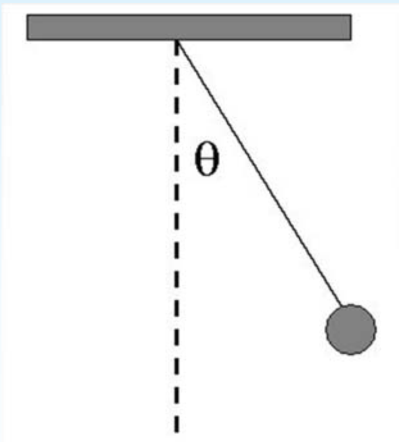
The spring in a watch is a harmonic oscillator.



$$\theta = \Theta \cos(\omega t + \phi)$$

The pendulum

The pendulum is a harmonic oscillator.



$$\theta = \Theta \cos(\omega t + \phi)$$

Harmonic oscillation: Equations of motion

Displacement: $x = x_0 \sin(\omega t) \longrightarrow x_{max} = x_0$

Velocity: $v = \frac{dx}{dt}$ $\quad v = \omega\, x_0 \cos(\omega t) \longrightarrow v_{max} = \omega x_0$

Acceleration: $a = \frac{dv}{dt}$ $\quad a = -\omega^2 x_0 \sin(\omega t) \longrightarrow a_{max} = \omega^2 x_0$

Rörelse av en massa hängande i en fjäder.

$x = 0$ när $t = 0$

Displacement: $x = A\cos(\omega t) \longrightarrow x_{max} = A$

Velocity: $v = \frac{dx}{dt}$ $\quad v = -\omega A \sin(\omega t) \longrightarrow v_{max} = \omega A$

Acceleration: $a = \frac{dv}{dt}$ $\quad a = -\omega^2 A\cos(\omega t) \longrightarrow a_{max} = \omega^2 A$

Massa i circulär rörelse.

$x = A$ när $t = 0$

Displacement: $x = A\cos(\omega t+\phi) \longrightarrow x_{max} = A$

Velocity: $v = \frac{dx}{dt}$ $\quad v = -\omega A \sin(\omega t+\phi) \longrightarrow v_{max} = \omega A$

Acceleration: $a = \frac{dv}{dt}$ $\quad a = -\omega^2 A\cos(\omega t+\phi) \longrightarrow a_{max} = \omega^2 A$

Harmonisk oscillation

$x = A\cos(\phi)$ när $t = 0$

ϕ = fasvinkeln

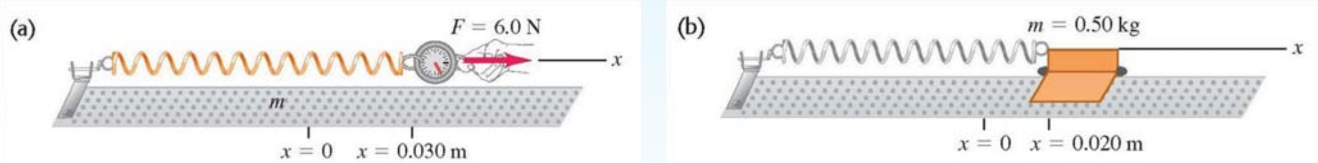
(avgör läget vid $t = 0$)

Harmonic oscillation: Problem

Problem solving

Harmonic oscillation: Problem

A spring is mounted horizontally, with its left end fixed. A spring balance attached to the free end and pulled toward the right (Fig. 14.8a) indicates that the stretching force is proportional to the displacement, and a force of 6.0 N causes a displacement of 0.030 m. We replace the spring balance with a 0.50-kg glider, pull it 0.020 m to the right along a frictionless air track, and release it from rest (Fig. 14.8b). (a) Find the force constant k of the spring. (b) Find the angular frequency ω, frequency f, and period T of the resulting oscillation.



$$k = -\frac{F_x}{x} = -\frac{-6.0\ \text{N}}{0.030\ \text{m}} = 200\ \text{N/m} = 200\ \text{kg/s}^2$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200\ \text{kg/s}^2}{0.50\ \text{kg}}} = 20\ \text{rad/s}$$

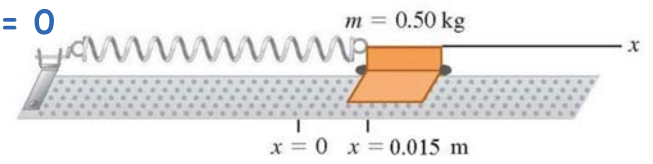
$$f = \frac{\omega}{2\pi} = \frac{20\ \text{rad/s}}{2\pi\ \text{rad/cycle}} = 3.2\ \text{cycle/s} = 3.2\ \text{Hz}$$

$$T = \frac{1}{f} = \frac{1}{3.2\ \text{cycle/s}} = 0.31\ \text{s}$$

Harmonic oscillation: Problem

We give the glider of Example 14.2 an initial displacement $x_0 = +0.015$ m and an initial velocity $v_{0x} = +0.40$ m/s. (a) Find the amplitude, and phase angle of the resulting motion.

t = 0



$\omega = 20\ \text{rad/s}$

Displacement: $x = A\cos(\omega t+\phi) \rightarrow x_{max} = A$

Velocity: $v = \frac{dx}{dt}$ $\quad v = -\omega A\sin(\omega t+\phi) \rightarrow v_{max} = \omega A$

Acceleration: $a = \frac{dv}{dt}$ $\quad a = -\omega^2 A\cos(\omega t+\phi) \rightarrow a_{max} = \omega^2 A$

t = 0

$$x_0 = A\cos\phi$$

$$v_{0x} = -\omega A\sin\phi$$

$$\frac{v_{0x}}{x_0} = \frac{-\omega A\sin\phi}{A\cos\phi} = -\omega\tan\phi$$

$$\phi = \arctan\left(-\frac{v_{0x}}{\omega x_0}\right) = \arctan\left(-\frac{0.40\ \text{m/s}}{(20\ \text{rad/s})(0.015\ \text{m})}\right) = -53^\circ = -0.93\ \text{rad}$$

$A = x_0 / \cos\phi = 0.015 / \cos(-0.93) = 0.025$ m

Harmonic oscillation: Problem

(b) Write equations for the displacement, velocity, and acceleration as functions of time.

ω = 20 rad/s
ϕ = -0.93 rad
A = 0.025 m

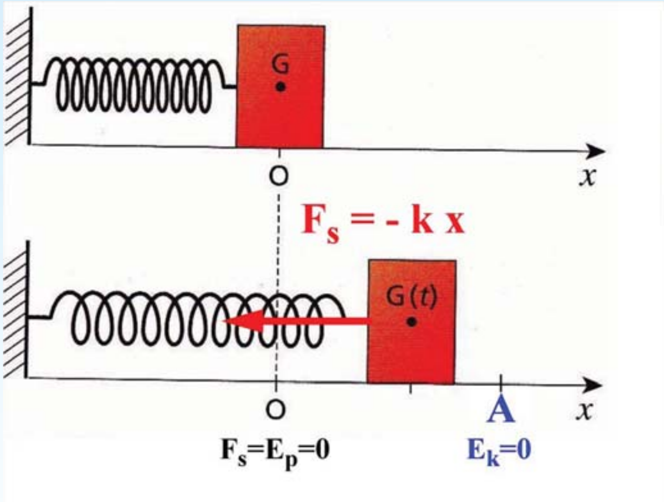
Displacement:	$x = A\cos(\omega t+\phi)$	$\longrightarrow x_{max} = A$
Velocity: $v = \frac{dx}{dt}$	$v = -\omega A\sin(\omega t+\phi)$	$\longrightarrow v_{max} = \omega A$
Acceleration: $a = \frac{dv}{dt}$	$a = -\omega^2 A\cos(\omega t+\phi)$	$\longrightarrow a_{max} = \omega^2 A$

$$x = (0.025\ \text{m})\cos[(20\ \text{rad/s})t - 0.93\ \text{rad}]$$
$$v_x = -(0.50\ \text{m/s})\sin[(20\ \text{rad/s})t - 0.93\ \text{rad}]$$
$$a_x = -(10\ \text{m/s}^2)\cos[(20\ \text{rad/s})t - 0.93\ \text{rad}]$$

Harmonic oscillation: Energy

Energy in harmonic oscillation

Harmonic oscillation: Energy

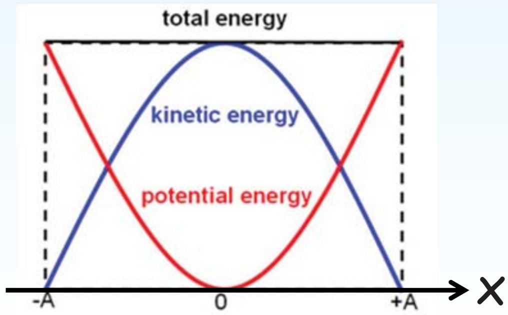


Kinetic energy: $E_k = \frac{mv^2}{2}$

Potential energy: $E_p = \frac{kx^2}{2}$

Total energy: $E_t = E_k + E_p = \frac{kA^2}{2}$ ($E_k = 0$ for $x = A$)

The total mechanical energy is constant



Harmonic oscillation: Energy

What is the total mechanical energy ?

Displacement: $x = A\cos(\omega t+\phi) \longrightarrow x_{max} = A$

Velocity: $v = \frac{dx}{dt}$ $\quad v = -\omega A\sin(\omega t+\phi) \longrightarrow v_{max} = \omega A$

Acceleration: $a = \frac{dv}{dt}$ $\quad a = -\omega^2 A\cos(\omega t+\phi) \longrightarrow a_{max} = \omega^2 A$

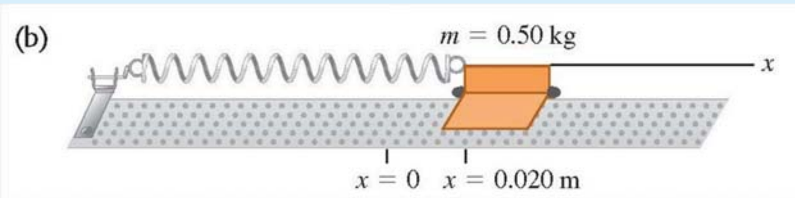
$$\omega = \sqrt{\frac{k}{m}}$$

$$\begin{aligned} E &= \tfrac{1}{2}mv_x^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}m[-\omega A\sin(\omega t+\phi)]^2 + \tfrac{1}{2}k[A\cos(\omega t+\phi)]^2 \\ &= \tfrac{1}{2}kA^2\sin^2(\omega t+\phi) + \tfrac{1}{2}kA^2\cos^2(\omega t+\phi) \\ &= \tfrac{1}{2}kA^2 \end{aligned}$$

Harmonic oscillation: Problem

Problem solving

Harmonic oscillation: Problem



$A = 0.020$ m

$k = 200$ N/m

$m = 0.50$ kg

What is v_{max}, v_{min}, a_{max} and a_{min} ?

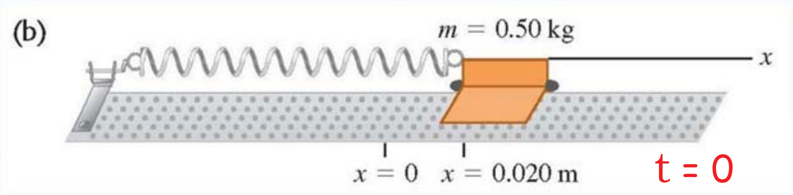
Displacement:	$x = A\cos(\omega t+\phi)$ →	$x_{max} = A$	$x_{min} = -A$
Velocity: $v = \frac{dx}{dt}$	$v = -\omega A\sin(\omega t+\phi)$ →	$v_{max} = \omega A$	$v_{min} = -\omega A$
Acceleration: $a = \frac{dv}{dt}$	$a = -\omega^2 A\cos(\omega t+\phi)$ →	$a_{max} = \omega^2 A$	$a_{min} = -\omega^2 A$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200\ \text{kg/s}^2}{0.50\ \text{kg}}} = 20\ \text{rad/s}$$

$$v_{max} = 20 \cdot 0.020 = 0.40\ \text{m/s}$$

$$a_{max} = 20 \cdot 20 \cdot 0.020 = 8\ \text{m/s}^2$$

Harmonic oscillation: Problem



A = 0.020 m

k = 200 N/m

m = 0.50 kg

What is the phase angle ?

Displacement:	$x = A\cos(\omega t+\phi)$ →	$x_{max} = A$	$x_{min} = -A$
Velocity: $v = \frac{dx}{dt}$	$v = -\omega A\sin(\omega t+\phi)$ →	$v_{max} = \omega A$	$v_{min} = -\omega A$
Acceleration: $a = \frac{dv}{dt}$	$a = -\omega^2 A\cos(\omega t+\phi)$ →	$a_{max} = \omega^2 A$	$a_{min} = -\omega^2 A$

Getting the phase angle:

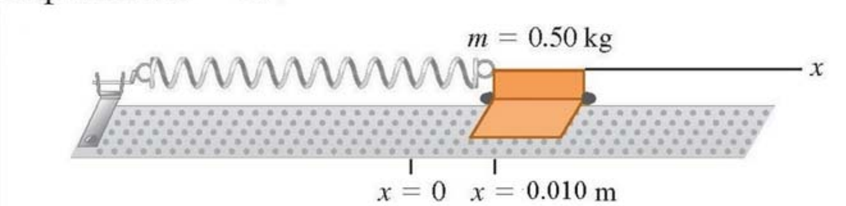
$x = A$ when $t = 0$

$A = A\cos(0 + \phi)$

$\phi = 0$

Harmonic oscillation: Problem

(c) Find the velocity v_x and acceleration a_x when the glider is halfway from its initial position to the equilibrium position $x = 0$.



A = 0.020 m

k = 200 N/m

m = 0.50 kg

ω = 20 rad/s

$\phi = 0$

Displacement:	$x = A\cos(\omega t)$
Velocity: $v = \frac{dx}{dt}$	$v = -\omega A\sin(\omega t)$
Acceleration: $a = \frac{dv}{dt}$	$a = -\omega^2 A\cos(\omega t)$

$x = A / 2$

$A / 2 = A\cos(\omega t)$

$\omega t = 1.047$ rad

$V = -20 \cdot 0.020 \sin(1.047) = -0.35$ m/s

$a = -20^2 \cdot 0.020 \cos(1.047) = -4.0$ m/s^2

Harmonic oscillation: Problem

(d) Find the total energy, potential energy, and kinetic energy at this position.

$x = A / 2 = 0.020 / 2 = 0.010$ m

$v = -0.35$ m/s

$A = 0.020$ m

$k = 200$ N/m

$m = 0.50$ kg

Kinetic energy: $E_k = \frac{mv^2}{2}$

Potential energy: $E_p = \frac{kx^2}{2}$

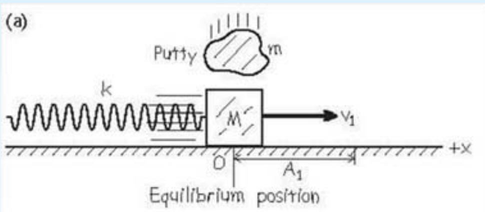
Total energy: $E_t = E_k + E_p = \frac{kA^2}{2}$ ($E_k = 0$ for $x = A$)

$$Ep = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(200\ \text{N/m})(0.010\ \text{m})^2 = 0.010\ \text{J}$$

$$Ek = \tfrac{1}{2}mv_x^2 = \tfrac{1}{2}(0.50\ \text{kg})(-0.35\ \text{m/s})^2 = 0.030\ \text{J}$$

Harmonic oscillation: Problem

A block of mass M attached to a horizontal spring with force constant k is moving in SHM with amplitude A_1. As the block passes through its equilibrium position, a lump of putty of mass m is dropped from a small height and sticks to it.



Calculate new period and amplitude !

$$\omega = \sqrt{\frac{k}{m}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{(simple harmonic motion)}$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \quad \text{(simple harmonic motion)}$$

Step 1. Calculate the new period T_2:

$T_2 = 2\pi / \omega = 2\pi (M+m)^{1/2} / k^{1/2}$

Step 2. The momentum in the x-direction stays the same (but not the energy):

$P_1 = P_2$

$M v_1 = (M + m) v_2$

$v_2 = v_1 M / (M + m)$

Step 3. Calculate new total energy with the putty at x = 0:

$E_{t2} = E_{k2} + 0 = ½(M + m) v_2^2 = ½ v_1^2 M^2 / (M + m)$

Step 4. Calculate new total energy at $x = A_2$:

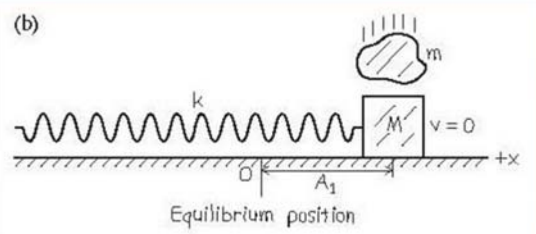
$E_{t2} = 0 + ½ k A_2^2$

Step 5. Combine to get A_2:

$A_2 = v_1 M / [(M + m)k]^{1/2}$

Harmonic oscillation: Problem

Drop at the end position:



Calculate new period and amplitude !

$$\omega = \sqrt{\frac{k}{m}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{(simple harmonic motion)}$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \quad \text{(simple harmonic motion)}$$

Step 1. Calculate the new period T_2:

$T_2 = 2\pi/\omega = 2\pi\,(M+m)^{½} / k^{½}$

Step 2. The momentum in the x-direction stays the same (= 0):

$P_1 = P_2 = 0$

Step 3. The energy stays the same because all energy is potential at x = A:

$E_{t1} = 0 + ½\,k\,A_1^2 = E_{t2} = 0 + ½\,k\,A_2^2$

Step 4. Combine to get A_2:

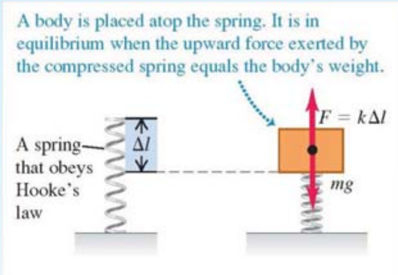
$A_2 = A_1$

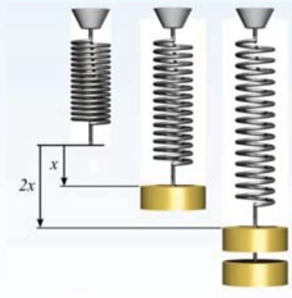
Step 5: If the total and potential energy is the same then the kinetic energy must also be the same with and without putty:

$E_{k1} = ½\,M\,v_1^2 = E_{k2} = ½(M+m)\,v_2^2$

Harmonic oscillation: Problem

The shock absorbers in an old car with mass 1000 kg are completely worn out. When a 980-N person climbs slowly into the car at its center of gravity, the car sinks 2.8 cm. The car (with the person aboard) hits a bump, and the car starts oscillating up and down in SHM. Model the car and person as a single body on a single spring, and find the period and frequency of the oscillation.





Hooke's law for a spring

$$F = -kX$$

$$k = -\frac{F_x}{x} = -\frac{980\ \text{N}}{-0.028\ \text{m}} = 3.5 \times 10^4\ \text{kg/s}^2$$

The person's mass is $w/g = (980\ \text{N})/(9.8\ \text{m/s}^2) = 100$ kg. The *total* oscillating mass is $m = 1000\ \text{kg} + 100\ \text{kg} = 1100$ kg. The period T is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1100\ \text{kg}}{3.5 \times 10^4\ \text{kg/s}^2}} = 1.11\ \text{s}$$

The frequency is $f = 1/T = 1/(1.11\ \text{s}) = 0.90$ Hz.

Vibration of molecules

Mathematics: The Binomial Theorem

$$(1+u)^n = 1 + nu + \frac{n(n-1)}{2!}u^2 + \frac{n(n-1)(n-2)}{3!}u^3 + \cdots$$

If u is small one can use the beginning of the series as an approximation:

$$(1+0.001)^{13} = 1.013078......$$

$$(1+0.001)^{13} \approx 1 + 13 \cdot 0.001 = 1.013$$

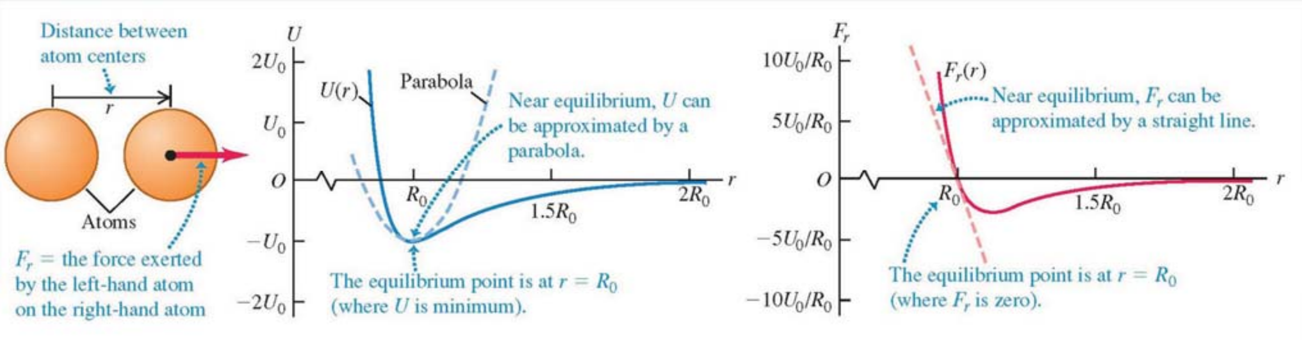
Harmonisk oscillation

Potential energy (U)

$$U = U_0\left[\left(\frac{R_0}{r}\right)^{12} - 2\left(\frac{R_0}{r}\right)^{6}\right]$$

The Force of one atom on the other (F)

$$F_r = -\frac{dU}{dr} = U_0\left[\frac{12R_0^{12}}{r^{13}} - 2\frac{6R_0^{6}}{r^{7}}\right] = 12\frac{U_0}{R_0}\left[\left(\frac{R_0}{r}\right)^{13} - \left(\frac{R_0}{r}\right)^{7}\right]$$



The equilibrium point is at r = R_0

The displacement from the equlibrium point is x = r – R_0

Harmonisk oscillation

$$F_r = -\frac{dU}{dr} = U_0\left[\frac{12R_0^{12}}{r^{13}} - 2\frac{6R_0^{6}}{r^{7}}\right] = 12\frac{U_0}{R_0}\left[\left(\frac{R_0}{r}\right)^{13} - \left(\frac{R_0}{r}\right)^{7}\right]$$

$$x = r - R_0$$

$$F_r = 12\frac{U_0}{R_0}\left[\left(\frac{R_0}{R_0+x}\right)^{13} - \left(\frac{R_0}{R_0+x}\right)^{7}\right] = 12\frac{U_0}{R_0}\left[\frac{1}{(1+x/R_0)^{13}} - \frac{1}{(1+x/R_0)^{7}}\right]$$

Assume that the vibrations are small so that x/R_0 is small !

We can then use the Binomial Theorem

$$\frac{1}{(1+x/R_0)^{13}} = (1+x/R_0)^{-13} \approx 1 + (-13)\frac{x}{R_0}$$

$$\frac{1}{(1+x/R_0)^{7}} = (1+x/R_0)^{-7} \approx 1 + (-7)\frac{x}{R_0}$$

$$F_r \approx 12\frac{U_0}{R_0}\left[\left(1 + (-13)\frac{x}{R_0}\right) - \left(1 + (-7)\frac{x}{R_0}\right)\right] = -\left(\frac{72U_0}{R_0^2}\right)x$$

This is just Hooke's law, with force constant $k = 72U_0/R_0^2$